Math 121 Review Test 3

1 Sequences

1. Determine whether the sequences below converge, if so find $\lim_{n\to\infty} a_n$.

   (a) $a_n = \dfrac{n^2+5}{2n^2+3n+1}$

   (b) $a_n = \dfrac{n^2-3n+7}{2^n-1}$

   (c) $a_n = \dfrac{2^n}{n^2+3}$

2. Determine whether the sequences below converge, if so find $\lim_{n\to\infty} a_n$.

   (a) $a_n = (-1)^n \dfrac{n^2+5}{2n^2+3n+1}$

   (b) $a_n = (-1)^n \dfrac{n^2-3n+7}{n^3+1}$

   (c) $a_n = \left(\dfrac{n+5}{n}\right)^{2n}$

2 Infinite Series

3. For the series below, determine if the divergence test can be used to draw a conclusion about the convergence of the series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n+1}$

4. Determine whether the geometric series below converges, if so find value of the series.

$$\sum_{n=1}^{\infty} 4 \cdot \frac{2^{3n+1}}{5^{2n}}$$

5. Use partial fractions to split the terms of the series. Use telescoping to find the value of the series.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}.$$

3 Integral Test

6. Use the integral test to determine whether the series below converge

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln(n)}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln(n))^2}$

4 Comparison Tests

7. Find an appropriate series to compare the following series to. Use the comparison test to decide if the series converges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n-\frac{1}{3}}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$

8. Find an appropriate series to limit compare the following series to. Use the limit comparison test to decide if the series converges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^4-n^2+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} n^3 e^{-2n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{n^2+3n-2}$

5 Ratio and Root Tests

9. Use the ratio test on the following series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{3^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n}$

10. Use the root test on the following series.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^3+n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

6 Absolute and Conditional Convergence

11. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{n^2+1}$ converges absolutely, converges conditionally or diverges.

12. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{n+1}$ converges absolutely, converges conditionally or diverges.

13. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ converges absolutely, converges conditionally or diverges.

7 Power Series

14. Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{2}{n^2 3^n}(x-1)^n$.

15. Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^n}(x-3)^n$.

16. Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{n}{(n+1)3^n}(x+5)^n$.