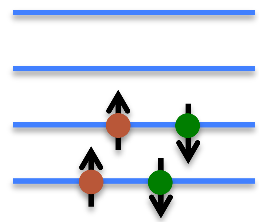
Application to the paring model:

In order to make sense of the equation written so far, we will start solving the Dyson Eq. for our pairing model.

To do this, we will exploit perturbation theory to calculate the self-energy. Thus, we start from our unperturbed solution for the ground state:

$|\Phi_0\rangle \equiv |$  $\rangle = \hat{P}^\dagger_{p=1}\,\hat{P}^\dagger_{p=0}|0\rangle$

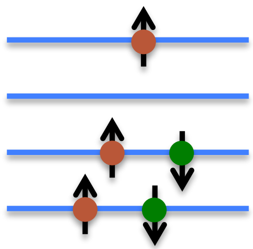
…starting from here, we will develop the static and dynamic contributions to the self-energy. For now we will to the first at 1st order and the latter at 2nd order.

Green's function theory is a prticle attached/particle removed theory, so we will be looking at 3-body and 5-body states. This means that the total spin of the intermediate states |A-1> and |A+1> is non zero and there will be one unpaired particle. We will only look at cases when *<u>all other nucleons are paired</u>*.

Application to the paring model:

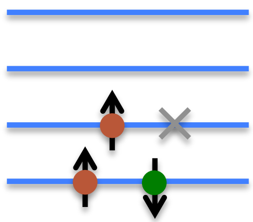
To calculate **<u>the self energy at first order</u>** in perturbations theory we can simple take the expectation value among the :

$$|\Phi^a;\uparrow\rangle \equiv \Big|$$



$$\Big\rangle = a^\dagger_{a\uparrow}|\Phi_0\rangle = a^\dagger_{a\uparrow}\hat{P}^\dagger_1\,\hat{P}^\dagger_0|0\rangle$$

$$|\Phi^j;\uparrow\rangle \equiv \Big|$$



$$\Big\rangle = a_{j\downarrow}|\Phi_0\rangle = a_{j\downarrow}\hat{P}^\dagger_1\,\hat{P}^\dagger_0|0\rangle$$

The 1st order self-energy $\Sigma^{(1)}_{\alpha\,\beta}$ (with $\alpha = i, j, k, \ldots, a, b, \ldots = 0\uparrow,\ 0\downarrow,\ 1\uparrow,\ 1\downarrow,\ 2\uparrow, \ldots$) is simply the perturbing interaction H_1:

$$\Sigma^{(1)}_{a\,b} = \langle\Phi^a; S_z|H_1|\Phi^b; S'_z\rangle\ ,$$

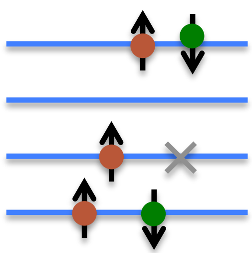
$$\Sigma^{(1)}_{a\,j} = \langle\Phi^a; S_z|H_1|\Phi^j; S'_z\rangle\ ,$$ etc....

Application to the paring model:

The $\Sigma^{(1)}_{\alpha\,\beta}$ is **static**, i.e. **energy independent**. It will have other contributions at higher orders in perturbation theory. But we don't worry about this because we can calculate the exact one (i.e. no PT at all). We will see this later, for now let's just stick to $\Sigma^{(1)}_{\alpha\,\beta}$ here, which is the first contribution. This is good enough for now.

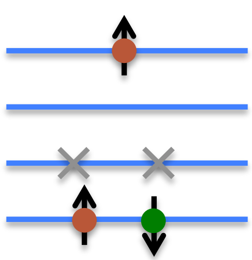
At **second order**, the interaction H1 can create/annihilate additional extra pairs, leading to an **energy dependent** contribution to the self energy. For this we will need 2p1h/2h1pintermediate states as follows:

$|\Phi^{ab}_i;\uparrow\rangle \equiv |$



$\rangle = a_i a^\dagger_a a^\dagger_b |\Phi_0\rangle = a_{i\downarrow} \hat{P}^\dagger_{a=b}\, \hat{P}^\dagger_1\, \hat{P}^\dagger_0 |0\rangle$

$|\Phi^{a}_{ij}\uparrow\rangle \equiv |$



$\rangle = a^\dagger_a a_i a_j |\Phi_0\rangle = a^\dagger_{a\uparrow} \hat{P}_{i=j}\, \hat{P}^\dagger_1\, \hat{P}^\dagger_0 |0\rangle$

Application to the paring model:

The **energy dependent second order self-energy** is then as follows as follows:

$$\Sigma^{(2)}_{\alpha\beta}(\omega) = \sum_{a\,b\,i} \langle \Phi^{\alpha} | H_1 | \Phi^{ab}_{i} \rangle \frac{1}{\omega - (\varepsilon^{(0)+}_{a} + \varepsilon^{(0)+}_{b} - \varepsilon^{(0)-}_{j}) + i\eta} \langle \Phi^{ab}_{i} | H_1 | \Phi^{\beta} \rangle$$

$$+ \sum_{a\,i\,j} \langle \Phi^{\alpha} | H_1 | \Phi^{a}_{ij} \rangle \frac{1}{\omega - (\varepsilon^{(0)-}_{i} + \varepsilon^{(0)-}_{j} - \varepsilon^{(0)+}_{a}) - i\eta} \langle \Phi^{a}_{ij} | H_1 | \Phi^{\beta} \rangle$$

The above equations define a first approximation of the self energy:

$$\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(1)}_{\alpha\beta} + \Sigma^{(2)}_{\alpha\beta}(\omega)$$

The dyson equation in energy domain is then written as follows (with implicit summations):

$$g_{\alpha\beta}(\omega) = g^{(0)}_{\alpha\beta}(\omega) \; + g^{(0)}_{\alpha\gamma}(\omega) \, \Sigma^{\star}_{\gamma\delta}(\omega) \, g_{\delta\beta}(\omega)$$