Master-No 0231310117

Faculty of Law, Economics and Finance

DISSERTATION

Presented on Aug. 20 in Luxembourg

to obtain the degree of

MASTER DE L'UNIVERSITÉ DU LUXEMBOURG

EN ÉCONOMIE

by

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POLICY MAKING IN A CHANGING WORLD: UNCERTAINTY DYNAMICS AND POLITICAL INSTITUTIONS

UNIVERSITÉ DU LUXEMBOURG

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Affidavit

I hereby confirm that the Master thesis entitled "POLICY MAKING IN A CHANGING WORLD: UNCERTAINTY DYNAMICS AND POLITICAL INSTITUTIONS" has been written independently and without any other sources than cited.

Luxembourg, ______________________ ______________________________

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Abstract: In today's policy-making landscape, decision-makers confront a multitude of challenges arising from ever-evolving uncertainties and the dynamic nature of collective decision-making processes. To address these complexities, I examine a dynamic legislative bargaining model with an infinite horizon, featuring an endogenous status quo and stochastic uncertainty. In this model, policies consist of pie divisions, where the choice of an alternative in one period influences the status quo for the next, and the size of the pie in each period follows a stochastic process. The model accommodates various voting rules that are proper, monotone, and noncollegial. Key findings include: (1) When players are short-sighted, there's no absorbing subgame perfect Nash equilibrium. (2) When players are sufficiently patient, I identify a class of pure strategy stationary Markov perfect equilibria. In these equilibria, the size principle from the Baron–Ferejohn model may fail, and avoid the rotating dictatorship noted by Kalandrakis 2004. (3) Policy stability is achievable with sufficient patient players, where increased uncertainty of the future requires even more patient players to sustain. (4) Policy persistence may come with downsides, such as persistent inefficiencies where some of the pie may be perpetually wasted or distributed among more than a minimal winning coalition. These findings provide insights into maintaining policy stability in the face of changing states. They also suggest that "patience" may explain two patterns observed in decision-making bodies: the persistence of economic policies even when their original rationale no longer applies, and the occasional emergence of reforms during times of crisis.

Key words: Legislative bargaining, endogenous status quo, stochastic uncertainty, policy persistence

Chapter 1

Introduction

> *"The geopolitical landscape is changing rapidly and dramatically. We are living in a completely different world compared with just three years ago... As policy-makers, we have to think ahead and prepare."*
>
> – *Executive Vice-President Dombrovskis, 2024* [1]

Following the seminal work by Baron and Ferejohn 1989, the focus on legislative bargaining (e.g., Baron 1996; J. s. Banks and Duggan 2000; J. S. Banks, Duggan, et al. 2006; Eraslan and Evdokimov 2019) has significantly increased, with numerous applications in decision making in committees, courts, and other small groups (Gehlbach 2021). Since influential contribution by Baron 1996 and the subsequent work by Kalandrakis 2004, a growing body of literature (e.g., Anesi and Duggan 2018; Eraslan, Evdokimov, and Zápal 2022) has highlighted the concept of the ”endogenous status quo” to address the dynamic nature of legislative policy making. In each period, a new policy is decided, and if no agreement is reached, the policy implemented is endogenously determined by the outcome of bargaining from the previous period. The institutional arrangements that govern how collective decision-making bodies can amend existing policies are thus critical to their ability to appropriately adjust policy instruments to changes in their operating environment, although they can sometimes lead to inefficient policy persistence.

[1] Speech by Executive Vice-President Dombrovskis at the European Parliamentary Week 2024: European Semester Conference

However, the ”endogenous status quo” is not the only consideration. Decision-makers must also navigate uncertainties about their surroundings and the future repercussions of their decisions. Legislative bargaining, being considered as a prominent example of negotiations in a changing environment, is influenced by stochastic shocks that affect the decision-making process (Dziuda and Loeper 2016). For instance, legislators' preferences over fiscal policies are shaped by heterogeneous ideologies and constituencies but are also impacted by shocks such as business cycles, changes in the country's credit rating, or shifts in public opinion. Similarly, in wage negotiations between labor unions and management, external factors such as business cycle fluctuations can impact a firm's ability to pay, posing critical uncertainties for unions (Oderanti, Li, and De Wilde 2012).

The starting point of my analysis stems from two observations about contracting and policy-making environments as illustrated above. These observations are frequently encountered in today's decision-making bodies. In most democracies, a vast array of fiscal policies rely on an endogenous status quo and are affected by shocks such as demographic transitions, financial crises, financial innovations, and natural disasters. These sources of risk contribute to the difficulty of maintaining political stability. A significant example is pensions. Over the past 30 years, all European Union (EU) member states have reformed their pension systems (Hinrichs 2021). These substantial reforms aim to enhance fiscal sustainability while maintaining adequate pension income.

Ongoing and intensifying population aging represents the most serious and enduring challenge for both developed and developing welfare states. Projected low birth rates and continued increases in life expectancy will result in a population that remains almost unchanged in size but becomes significantly older by 2060. The EU is expected to shift from having about four working-age people (aged 15-64) for every person aged over 65 to a ratio of only two to one (Carone et al. 2016). The fiscal impact of population aging extends beyond the pension system, which is almost universally the largest item of welfare state expenditure. It competes for public funding with other elderly-heavy policy areas, such as long-term care and healthcare, where senior individuals are the primary beneficiaries.

Another challenge emerged during and after the Great Recession that hit the EU in 2008-

09 in the wake of the financial market crisis (Bakker and Klingen 2012). The reform process gained additional momentum through the adoption of further measures, which sometimes had short-term impacts, particularly in countries facing serious sovereign debt problems and consequently requiring financial aid from supranational organizations. In some instances, partial or full reversals of past systemic reforms were observed. Moreover, the financial market crisis significantly altered the landscape for pre-funded, private pensions, which had been the focus of previous reform efforts in several EU member states (Hinrichs 2021). The yet remaining impacts of the COVID-19 pandemic (consider rare disaster) on labor markets and public finances may prompt even more substantial changes to pension systems than those seen following the exogenous shock of 2008.

My research goal is to develop a new theoretical framework that both integrates and extends existing literature, allowing for a tractable analysis of how policy persistence can be sustained in a changing world. I seek to answer key questions: (1) Under what conditions do collective policymaking bodies frequently amend existing status quo policies in response to stochastic uncertainties, or, despite the volatility of risks, can they still sustain stability? (2) If political stability can be ensured, would it exhibit inefficient persistence? (3) Must each payoff be divided among a minimal winning majority, as predicted by the size principle? From a technical perspective, I aim to address these questions investigating the following: (4) When do stationary Markov perfect equilibria (SMPEs) exist, and if they do, (5) are equilibrium payoffs unique?

My contribution is to introduce risk into the existing literature on dynamic legislative bargaining, enabling me to test the robustness of previous findings. This approach raises additional questions and explores the interplay between various components, which I address by providing the following answers:

No absorbing SPNE whenever players are myopic. I prove that there is no absorbing Subgame Perfect Nash Equilibrium (absorbing SPNE) in which all players use strategies that result in the same policy being implemented in each time period (a property called *absorbing*), whenever the players are sufficiently short-sighted.

Equilibrium existence and multiplicity of equilibria whenever players are patient. I construct pure strategy SMPEs for any game with voting rules that are proper, monotone, noncollegial, and involve sufficiently patient players, assuming that realized states have an upper bound. Consider any point in the policy space where at least a winning coalition has a positive share of the pie. If players are sufficiently patient, then it is possible to construct a pure strategy SMPE in which that policy is implemented in the first period and never amended (a property which I call *no-delay*).

Moreover, when players are sufficiently patient, my model avoids the rotating dictatorship scenario described by Kalandrakis 2004. In such a scenario, any proposer would, in each period, propose to allocate the entire share of the pie to themselves, and this proposal would be implemented.

The size principle. The size principle (Baron and Ferejohn 1989) predicts that only minimal winning coalitions may receive a positive share of the pie. This principle has been central to the study of legislatures since Riker 1962, even though majorities in legislatures are often supraminimal (Anesi and Seidmann 2015). In the class of solutions I construct, SMPEs can involve sharing the pie among more than just a minimal winning coalition.

Effect of stochastic uncertainty. I demonstrate that stochastic uncertainty imposes a stricter constraint on players. As the pie follows a stochastic process with higher variance, the level of patience required from players increases in order to maintain the equilibrium. This may explain the pensions reform observation described above: as the level of uncertainty increases, current members may not be patient enough to sustain the equilibrium.

Waste. I demonstrate that policy persistence can come at the cost of wasting some of the pie when all players are sufficiently patient. The interaction between an endogenous status quo and the fear of being excluded from the winning coalition and receiving a low payoff in future periods can lead to static inefficiencies in equilibrium. Specifically, for every $\varepsilon \in (0, 1)$, one can construct a SMPE in which a policy that wastes a proportion ε of the pie is agreed upon in the first period and never amended.

By linking waste to the existence of equilibrium, this supports the conventional wisdom in political economy that, once an economic policy is introduced, it tends to persist, even when its original rationale is no longer valid or has been disproven (Coate and Morris 1999). My model offers a plausible explanation for this persistence: it can be attributed to the sufficient patience of legislators.

Furthermore, in games that do not adhere to the size principle, players can perpetually waste any proportion of the pie in SMPEs. My model thus aligns with features commonly observed in pork barrel politics (Evans 2011).

Related literature. Even though dynamic bargaining with an endogenous status quo in a stochastic environment is central to many economically relevant situations, the existing literature on this topic is scarce. Most studies have considered these two aspects of dynamic policy making—endogenous status quo and stochastic environment—independently, rather than in conjunction. This may be a consequence of the relative intractability of these games (Dziuda and Loeper 2016) and the inherent difficulty in defining the status quo policy when the size of the pie is stochastic. Recent contributions include Bils and Izzo 2023 consider stochastic preferences to model how players anticipate ideological polarization and stochastic shocks related to payoffs under a spatial policy, where I allow for a richer policy space that encompasses spatial or distributive policy; and Agranov, Eraslan, and Tergiman 2024 address uncertainty through the stochastic process that governs the realization of budget size and the selection of the proposer, their focus is primarily on laboratory experiment using majority and unanimity voting rules. I concentrate more on theoretical model, aiming to build a broader framework that accommodates a variety of voting rule structures.

More generally, one literature has focused on the consequences stochastic uncertainty on efficiency and delays in decision-making process, assuming that a policy cannot be revised once implemented (e.g., Merlo and Wilson 1995; Merlo and Wilson 1998; Eraslan and Merlo 2002). A noteworthy example is Eraslan and Merlo 2017 study a stochastic bargaining model to address fairness issues. In their model, players are heterogeneous with respect to the potential surplus they bring to the bargaining table. They demonstrate that when players are sufficiently patient, the equilibrium allocations become less equitable as the voting rules become more inclusive; another (fast-growing) literature has instead focused on policy persistence in contexts where laws remain in effect only if and when no decisive coalition of policymakers is willing to amend them (e.g., Baron 1996; Kalandrakis 2004; Anesi and Seidmann 2015; Anesi and Duggan 2018; Eraslan, Evdokimov, and Zápal 2022). While these frameworks have shed light on contemporary policy issues like mandatory spending programs (Bowen, Chen, and Eraslan 2014) or budgetary institutions (Bowen, Chen, Eraslan, and Zápal 2017), they typically assume that the agents' preferences, their knowledge, and/or the policy making environment are static; or their dynamics is too trivial to capture stochastic uncertainties of current real-world policy problems, like those described above.

Thus, one literature addresses how dynamic uncertainties impact static policy making but remains silent about long-term adjustments; another examines the role of political institutions in collectively overcoming status-quo persistence and sustaining efficient reforms in the long run, but overlooks real-world complexities of stochastic uncertainty in policy challenges. There remains a gap in understanding the interdependence between long-term stochastic uncertainty and dynamic policy-making, and especially the role of political institutions in that interdependence.

Structure of the paper. The structure of the paper is outlined as follows: Chapter 2 defines the general bargaining framework and the equilibrium concept, and presents a toy example with $n = 3$ and majority rules when the "pie" is drawn from a series of uniform distributions to illustrate the main intuitions. Chapter 3 offers a proposition that ensures the existence of equilibrium within the general framework. Finally, Chapter 4 summarizes these observations, discusses my findings on how policy persistence can be ensured in a changing world, and concludes by addressing the broader implications of these results for political stability. The Appendix contains the formal proofs omitted from the main text.

Chapter 2

General framework

2.1 Stochastic dynamic bargaining framework

As explained above, existing models of dynamic collective decision making with evolving status quo fail to integrate policy evolution and stochastic shocks. Neither those models nor their results can be directly “taken off the shelf” for my purposes, necessitating the development of a new framework. More specifically, I develop a legislative bargaining framework with endogenous status quo and stochastic uncertainty about the dynamics of policy-relevant states. Building upon insights from prior works by Merlo and Wilson 1995; Anesi and Seidmann 2015; Anesi and Duggan 2018; Eraslan, Evdokimov, and Zápal 2022, my model is as follows.

The game. A finite set of legislators in set $N \equiv \{1, 2, \cdots, n\}$, where $n \geq 3$, must choose a policy x^t in each of an infinite number of discrete periods t, i.e., $(t = 0, 1, \cdots, \infty)$, to bargain over the split of a ”pie”. The size of the pie S^t in each t is denoted as a *state*, where the state space is $\mathbf{S} \ni S^t$. Let $\{S^t\}_{t=0}^{\infty}$ be a stochastic process drawn independently and identically from a distribution function F with a mean of 1 (i.e., $E[S^t] = 1$) and an upper bound $\overline{s} < \infty$. Furthermore, to ensure there is some pie available to be divided, assume $supp(\mathrm{F}) \subseteq \mathbb{R}_{++}$. For each t, denote the t-period state-history by $\mathcal{S}^t \equiv (S^0, S^1, \cdots, S^t)$, with a typical realization $\boldsymbol{s}^t \equiv (s^0, s^1, \cdots, s^t)$.

In most literature on legislative bargaining, the size of the pie is standardized to one unit. However, in my model, S^t is a random variable. I generalize both settings by interpreting the bargaining process as negotiating over the *share* of the pie. To be precise, let the policy space (the set of possible alternatives) be denoted by $\mathbf{X} \subseteq \mathbb{R}^d$, with nonempty interior. Define policy as $x^t \equiv (x_1^t, x_2^t, \cdots, x_n^t) \in [0,1]^n$, which player i $(i \in N)$, receives x_i share of the pie. One immediate constraint is that the policy space $\mathbf{X}$ should be feasible i.e., $\mathbf{X} = \big\{(x_1, x_2, \cdots, x_n) \in [0,1]^n : \sum_{i \in N} x_i \leq 1\big\}$, which aligns with the pie-division with free disposal setting.

Bargaining takes place as follows. Each period t begins with a with a realized state s^t to the public, and a status quo alternative x^{t-1}, which is in place from the previous period. Player $i \in N$ is selected with probability $p_i \in (0,1)$ to propose a policy $y \in \mathbf{X}(s^t)$, $\sum_{i \in N} p_i = 1$; all players then simultaneously vote to accept or to reject the chosen proposal. It is accepted if a coalition $C \in \mathcal{D}$ of players vote to accept, and it is rejected otherwise, where $\mathcal{D} \subseteq 2^N \setminus \{\emptyset\}$ is the nonempty collection of decisive coalitions, which have the authority to decide policy in a given period. If proposal y is accepted, then it is implemented in period t and becomes the status quo next period (i.e., $x^t = y$); otherwise, the previous status quo, x_{t-1}, is implemented and remains the status quo in period $t+1$ (i.e., $x^t = x^{t-1}$). This process continues ad infinitum. The initial status quo, $x^0 \in \mathbf{X}$, is exogenously given. The sequence of events is summarized in Figure 2.1.

Utility. Players maximize the expected discounted sum of their utility over time. Stage utility of player i from policy $x_i^t \in x^t$ in any period t when the state is s^t is given by $u_i(s^t, x_i^t)$ with $u_i(s^t, 0) = u_i(0, x_i^t) = 0$, where I assume $u_i(s^t, x_i^t) : \mathbf{S} \times [0,1] \to \mathbb{R}$ is strictly increasing in both s^t and x_i^t, twice continuously differentiable with respect to both s^t and x_i^t, concave in

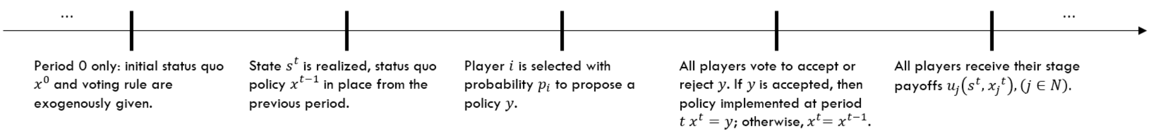

Figure 2.1: Timing in each period $t = 0, 1, 2, \cdots$

x_i^t, and $\frac{\partial^2 u_i}{\partial s^t \partial x_i^t} > 0$ (e.g., $u_i(s^t, x_i^t) = s^t x_i^t$). Given a sequence of alternatives $\{x^t\} \in \mathbf{X}^\infty$ and realized states $\{s^t\} \in \mathbf{S}^\infty$, player i's discounted utility is given by $(1-\delta_i)\sum_{t=0}^{\infty} \delta_i^t \cdot u_i(s^t, x_i^t)$, $\delta_i \in [0,1)$ is the discount factor of player i. The normalization factor $(1-\delta_i)$ serves to measure the repeated game payoffs and the stage game payoffs in the same units.

Voting rules. Other than restriction forms of utility function, need to add reasonable hypotheses to voting rules to make the framework feasible. I assume the voting rule $\mathcal{D}$ is proper, i.e., every pair of decisive coalitions has nonempty intersection: $C, C' \in \mathcal{D}$ implies $C \cap C' \neq \emptyset$. What's more, I assume $\mathcal{D}$ is monotonic, i.e., any superset of a decisive coalition is itself decisive: $C \in \mathcal{D}$ and $C' \subseteq C$ imply $C' \in \mathcal{D}$. Further, I assume $\mathcal{D}$ is noncollegial, i.e., no player has a veto: I have $N \backslash \{i\} \subseteq \mathcal{D}$ for all $i \in N$. Thus, I allow for any quota rule defined by $\mathcal{D} \equiv \{C : |C| \geq q\}$, with the only restriction on the quota q being $\frac{n}{2} < q < n$, which falls between a simple majority and less than unanimity.

Equilibrium concept. I restrict attention to pure stationary Markov strategies and equilibria to make behavior in any period dependent on only a relatively small set of variables rather than on the entire history of play (Maskin and Tirole 2001). A stationary Markov perfect equilibrium (SMPE) is a subgame perfect Nash equilibrium (SPNE) in which all players use stationary Markov strategies. For any player $i \in N$, a stationary Markov strategy $\sigma_i = (\pi_i, \alpha_i)$ consists of a proposal strategy $\pi_i : \mathbf{S} \times \mathbf{X} \to \mathbf{X}$, where $\pi_i(S, x)$ is the proposal made by player i when the current status quo and state is x and S respectively (conditional on her being selected to propose); and a voting strategy $\alpha_j : \mathbf{S} \times \mathbf{X}^2 \to \{0, 1\}$, where $\alpha_j(S, x, y)$ is the (degenerate) probability that player $j \in N$ votes to accept a proposal y when the current status quo is x at state S. A strategy profile thus can be denoted as $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_n)$. I drop the time subscript since infinite horizon models further restrict attention to stationary strategies, which do not allow dependence on time t. Referred by Anesi and Duggan 2018, for succinctly, I use a *stationary bargaining equilibrium* i.e., a pure stationary Markov perfect equilibrium in stage-undominated voting strategies.

Absorbing points and no-delay strategies. Every stationary Markov strategy profile σ (in conjunction with recognition probabilities) generates a transition function $P^\sigma : \mathbf{S} \times \mathbf{X}^2 \to [0, 1]$, where $P^\sigma(S, x, y)$ is the probability, given σ, that the alternative implemented in the next period is y, when current status quo is x, at state S.

Define $x \in \mathbf{X}$ as an *absorbing point* of σ if and only if $P^\sigma(S, x, x) = 1$ for all states $S \in \mathbf{S}$. This means that, given σ, with probability one, the current policy x will be implemented again in the next period regardless of the state. Denote $A(\sigma)$ the set of absorbing points i.e., $A(\sigma) \equiv \{x \in \mathbf{X} : \; P^\sigma(S, x, x) = 1\}$. An absorbing SPNE is thus SPNE in which all players use strategies within $A(\sigma)$.

What's more, we say σ is *no-delay* if and only if $A(\sigma)$ is non-empty and for all $x \in \mathbf{X}$ and $S \in \mathbf{S}$ there is $y \in A(\sigma)$ such that $P^\sigma(S, x, y) = 1$. In words, a strategy profile σ is characterized as *no-delay* if, in every period and every state (both on and off the equilibrium path), an absorbing point is implemented.

2.2 Simplifications and observations

2.2.1 Preliminary intuitions

There are two preliminary observations worth discussing. For myopic (i.e., δ_i is small) players, "Observation 1" in Anesi and Seidmann 2015 remains valid. I discuss this observation through the following two points. Furthermore, when players are sufficiently patient (i.e., δ_i is large), a no-delay stationary bargaining equilibrium exists (not necessary unique). The formal proof for the general case is provided in the next section; here, I offer a toy example to better illustrate the intuition.

Observation 1. Given voting rule $\mathcal{D}$ defined as in section 2.1, if $\delta_i = 0$ for each $i \in N$, then it is a "rotating dictator" equilibrium (Kalandrakis 2004), i.e., each period's proposer receives the entire pie at the revealing state in every SPNE.

This is because $\mathcal{D}$ is not unanimity, and when $\delta_i = 0$, the game behaves like an "ultimatum" game. In each period t, any player only cares about their stage payoff, given the state realization s^t and status quo x^{t-1}, which assigns everyone 0 except for the proposer j at $t-1$ when $t > 0$ (for $t = 0$, the status quo is 0 for everyone thus any policy can be implemented). Consequently, the proposer k at period t make an alternative $x^t = (0, \cdots, 0, 1, 0, \cdots, 0)$ (1 in the j-th coordinate and 0 otherwise), i.e., allocates the entire pie s^t to herself, giving 0 to everyone else to make them indifferent, except for k if $k \neq j$ (propose the same policy if $k = j$). Consider it's non-unanimity, k herself is indecisive, thus x^t is implemented.

Observation 2. There exists $\underline{\delta} \in (0, 1)$ such that there is no absorbing SPNE whenever $\max_{i \in N} \delta_i < \underline{\delta}$.

This observation, documented in Anesi and Seidmann 2015, is extended here to incorporate a stochastic component, and the observation remains valid. A formal proof is provided in Appendix A. The proof is by contradiction: suppose there exists an absorbing policy x. Then, players other than the one with the highest payoffs can form a new winning coalition (recall that $\mathcal{D}$ is noncollegial,meaning any coalition with $n - 1$ players is a winning coalition) and redistribute the excluded player's values among themselves. Such a new proposal would always be implemented if the players are sufficiently short-sighted. Therefore, no policy can be an absorbing point.

In summary, I demonstrate that the core is empty—there is always a winning coalition that can make all its short-sighted members strictly better off by amending any potential absorbing point to another policy under some realization of the state.

2.2.2 Simple solutions

Followed by Anesi and Seidmann 2015, I further refined the equilibrium concept by constructing a class of pure strategy no-delay SMPEs, in which each player $j \in N$ is offered only two different shares of the pie: a "high" offer $x_j > 0$, and a "low" offer $y_j < x_j$, after any history.

In each period and for any ongoing default, each proposer i (conditional on being recognized to make an offer), implicitly selects a winning coalition $C_i \ni i$ by making high offers to the members of C_i and low offers to the remaining members outside coalition $N \setminus C_i$. If each player receives a low offer from at least one proposer, I refer to the set of such proposals (one for each player) as a simple solution. Formally:

Definition 1. Let $\mathcal{C} \equiv \{C_i\}_{i \in N} \subseteq \mathcal{D}$ be a class of coalitions such that, for each $i \in N$, $i \in C_i$ and $i \notin C_j$ for some $j \in N \setminus \{i\}$. Let $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$ be two vectors in $[0,1]^n$, for all $i \in N$:

$$\begin{aligned} \textstyle\sum_{j \in C_i} x_j + \sum_{j \notin C_i} y_j \leq 1 \\ x_i > y_i \end{aligned} \tag{2.1}$$

The simple solution induced by $(\mathcal{C}, x, y)$ is the set of policies $X \equiv \left\{x^{C_i}\right\}_{i \in N}$, where

$$x^{C_i} \equiv \begin{cases} x_j \ if \ j \in C_i \\ y_j \ if \ j \notin C_i \end{cases} for \ all \ i, j \in N \tag{2.2}$$

A set of policies $X \subseteq \mathbf{X}$ is a simple solution if there exists a triplet $(\mathcal{C}, x, y)$ (as defined above) such that X is a simple solution induced by $(\mathcal{C}, x, y)$.

The following example demonstrates some key mechanisms behind my equilibrium construction.

2.2.3 An example

For a simplified case, restrict $N = \{1, 2, 3\}$, i.e., three homogeneous legislators. The primary distinction from Kalandrakis 2004; Anesi and Seidmann 2015 is the incorporation of a stochastic process that independent and identically distributed random variables draws from F. Here let F be a uniform distribution, defined as $\mathrm{F} \stackrel{\text{def}}{=} \mathrm{Unif}\,[1-\rho, 1+\rho]$, where $\rho \in [0, 1)$. An immediate observation is that this simplification maintains the expected value of the state at 1, while ρ as a parameter serves to measure the variance or risk, thus the uncertainty i.e., $E[S^t] = 1, Var(S^t) = \frac{\rho^2}{3}, \left\{S^t\right\} \stackrel{i.i.d.}{\sim} \mathrm{F}$.

Assume that legislators are risk-neutral and care only about the payoff they receive, i.e., $u_i(s, x_i) = s \cdot x_i, i \in N$ with a common discount factor δ. Each legislator has an equal probability of being selected as a proposer in each period, i.e., $p_i \equiv \frac{1}{3}$. The decisive coalitions are determined under simple majority rule. Then the coalition is given by $C_1 = \{1,2\}, C_2 = \{2,3\}, C_3 = \{1,3\}$.

Assume the "high" offer $x_j \equiv h$, and the "low" offer $y_j \equiv l$ for every player $j \in N$, where $0 \le l < h \le 1$. The simple solution can be derived from $X = \{(h,h,l),(l,h,h),(h,l,h)\}$. If $\delta > 1 - \frac{h-l}{(h-l)+3(1+\rho)(1-h)} =: \bar{\delta}$, then the following strategy profile forms a stationary bargaining equilibrium (pure strategy, no-delay SMPE) whose set of absorbing points is X:

- Player i always offers h to the players in C_i and l to the player outside C_i for any realizations of $S \in$ **S**, if the status quo does not belong to X, and proposes the existing status quo otherwise (I call this scenario as *passes*);
- Player i accepts proposal z when the status quo is w if and only if one of the following conditions holds: (1) $w \in X$ and $w = l$; (2) $w \notin X$, $z \in X$, and $((1-\delta)S + \delta)z_i \ge (1-\delta)Sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l)$; or (3) $w, z \notin X$, and $w_i \le z_i$.

The intuition behind this is as follows: For sufficient patient players, this (pure) strategy profile is no-delay and that X is the set of absorbing points: each policy $x^{C_i} \in X$ is proposed by player i with probability $\frac{1}{3}$. Once proposed, it is accepted by the two members of the majority coalition C_i and is never amended.

To see why this is an SMPE, observe first that in the long run, each player $i \in \{1,2,3\}$ can only end up receiving two possible shares of the pie: either receives h when in a winning coalition, or l otherwise. Denote current state as S with realization s, and the next period state as S'. Indeed, any ongoing status quo w is either (1) an absorbing point in itself or (2) will immediately lead to some absorbing point $x^{C_j} \in X$, where $x_i^{C_j} \in \{h, l\}$. In (1), player i's expected payoff is $(1-\delta)s \cdot h + \delta E[S'h] = (1-\delta)sh + \delta h$ if $i \in C_j$, and $(1-\delta)s \cdot l + \delta E[S'l] = (1-\delta)sl + \delta l$ if $i \notin C_j$. In (2) e.g., if the current period's proposer fails to amend w, player i receives $s \cdot w_i$ in the current period and $S'(\frac{2}{3}h + \frac{1}{3}l)$ in the next period ($i \in C_j$ with probability $\frac{2}{3}$). Her expected payoff is therefore $(1-\delta)s \cdot w_i + \delta E[S'(\frac{2}{3}h + \frac{1}{3}l)] = (1-\delta)sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l)$.

For $\delta > \bar{\delta}$, $(1-\delta)sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l) < (1-\delta)sh + \delta h$ holds for all $w_i \in [0,1]$. The proof is shown in Appendix B. As we know $\bar{\delta} \in [0,1)$ by construction. For all h, l, ρ in their support, we can always find such a $\delta \in (\bar{\delta}, 1)$. This correspond to a general case of Kalandrakis 2004 where $\rho = 0, l = \frac{1}{6}, h = \frac{1}{3}$.

In every voting stage, players know that if the current status quo is not in X, the proposer i will successfully offer the absorbing point $x^{C_i} \in X$, and pass if the current status quo is already in X. As $x_j^{C_i} = \{l, h\}$ for all $i, j \in N$, every player j anticipates that her shares of the pie in all future periods will be either l or h, and receive stage payoff $S'l$ or $S'h$, respectively. For risk-neutral, farsighted players, in expectation, the future stage payoff for each player j is bounded above by $E[u_j(S', h)] = h$. It's thus optimal for player j to reject any proposal to amend the status quo x^{C_i} whenever $j \in C_i$, and accept if $j \notin C_i$ (in this case she receives the lowest possible expected payoff). For any realization of the current state s, changing x^{C_i} to another policy can only decrease the long-run payoffs.

Since C_i are winning coalitions, this ensures that it is impossible to amend the policies x^{C_i} once they have been implemented. If the current status quo is not in X, then, by the same logic, it is optimal for each member of coalition C_i to accept x^{C_i} and for any other player to reject it. These voting strategies imply that there is no profitable deviation from the proposed strategies. For proposal strategies, if the current status quo is outside X, the optimal strategy for proposer i is to offer x^{C_i}. This proposal will be accepted by all members of the winning coalition C_i from the above voting strategies and guarantees the proposer the highest possible expected long-term payoff. Since any attempt to amend the status quo when it is in X will be unsuccessful, passing is the optimal choice. Thus, it is indeed a SMPE.

Two key observations can be derived from the above example. Firstly, it explains why shares of the pie can be perpetually wasted and/or shared among more than a minimal winning coalition in equilibrium. Players can be locked into equilibria where any deviation to proposing a Pareto superior policy would be rejected (as in Anesi and Seidmann 2015; Anesi and Duggan 2018). In the above example, farsighted player 1 (as a representative) would be blocked from accepting, for instance, $(\frac{1}{2}, \frac{1}{2}, 0) \in \mathbf{X}$ when current status quo is

$x^{C_1} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{6})$. This is because there is a possibility $p_2 > 0$, that player 2 could be the proposer in the next period and propose $x^{C_2} = (\frac{1}{6}, \frac{1}{3}, \frac{1}{3})$ to form another winning coalition that excludes player 1. It reflects uncertainty aversion: rather than opting for a higher payoff in the short term, a patient player chooses the long-term sustainable gain. Secondly, $\bar{\delta}$ increases as ρ increases ($\frac{\partial \bar{\delta}(\rho)}{\partial \rho} > 0$). The example degenerates into a deterministic case within the dynamic bargaining framework for $\rho = 0$, allowing for a relatively small δ to sustain the equilibrium. As ρ approaches 1, indicating a high level of uncertainty and risk, $\bar{\delta}$ must also increase accordingly, imposing a stricter constraint on δ that requires players to be more farsighted. This implies that, in order to ensure policy stability for sustainable development in a more volatile environment, legislators must exhibit greater patience.

To conclude the example, a sustainability-efficiency trade-off can be detected: high δ (close to 1) can lead to stable policies, which benefit e.g., steady growth, low inflation, and low levels of unemployment. However, it may also result in the persistence of inefficient resource allocation.

Chapter 3

Results

3.1 Existence of equilibrium in the general framework

In this section, I begin by examining the existence of equilibrium in the general case, focusing on the form of the utility function given by $u_i(s^t, x_i^t) \equiv s^t \cdot \tilde{u}_i(x_i^t)$, where $\tilde{u}_i(x_i^t) : [0,1] \to \mathbb{R}$ is a strictly increasing, continuously differentiable concave utility function. In words, suppose players only care about their received share of the pie, and their utility is proportional to the size of the pie, meaning they receive more as the total pie increases.

By Observation 2, we know that there is no equilibrium when players are short-sighted. However, when players are sufficiently patient, an equilibrium can exist. I describe a no-delay stationary bargaining equilibrium in which each policy in a simple solution is proposed by some player, and no other policy is proposed after any history.

Theorem 1. Let X be a simple solution. There exists a threshold $\bar{\delta} = \max_{i \in N} \bar{\delta}_i \in (0,1)$ such that whenever $\min_{i \in N} \delta_i > \bar{\delta}$, there exists a no-delay stationary bargaining equilibrium for the stochastic dynamic bargaining game described in Section 2.1, where the set of absorbing points is X.

The proof is provided in Appendix C.

To be specific, $\bar{\delta}_i$ is defined as

$\bar{\delta}_i \equiv \max\{\frac{\tilde{u}_i(1)-\tilde{u}_i(x_i)}{\tilde{u}_i(1)-\tilde{u}_i(x_i)+\frac{p^{min}}{\bar{s}}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}, \frac{\tilde{u}_i(y_i)-\tilde{u}_i(0)}{\tilde{u}_i(y_i)-\tilde{u}_i(0)+\frac{p^{min}}{\bar{s}}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}\}$, $p^{min} = \min_{i \in N} p_i > 0$.

3.2 Positive results

Under Observation 1, 2 and Theorem 1, I derive the following implications.

Multiplicity of SMPE payoffs. One immediate result from the proof is that the no-delay stationary bargaining equilibrium payoff need not be unique. By the same logic as in the example, any policy z which assigns a positive share to some winning coalition is part of a simple solution. Theorem 1 therefore implies that z is an absorbing point of an no-delay stationary bargaining equilibrium in any game with non-unanimity voting rules and sufficiently patient players. In that SPME, the proposer i successfully propose z, and never amended regardless the state of the world.

Two points are worth highlighting: (1) This argument does not apply to policies that assign a positive share only to players who do not form a winning coalition (e.g., dictatorship) and, therefore, such policies cannot be part of a simple solution. (2) Policies that assign a zero share to some winning coalition (including the initial default $(0, 0, \cdots, 0)$) cannot be absorbing points of an SMPE because every member of such a coalition could profitably deviate as a proposer.

"Rotating dictator". In Kalandrakis 2004, it is concluded that in a three-player dynamic legislative bargaining model, regardless of δ_i, the proposer obtains the entile pie with probability one in the long run. However, given Observation 1 and Theorem 1, their rotating dictatorship result only holds in my model when $\delta_i = 0$. My model naturally prevents the existence of a dictator because $\mathcal{D}$ is noncollegial, i.e., $N \backslash \{i\} \subseteq \mathcal{D}$ for all $i \in N$. By the same logic as the above (2), if player i were to take the entire payoff for herself, then $N \backslash \{i\}$ would form a coalition and could profitably deviate as a proposer.

Minimal winning coalitions. The Baron–Ferejohn model (Baron and Ferejohn 1989) predicts that the size principle holds: only minimal winning coalitions share the pie in any stationary SPE. Theorem 1 immediately implies that this property may fail in my model for endogenous status quo, which in lines with Anesi and Seidmann 2015. As mentioned earlier, policies in a simple solution may all assign a positive share to any winning coalition (e.g., $N \backslash \{1\}$).

Policy persistence. Political stability aligns with sustainable development goals. Theorem 1 implies that there are no-delay stationary bargaining equilibria in which statically policies are retained indefinitely. Regardless of risk, policy persistence can still be achieved by selecting sufficiently patient legislators, such as those who are concerned about future generations. These legislators are willing to prioritize long-term outcomes over immediate gains, thereby supporting consistent and sustainable policies.

Uncertainty introduces a stricter constraint on this threshold. As the state of the world becomes more volatile (i.e., as $\bar{s}$ increases), the need for even more patient legislators grows. A greater volatility requires legislators who are capable of withstanding the pressures and challenges during difficult times. Consequently, maintaining policy stability in the face of increased uncertainty demands a higher level of patience and commitment from legislators.

Waste. Despite the allure of political stability, there is a possibility of persistent inefficiency. As illustrated in the example, the interplay between an endogenous status quo and uncertainty aversion (where uncertainty pertains to the fear of being excluded from the winning coalition and receiving a low payoff in all future periods) can lead to static inefficiencies in equilibrium.

For any $\varepsilon \in (0, 1)$, Let $\mathbf{X}_\varepsilon$ be the set of policies where the committee "wastes" more than ε i.e., $\mathbf{X}_\varepsilon \equiv \{x \in \mathbf{X} : \sum_{i \in N} x_i < 1 - \varepsilon\}$. Consider simple solutions induced by $(\mathcal{C}, x, y)$ where $x_i = \frac{1-\varepsilon}{2(n-1)}$, $y_i = 0$, and C_i is a winning coalition that includes i, for all $i \in N$. Given that $\mathcal{D}$ is non-unanimity, the maximum number of players in a winning coalition is $n - 1$. This confirms that such constructions of simple solutions are indeed subsets of $\mathbf{X}_\varepsilon$. Theorem 1 implies that in any non-unanimity game with upper-bounded risks, there exists a pure-strategy no-delay SMPE for sufficiently patient players in which all absorbing points belong to $\mathbf{X}_\varepsilon$. This means that, along the equilibrium path, the committee wastes at least $\varepsilon > 0$ shares of the pie in every period. This finding aligns with Anesi and Seidmann 2015, which demonstrates static inefficiencies in equilibrium; in my model, this result is extended to a stochastic setting.

Theorem 1 also implies that inefficiencies can persist without requiring players to unilaterally invest in maintaining policies, unlike the scenarios discussed by Coate and Morris 1999 and Acemoglu and Robinson 2008. This finding aligns with Brainard and Verdier 1997:

The tendency for protection to persist once it is instituted is one of the central stylized facts in trade.

Pork barrel politics. Theorem 1 illustrates that it is possible to avoid a rotating dictator equilibrium and maintain political persistence in a changing world. However, it does not eliminate the possibility of static inefficiencies or the breakdown of the size principle. In fact, both issues can occur simultaneously in one equilibrium. Baron 1991 points out that distributive legislation frequently exhibits this combination of problems. While models of ad hoc committees can account for pork barrel politics, they fail to address the issue of size principle violations. In contrast, Theorem 1 demonstrates that a equilibrium agreements in a standing committee can exhibit both properties without relying on a norm of universalism.

I record the implications above as the following:

Corollary 1 For voting rules where $\mathcal{D}$ is proper, monotonic, and noncollegial, and for a pie with size drawn independently and identically from a distribution function with an upper bound $\bar{s} < \infty$. The following statements hold: there exists a threshold $\bar{\delta} \in (0, 1)$, such that each statement is true whenever $\min_{i \in N} \delta_i > \bar{\delta}$.

1. There exist multiple no-delay stationary bargaining equilibria, for any realization of the pie size;
2. Any policy that allocates a positive share to some players within a winning coalition is an absorbing point in some no-delay stationary bargaining equilibrium;
3. A proposal strategy that allocates the entire pie to a single player is not in a no-delay stationary bargaining equilibrium;
4. There are no-delay stationary bargaining equilibria which fail the size principle;
5. For any $\varepsilon \in (0, 1)$, there is no-delay stationary bargaining equilibrium σ such that $P^{\sigma}(S, x, x) = 1$ for all $x \in \mathbf{X}_{\varepsilon}$;
6. There exist no-delay stationary bargaining equilibria in which the agreement wastes a portion of the pie and violates the size principle.

Chapter 4

Discussion and perspectives

I analyze a dynamic legislative bargaining model with an infinite horizon, featuring an endogenous status quo, stochastic uncertainty, non-unanimity committees, and sufficiently patient players. I identify a class of no-delay stationary bargaining equilibria (pure strategy SMPEs). This analysis advances existing models by integrating two previously separate but prominent topics in bargaining theory. It offers new insights into how policy persistence is maintained within standing committees and highlights the effects of an endogenous status quo under stochastic uncertainty.

The equilibria identified in this model exhibit a no-delay property: the first policy proposal is accepted and remains unchanged in all subsequent periods. This ensures that policy persistence, though not necessarily efficient, is maintained regardless of the state's realization or future uncertainty. The only requirement for this to hold is that legislators must be sufficiently patient. As uncertainty increases, greater patience among legislators is needed to sustain policy stability.

My results provide two contributions to the literature. First, I extend the Anesi and Seidmann 2015 model for non-unanimity cases to hold in a broader setting. Specifically, I identify no-delay stationary bargaining equilibria when players are patient enough, which suggests that laws can remain stable even if precedent is not explicitly recognized and for any state of the world. Second, I've shown that when players are sufficiently short-sighted, an absorbing SPNE does not exist, leading to frequent amendments of existing status-quo policies in response to stochastic uncertainties.

One of the advantages of maintaining policy stability is its positive impact on trade. Stable trade policies are crucial for both the economic welfare of the countries involved and the preservation of the multilateral trading system (Drabek and Brada 1998). Policy stability fosters trust between trading partners, making countries and businesses more likely to engage in trade agreements and long-term contracts when they believe the rules will not change unexpectedly. A stable policy environment reduces the costs associated with frequent changes, such as re-negotiating contracts, adjusting to new regulations, and managing compliance issues. Additionally, during ”bad” times (e.g., financial crisis), consistent policies play a crucial role in controlling inflation and stabilizing interest rates, which are vital for economic stability during a crisis (De Mendonça and Souza 2012). Once the immediate crisis is resolved, stable policies provide a solid foundation for economic recovery and growth (Walsh et al. 2009).

However, maintaining policy stability comes with costs. My findings indicate that, in equilibrium, policy allocations can result in persistent waste and/ or fail to adhere to the size principle. This observation is supported by empirical evidence. For instance, in the United States, farm programs originally designed to support impoverished farmers continue long after their beneficiaries have become significantly wealthier than the taxpayers funding them (Rausser 1992). Similarly, in developing countries, tariff programs intended to support import-substituting industries persist even after such development strategies have been discredited (Krueger 2002). This aligns with conventional wisdom in political economy, which argues that once an economic policy is established, it tends to persist even when its original rationale no longer holds (Coate and Morris 1999). My model thus also offers one possible explanation for this enduring nature of policies: the patience of legislators.

On the other hand, policy persistence is harder to maintain in ”bad” times, as current legislators may not be patient enough to stay in the equilibrium. My findings provide an explanation for the phenomenon where a time of crisis occasionally enables radical reforms that would have been unthinkable in calmer times. This aligns with the observation noted by Rodrik 1992, that in some key cases of radical trade reform—such as Bolivia (in 1985), Mexico (since 1987), Poland (1990), and Peru (1990)—a macroeconomic crisis of unprecedented proportions led the leadership to embrace a wide range of reforms, of which trade liberalization was one component.

This model represents a preliminary step that introduces novel ideas to the literature by integrating an endogenous status quo with stochastic uncertainty. It paves the way for future research in several areas. For example, an immediate observation of dealing with waste is to consider a pie-division with no disposal setting, in which the pie must be fully divided i.e., $\mathbf{X} = \left\{(x_1, x_2, \cdots, x_{n-1}) \in [0,1]^{n-1} : \sum_{i=1}^{n-1} x_i \leq 1\right\}$. Such construction is worth discussing in future studies. Additionally, future studies could also investigate cases where stochastic states themselves are endogenous, such as when a fair or efficient policy positively influences future states. There is also potential to further generalize the utility function, as in Anesi and Duggan 2018, to include human factors such as envy, altruism, or the effect of externalities, which player's utility might be affected by the entire allocation x.

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Chapter 5

Appendices

5.1 Appendix A

NTS. There exists $\underline{\delta} \in (0,1)$ such that there is no absorbing SPNE whenever $\max_{i\in N} \delta_i < \underline{\delta}$, $i \in N$.

Proof. Define $\underline{\delta}_i = \frac{u_i(1,1)-u_i(1,1-\frac{1}{n(n-1)})}{u_i(1,1)-u_i(1,1-\frac{1}{n(n-1)})+u_i(\bar{s},1)}$ and $\underline{\delta} = \min_{i\in N}\{\underline{\delta}_i\}$, where $\bar{s}$ is the upper bound of the state. By construction, $\underline{\delta}_i \in (0,1)$ thus $\underline{\delta} \in (0,1)$ as well. Suppose towards a contradiction that exists an absorbing policy $\bar{x} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)$. Without loss of generality, let $\bar{x}_1 = \max_{i\in N}\{\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n\}$. Given $\mathcal{D}$ is noncollegial, let coalition $C = \{2, 3, \cdots, n\} \in \mathcal{D}$.

Before proceeding, I first prove two claims that will be used later:

Claim 1. $\sum_{i\neq 1}\bar{x}_i \leq \frac{n-1}{n}$.

Proof. This is true by the feasible constraint of the policy and the above construction.

$$\begin{aligned}
&\bar{x}_1 = \max_{i\in N}\{\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n\} \\
&\Rightarrow \bar{x}_1 \geq \bar{x}_i,\ \forall i \in N \\
&\Rightarrow (n-1)\bar{x}_1 \geq \textstyle\sum_{i\neq 1}\bar{x}_i \\
&\overset{\sum_{i\in N}\bar{x}_i\leq 1}{\Rightarrow} \frac{1}{(n-1)}\textstyle\sum_{i\neq 1}\bar{x}_i + \sum_{i\neq 1}\bar{x}_i \leq \bar{x}_1 + \sum_{i\neq 1}\bar{x}_i \leq 1 \\
&\Rightarrow \textstyle\sum_{i\neq 1}\bar{x}_i \leq \frac{n-1}{n}
\end{aligned} \tag{5.1}$$

Claim 2. Define $f(s,x_i) = u_i(s,x_i+c) - u_i(s,x_i)$, where $c > 0$ is a constant. Then (1) $\frac{\partial f(s,x_i)}{\partial s} > 0$, and (2) $\frac{\partial f(s,x_i)}{\partial x_i} \leq 0$.

Proof. Recall the definition of $u_i(s,x_i)$. We are equipped with conditions that u_i is twice continuously differentiable with respect to both s and x_i, concave in x_i (i.e., $\frac{\partial^2 u_i}{\partial x_i^2} \leq 0$), and $\frac{\partial^2 u_i}{\partial s \partial x_i} > 0$.

$$\frac{\partial f(s,x_i)}{\partial s} = \frac{\partial u_i(s,x_i+c)}{\partial s} - \frac{\partial u_i(s,x_i)}{\partial s} > 0 \tag{5.2}$$

this is because $\frac{\partial^2 u_i}{\partial s \partial x_i} > 0$, for any $c > 0$, $\frac{\partial u_i(s,x_i+c)}{\partial s} > \frac{\partial u_i(s,x_i)}{\partial s}$.

$$\frac{\partial f(s,x_i)}{\partial x_i} = \frac{\partial u_i(s,x_i+c)}{\partial x_i} - \frac{\partial u_i(s,x_i)}{\partial x_i} \leq 0 \tag{5.3}$$

this is because $\frac{\partial^2 u_i}{\partial x_i^2} \leq 0$, for any $c > 0$, $\frac{\partial u_i(s,x_i+c)}{\partial x_i} \leq \frac{\partial u_i(s,x_i)}{\partial x_i}$.

Now, take a subgame in which $\bar{x}$ is the status quo policy, $s \in$ **S** is the realization of the state. Let s be within range $[1,\bar{s}]$ (recall $E[S] = 1$; if $\bar{s} = 1$, then it is a point mass at 1 with probability 1). This occurs with probability $(1 - \Pr(S < 1)) > 0$. Suppose some player $j \in C$ has been recognized with probability $p_j > 0$ to propose.

If j offers $y = (y_1, y_2, \cdots, y_n)$, where $y_i = \begin{cases} \bar{x}_i + \frac{1}{n(n-1)} & if\ i \in C \\ 0 & if\ i \notin C \end{cases}$

By Claim 1, y is feasible i.e., $\sum_{i \in N} y_i = \sum_{i \neq 1} \bar{x}_i + \frac{1}{n} \leq 1$.

Each $i \in C$ receives $(1-\delta_i)u_i(s,y_i) + \delta_i V_i^a$ if y has been implemented, where V_i^a is i's continuation value in equilibrium from accepting. A key observation is that the lower bound for V_i^a is 0 (i.e., receive nothing).

If y is not been voted up, then the status quo $\bar{x}$ is implemented, and each $i \in C$ receives $(1-\delta_i)u_i(s,\bar{x}_i) + \delta_i V_i^r$, where $V_i^r = E[u_i(S,\bar{x}_i)] \leq u_i(\bar{s},1)$ is i's continuation value in equilibrium from rejection.

$i \in C$ is better off accepting whenever

$$(1-\delta_i)u_i(s,y_i) + \delta_i V_i^a > (1-\delta_i)u_i(s,\bar{x}_i) + \delta_i V_i^r \tag{5.4}$$

which holds for $\delta_i < \underline{\delta}$ because

$$
\begin{aligned}
&\delta_i < \underline{\delta}_i = \frac{u_i(1,1) - u_i(1,1-\frac{1}{n(n-1)})}{u_i(1,1) - u_i(1,1-\frac{1}{n(n-1)}) + u_i(\bar{s},1)} = \frac{1}{1 + \frac{u_i(\bar{s},1)}{u_i(1,1)-u_i(1,1-\frac{1}{n(n-1)})}} \\
&\overset{Claim2\ (2)}{\Rightarrow} \delta_i < \underline{\delta}_i = \frac{1}{1 + \frac{u_i(\bar{s},1)}{u_i(1,1)-u_i(1,1-\frac{1}{n(n-1)})}} \leq \frac{1}{1 + \frac{u_i(\bar{s},1)}{u_i(1,\bar{x}_i+\frac{1}{n(n-1)})-u_i(1,\bar{x}_i)}} \\
&\overset{Claim2\ (1)}{\Rightarrow} \delta_i < \frac{1}{1 + \frac{u_i(\bar{s},1)}{u_i(s,\bar{x}_i+\frac{1}{n(n-1)})-u_i(s,\bar{x}_i)}} \\
&\Rightarrow (1-\delta_i)u_i(s,\bar{x}_i + \frac{1}{n(n-1)}) > (1-\delta_i)u_i(s,\bar{x}_i) + \delta_i u_i(\bar{s},1) \\
&\Rightarrow (1-\delta_i)u_i(s,\bar{x}_i + \frac{1}{n(n-1)}) + \delta_i V_i^a \geq (1-\delta_i)u_i(s,\bar{x}_i + \frac{1}{n(n-1)}) + \delta_i \cdot 0 > \\
&\quad (1-\delta_i)u_i(s,\bar{x}_i) + \delta_i u_i(\bar{s},1) \geq (1-\delta_i)u_i(s,\bar{x}_i) + \delta_i V_i^r \\
&\Rightarrow (1-\delta_i)u_i(s,y_i) + \delta_i V_i^a > (1-\delta_i)u_i(s,\bar{x}_i) + \delta_i V_i^r
\end{aligned}
\tag{5.5}
$$

Thus, for short-sighted players, y must be accepted in equilibrium. Proposer j (if selected and the realized state is greater than 1, e.g., in "good" times) can profitably deviate from the equilibrium by successfully proposing y when the status quo is $\bar{x}$. For $\bar{x}$ to be an absorbing policy, the policy must be stable under any realization of the state. This leads to a contradiction.

Therefore, there exists $\underline{\delta} \in (0,1)$ such that the core is empty, meaning that, there is always a winning coalition that can make all its short-sighted members strictly better off by amending any potential absorbing point to another policy at some realization of the state.

5.2 Appendix B

Setup. $N = \{1,2,3\}$, $\mathrm{F} \overset{\mathrm{def}}{=} \mathrm{Unif}\,[1-\rho, 1+\rho]$, where $\rho \in [0,1)$. Denote current state as S with realization s, and the next period state as S'. $u_i(s,x_i) = s \cdot x_i$, $\delta_i = \delta \in [0,1)$. $p_i = \frac{1}{3}$, $i \in N$. $x_j \equiv h$, $y_j \equiv l$, $j \in N$, where $0 \leq l < h \leq 1$. $\bar{\delta} \equiv 1 - \frac{h-l}{(h-l)+3(1+\rho)(1-h)} < 1$.

The simple solution can be derived from $\mathcal{C} = \{(h,h,l),(l,h,h),(h,l,h)\}$, where $C_1 = \{1,2\}, C_2 = \{2,3\}, C_3 = \{1,3\}$.

NTS. For $\delta > \bar{\delta}$, $(1-\delta)sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l) < (1-\delta)sh + \delta h$ holds for all $w_i \in [0,1]$.

Proof. The proof is straightforward:

$$\begin{aligned}
&\delta > 1 - \frac{h-l}{(h-l)+3(1+\rho)(1-h)},\ 0 \le l < h \le 1 \\
\Rightarrow &\delta > 1 - \frac{h-l}{(h-l)+3s(1-h)},\ s \in [1-\rho, 1+\rho], \rho \in [0,1) \\
\Rightarrow &\delta > \frac{s(1-h)}{\frac{1}{3}(h-l)+s(1-h)} \\
\Rightarrow &s(1-h) < (\frac{1}{3}(h-l)+s(1-h))\delta \\
\Rightarrow &(1-\delta)s(1-h) < \frac{1}{3}\delta(h-l) \\
\Rightarrow &(1-\delta)s + \delta(\frac{2}{3}h + \frac{1}{3}l) < (1-\delta)sh + \delta h \\
\Rightarrow &(1-\delta)sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l) < (1-\delta)sh + \delta h, \forall w_i \in [0,1]
\end{aligned} \tag{5.6}$$

5.3 Appendix C

Theorem 1. Let X be a simple solution. There exists a threshold $\bar{\delta} \in (0,1)$ such that whenever $\min_{i\in N}\delta_i > \bar{\delta}$, there exists a no-delay stationary bargaining equilibrium for the stochastic dynamic bargaining game described in Section 2.1, where the set of absorbing points is X.

Proof. For $u_i(s, x_i) \equiv s \cdot \tilde{u}_i(x_i)$, $i \in N$, let $\mathcal{C} \equiv \{C_i\}_{i\in N} \subseteq \mathcal{D}$ and $x = (x_1, x_2, \cdots, x_n), y = (y_1, y_2, \cdots, y_n) \in [0,1]^n$ satisfy the conditions in Definition 1, let $X \equiv \{x^{C_i}\}_{i\in N}$ be the simple solution induced by $(\mathcal{C}, x, y)$.

Define $\bar{\delta}_i \equiv \max\{\frac{\tilde{u}_i(1)-\tilde{u}_i(x_i)}{\tilde{u}_i(1)-\tilde{u}_i(x_i)+\frac{p^{min}}{\bar{s}}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}, \frac{\tilde{u}_i(y_i)-\tilde{u}_i(0)}{\tilde{u}_i(y_i)-\tilde{u}_i(0)+\frac{p^{min}}{\bar{s}}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}\}$ as the threshold for player $i \in N$ to sustain the equilibrium, where p^{min} is the minimal probability of being selected as a proposer i.e., $p^{min} = \min_{i\in N} p_i > 0$. The threshold for all players thus can be defined as $\bar{\delta} = \max_{i\in N}\bar{\delta}_i$.

For $\min_{i\in N}\delta_i > \bar{\delta}$, construct a stationary Markov strategy profile as $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_n)$. For each $i \in N$, define the function $\phi^i : \mathbf{S} \times \mathbf{X} \to X$ which maps the product of state space and policy space to absorbing points as follows:

- if $w \in X$, then $\phi^i(S,w) = w$;
- if $w \notin X$, then $\phi^i(S,w) = (\phi^i_1(S,w), \phi^i_2(S,w), \cdots, \phi^i_n(S,w)) = (x_1^{C_i}, x_2^{C_i}, \cdots, x_n^{C_i})$.

Equipped with a collection of function $(\phi^i)_{i\in N}$, $\sigma_i \equiv (\pi_i, \alpha_i)$ is defined as the following:

1. In the proposal stage of any period t with status quo w and state realization $s \in \mathbf{S}$, i's proposal (conditional on being selected to make a proposal) is $\pi_i(s,w) = \phi^i(s,w)$;
2. In the voting stage of any period t with with status quo w and state realization s, player i accepts proposal $z \in \mathbf{X} \setminus \{w\}$ (i.e., $\alpha_i(s,w,z) = 1$) if and only if: either (a) $w \in X$ and $w_i = y_i$; or (b) $w \notin X$ and $(1-\delta_i)u_i(s,z_i) + \delta_i E[\sum_{j\in N} p_j u_i(S, \phi^j_i(S,z))] \geq (1-\delta_i)u_i(s,w_i) + \delta_i E[\sum_{j\in N} p_j u_i(S, \phi^j_i(S,w))]$.

By construction, $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_n)$ is a pure strategy stationary Markov strategy profile. Before proceeding, I will first prove four claims that will be used later:

Claim 1. For all $i \in N$, $w \notin X$, $s \in \mathbf{S}$, $(1-\delta_i)u_i(s,w_i) + \delta_i E[\sum_{j\in N} p_j u_i(S, \phi^j_i(S,w))] \leq (1-\delta_i)u_i(s,x_i) + \delta_i E[u_i(S,x_i)]$.

Proof.

The upper bound essentially arises from the best-case scenario. Followed by the construction of $\bar{\delta}_i$ and $\delta_i \geq \bar{\delta} \geq \bar{\delta}_i$ and recall $E[S] = 1$, thus $\bar{s} > 1$, it can be shown from the following:

$$
\begin{aligned}
&(1-\delta_i)u_i(s,w_i) + \delta_i E[\textstyle\sum_{j\in N} p_j u_i(S, \phi^j_i(S,w))] = (1-\delta_i)s\tilde{u}_i(w_i) + \delta_i E[\sum_{j\in N} p_j S \tilde{u}_i(\phi^j_i(S,w))] \\
&\overset{E[S]=1}{=} (1-\delta_i)s\tilde{u}_i(w_i) + \delta_i E[\textstyle\sum_{j\in N} p_j \tilde{u}_i(\phi^j_i(S,w))] \\
&= (1-\delta_i)s\tilde{u}_i(w_i) + \delta_i E[\tilde{u}_i(x_i)\textstyle\sum_{j:i\in C_j} p_j + \tilde{u}_i(y_i)\sum_{j:i\notin C_j} p_j] \\
&\leq (1-\delta_i)s\tilde{u}_i(1) + \delta_i[(1-p^{min})\tilde{u}_i(x_i) + p^{min}\tilde{u}_i(y_i)] \\
&\leq (1-\delta_i)s\tilde{u}_i(x_i) + \delta_i\tilde{u}_i(x_i) \\
&= (1-\delta_i)u_i(s,x_i) + \delta_i E[u_i(S,x_i)]
\end{aligned}
\tag{5.7}
$$

The last inequality comes from the fact that $\delta_i > \bar{\delta} \geq \bar{\delta}_i \geq \frac{\tilde{u}_i(1)-\tilde{u}_i(x_i)}{\tilde{u}_i(1)-\tilde{u}_i(x_i)+\frac{p^{min}}{\bar{s}}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))} \geq \frac{\tilde{u}_i(1)-\tilde{u}_i(x_i)}{\tilde{u}_i(1)-\tilde{u}_i(x_i)+\frac{p^{min}}{s}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}$ for all $s \in \mathbf{S}$ i.e., $\left(\frac{\partial \frac{\tilde{u}_i(1)-\tilde{u}_i(x_i)}{\tilde{u}_i(1)-\tilde{u}_i(x_i)+\frac{p^{min}}{s}:(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}}{\partial s} > 0\right)$.

Claim 2.

1. According to σ, in every period that starts with status quo $w \notin X$, each proposer j (if been selected to make a proposal) successfully offers $\phi^j(S, w) \in X$, which is never amended; σ is no-delay with $A(\sigma) = X$.

2. Given σ, for all $w \in \mathbf{X}$, $i \in N$ and $s \in \mathbf{S}$, i's continuation value (i.e., the expected payoff that player i receives from w being implemented at s onwards) satisfies

$$V_i^{\sigma}(s, w) = (1 - \delta_i)u_i(s, w_i) + \delta_i E[\textstyle\sum_{j \in N} p_j u_i(S, \phi_i^j(S, w))] \tag{5.8}$$

Proof.

(1) Consider a period that starts with with status quo $w \notin X$ and state realization s. Each player $j \in N$ is selected with probability $p_j > 0$ to make a proposal. From the definition of $\pi_j(s, w)$, she proposes $z = \phi^j(s, w)$. $\phi^j \in X$. By definition of ϕ^i, $\phi^i(s, z) = z$ for all $i \in N$: all players pass when the status quo is z. Proposer j's offer remains in effect, never amended in all future periods if it is voted up.

Since $w \notin X$, by part (b) of the definition of voting strategies, proposal z is voted up if there is a winning coalition of players i for which $(1-\delta_i)u_i(s, z_i)+\delta_i E[\sum_{j \in N} p_j u_i(S, \phi_i^j(S, z))] \geq (1 - \delta_i)u_i(s, w_i) + \delta_i E[\sum_{j \in N} p_j u_i(S, \phi_i^j(S, w))]$.

This inequality holds for a winning coalition C_j because of the following, for all $i \in C_j$, $\phi_i^j(S, z) \overset{z \in X}{=} z_i \overset{w \notin X}{=} \phi_i^j(S, w) = x_i^{C_j} = x_i$.

$$
\begin{aligned}
(1-\delta_i)u_i(s,z_i)+\delta_i E[\textstyle\sum_{j\in N}p_j u_i(S,\phi_i^j(S,z))] &= (1-\delta_i)u_i(s,z_i)+\delta_i E[\textstyle\sum_{j\in N}p_j u_i(S,x_i)] \\
&= (1-\delta_i)s\tilde{u}_i(x_i)+\delta_i E[\tilde{u}_i(x_i)\textstyle\sum_{j\in N}p_j] \\
&= (1-\delta_i)s\tilde{u}_i(x_i)+\delta_i\tilde{u}_i(x_i) \\
&\overset{Claim 1.}{\geq} (1-\delta_i)u_i(s,w_i)+\delta_i E[\textstyle\sum_{j\in N}p_j u_i(S,\phi_i^j(S,w))]
\end{aligned} \tag{5.9}
$$

Thus, each player i in $C_j \in \mathcal{C}$ vote "yes" to z, which is then implemented (and never amended since $z \in X$). This also implies that $P^\sigma(S,w,z)=1$ for all $w \notin X$ and $z \in X$. As σ prescribes all proposers to pass at status quo $w \in X$, $P^\sigma(S,w,w)=1$ for all $w \in X$. This proves part 1 of Claim 2.

(2) Firstly, suppose that $w \notin X$ is implemented in the current period with state s. Every player $i \in N$ receives $(1-\delta_i)u_i(s,w_i)$ in the current period. From the discussion above, their continuation value from the next period on will be $E[\sum_{j\in N}p_j u_i(S,\phi_i^j(S,w))]$, the expected payoff would thus be $(1-\delta_i)u_i(s,w_i)+\delta_i E[\sum_{j\in N}p_j u_i(S,\phi_i^j(S,w))]$. Hence, equation (5.8) holds for $w \notin X$.

On the other hand, when $w \in X$ is implemented, according to the definition of proposal strategies, all proposers will pass in future periods i.e., $w_i = \phi_i^j(S,w)$ for all $i,j \in N$. i's continuation value would thus be $(1-\delta_i)u_i(s,w_i)+\delta_i E[u_i(S,w_i)]$. Thus, $V_i^\sigma(s,w) = (1-\delta_i)u_i(s,w_i)+\delta_i E[\sum_{j\in N}p_j u_i(S,\phi_i^j(S,w))]$ holds for $w \in X$. Combining these results, (2) has been proved.

Claim 3. Given status quo w and proposal z, each voter $i \in N$ votes "yes" at $s \in \mathbf{S}$ only if $V_i^\sigma(s,z) \geq V_i^\sigma(s,w)$ and vote no only if $V_i^\sigma(s,w) \geq V_i^\sigma(s,z)$.

Proof. If $w \notin X$, by Claim 2 and the definition of voting strategies, the argument is satisfied.

Now suppose that $w \in X$, $V_i^\sigma(s,w) = (1-\delta_i)u_i(s,w_i)+\delta_i E[u_i(S,w_i)]$. If $z \in X$, by the definition of simple solution, $V_i^\sigma(s,w) = (1-\delta_i)u_i(s,x_i)+\delta_i E[u_i(S,x_i)] = (1-\delta_i)s\tilde{u}_i(x_i)+\delta_i\tilde{u}_i(x_i)$ if i in the winning coalition, and $V_i^\sigma(s,w) = (1-\delta_i)u_i(s,y_i)+\delta_i E[u_i(S,y_i)] = (1-\delta_i)s\tilde{u}_i(y_i)+\delta_i\tilde{u}_i(y_i)$ if not. By the construction of σ, if i votes "yes" then $w_i = y_i$, and $V_i^\sigma(s,w)$

reaches the lower bound. Therefore, $V_i^\sigma(s,w) \leq V_i^\sigma(s,z)$ for all $z \in X$; if i votes "no" then $w_i = x_i$, and $V_i^\sigma(s,w)$ reaches the upper bound. $V_i^\sigma(s,z) \leq V_i^\sigma(s,w)$ for all $z \in X$.

If $z \notin X$, according to σ, if i accepts then $w_i = y_i$

$$\begin{aligned} V_i^\sigma(s,w) &= (1-\delta_i)u_i(s,y_i) + \delta_i E[u_i(S,y_i)] \\ &\overset{(a)}{=} (1-\delta_i)s\tilde{u}_i(y_i) + \delta_i \tilde{u}_i(y_i) \\ &\overset{(b)}{\leq} (1-\delta_i)s\tilde{u}_i(0) + \delta_i[p^{min}\tilde{u}_i(x_i) + (1-p^{min})\tilde{u}_i(y_i)] \\ &\overset{(c)}{\leq} (1-\delta_i)s\tilde{u}_i(z_i) + \delta_i \textstyle\sum_{j\in N} p_j \tilde{u}_i(\phi_i^j(S,z)) \\ &= (1-\delta_i)s\tilde{u}_i(z_i) + \delta_i E[\textstyle\sum_{j\in N} p_j S\tilde{u}_i(\phi_i^j(S,z))] \\ &= (1-\delta_i)u_i(s,z_i) + \delta_i E[\textstyle\sum_{j\in N} p_j u_i(S,\phi_i^j(S,z))] \end{aligned} \tag{5.10}$$

(a) $E[S] = 1$; (b) $\delta_i \geq \bar{\delta} \geq \bar{\delta}_i \geq \frac{\tilde{u}_i(y_i)-\tilde{u}_i(0)}{\tilde{u}_i(y_i)-\tilde{u}_i(0)+\frac{p^{min}}{\bar{s}}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))} \geq \frac{\tilde{u}_i(y_i)-\tilde{u}_i(0)}{\tilde{u}_i(y_i)-\tilde{u}_i(0)+\frac{p^{min}}{s}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}$ for all $s \in$ **S** i.e., $\left(\frac{\partial \frac{\tilde{u}_i(y_i)-\tilde{u}_i(0)}{\tilde{u}_i(y_i)-\tilde{u}_i(0)+\frac{p^{min}}{s}(\tilde{u}_i(x_i)-\tilde{u}_i(y_i))}}{\partial s} > 0\right)$; (c) $z_i \geq 0$, $p^{min}\tilde{u}_i(x_i)+(1-p^{min})\tilde{u}_i(y_i) \leq \sum_{j\in N} p_j \tilde{u}_i(\phi_i^j(S,z))$ (LHS is the worst-case scenario expected payoff for the next period).

If i votes no then $w_i = x_i$; so that $V_i^\sigma(s,w) = (1-\delta_i)u_i(s,x_i) + \delta_i E[u_i(S,x_i)]$. Moreover, Claim 2 implies that $V_i^\sigma(s,z) = (1-\delta_i)u_i(s,z_i) + \delta_i E[\sum_{j\in N} p_j u_i(S,\phi_i^j(S,z))]$. Here $z \notin X$, we have

$$\begin{aligned} V_i^\sigma(s,z) &= (1-\delta_i)u_i(s,z_i) + \delta_i E[\textstyle\sum_{j\in N} p_j u_i(S,\phi_i^j(S,z))] \\ &\overset{Claim1.}{\leq} (1-\delta_i)u_i(s,x_i) + \delta_i E[u_i(S,x_i)] = V_i^\sigma(s,w) \end{aligned} \tag{5.11}$$

thus completing the proof of Claim 3.

Claim 4. There is no profitable one-shot deviation from σ in the proposal stage of any period.

Proof. Suppose $w \in X$. Passing is indeed one of the optimal strategies for the proposer: members of some winning coalition who receive their "high" offer i.e., $j \in N$ who receive $w_j = x_j$, would reject any proposal in X to to sustain the implementation of w.

For $w \notin X$, if proposer i follows the prescription of σ_i, then by Claim 2, her proposal

$\phi^i(S, w)$ would be accepted, and she would receive her highest expected payoff $V_i^\sigma(S, x)$ she can obtain by making a successful proposal in X. Therefore, if she is to profitably deviate, she must either (a) make an unsuccessful proposal, obtaining payoff $V_i^\sigma(S, w)$, or (b) successfully propose some $z \notin X$, and obtaining $V_i^\sigma(S, z)$. In (a), $V_i^\sigma(S, w) \leq V_i^\sigma(S, x)$ by Claim 1 and $w \notin X$ and definition of $V_i^\sigma(\cdot, \cdot)$ prescribed in Claim 2; in (b), $V_i^\sigma(S, z) \leq V_i^\sigma(S, x)$ by the same argument as in (a) and $z \notin X$. This demonstrates that no proposer has a profitable one-shot deviation from σ.

Combining Claims 1-4, the theorem is proved. To recap, Claims 1 and 2 determine the continuation value functions induced by σ and show that σ is a no-delay strategy profile with absorbing set X. Claim 3 demonstrates that, in every voting stage, no player uses a weakly dominated strategy. Finally, Claim 4 proves that there is no profitable one-shot deviation from σ in the proposal stage of any period. By the one-shot deviation principle, Claims 3 and 4 ensures that σ is a stage-undominated SPNE, thus completing the proof of the theorem.

It's worth noting that Theorem 1 still applies in a more general setting of $u_i(\cdot, \cdot)$, one just needs to adjust $\bar{\delta}_i$ accordingly. However, including this generalization would make the results less intuitive, so I have opted to omit it.