

# Projects and how do develop a numerical project

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## Plan for the Talent course

An essential element of the Talent courses is to develop a large project(s) which allows you to study and understand teoretical concepts in nuclear physics. These concepts will in turn allow you to interpret results from experiments and understand the pertinent physics in terms of the underlying forces and laws of motion.

Together with the regular lectures in the morning, the hope is that during these three weeks you will be able to write and run a program which implements at least one of the methods discussed during the lectures. The lectures will also cover additional material which aims at giving you a broader view on what can be achieved with the methods to be discussed. Combined with the 'hands-on' afternoon sessions, the hope is that the lectures and the computational projects will together allow you to achieve these goals. For those of you who would like to get credits to be transferred to your home university, the project(s) can be extended upon allowing you to include further elements to the many-body methods. The load of the final project is estimated to be 80 hours. In total, attendance at the course and doing the final project amounts to seven ECTS. There is no credit transfer for Northern-American students.

## Outline of the project(s)

What is outlined here are several paths that will allow you to develop a program which can be used to study nuclear systems. In particular, we will focus on two widely used many-body methods, coupled cluster theory with its various approximations and Green's function theory.

Before we begin, we would however like to present a warm-up problem which contains many of the basic elements of the many-body methods exposed in this course. Furthermore, the simple pairing model described below, provides us with benchmark results to which we can compare the different many-body methods. The basic elements of the model (single-particle basis and Hamiltonian) can easily be (if structured properly in your program) extended to studies of more realistic systems, either finite systems like nuclei or infinite nuclear matter.

The first part of the project is a traditional paper and pencil part, where the aim is to apply second quantization in order to set up the Hamiltonian matrix to be diagonalized. We will study this model the first two afternoons of the school.

At most, with four particles in four doubly degenerate single particle states and no broken pairs, you will have a  $6 \times 6$  Hamiltonian matrix to diagonalize. Below, you will find a simple python program which performs the diagonalization and plots the eigenvalues. Parts of this warm-up exercise are solved below. These aspects will be discussed partly during the lectures but also during some of the afternoon sessions. Practical guidelines for code writing will also be discussed.

This exercise will allow us to start writing our first skeleton for a coupled cluster theory program at the so-called doubles level of truncation. It will also, for those of you who will prefer to study Green's functions, serve as a starting point for developing the skeleton of a program for this method. The aim is to finish this part of the program during the first week.

With a program which then benchmarks the simple pairing model, we will turn our attention to more realistic systems. Here we propose the following paths that can be studied with either coupled cluster theory or Green's function theory. Many of the technicalities will be discussed at the lectures and the afternoon sessions during the last two weeks.

1. Studies of nuclear matter with a cartesian basis using coupled cluster theory at the level of doubles excitations
  - During the three weeks of the course we will mainly focus on two-particle and two-hole excitations
  - Focusing only on two-particle correlations in a finite cartesian basis, these can be compared with Brueckner-Hartree-Fock calculations in the infinite limit. The latter should represent numerically exactly two-particle correlations to infinite order. A simple model for the nuclear force will be used.
  - For the final project the code can be extended to include particle-hole correlations as well

2. Studies of finite nuclei using coupled cluster theory at the level of doubles excitations only
  - Here we will provide Hartree-Fock based single-particle states and two-body interactions in an uncoupled scheme.
  - The system to be studied is that of neutrons in harmonic oscillator traps.
  - For the final project, one can include singles excitations as well.
3. The two above paths can be performed using Green's function as well.
  - Studies of infinite nuclear matter including two-particle and two-hole correlations using a cartesian basis.
  - Comparison of two-particle correlations from Brueckner-Hartree-Fock calculations in the infinite limit
4. Studies of finite nuclei with the Green's function method
  - Again we will provide Hartree-Fock based single-particle states and two-body interactions in an uncoupled scheme.
  - The system to be studied is that of neutrons in harmonic oscillator traps.
  - For the final project, one can include self-consistently calculated single-particle energies as well.

When working on the projects, we recommend strongly that you form teams of two-three participants. Every team **must** have a *github account* where we can monitor your progress and give you appropriate feedback.

If you have not used version control before now, it is time to do so. Proper version control is central to a good ethical scientific conduct. We do require that you use some kind of version control software when working on the projects. We recommend strongly [github](#).

Furthermore, before coming to the course, we recommend that you refresh your knowledge on second quantization. If your background in second quantization is rudimentary and mentioning of Wick's leave you gazing at the stars, we recommend that you study the material at the course web site on [second quantization](#). Try in particular to do some of the exercises.

We will also require that you come with your own laptop and have installed either a

- Fortran compiler or a
- c++ compiler

You can use Python as programming language as well, but normally the efficiency of Python for the problems addressed in this course is lower than for codes written in Fortran or c++. We recommend however that use Python as a scripting language for running codes and making plots, as well as using the ipython notebooks provided by us.

## Some basic ingredients for a successful numerical project

In when building up a numerical project there are several elements you should think of

1. How to structure a code in terms of functions
2. How to make a module
3. How to read input data flexibly from the command line
4. How to create graphical/web user interfaces
5. How to write unit tests (test functions or doctests)
6. How to refactor code in terms of classes (instead of functions only), in our case you think of a system and a solver class
7. How to conduct and automate large-scale numerical experiments
8. How to write scientific reports in various formats (L<sup>A</sup>T<sub>E</sub>X, HTML)

The conventions and techniques outlined here will save you a lot of time when you incrementally extend software over time from simpler to more complicated problems. In particular, you will benefit from many good habits:

1. New code is added in a modular fashion to a library (modules)
2. Programs are run through convenient user interfaces
3. It takes one quick command to let all your code undergo heavy testing
4. Tedious manual work with running programs is automated,
5. Your scientific investigations are reproducible, scientific reports with top quality typesetting are produced both for paper and electronic devices.

## The pairing model as warm-up problem

We present a simplified Hamiltonian consisting of an unperturbed Hamiltonian and a so-called pairing interaction term. It is a model which to a large extent mimicks some central features of atomic nuclei, certain atoms and systems which exhibit superfluidity or superconductivity. To study this system, we will use a mix of many-body perturbation theory (MBPT), Hartree-Fock (HF) theory and full configuration interaction (FCI) theory. The latter will also provide us with the exact answer. When setting up the Hamiltonian matrix you will need to solve an eigenvalue problem.

We define first the Hamiltonian, with a definition of the model space and the single-particle basis. Thereafter, we present the various exercises.

The Hamiltonian acting in the complete Hilbert space (usually infinite dimensional) consists of an unperturbed one-body part,  $\hat{H}_0$ , and a perturbation  $\hat{V}$ .

We limit ourselves to at most two-body interactions, our Hamiltonian is then represented by the following operators

$$\hat{H} = \sum_{\alpha\beta} \langle \alpha | h_0 | \beta \rangle a_\alpha^\dagger a_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma,$$

where  $a_\alpha^\dagger$  and  $a_\alpha$  etc. are standard fermion creation and annihilation operators, respectively, and  $\alpha\beta\gamma\delta$  represent all possible single-particle quantum numbers. The full single-particle space is defined by the completeness relation

$$\hat{\mathbf{1}} = \sum_{\alpha=1}^{\infty} |\alpha\rangle \langle \alpha|.$$

In our calculations we will let the single-particle states  $|\alpha\rangle$  be eigenfunctions of the one-particle operator  $\hat{h}_0$ . Note that the two-body part of the Hamiltonian contains anti-symmetrized matrix elements.

The above Hamiltonian acts in turn on various many-body Slater determinants constructed from the single-basis defined by the one-body operator  $\hat{h}_0$ . As an example, the two-particle model space  $\mathcal{P}$  is defined by an operator

$$\hat{P} = \sum_{\alpha\beta=1}^m |\alpha\beta\rangle \langle \alpha\beta|,$$

where we assume that  $m = \dim(\mathcal{P})$  and the full space is defined by

$$\hat{P} + \hat{Q} = \hat{\mathbf{1}},$$

with the projection operator

$$\hat{Q} = \sum_{\alpha\beta=m+1}^{\infty} |\alpha\beta\rangle \langle \alpha\beta|,$$

being the complement of  $\hat{P}$ .

Our specific model consists of  $N$  doubly-degenerate and equally spaced single-particle levels labelled by  $p = 1, 2, \dots$  and spin  $\sigma = \pm 1$ . These states are schematically portrayed in Fig. reffig:schematic. The first two single-particle levels define a possible model space, indicated by the label  $\mathcal{P}$ . The remaining states span the excluded space  $\mathcal{Q}$ .

We write the Hamiltonian as

$$\hat{H} = \hat{H}_0 + \hat{V},$$

where

$$\hat{H}_0 = \xi \sum_{p\sigma} (p-1) a_{p\sigma}^\dagger a_{p\sigma}$$

and

$$\hat{V} = -\frac{1}{2}g \sum_{pq} a_{p+}^\dagger a_{p-}^\dagger a_{q-} a_{q+}.$$

Here,  $H_0$  is the unperturbed Hamiltonian with a spacing between successive single-particle states given by  $\xi$ , which we will set to a constant value  $\xi = 1$  without loss of generality. The two-body operator  $\hat{V}$  has one term only. It represents the pairing contribution and carries a constant strength  $g$ .

The indices  $\sigma = \pm$  represent the two possible spin values. The interaction can only couple pairs and excites therefore only two particles at the time, as indicated by the rightmost four-particle state in Fig. reffig:schematic. There one of the pairs is excited to the state with  $p = 9$  and the other to the state  $p = 7$ . The two middle possibilities are not possible with the present model. We label single-particle states within the model space as hole-states. The single-particle states outside the model space are then particle states.

In our model we have kept both the interaction strength and the single-particle level as constants. In a realistic system like an atom or the atomic nucleus this is not the case.

### Exercise 1: Pairing Hamiltonian

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a) Show that the unperturbed Hamiltonian  $\hat{H}_0$  and  $\hat{V}$  commute with both the spin projection  $\hat{S}_z$  and the total spin  $\hat{S}^2$ , given by

$$\hat{S}_z := \frac{1}{2} \sum_{p\sigma} \sigma a_{p\sigma}^\dagger a_{p\sigma}$$

and

$$\hat{S}^2 := \hat{S}_z^2 + \frac{1}{2}(\hat{S}_+ \hat{S}_- + \hat{S}_- \hat{S}_+),$$

where

$$\hat{S}_\pm := \sum_p a_{p\pm}^\dagger a_{p\mp}.$$

This is an important feature of our system that allows us to block-diagonalize the full Hamiltonian. We will focus on total spin  $S = 0$ . In this case, it is convenient to define the so-called pair creation and pair annihilation operators

$$\hat{P}_p^+ = a_{p+}^\dagger a_{p-}^\dagger,$$

and

$$\hat{P}_p^- = a_{p-} a_{p+},$$

respectively.

Show that you can rewrite the Hamiltonian (with  $\xi = 1$ ) as

$$\hat{H} = \sum_{p\sigma} (p-1) a_{p\sigma}^\dagger a_{p\sigma} - \frac{1}{2}g \sum_{pq} \hat{P}_p^+ \hat{P}_q^-.$$

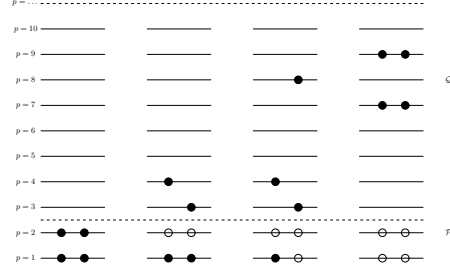


Figure 1: Schematic plot of the possible single-particle levels with double degeneracy. The filled circles indicate occupied particle states while the empty circles represent vacant particle(hole) states. The spacing between each level  $p$  is constant in this picture. The first two single-particle levels define our possible model space, indicated by the label  $\mathcal{P}$ . The remaining states span the excluded space  $\mathcal{Q}$ . The first state to the left represents a possible ground state representation for a four-fermion system. In the second state to the left, one pair is broken. This possibility is however not included in our interaction.

Show also that Hamiltonian commutes with the product of the pair creation and annihilation operators. This model corresponds to a system with no broken pairs. This means that the Hamiltonian can only link two-particle states in so-called spin-reversed states.

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b) Construct thereafter the Hamiltonian matrix for a system with no broken pairs and total spin  $S = 0$  for the case of the four lowest single-particle levels indicated in the Fig. reffig:schematic. Our system consists of four particles only. Our single-particle space consists of only the four lowest levels  $p = 1, 2, 3, 4$ . You need to set up all possible Slater determinants. Find all eigenvalues by diagonalizing the Hamiltonian matrix. Vary your results for values of  $g \in [-1, 1]$ .

We refer to this as the exact calculation. Comment the behavior of the ground state as function of  $g$ .

**Solution.** When calculating the matrix elements, we see that we can either have no non-coincidences, two non-coincidences or four non-coincidences. Another way to state this is to say that there can a mismatch of zero, one or two pairs.

The following python program diagonalizes the above Hamiltonian matrix for a given span of interaction strength values.

```
from numpy import *
from sympy import *
from matplotlib.pyplot import *

g_array = linspace(-1, 1, 1001)
e1_array = []
e2_array = []

for g in g_array:
    H1 = matrix([[2-g, -g/2., -g/2., -g/2., -g/2., 0],
                 [-g/2., 4-g, -g/2., -g/2., 0., -g/2.],
                 [-g/2., -g/2., 6-g, 0, -g/2., -g/2.],
                 [-g/2., -g/2., 0, 6-g, -g/2., -g/2.],
                 [-g/2., 0, -g/2., -g/2., 8-g, -g/2.],
                 [0, -g/2., -g/2., -g/2., -g/2., 10-g]])

    H2 = matrix([[2-g, -g/2., -g/2., -g/2., -g/2.],
                 [-g/2., 4-g, -g/2., -g/2., 0.],
                 [-g/2., -g/2., 6-g, 0, -g/2.],
                 [-g/2., -g/2., 0, 6-g, -g/2.],
                 [-g/2., 0, -g/2., -g/2., 8-g]])

    u1, v1 = linalg.eig(H1)
    u2, v2 = linalg.eig(H2)

    if g == 1./2:
        print argmin(u1)

        for i in range(5):
            print "%.3f " % v2[i,0],

    e1_array.append(min(u1))
    e2_array.append(min(u2))

plot(g_array, e1_array, linewidth=2.0)
#plot(g_array, e2_array, linewidth=2.0)
plot(g_array, (2-g_array), linewidth=2.0)
grid()
xlabel(r"Strength of interaction, $g$", fontsize=16)
ylabel(r'Ground state energy', fontsize=16)
#axis([-1,1,-0.4,0.05])
legend(['FCI -- Exact', 'Reference energy'])
savefig("pairing.pdf")
show()
```



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c) Instead of setting up all possible Slater determinants, construct only an approximation to the ground state (where we assume that the four particles are in the two lowest single-particle orbits only) which includes at most two-particle-two-hole excitations. Diagonalize this matrix and compare with the exact calculation and comment your results. Can you set up which diagrams this approximation corresponds to?

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d) We switch now to approximative methods, in our case Hartree-Fock theory and many-body perturbation theory. Hereafter we will define our model space to consist of the single-particle levels  $p = 1, 2$ . The remaining levels  $p = 3, 4$  define our excluded space. This means that our ground state Slater determinant consists of four particles which can be placed in the doubly degenerate orbits  $p = 1$  and  $p = 2$ . Our first step is to perform a Hartree-Fock calculation with the pairing Hamiltonian. Write first the normal-ordered Hamiltonian with respect to the above reference state given by four spin 1/2 fermions in the single-particle levels  $p = 1, 2$ . Define what is meant by a canonical Hartree-Fock case, a non-canonical case and a general case. For all three cases, write down the normal-ordered Hamiltonian and draw the diagrammatic form of the Hamiltonian for all three cases.

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e) We will now set up the Hartree-Fock equations by varying the coefficients of the single-particle functions. The single-particle basis functions are defined as

$$\psi_p = \sum_{\lambda} C_{p\lambda} \psi_{\lambda}.$$

where in our case  $p = 1, 2, 3, 4$  and  $\lambda = 1, 2, 3, 4$ , that is the first four lowest single-particle orbits of Fig. reffig:schematic. Set up the Hartree-Fock equations for this system by varying the coefficients  $C_{p\lambda}$  and solve them for values of  $g \in [-1, 1]$ . Comment your results and compare with the exact solution. Discuss also which diagrams in Fig. reffig:diagrams that can be affected by a Hartree-Fock basis. Compute the total binding energy using a Hartree-Fock basis and comment your results.

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f) We will now study the system using non-degenerate Rayleigh-Schroedinger perturbation theory to third order in the interaction. If we exclude the first order contribution, all possible diagrams (so-called anti-symmetric Goldstone diagrams) are shown in Fig. reffig:diagrams.

Based on the form of the interaction, which diagrams contribute to the binding energy of the ground state? Write down the expressions for the diagrams that contribute and find the contribution to the ground state energy as function  $g \in [-1, 1]$ . Comment your results. Compare these results with those you obtained from the exact diagonalization with and without the  $4p - 4h$  state.

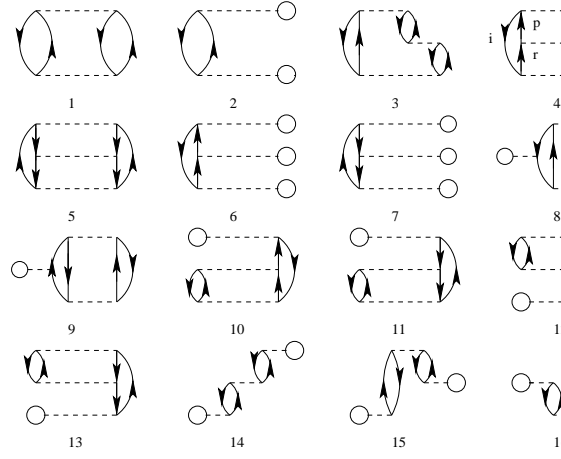


Figure 2: Diagrams to third order in the interaction, including also non-canonical Hartree-Fock diagrams. The first order term is excluded. All interaction vertices represent anti-symmetrized matrix elements.

Discuss your results for a canonical Hartree-Fock basis and a non-canonical Hartree-Fock basis.

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g) Diagram 1 in Fig. reffig:diagrams represents a second-order contribution to the energy and a so-called  $2p - 2h$  contribution to the intermediate states. Write down the expression for the wave operator in this case and compare the possible contributions with the configuration interaction calculations without the  $4p - 4h$  Slater determinant. Comment your results for various values of  $g \in [-1, 1]$ .

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h) We limit now the discussion to the canonical Hartree-Fock case only. To fourth order in perturbation theory we can produce diagrams with  $1p - 1h$  intermediate excitations as shown in Fig. reffig:fourthorder1p1h,  $2p - 2h$  excitations, see Fig. reffig:fourthorder2p2h,  $3p - 3h$  excitations as shown in Fig. reffig:fourthorder3p3h and finally so-called diagrams with intermediate four-particle-four-hole excitations, see Fig. reffig:fourthorder4p4h.

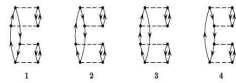


Figure 3: One-particle-one-hole excitations to fourth order.

Calculate the energy to fourth order with a canonical Hartree-Fock basis for  $g \in [-1, 1]$  and compare with the full diagonalization case in exercise 2). Discuss the results.

**Solution.** The following Python program gives us the final results.

```
from sympy import *
from pylab import *

below_fermi = (0,1,2,3)
above_fermi = (4,5,6,7)

states = [(1,1),(1,-1),(2,1),(2,-1),(3,1),(3,-1),(4,1),(4,-1)]
N = 8
g = Symbol('g')

def h0(p,q):
    if p == q:
        p1, s1 = states[p]
        return (p1 - 1)
    else:
        return 0
```

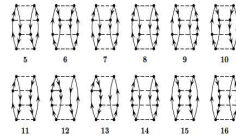


Figure 4: Two-particle-two-hole excitations to fourth order.

```
def f(p,q):
    if p == q:
        return 0

    s = h0(p,q)
    for i in below_fermi:
        s += assym(p,i,q,i)
    return s

def assym(p,q,r,s):
    p1, s1 = states[p]
    p2, s2 = states[q]
    p3, s3 = states[r]
    p4, s4 = states[s]

    if p1 != p2 or p3 != p4:
        return 0
    if s1 == s2 or s3 == s4:
        return 0
    if s1 == s3 and s2 == s4:
        return -g/2.
    if s1 == s4 and s2 == s3:
        return g/2.

def eps(holes, particles):
```

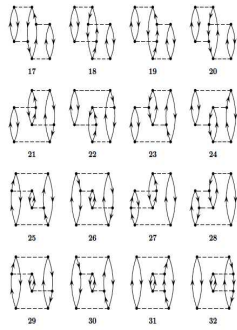


Figure 5: Three-particle-three-hole excitations to fourth order.

```

E = 0
for h in holes:
    p, s = states[h]
    E += (p-1)
for p in particles:
    p, s = states[p]
    E -= (p-1)
return E

# Diagram 3
# s = 0
# for a in above_fermi:
#     for b in above_fermi:
#         for c in above_fermi:
#             for i in below_fermi:
#                 for j in below_fermi:
#                     for k in below_fermi:
#                         s += assym(i, j, a, b)*assym(a, c, j, k)*assym(b, k, c, i)/eps
# print s

# ga = linspace(-1,1,101)
# corr2 = []
# corr3 = []
# corrx = []

```

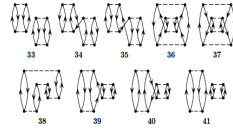


Figure 6: Four-particle-four-hole excitations to fourth order. Define first linked and unlinked diagrams and explain briefly Goldstone's linked diagram theorem. Based on the linked diagram theorem and the form of the pairing Hamiltonian, which diagrams will contribute to fourth order?

```
# Diagram 1
s1 = 0
for a in above_fermi:
    for b in above_fermi:
        for i in below_fermi:
            for j in below_fermi:
                s1 += 0.25*assym(a,b,i,j)*assym(i,j,a,b)/eps((i,j),(a,b))

# Diagram 4
s4 = 0
for a in above_fermi:
    for b in above_fermi:
        for c in above_fermi:
            for d in above_fermi:
                for i in below_fermi:
                    for j in below_fermi:
                        s4 += 0.125*assym(i,j,a,b)*assym(a,b,c,d)*as

# Diagram 5
s5 = 0
```

```

for a in above_fermi:
    for b in above_fermi:
        for i in below_fermi:
            for j in below_fermi:
                for k in below_fermi:
                    for l in below_fermi:
                        s5 += 0.125*assym(i,j,a,b)*assym(k,l,i,j)*assym(a,b,k,l)

# Diagram 8 (simplified)
s8 = 0
for a in above_fermi:
    for b in above_fermi:
        for i in below_fermi:
            for j in below_fermi:
                for k in below_fermi:
                    s8 -= 0.5*assym(i,j,a,b)*assym(a,b,i,k)*f(k,j)/eps((i,j),(a,b))

# Diagram 9 (simplified)
s9 = 0
for a in above_fermi:
    for b in above_fermi:
        for c in above_fermi:
            for i in below_fermi:
                for j in below_fermi:
                    s9 += 0.5*assym(i,j,a,b)*assym(a,c,i,j)*f(b,c)/eps((i,j),(a,b))

print s1
print s4
print s5
print s8
print s9

s_5 = -0.0291521990740741*g**4
s14 = -0.0308883101851853*g**4
s34 = 0.0163049768518519*g**4
s36 = -0.0145760995370371*g**4
s38 = -0.0201099537037037*g**4
s39 = 0.0176938657407407*g**4

ga = linspace(-1,1,10001)
e1 = []
corr2 = []
corr3 = []
corr4 = []
for g_val in ga:
    H1 = matrix([[2-g_val, -g_val/2., -g_val/2., -g_val/2., -g_val/2., 0],
                 [-g_val/2., 4-g_val, -g_val/2., -g_val/2., 0., -g_val/2.],
                 [-g_val/2., -g_val/2., 6-g_val, 0, -g_val/2., -g_val/2.],
                 [-g_val/2., -g_val/2., 0, 6-g_val, -g_val/2., -g_val/2.],
                 [-g_val/2., 0, -g_val/2., -g_val/2., 8-g_val, -g_val/2.],
                 [0, -g_val/2., -g_val/2., -g_val/2., -g_val/2., 10-g_val]])

    u1, v1 = linalg.eig(H1)
    e1.append(min(u1))

    corr2.append((s1).subs(g,g_val))
    corr3.append((s1+s4+s5).subs(g,g_val))
    corr4.append((s1+s4+s5+2*s_5+2*s14+2*s34+2*s36+s38+2*s39).subs(g,g_val))

exact = e1 - (2-ga)

```

```

plot(ga, exact, linewidth=2.0)
plot(ga, corr2, linewidth=2.0)
plot(ga, corr3, linewidth=2.0)
plot(ga, corr4, linewidth=2.0)
xlabel(r'Interaction strength, $g$', fontsize=16)
ylabel(r'Correlation energy', fontsize=16)
axis([-1,1,-0.5,0.05])
grid()
legend(["Exact", "2. order MPBT", "3. order MPBT", "4. order MPBT"], 'lower left')
savefig("perturbationtheory.pdf")
show()
error1 = zeros(len(exact))
error2 = zeros(len(exact))
error3 = zeros(len(exact))

for i in range(len(exact)):
    error1[i] = abs(float(exact[i]-corr2[i]))
    error2[i] = abs(float(exact[i]-corr3[i]))
    error3[i] = abs(float(exact[i]-corr4[i]))

error1 = array(error1)
error2 = array(error2)
error3 = array(error3)
print type(error1)

plot(ga, log10(error1))
plot(ga, log10(error2))
plot(ga, log10(error3))
xlabel(r"Strength of interaction, $g$", fontsize=16)
ylabel(r"Error, $\log_{10}(\text{abs}(\text{error}))$", fontsize=16)
legend(["2. order MPBT", "3. order MPBT", "4. order MPBT"], 'lower left')
grid()
savefig("logerror.pdf")
show()

```

**Project 2:** Coupled cluster calculations with doubles excitations only for the pairing model

**Project 3:** Coupled cluster calculations with doubles excitations only for infinite nuclear matter

**Project 4:** Coupled cluster calculations with doubles excitations only for finite nuclei

**Project 5:** Green's function calculations with doubles excitations only for the pairing model

**Project 6:** Green's function calculations with doubles excitations only for infinite nuclear matter

**Project 7:** Green's function calculations with doubles excitations only for finite nuclei