DEVELOPMENT AND FUTURE OF WAVELET ANALYSIS

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Abstract:

Here, we want to look ahead and see what the future can bring to wavelet research. We try to find a common denominator for ”wavelet” and identify promising research directions and challenging problems.

Keywords:

Wavelet analysis, development, future

1. Introduction

As so many papers in this special issue show, wavelet analysis has been successfully applied in a wide variety of research areas. These articles prove that wavelet provides a common framework to study problems that at first sight seem unrelated. Part of the power of wavelet comes from the fact that they lie at the crossroads of a wide variety of research areas.

We want to look ahead and ask ourselves the question: What is the future of wavelet? We point out in which direction the wavelet research front is moving and identify challenging problems.

Predicting is hard, especially about the future. There is no need to take these writings too seriously: let us relax and enjoy discussing current issues in wavelet research.

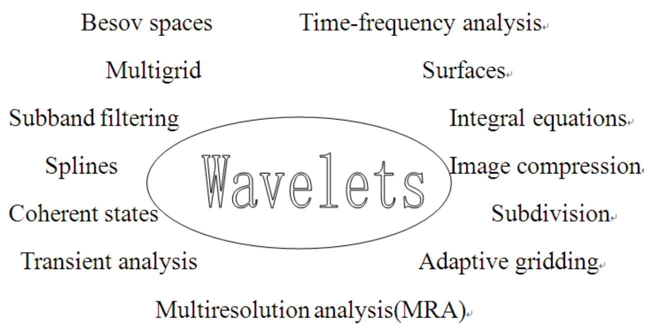

Fig.1 Many areas of science, engineering, and mathematics have contributed to the development of wavelets. Some of these are indicated surrounding the center bubble

2. What are wavelets[1-7] ?

In this section we give an idea of what makes a function a wavelet and wavelets are desirable in certain applications. Given that the wavelet fields keeps growing, the definition of a wavelet continuously changes. Therefore it is almost impossible to rigorously define a wavelet. As in most areas of research, the wavelet knowledge front advance like an infinite dimensional fractal, sometimes taking off in isolated directions but also many times folding back onto itself. Finding a definition of a wavelet is like approximating this fractal with a ball.

Following we give a definition of wavelet, this definition is not rigorous.

Definition:

(A) Wavelets are building blocks for general functions.

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$$f(x) = \sum_{j,k} c_{j,k} 2^{j/2} \Psi(2^j x - k)$$

(B) Wavelets have time-frequency localization. Locality in time implies that most of the energy of a wavelet is restricted to a finite interval.

Locality in frequency means that a wavelet has smoothness and vanishing moments.

(C) Wavelets have fast transform algorithms.

3. Why do wavelets work?

$$f(x) = \sum_{j,k=-\infty}^{+\infty} c_{j,k} 2^{j/2} \Psi(2^j x - k)$$

$$f_M(x) = \sum_{(j,k)\in\Lambda_M} c_{j,k} 2^{j/2} \Psi(2^j x - k)$$

$$\downarrow$$

$$\|f(x) - f_M(x)\| = O(M^{-\alpha})$$

This is the key to applications of wavelets.

4. Instances of wavelets

(1) Dyadic translates and dilates of one function
(2) Wavelet packets
(3) Local trigonometric base
(4) Multiwavelets
(5) Second generation wavelets

5. Questionnaire

(1) In your opinion, which are the most promising developments in wavelets?
(2) Where do you feel the field is moving?
(3) What kind of theoretical developments(theorems) Are needed to further push forward the field?
(4) What kind of wavelet functions should be further be sought after?
(5) Which application domains have not yet been sufficiently explored with wavelets?
(6) Where will wavelets be in 10 years from today?
(7) Have wavelets kept their promises?
(8) Are there any dangers threatening the wavelet field?

3. Answers

(1) Wavelets and PDEs

On the practical side significant progress has been made in boundary element methods since the Beylkin-Coifman-Rokhlin paper. Wavelets seem to be more general than multi-pole methods. We still need fast quadrature formulas for the discretization of integral operators.

In case of elliptic PDEs the existing algorithms are good. Wavelets will not necessarily outperform them, but they could be equal. Multigrid is very efficient, especially on the simple problems done so far with wavelets. We should combine wavelet and finite element methods. For example, we can now prove that finite element method codes work using the norm equivalence of unconditional wavelet expansions.

Classical wavelets seems to be good in computing low frequency scattering and antenna problems. However they fail in case of very high frequencies. Additional engineering expertise is needed to solve the problem. High frequency oscillatory integral kernels need local cosines, not classical wavelets.

In the context of building solvers for integral equations, adaptive methods are crucial. There is no point in first building a full N*N matrix at a cost of O(N 2), and then using wavelets to invert the matrix in O(N) steps. One should immediately build the compressed matrix in the wavelet basis. This approach needs of fast and accurate error estimators.

(2) Interesting Recent Wavelet Developments

Denoising has both opened up other fields and imported techniques such as dictionaries and nonlinear approximation from other fields. Nonlinear approximation, smoothing, and reduction to small optimization problems, are real achievements.

Wavelets have had a big psychological impact. People from many different areas became interested in time-frequency and time-scale transforms. There has been a revolution in signal processing . There is less specialization, and the subject is now opened to new problems. More than just a simple tool, wavelet ideas prompt new points of view. Some of the best ideas are not written down. The big influence will come from the new generation researchers now growing up amidst wavelet ideas.

Wavelets have advanced our understanding of singularities. The singularity spectrum completely characterizes the complexity of the underlying phenomena, to get the equation from the solution. Wavelets do not give all the answers but they force us to ask right questions.

Wavelets can be used to distinguish coherent versus incoherent parts of turbulence in fluid flow. They give some information but do not entirely solve the problem. Experiments provide high Reynolds' numbers with few measurements, while simulations provide many measurements but are restricted to low Reynolds' numbers.

In Japan, wavelets resulted in a fruitful collaboration between academic research and industrial development.

The results on regularity, approximation power, and wavelet design techniques , have led to significant developments in signal and image processing. For instance, some of the very best filter banks for image coding, including biorthogonal spline filter banks, have been designed from a wavelet point of view rather than the more traditional subband filtering point of view.

(3) Theorems needed to advance the field

We need good theoretical models on how to distinguish signal from noise.

We need to further study nonlinear multiscale methods in order to understand self-organizing multiscale systems with nonlinear relations between scales. Test cases are caricatures of turbulence, the stock market, or fractal piles of sand. There is no unifying theory at this moment, but maybe wavelets can provide one. We should not use linear multiscale modles in these cases but instead look for nonlinear approximation with fast convergence. We need external measures to determine internal simplicity. The currently known tools are time-frequency analysis, smoothness, and L p spaces.

We need to understand simplicity in high-dimensional phenomena. We have tools and talent here to advance our understanding of such problems. Such problems will become more common as we collect more and more measurements for even simple physical systems.

It is important to understand the right balance between simple building blocks and complex representations, or complex building blocks and simple representations. Only focusing on one is naive.

We need to work on dictionaries as methods to representing complex objects in terms of simpler components.

Many results in higher dimensions are still incomplete. Theoretical advances in higher dimensional signal approximation bounds, regularity, design techniques, would be very useful in image processing applications. There is room for substantial improvement of the current state of the art.

(4) What kind of wavelets do we still need?

We would like to have isotropic, compactly supported, and orthogonal wavelets, but unfortunately they don't exist. Multi-wavelets can provide an answer here.

We need to work on spanning the gap between eigenfunctions and wavelets.

There seems to be a debate on the relationship between splines and wavelets. Some people feel that splines and spline based wavelets will be able to span the gap between eigenfunctions and wavelets. For example, splines already are exact solutions to variational problems. Other people feel that there is no fundamental difference between splines and (non spline based) wavelets and that each exact spline solution has a corresponding almost-solution in wavelets.

(5) Problems not sufficiently explored with wavelets

Prediction: The stock market, earthquakes, weather.

We should use wavelets to build models based on previously observed data.

Physics: We can use wavelets to define the scale of the observer. Wavelet techniques can be applied to quantum physics, atoms, and lasers.

Scientific computing and in particular computationalchemistry. We need to look for problems that need multiscale and nonlinear approximation methods.

Wavelets and filter banks should be systematically applied to problems in communications: forexample, ansmultiplexers to map many different signals onto one channel. Wavelets and filter banks provide a broad, rich class of new transforms with which to do this transmultiplexing, offering the familiar advantages of time localization and frequency selectivity. Because wavelets were initially developed in conversation with the subband coding community, these applications have gotten less attention. However, because of the large global telecommunications market, telecommunication applications of wavelets could be the sleeper that surprises everybody.

Really make wavelet methods work for video compression. While MPEG's success and proliferation make medium-bitrate wavelet video a purely academic endeavor, low-bitrate compression remains a place where wavelet methods can prevail.

We should develop a wavelet image compression standard more general than just fingerprints.

The idea of smoothness of functions should really be developed and applied as a tool for signal processing-incorporated into models of filtering and distortion. Donoho has already shown how smoothness and wavelets fit together for statistical denoising; similar attention should be given to the role of smoothness in image compression and restoration. Given the characterizations of Sobolev and Holder smoothness in terms of wavelet coefficients, this problem area begs for the use of wavelet methods.

There is a big need for wavelet constructions adapted to irregular two-dimensional sampling.

Wavelets have just opened the door to the field of non-stationary, non-uniform, non-time invariant signal processing. This field is much larger than the field of time-invariant processing where the Fourier Transform rules. It is like going from linear to nonlinear operators. In this huge domain, wavelets are one important tools but many others are needed, wavelet packets, local cosine and may other transforms will come. Important orientations are: adaptive transforms, the use of other operators than convolution operators in signal processing applications, and development of stochastic models for non-stationary processes. Right now most things have been done on signals, but the same techniques can be used for numerical computations.

(6) Dangers facing the field

There is too much media distortion concerning wavelets. Wavelets are oversold and in danger from their fashionability. We must say no to certain contracts. Claims by wavelet proponents, such as picture compression people, can not always be met. We should be more conservative in our claims.

There is a danger of underselling wavelets! In the JPEG and MPEG standards, there are no wavelets.

There is a danger that wavelet people will split off from their previous specialties and start inbreeding. Wavelet people should preserve their roots.

The fact that wavelets have a solid theoretical foundation will prevent them from burning out after 10 years.

6. Conclusion

We have tried to give an idea of what people feel wavelets are, why they work, and in which direction they are moving. Obviously this is only a snapshot taken from one point of view of a continuously evolving story and the only way to find out how it ends is to wait and see[1-7].

Acknowledgments

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