Introduce gap equation

- Many nontrivial ways to introduce gap equation
  - Variational ansatz BCS wave function (violates particle number)
  - Nambu derivation adding a pair adding and removing auxiliary Hamiltonian (violates particle number)
  - Propagator formulation with anomalous propagators (violates particle number) —> has analogy to Bose condensation —> Gorkov formulation
  - Average particle number can and must be restored
- State without derivation but it should be clear what it accomplishes
- Motivation first

Pairing in neutron stars...

- 1959 Migdal suggests pairing in neutron stars before they are even observed...
- 1969 Vela and Crab pulsars exhibit sudden spin-ups (glitches)
- Relaxation to constant rate of slowing down too slow to be explained in terms of viscous processes of normal matter --> glitches --> superfluidity (Pines)
- Critical information: pairing gap as a function of temperature
- BCS yields--->
- Lots and lots of BCS calculations of neutron matter
- Also calculations of pairing in symmetric matter --> puzzle

1.C: 1.E.6 Nuclear Physics 13 (1959) 655—674; © North-Holland Publishing Co., Amsterdam
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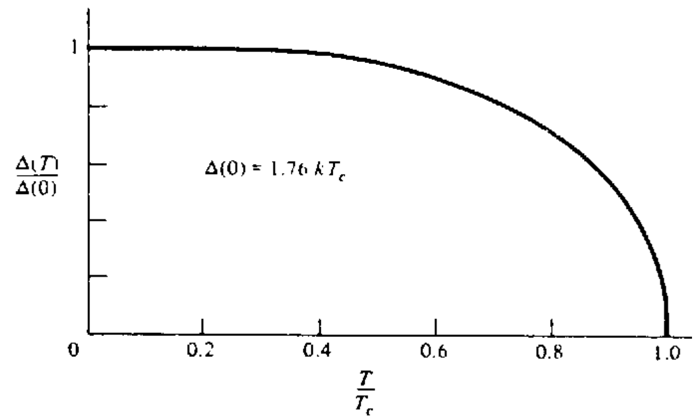
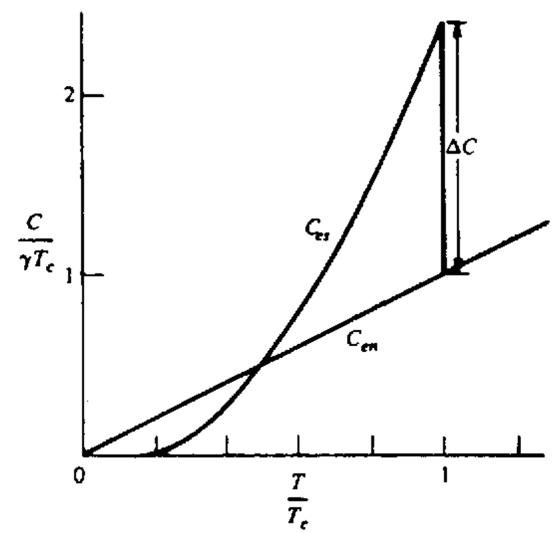
SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL
Atomic Energy Institute of USSR, Academy of Sciences, Moscow

Received 11 April 1959





Pairing N*

Cassiopeia A

Cooling observations

Vol 462 | 5 November 2009 | doi:10.1038/nature08525 nature

LETTERS

A neutron star with a carbon atmosphere in the Cassiopeia A supernova remnant

Wynn C. G. Ho[1] & Craig O. Heinke[2]

Mon. Not. R. Astron. Soc. **412**, L108–L112 (2011) doi:10.1111/j.1745-3933.2011.01015.x

Cooling neutron star in the Cassiopeia A supernova remnant: evidence for superfluidity in the core

Peter S. Shternin,[1,2★] Dmitry G. Yakovlev,[1] Craig O. Heinke,[3] Wynn C. G. Ho[4★] and Daniel J. Patnaude[5]

Pairing N*

Fingerprint of P-wave pairing?

- Rapid cooling
- Small gap P-wave or …?

Selected for a Viewpoint in *Physics*

PRL 106, 081101 (2011) PHYSICAL REVIEW LETTERS week ending 25 FEBRUARY 2011

Rapid Cooling of the Neutron Star in Cassiopeia A Triggered by Neutron Superfluidity in Dense Matter

Dany Page,[1] Madappa Prakash,[2] James M. Lattimer,[3] and Andrew W. Steiner[4]

PHYSICAL REVIEW C 85, 022802(R) (2012)

Cooling of the neutron star in Cassiopeia A

D. Blaschke,[1,2] H. Grigorian,[3] D. N. Voskresensky,[4,5] and F. Weber[6]

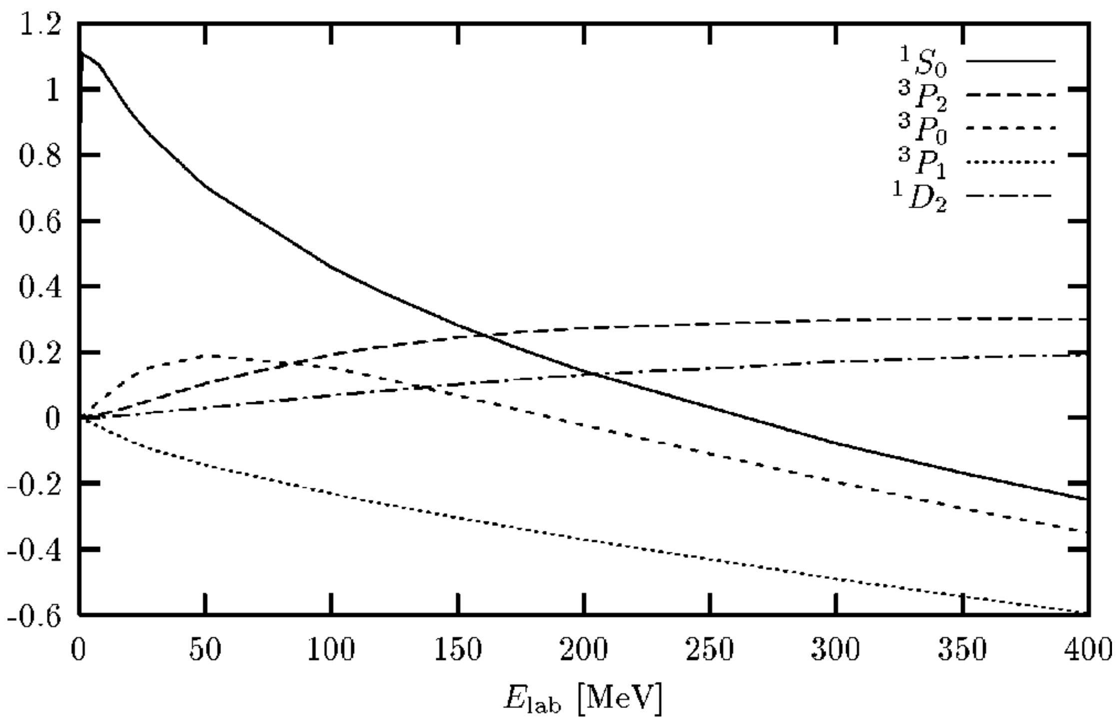
PHYSICAL REVIEW C 88, 065805 (2013)

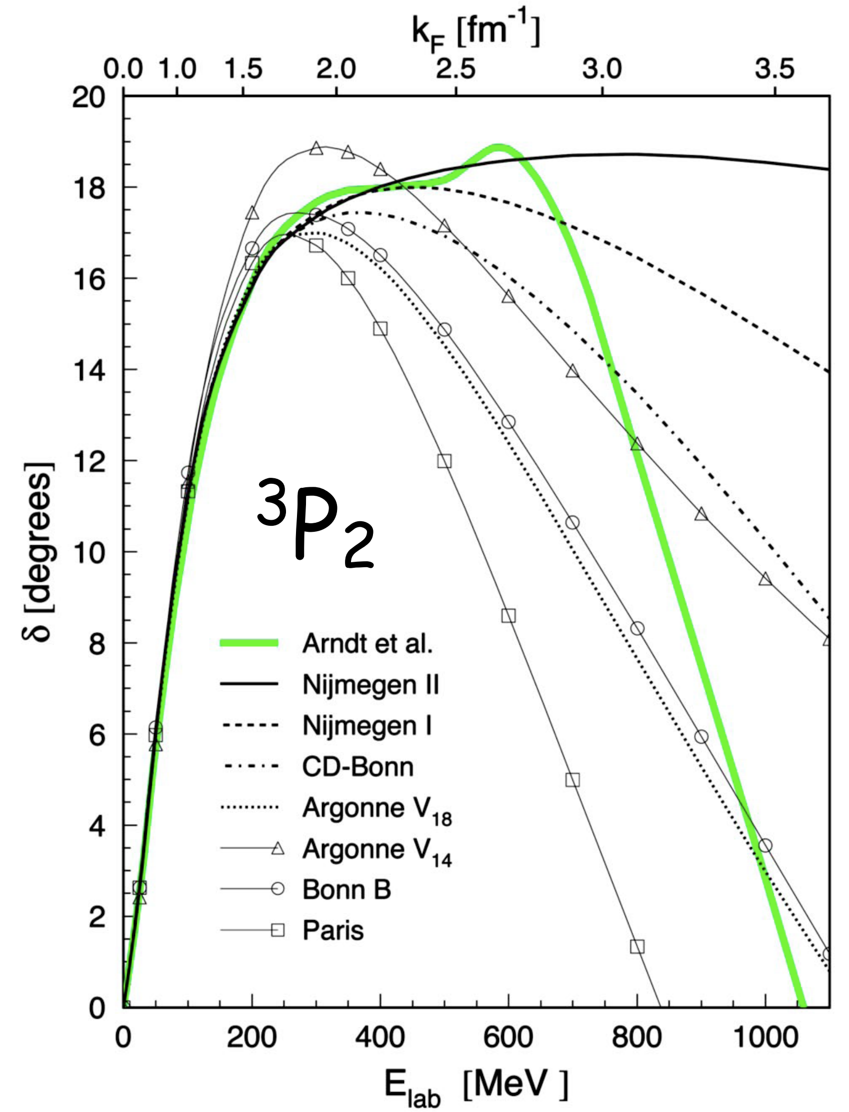
Nuclear medium cooling scenario in light of new Cas A cooling data and the $2M_\odot$ pulsar mass measurements

D. Blaschke,[1,2] H. Grigorian,[3,4] and D. N. Voskresensky[5]

NN interaction and phase shifts for T=1

- L+S+T --> odd (Pauli)
- T=1 --> L+S even
- Attraction: positive phase shift



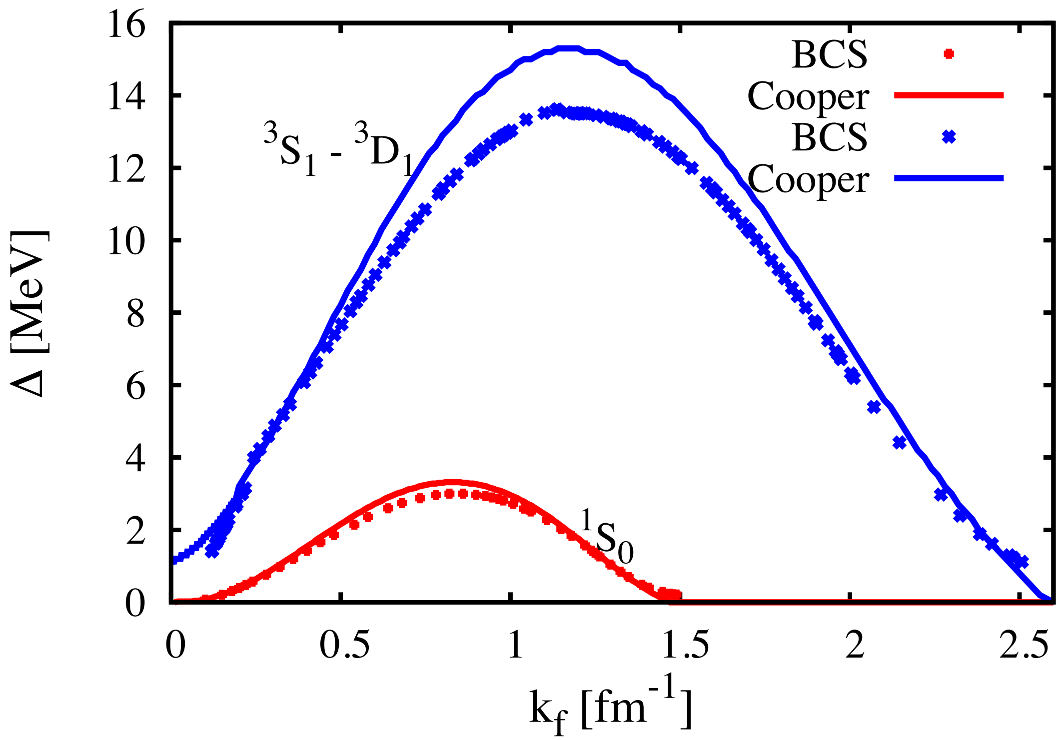


- --> low density $^1S_0$ dominates with $^3P_2$ possibly at higher density

Review: e.g. Dean & Hjorth-Jensen, Rev.Mod.Phys.75, 607 (2003)

BCS for $^3S_1$-$^3D_1$ in symmetric nuclear matter

- Puzzle



Mean-field particles

Early nineties: BCS gaps ~ 10 MeV

Alm et al. Z.Phys.A337,355 (1990)
Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)

Dressing nucleons is expected to reduce pairing strength as suggested by in-medium scattering

Removal probability for valence protons from NIKHEF data

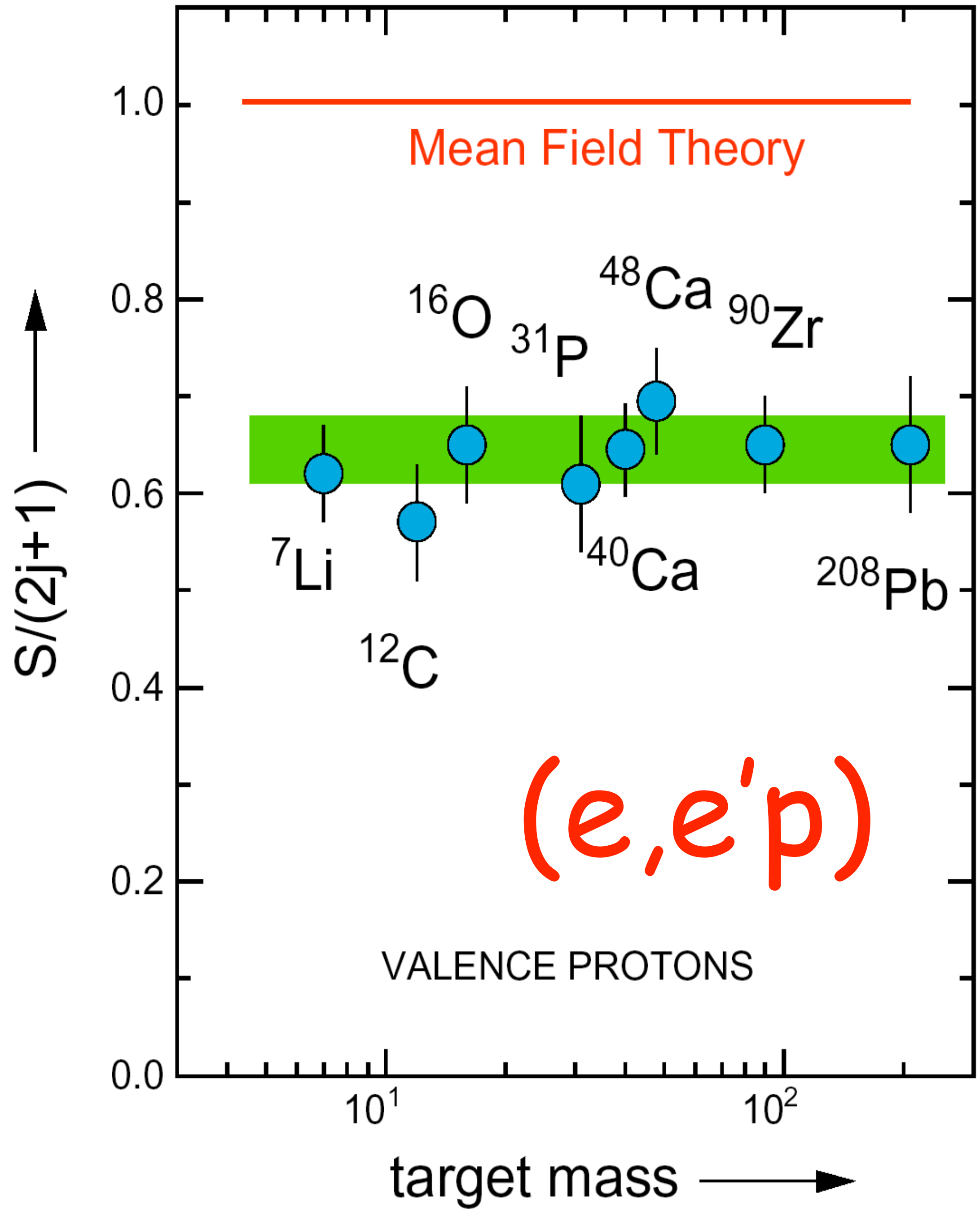
L. Lapikás, Nucl. Phys. A553,297c (1993)

S ≈ 0.65 for valence protons

Reduction ⇒

- SRC about 15%
- LRC another 15-20%

In matter —> SRC can be included

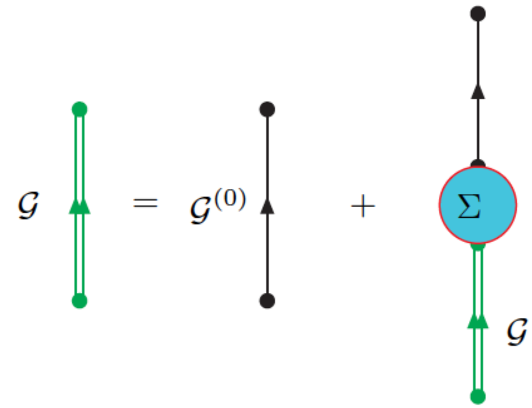


Pairing N*

Green´s function and Γ-matrix approach (ladders)

Single-particle Green´s function $\mathcal{G}$

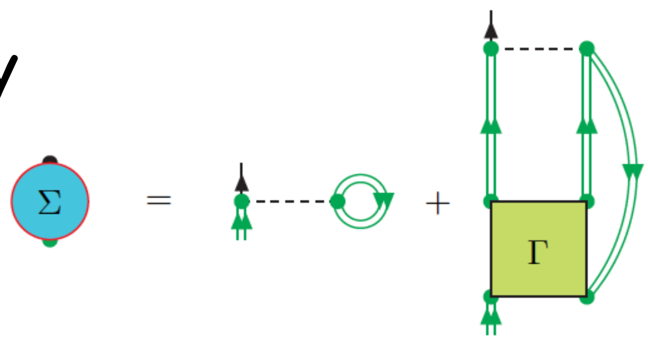
Dyson equation: $\mathcal{G} = \mathcal{G}^{(0)} + \mathcal{G}^{(0)} \Sigma \, \mathcal{G}$



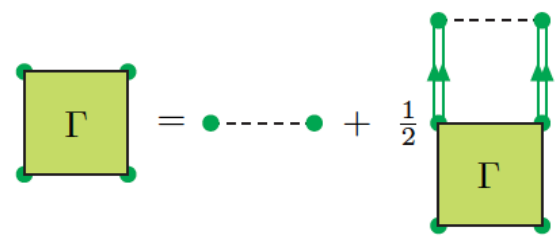
$$\mathcal{G}(k, E) = \frac{1}{E - \varepsilon_k - \Sigma(k, E)}$$

spectral function ~ $\mathrm{Im}\, \mathcal{G}(k, E)$

Self-energy



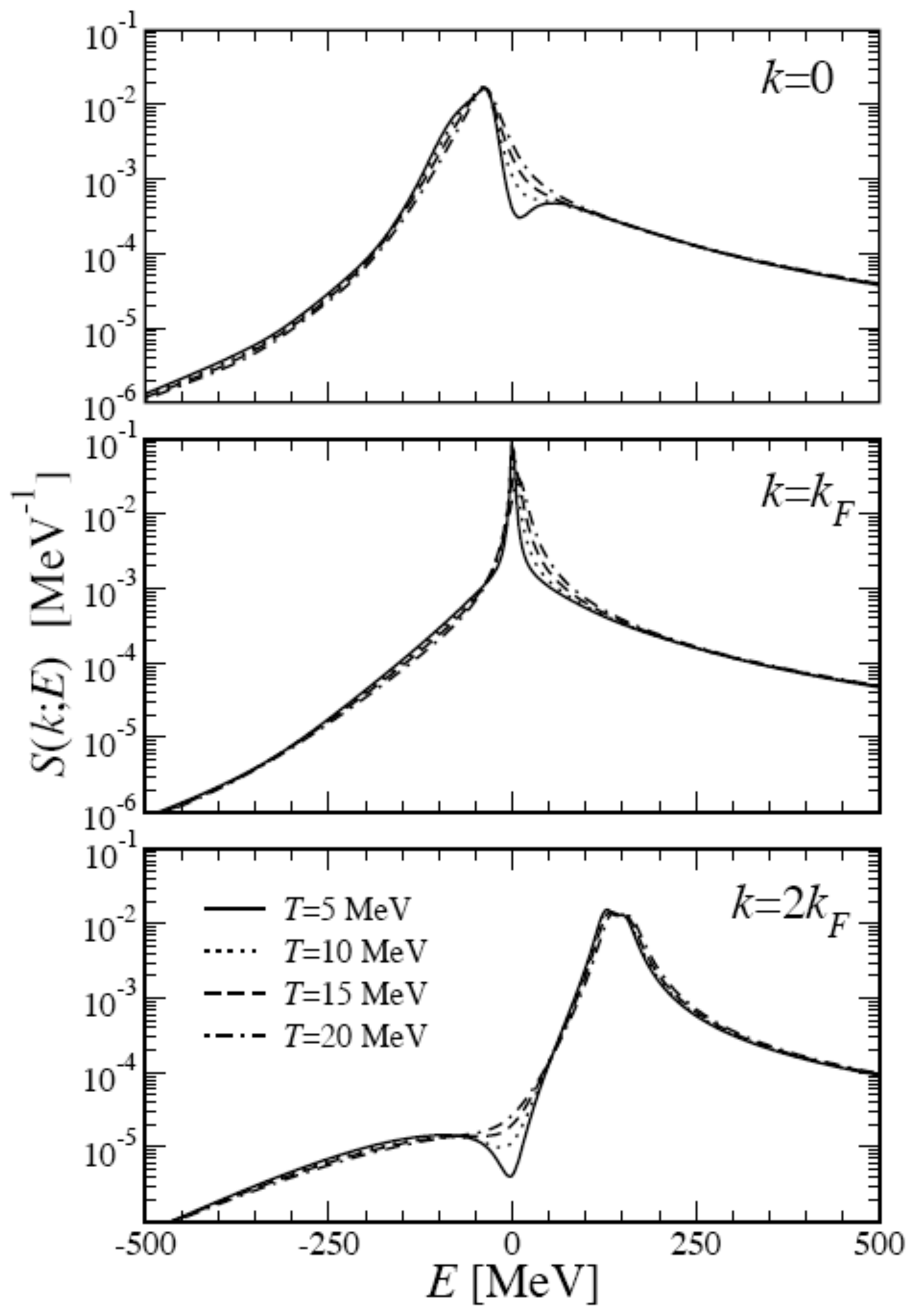
Γ-matrix



- Pairing instability possible
- Finite temperature calculation can avoid this

Temperature dependence

- Arnau Rios thesis
- Density $\rho = 0.16\ \mathrm{fm}^{-3}$
- Width computationally helpful compared to sharp features at T=0
- Zero at Fermi energy for T=0 disappears --> at most a dip
- Tails hardly T-dependent



BCS: a primer

$$\varepsilon_{p\mu} = \varepsilon(p) - \mu = \chi_p$$

NN correlations on top of Hartree-Fock:

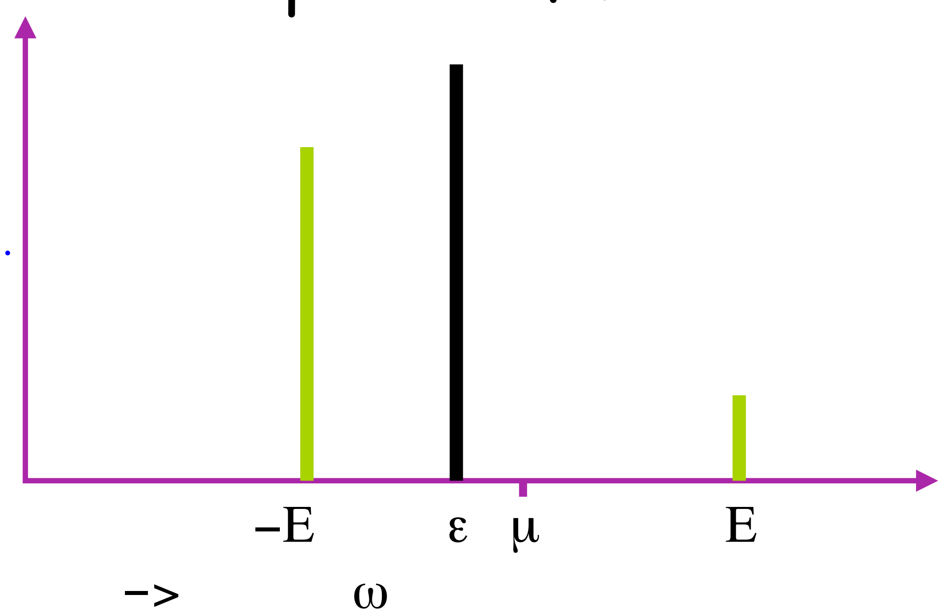
Bogoliubov transformation

$$\alpha^\dagger_{p\uparrow} = u_p a^\dagger_{p\uparrow} + v_p a_{-p\downarrow}$$

with
$$u_p^2 = \frac{1}{2}\left(1 + \frac{\chi_p}{E_p}\right)$$
$$v_p^2 = \frac{1}{2}\left(1 - \frac{\chi_p}{E_p}\right)$$

$$E_p = \left[\chi_p^2 + \Delta_p^2\right]^{1/2}$$

Spectral function



Gap equation

$$\Delta_\ell^{JST}(p) = -\sum_{\ell'} \int_0^\infty dk\, k^2 \,\langle p\ell | V_{\ell\ell'}^{JST} | k\ell' \rangle \, \frac{\Delta_{\ell'}^{JST}(k)}{2E_k}\,.$$

Solution of the gap equation

$$\Delta(k) = \sum_{k'} \langle k|V|k'\rangle \frac{\Delta(k')}{\omega - 2E(k)}$$ with $E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}$ and $\omega = 0$

Define: $\delta(k) = \dfrac{\Delta(k)}{\omega - 2E(k)}$ then

$$\begin{pmatrix} 2E(k_1) + \langle k_1|V|k_1\rangle & \dots & \langle k_1|V|k_N\rangle \\ \vdots & \ddots & \vdots \\ \langle k_N|V|k_1\rangle & \dots & 2E(k_N) + \langle k_N|V|k_N\rangle \end{pmatrix} \begin{pmatrix} \delta_{k_1} \\ \vdots \\ \delta_{k_N} \end{pmatrix} = \omega \begin{pmatrix} \delta_{k_1} \\ \vdots \\ \delta_{k_N} \end{pmatrix}$$

Eigenvalue problem for a pair of nucleons at $\omega = 0$

Steps of the calculation:

- Assume Δ(k) and determine E(k)
- Solve eigenvalue equation and evaluate new Δ(k)
  - If lowest eigenvalue ω<0 enhance Δ(k) (resp. δ(k))
  - If lowest eigenvalue ω>0 reduce Δ(k)
- Repeat until convergence

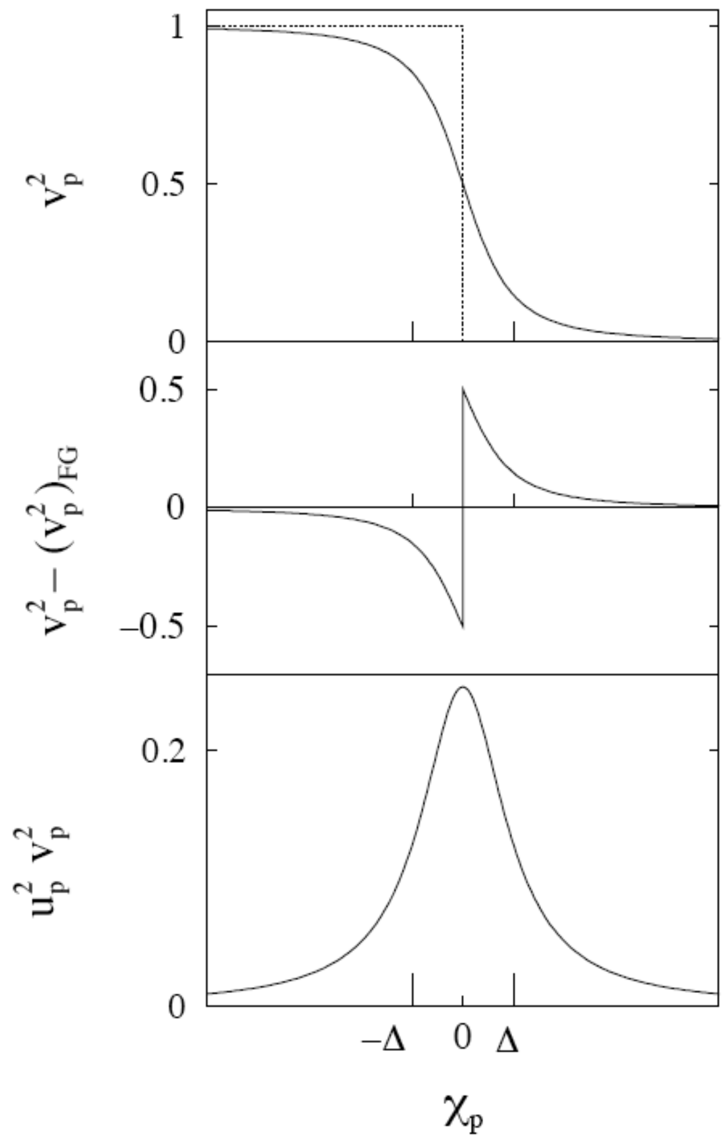
Gap!

- BCS amplitudes

$$u_p^2 = \frac{1}{2}\left(1 + \frac{\chi_p}{E_p}\right), \quad v_p^2 = \frac{1}{2}\left(1 - \frac{\chi_p}{E_p}\right)$$

- Add up to 1
- Relative sign fixed by $u_p v_p = \dfrac{\Delta_p}{2E_p}$
- For a constant gap one finds -------->
- Density from propagator

$$\begin{aligned} n_p &= v_p^2 = \frac{1}{2}\left(1 - \frac{\chi_p}{E_p}\right) \\ &= \frac{1}{2}\left(\frac{\sqrt{\chi_p^2 + \Delta^2} - \chi_p}{\sqrt{\chi_p^2 + \Delta^2}}\right) \end{aligned}$$



- Anomalous propagators necessary

Beyond BCS in the framework of SCGF

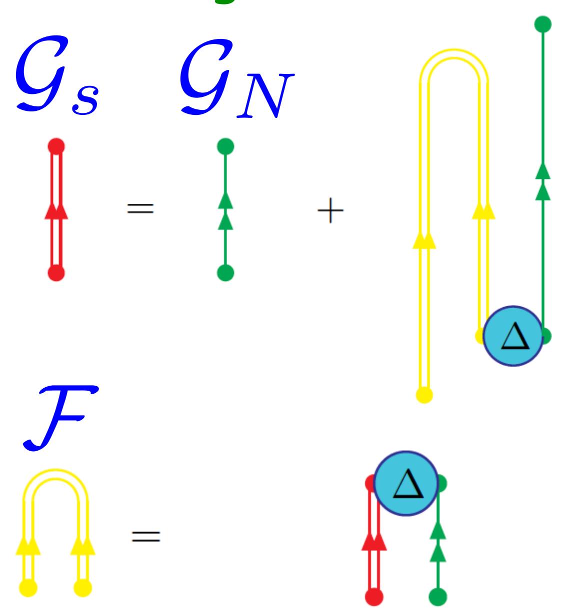
Generalized Green's functions: Extend normal propagator

—>Anomalous propagators

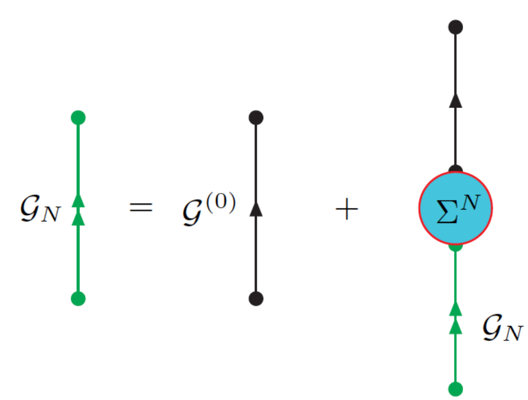
Leads to generalized Dyson equation: Gorkov equations

$$\begin{bmatrix} E-\varepsilon_{p\mu}-\Sigma(\boldsymbol{p},E) & -\Delta(\boldsymbol{p},E) \\ -\Delta^{+}(\boldsymbol{p},E) & E+\varepsilon_{p\mu}+\Sigma(\boldsymbol{p},E) \end{bmatrix} \begin{bmatrix} \mathcal{G}_s(\boldsymbol{p},E) & \mathcal{F}(\boldsymbol{p},E) \\ \mathcal{F}^{+}(\boldsymbol{p},E) & \overline{\mathcal{G}}_s(\boldsymbol{p},E) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Leads to e.g.



$\mathcal{G}_N$ includes all normal self-energy terms



Pairing N*

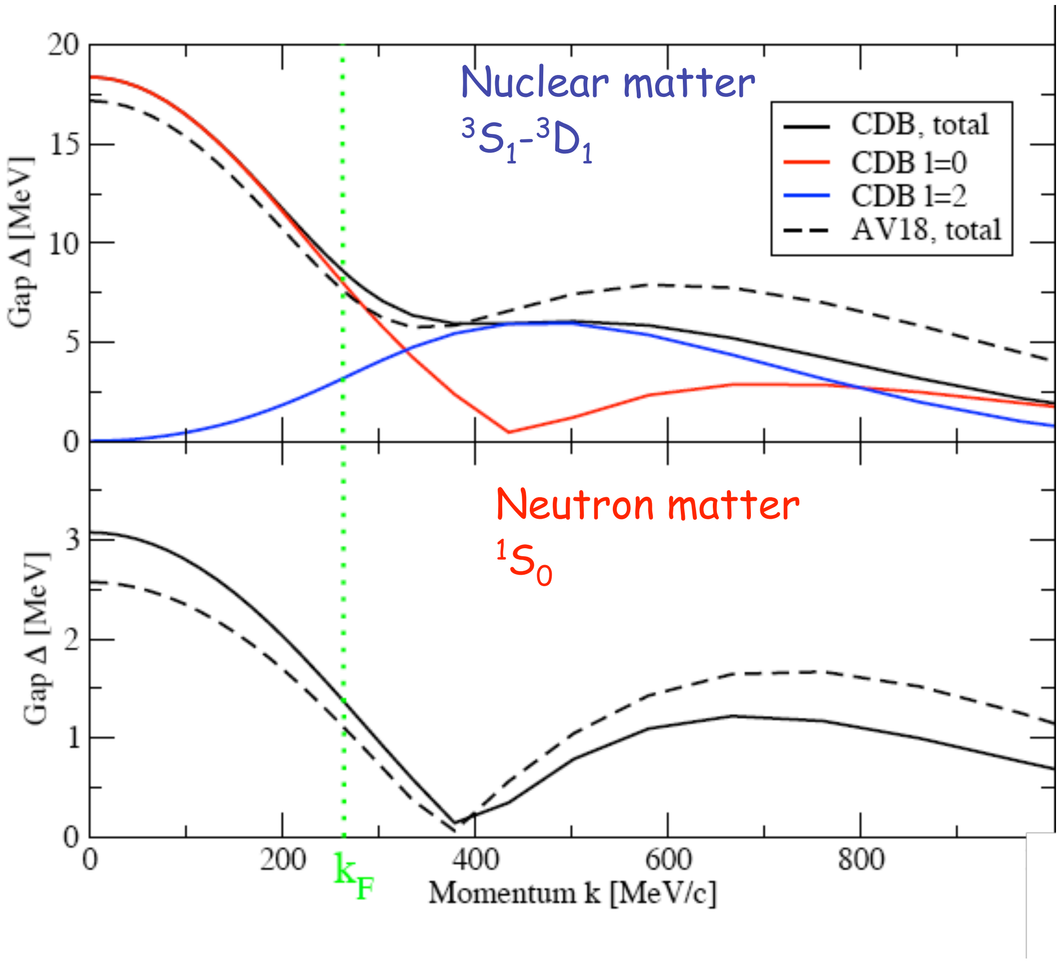
Some pairing issues in infinite matter

- Gap size in nuclear matter & neutron matter
- Density & temperature range of superfluidity
- Resolution of $^3S_1$-$^3D_1$ puzzle (size of pn pairing gap)
- Influence of short-range correlations (SRC)
- Influence of polarization contributions
- Relation of infinite matter results & finite nuclei

Review: e.g. Dean & Hjorth-Jensen, RMP75, 607 (2003)

Some results from: H. Müther and WHD, Phys. Rev. C72, 054313 (2005)
and D. Ding, A. Rios, WHD, H. Dussan, A. Polls, and S. J. Witte to be published

Gaps from BCS for realistic interactions



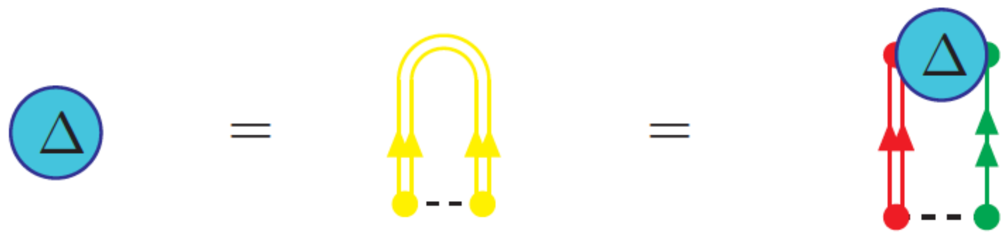
T = 0
Mean-field particles

- momentum dependence $\Delta(k)$
- different NN interactions
- very similar to pairing gaps in finite nuclei for like particles…!?
- for np pairing no strong empirical evidence…?!

Early nineties: BCS gaps ~ 10 MeV

Alm et al. Z.Phys.A337,355 (1990)
Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)

Anomalous self-energy: Δ & generalized Gap equation



$$\Delta_\ell^{JST}(p) = \sum_{\ell'} \int_0^\infty \mathrm{d}k\, k^2 \,\langle p | V_{\ell\ell'}^{JST} | k \rangle \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi} A(k,\omega) A_s(k,\omega') \frac{1 - f(\omega) - f(\omega')}{-\omega - \omega'} \Delta_{\ell'}^{JST}(k)\,.$$

Fermi function $f(\omega) = \dfrac{1}{e^{\beta\omega} + 1}$

If we replace $A(k,\omega)$ by "HF" approximation and $A_s(k,\omega)$ by BCS form:

⇨ Usual Gap equation

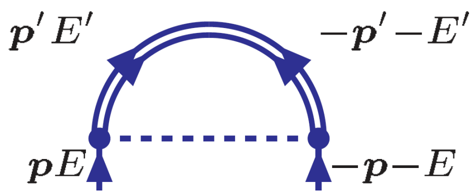
If we take $A_s(k,\omega) = A(k,\omega)$:

⇨ Corresponds to the homogeneous solution of Γ-matrix eq.

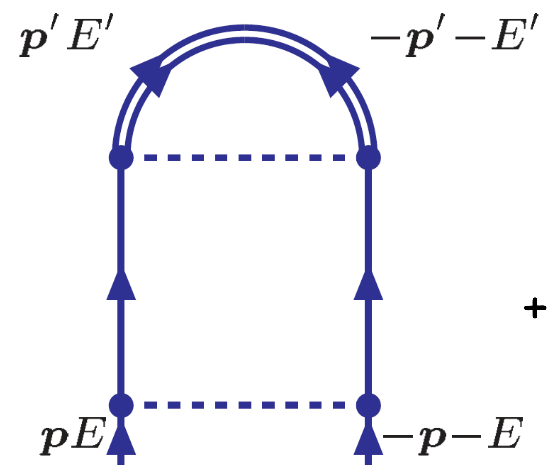
With $A_s(k,\omega)$:

⇨ The above and self-consistency

Consistency of Gap equation (anomalous self-energy) and Ladder diagrams



Iteration of Gorkov equations for anomalous propagator generates $\Rightarrow$



+ ...

... and all other ladder diagrams at
total momentum and energy zero (w.r.t. 2μ)
plus anomalous self-energy terms in normal part of propagator

So truly consistent with inclusion of ladder diagrams at other total momenta and energies

Generalized gap equation with bare interaction

- Coupled-channel —> angle-averaged gap

$$\Delta_\ell^{JST}(p) = \sum_{\ell'} \int_0^\infty \mathrm{d}k\, k^2 \,\langle p|V_{\ell\ell'}^{JST}|k\rangle \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi} A(k,\omega) A_s(k,\omega') \frac{1-f(\omega)-f(\omega')}{-\omega-\omega'} \Delta_{\ell'}^{JST}(k)\,.$$

- with $f(\omega) = \dfrac{1}{e^{\beta\omega}+1}$
- Convolution

$$\frac{1}{-2E_k} \quad \rightarrow \quad \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi} A(k,\omega) A_s(k,\omega') \frac{1-f(\omega)-f(\omega')}{-\omega-\omega'}$$

- Still "simple" gap equation

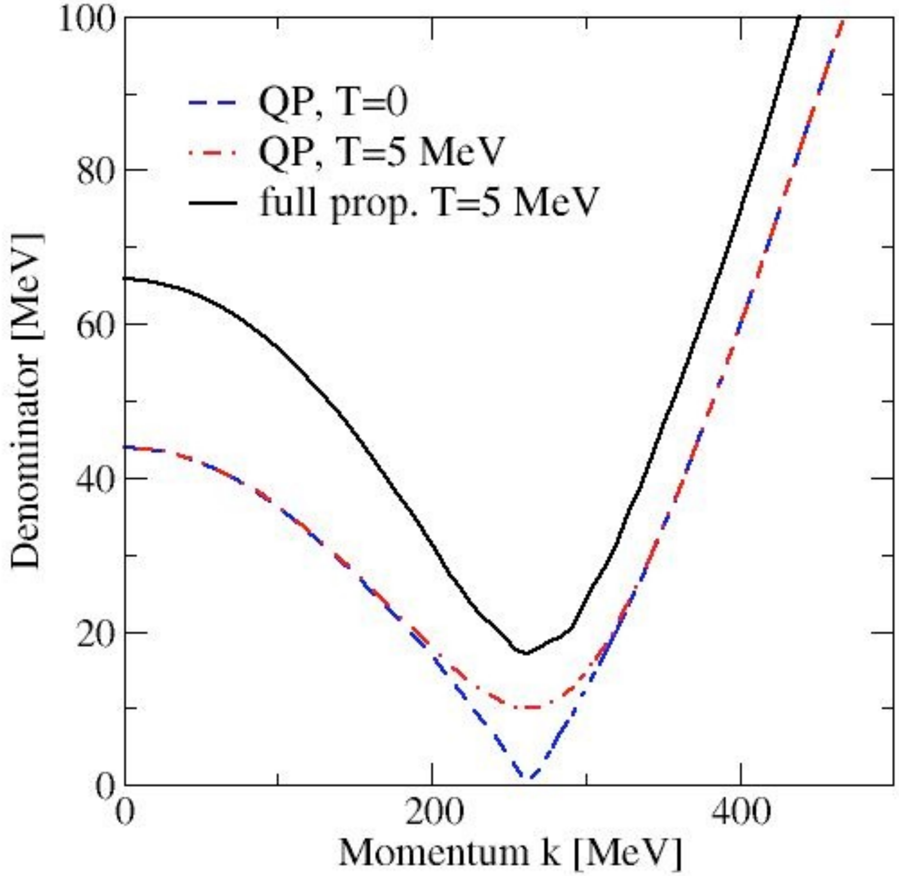
$$\Delta_\ell^{JST}(p) = -\sum_{\ell'} \int_0^\infty dk\, k^2 \,\langle p\ell|V_{\ell\ell'}^{JST}|k\ell'\rangle \frac{\Delta_{\ell'}^{JST}(k)}{2E_k}\,.$$

Features of generalized gap equation

$$\Delta_\ell^{JST}(p) = \sum_{\ell'} \int_0^\infty \mathrm{d}k\, k^2 \, \langle p | V_{\ell\ell'}^{JST} | k \rangle \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi} A(k,\omega) A_{\cancel{s}}(k,\omega') \frac{1 - f(\omega) - f(\omega')}{-\omega - \omega'} \Delta_{\ell'}^{JST}(k)\,.$$

$$-\frac{1}{2\tilde{\chi}_k}$$

$\tilde{\chi}_k$



Dashed:

Spectral strength only at 1 energy

Dashed-dot:

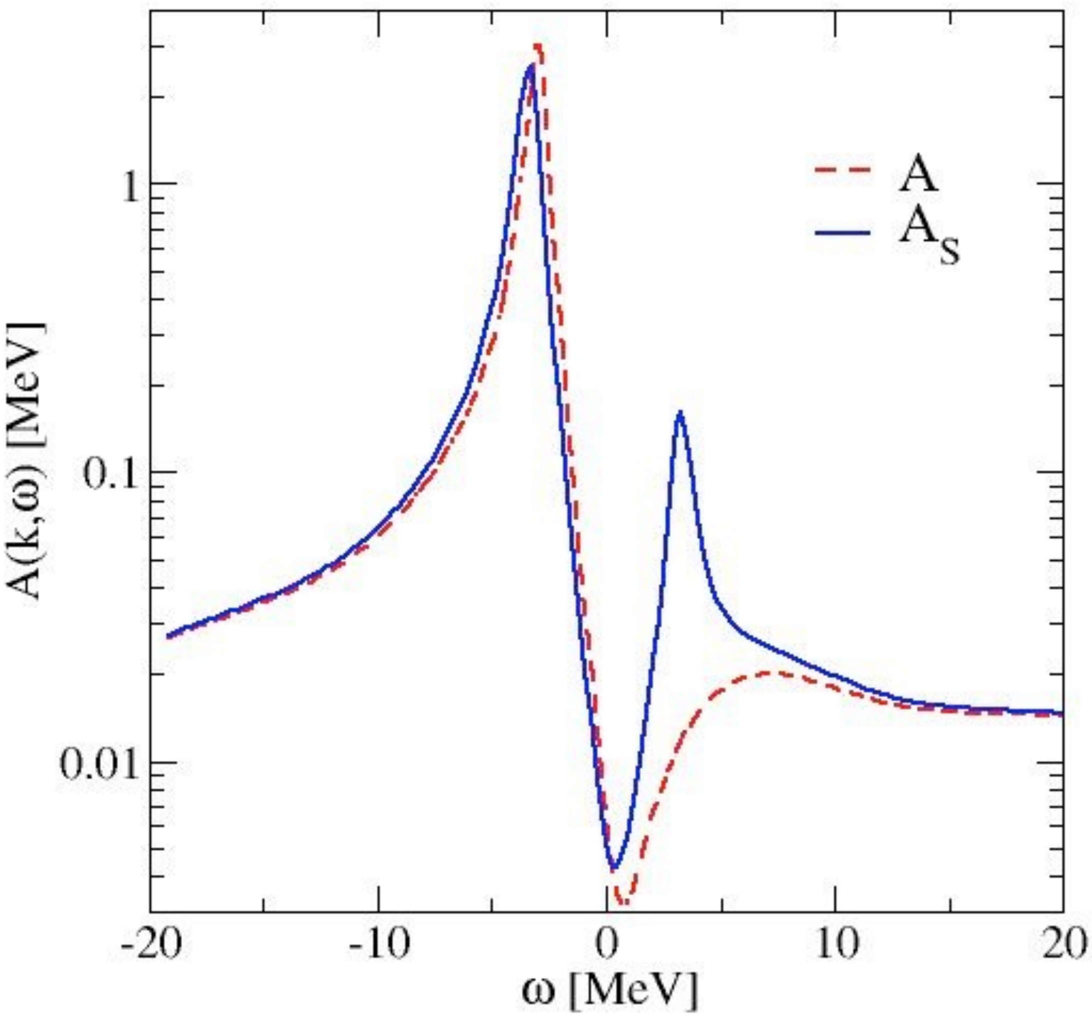
Effect of temperature (5 MeV)

Solid:

Includes complete strength distribution due to SRC

Related studies by Baldo, Lombardo, Schuck et al. use BHF self-energy

Pairing and spectral functions



Spectral functions

$S(k,\omega)$ dashed = $A(k,\omega)$

$S_{pair}(k,\omega)$ solid = $A_S(k, \omega)$

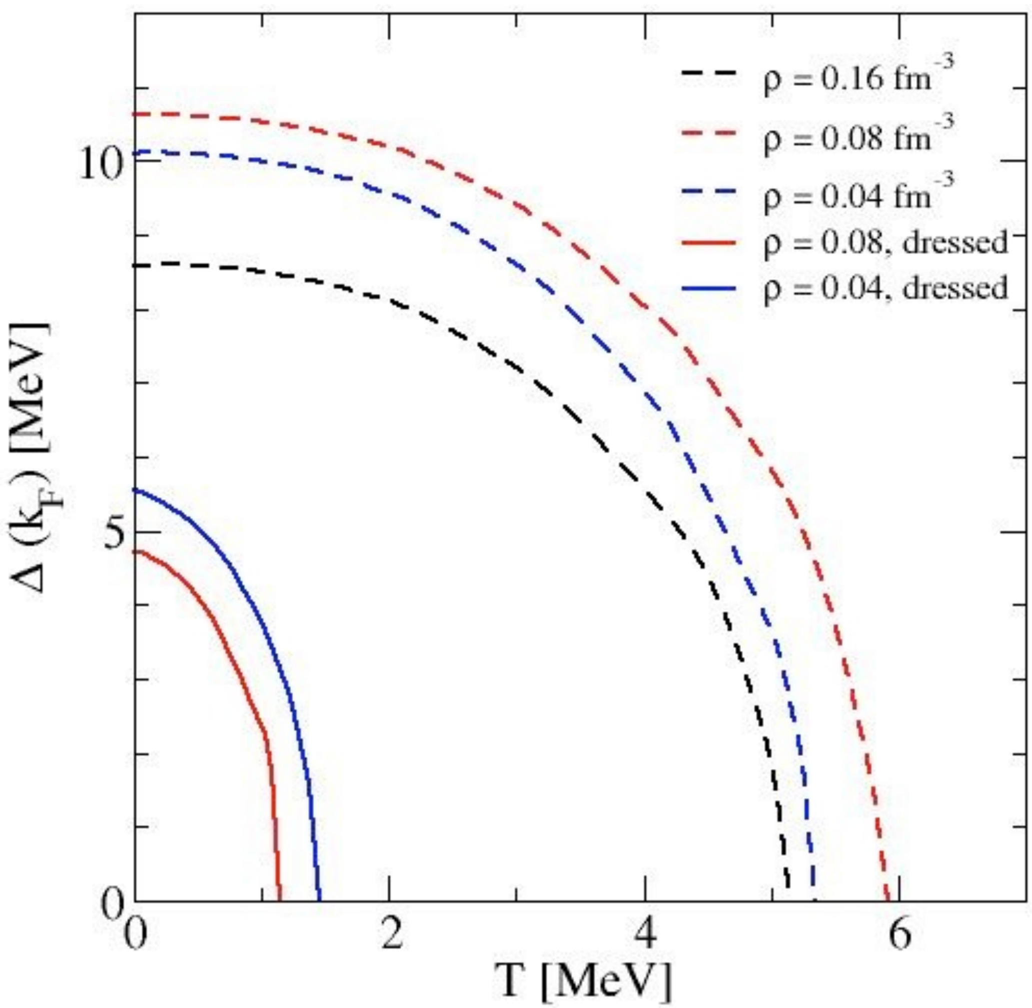
$\rho = 0.08\ \mathrm{fm}^{-3}$

T = 0.5 MeV

k = 193 MeV/c   0.9 k_F

Expected effect

Proton-neutron pairing in symmetric nuclear matter
$^3S_1$-$^3D_1$



Using CDBonn

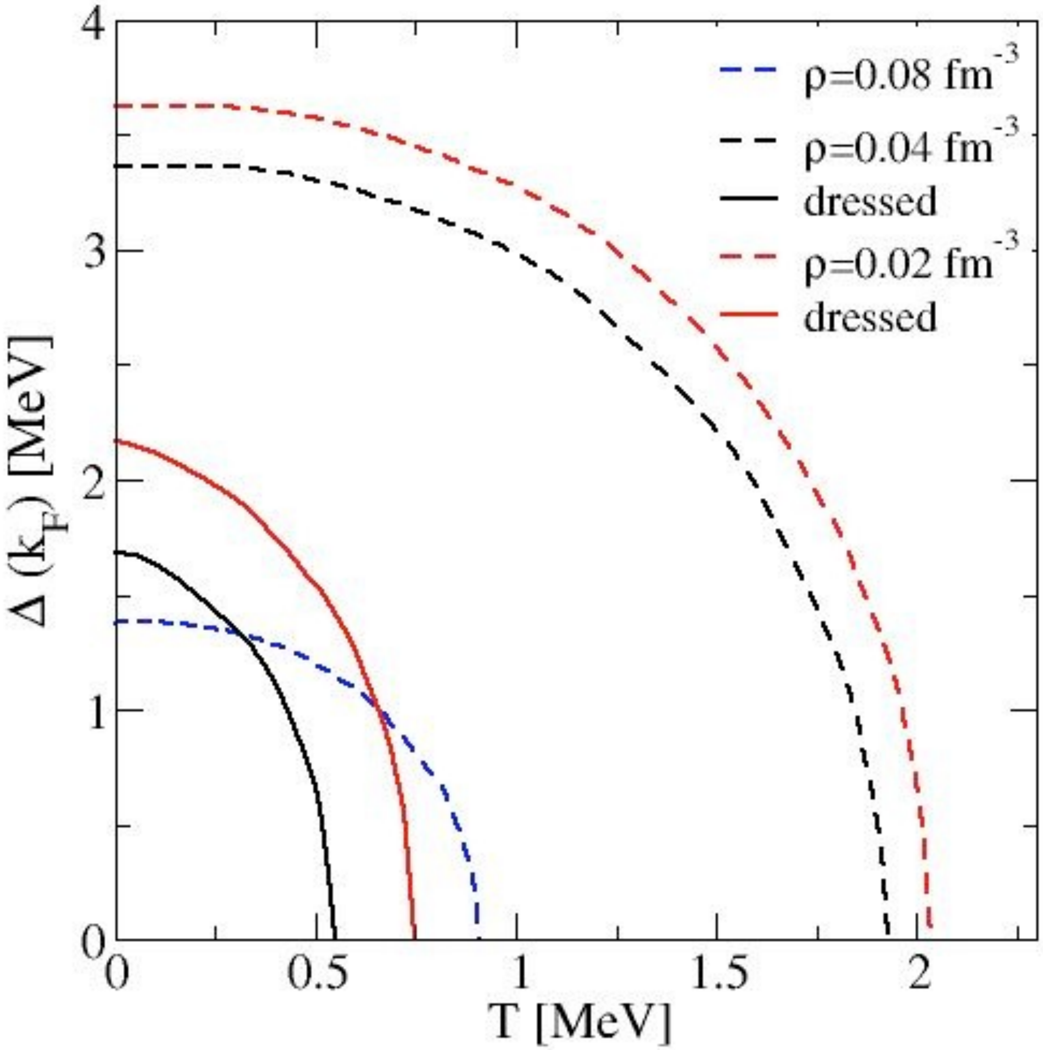
Dashed lines:
quasiparticle poles

Solid lines:
dressed nucleons

No pairing at saturation density!!!!

Solves puzzle!

Pairing in neutron matter --> $^1S_0$



Dressing effects weaker, but non-negligible

CDBonn

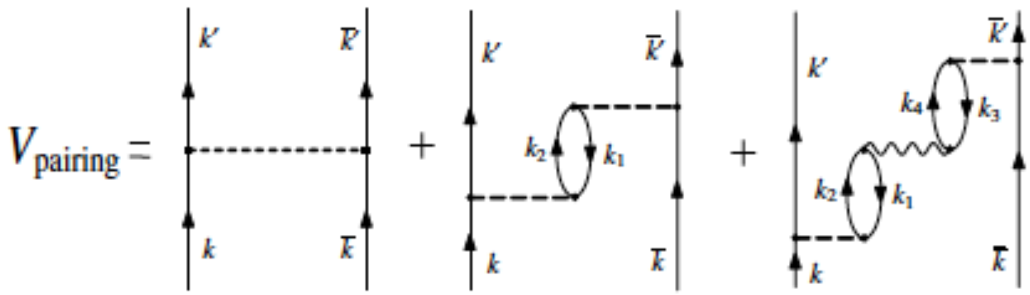
Present effort

- Dong Ding graduate student is here now
- Sam Witte undergraduate student now at UCLA
- Helber Dussan postdoc
- Arnau Rios, Surrey
- Arturo Polls, Barcelona here now

- SRC for 3 realistic interactions in neutron matter
- $^1S_0$
- $^3P_2$-$^3F_2$
- Polarization —> LRC according to PRC74, 064301 (2006)

Inclusion of polarization

- Possible approach to improve the pairing interaction by including density and spin fluctuations governed by Landau parameters



(a) (b) (c)

PHYSICAL REVIEW C **74**, 064301 (2006)

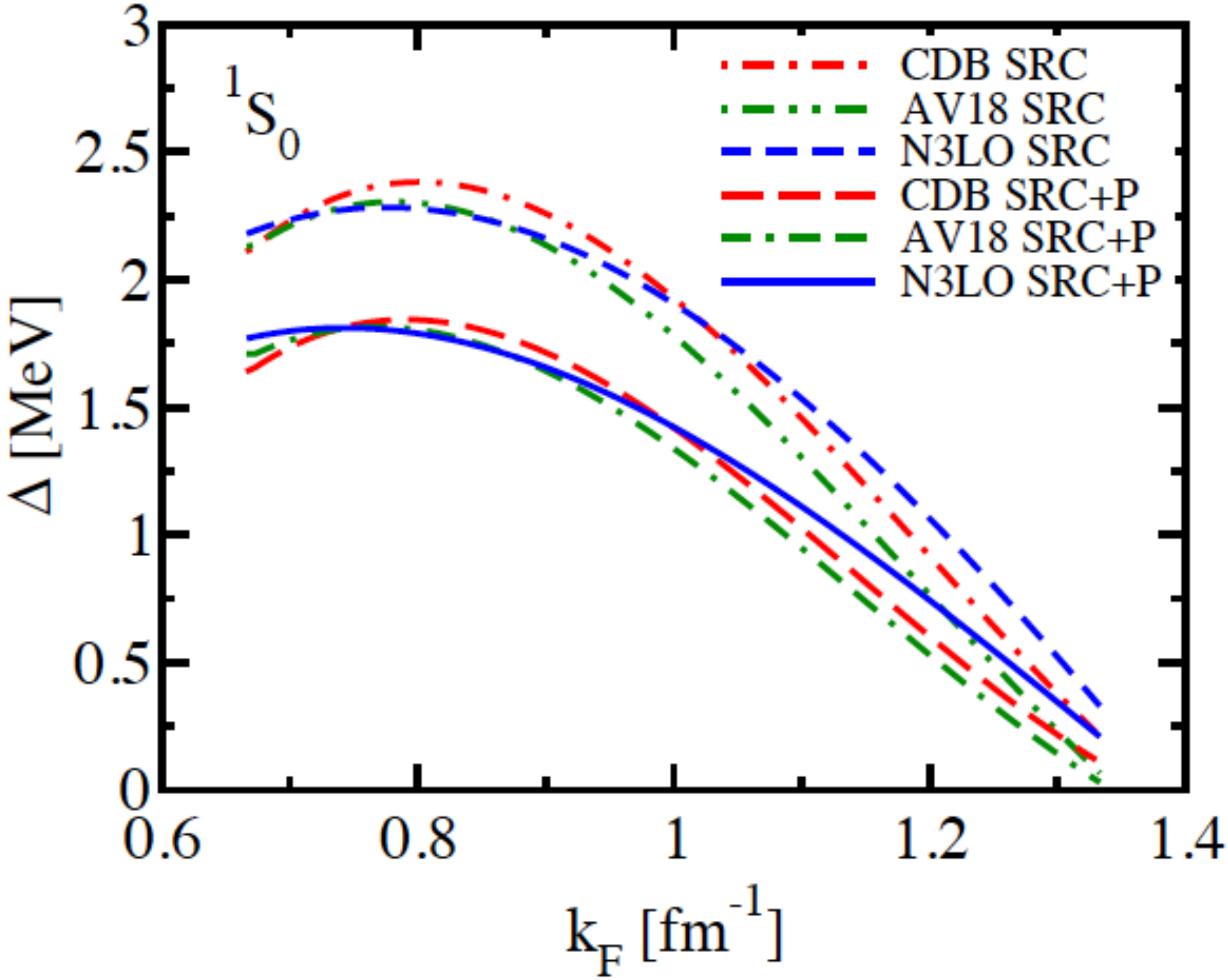
Screening effects in superfluid nuclear and neutron matter within Brueckner theory

L. G. Cao,[1] U. Lombardo,[1,2] and P. Schuck[3]
[1]*Laboratori Nazionali del Sud, INFN, Via Santa Sofia 62, I-95123 Catania, Italy*
[2]*Dipartimento di Fisica dell'Università, Viale Andrea Doria 6, I-95123 Catania, Italy*
[3]*Institut de Physique Nucléaire, Université Paris-Sudd, F-91406 Orsay Cedex, France*
(Received 1 August 2006; published 1 December 2006)

Effects of medium polarization are studied for $^1S_0$ pairing in neutron and nuclear matter. The screening potential is calculated in the RPA limit, suitably renormalized to cure the low density mechanical instability of nuclear matter. The self-energy corrections are consistently included resulting in a strong depletion of the Fermi surface. All medium effects are calculated based on the Brueckner theory. The $^1S_0$ gap is determined from the generalized gap equation. The self-energy corrections always lead to a quenching of the gap, which is enhanced by the screening effect of the pairing potential in neutron matter, whereas it is almost completely compensated by the antiscreening effect in nuclear matter.

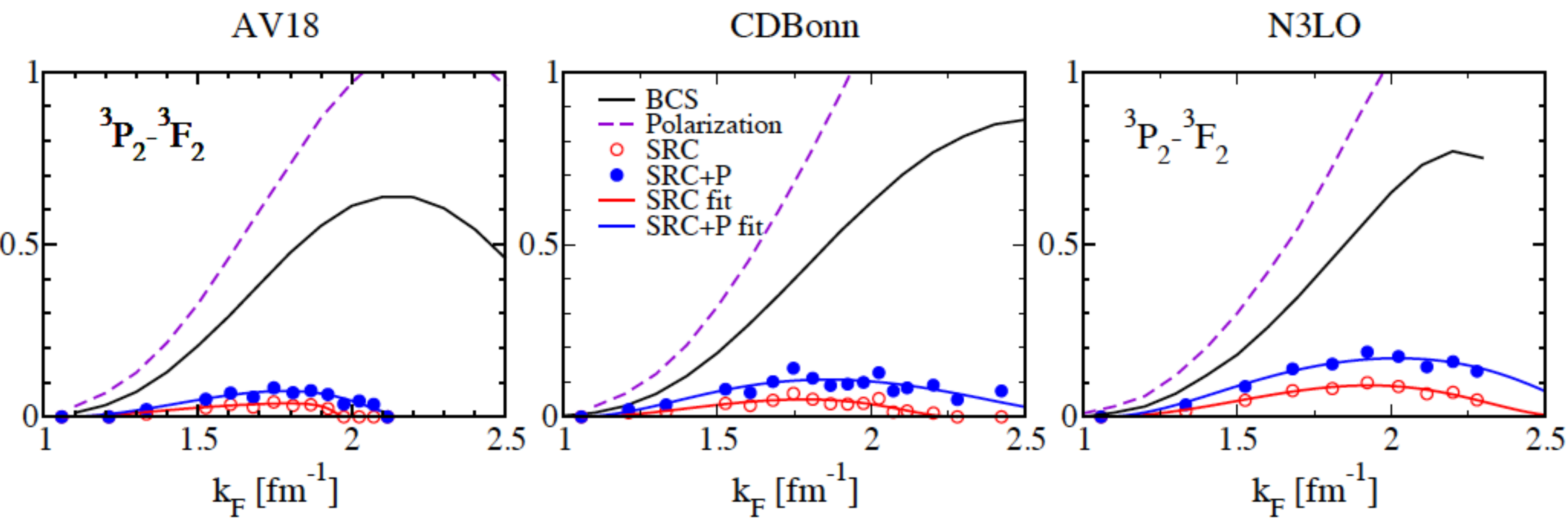
Low-density neutron matter

- SRC
- SRC +LRC



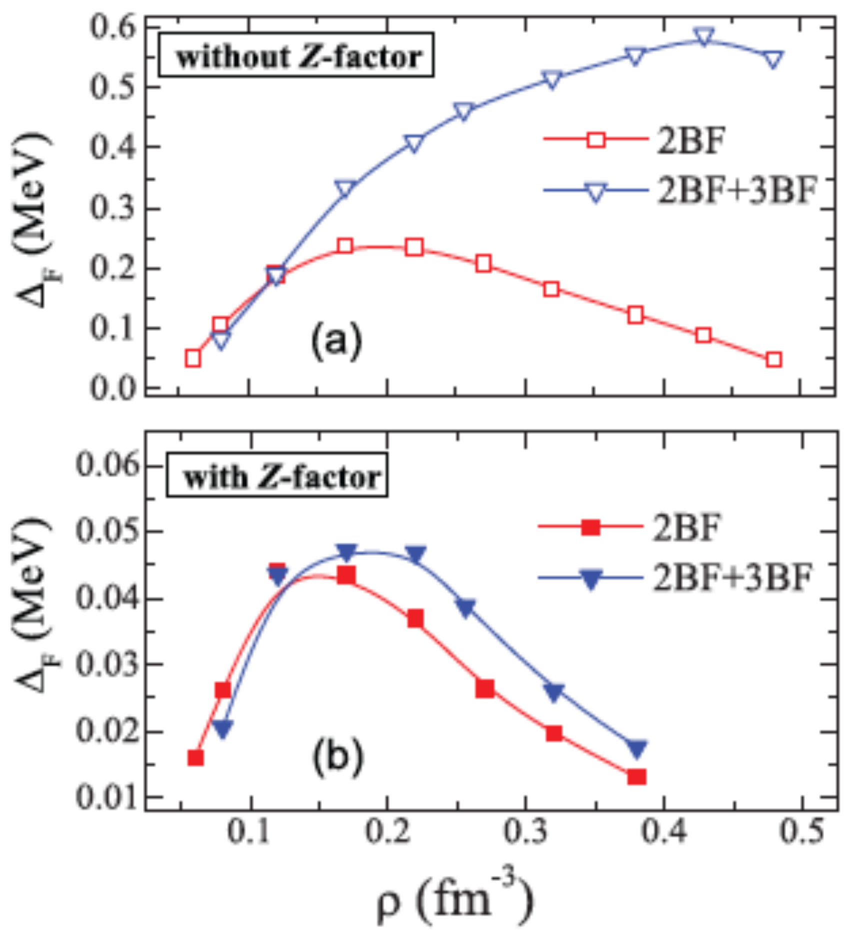
High-density neutron matter pairing

- SRC
- LRC
- SRC+ LRC



Improvements

- Polarization
  - not a proper treatment of pion exchange
  - Lindhard functions modified by gap and the effect of SRC
  - proper treatment yields energy dependent (complex) gap
- 3N forces
  - Z factors?
  - No LRC
  - attraction?



2013)

n matter

Zuo[1]
Lanzhou 730000, China
FN), Catania 95123, Italy
e 2013)

died within the BCS framework with
in the gap equation, the pairing gap
a peak value a bit less than 0.05 MeV.
ation of phenomena occurring in the

Conclusions and outlook

- Effect of SRC on pairing correlations can be accurately included in bulk nuclear or neutron matter
- Nuclear matter pairing puzzle is thereby resolved
- Neutron matter $^1S_0$ gap reduced by SRC for all interactions
- Inclusion of polarization (minimal) yields a maximum gap of about 1.5 MeV for all considered interactions around a density corresponding to k_F = 0.8 fm^{-1}
- At higher density SRC reduces $^3P_2$-$^3F_2$ gap for all interactions
- Polarization exhibits antiscreening and enhances pairing slightly

- Treatment of polarization can be much improved
- 3N force as a density dependent 2N force can be included