

Policy Making in a Changing World: Uncertainty Dynamics and Political Institutions

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Introduction

- **Legislative bargaining** is a prominent example of negotiations in a dynamic, ever-changing environment. Policies consist of pie divisions.
- **Two key ingredients**
 - **Endogenous status quo** In each period, a new policy is negotiated. If no agreement is reached, the policy that gets implemented is determined by the outcome of the previous period's bargaining, making it "endogenous."
 - **Stochastic uncertainty** The resources or pie being divided in each period are uncertain, adding an element of unpredictability to the negotiations.
- **Examples** This framework is evident in areas such as fiscal policies, pensions, and wage negotiations.

Literature Review

- **Fundamental legislative bargaining** Baron and Ferejohn 1989; Baron 1996; J. S. Banks and Duggan 2000; J. S. Banks, Duggan, et al. 2006; Eraslan and Evdokimov 2019.
- **Endogenous status quo** Baron 1996; Kalandrakis 2004; Bowen, Chen, and Eraslan 2014; Anesi and Seidmann 2015; Bowen, Chen, Eraslan, and Zapal 2017; Anesi and Duggan 2018; Eraslan, Evdokimov, and Zapal 2022.
- **Stochastic uncertainty** Merlo and Wilson 1995; Merlo and Wilson 1998; Eraslan and Merlo 2002; Eraslan and Merlo 2017.
- **Recent contributions** Bils and Izzo 2023; Agranov, Eraslan, and Tergiman 2024.

The Game

Setup

- A finite set of players $N \equiv \{1, 2, \dots, n\}$, $n \geq 3$.
- Time periods $t = 0, 1, \dots, \infty$.
- Policy in each time period $x^t \equiv (x_1^t, x_2^t, \dots, x_n^t) \in \mathbf{X}$, consist of pie division.
- Policy space $\mathbf{X} = \{(x_1, x_2, \dots, x_n) \in [0, 1]^n : \sum_{i \in N} x_i \leq 1\}$.
- State (the size of the pie) S^t , with realization s^t at the beginning of t .
- A known distribution F , $\{S^t\} \stackrel{i.i.d.}{\sim} F$, with a standardized mean $E[S^t] = 1$, an upper bound $\bar{s} < \infty$, and strictly positive support $\text{supp}(F) \subseteq \mathbb{R}_{++}$.
- See: Voting rule \mathcal{D} and stage utility $u_i(s^t, x_i^t)$.

The Game

- n players bargaining over the *share* of a pie in each time period. Bargaining takes place as follows.
- See: *Equilibrium concept*.
- See: *Simple solutions*.

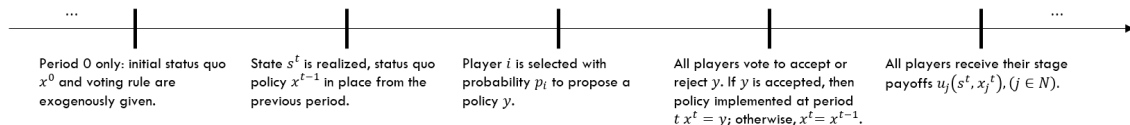


Figure: Timing in each period $t = 0, 1, 2, \dots$

An example

Setup

- 3 Players $N = \{1, 2, 3\}$.
- States $F \stackrel{\text{def}}{=} \text{Unif}[1 - \rho, 1 + \rho]$, where $\rho \in [0, 1)$ is the measurement of risk.
 $E[S^t] = 1$, $\text{Var}(S^t) = \frac{\rho^2}{3}$, $\{S^t\} \stackrel{i.i.d.}{\sim} F$.
- Utility $u_i(s, x_i) = s \cdot x_i$, $\delta_i \equiv \delta$, $i \in N$.
- Probability of being selected as a proposer $p_i \equiv \frac{1}{3}$.
- Simple majority rule $\mathcal{D} \equiv \{C : |C| \geq 2\}$.
- Coalition $C_1 = \{1, 2\}$, $C_2 = \{2, 3\}$, $C_3 = \{1, 3\}$.
- Two offers: "high" offer $x_j \equiv h$, "low" offer $y_j \equiv l$.

An example

Strategy profile

- If $\delta > 1 - \frac{h-l}{(h-l)+3(1+\rho)(1-h)} =: \bar{\delta}$, then the following strategy profile forms a pure strategy, no-delay SMPE whose set of absorbing points is $X = \{(h, h, l), (l, h, h), (h, l, h)\}$:
 - Player i always offers h to the players in C_i and l to the player outside C_i for any realizations of $S \in \mathbf{S}$, if the status quo does not belong to X , and proposes the existing status quo otherwise (I call this scenario as *passes*);
 - Player i accepts proposal z when the status quo is w if and only if one of the following conditions holds:
 - 1 $w \in X$ and $w = l$
 - 2 $w \notin X$, $z \in X$, and $((1 - \delta)S + \delta)z_i \geq (1 - \delta)Sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l)$
 - 3 $w, z \notin X$, and $w_i \leq z_i$

An example

Proof

- Current state S with realization s , and the next period state S' .
- For any ongoing status quo $w \in \mathbf{X}$ is either
 - ① an absorbing point in itself, or
 - ② will immediately lead to some absorbing point $x^{C_j} \in X$, where C_j is the winning coalition.

An example

Proof

- 1 player i 's expected payoff is

$$(1 - \delta)s \cdot x_i^{C_j} + \delta E[S' x_i^{C_j}] = \begin{cases} (1 - \delta)sh + \delta h & \text{if } i \in C_j \\ (1 - \delta)sl + \delta l & \text{if } i \notin C_j \end{cases} \quad (1)$$

- 2 e.g., if the current period's proposer fails to amend w , player i 's expected payoff is

$$\overbrace{(1 - \delta)s \cdot w_i}^{\text{today}} + \overbrace{\delta E[S'(\frac{2}{3}h + \frac{1}{3}l)]}^{\text{discounted tomorrow}} = (1 - \delta)sw_i + \delta(\frac{2}{3}h + \frac{1}{3}l)$$

An example

Proof

- For $\delta > \bar{\delta}$, $\overbrace{(1 - \delta)s(w_i - h)}^{\text{benefit for deviating}} < \overbrace{\delta(h - (\frac{2}{3}h + \frac{1}{3}l))}^{\text{cost}}$ holds for all $w_i \in [0, 1]$ (See Appendix B).
- x^{C_j} would be voted up by 2 players in C_j , and voted down for 1 player not in C_j .
- Since C_j are winning coalitions, it is impossible to amend the policies X^{C_j} once they have been implemented.

An example

Proof

- **Voting Strategies** The voting strategies ensure that there is no profitable deviation from the proposed strategies.
- **Proposal strategies**
 - If the current status quo is outside X , the optimal strategy for proposer i is to offer x^{C_i} and would be implemented by the simple majority rule.
 - If the current status quo is inside X , passing would be the optimal choice since it either guarantees the highest possible expected long-term payoff if in the winning coalition, or any other unsuccessful proposal, making the proposer indifferent.
- Thus, it is indeed a SMPE.

An example

Waste

- Farsighted player 1 (as a representative) would be blocked from accepting, for instance, $(\frac{1}{2}, \frac{1}{2}, 0) \in \mathbf{X}$ when current status quo is $x^{C_1} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{6})$.
- This is because there is a possibility $p_2 > 0$, that player 2 could be the proposer in the next period and propose $x^{C_2} = (\frac{1}{6}, \frac{1}{3}, \frac{1}{3})$ to form another winning coalition that excludes player 1.

An example

Uncertainty

- $\bar{\delta} := 1 - \frac{h-l}{(h-l)+3(1+\rho)(1-h)}$ increases as ρ increases.
- In order to ensure policy stability for sustainable development in a more volatile environment, players must exhibit greater patience.
- **Sustainability-efficiency trade-off** High δ (close to 1) can lead to stable policies, which benefit e.g., steady growth, low inflation, and low levels of unemployment. However, it may also result in the persistence of inefficient resource allocation.

Results

- **Observation 1.** Given voting rule \mathcal{D} , if $\delta_i = 0$ for each $i \in N$, then it is a "rotating dictator" equilibrium, i.e., each period's proposer receives the entire pie at the revealing state in every SPNE.
 - Hint: voting rule is non-unanimity.
- **Observation 2.** There exists $\underline{\delta} \in (0, 1)$ such that there is no absorbing SPNE whenever $\max_{i \in N} \delta_i < \underline{\delta}$.
 - Hint: here the core is empty (See Appendix A).

Results

- Theorem 1.** Let X be a simple solution. There exists a threshold $\bar{\delta} = \max_{i \in N} \bar{\delta}_i \in (0, 1)$ such that whenever $\min_{i \in N} \delta_i > \bar{\delta}$, there exists a no-delay stationary bargaining equilibrium for the stochastic dynamic bargaining game described in Section 2.1, where the set of absorbing points is X .
- To be specific, $\bar{\delta} := \max_{i \in N} \bar{\delta}_i$, where

$$\bar{\delta}_i \equiv \max \left\{ \frac{\tilde{u}_i(1) - \tilde{u}_i(x_i)}{\tilde{u}_i(1) - \tilde{u}_i(x_i) + \frac{p^{min}}{\delta} (\tilde{u}_i(x_i) - \tilde{u}_i(y_i))}, \frac{\tilde{u}_i(y_i) - \tilde{u}_i(0)}{\tilde{u}_i(y_i) - \tilde{u}_i(0) + \frac{p^{min}}{\delta} (\tilde{u}_i(x_i) - \tilde{u}_i(y_i))} \right\}.$$

Results

In words

- **No absorbing SPNE** whenever players are myopic.
- **Equilibrium existence and multiplicity of equilibria** whenever players are patient.
- **The size principle** The size principle predicts that only minimal winning coalitions may receive a positive share of the pie may fail.
- **Effect of stochastic uncertainty** As the volatility in the size of the pie increases, players need to exhibit a higher level of patience to sustain the equilibrium.
- **Waste** Players can be locked into equilibria where any deviation to proposing a Pareto superior policy would be rejected.

See: Corollary 1.



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Discussions

- **Policy persistence** can be maintained regardless of the state's realization or future uncertainty.
- **Pork barrel politics** The failure of the size principle and the waste aligns with features commonly observed in pork barrel politics.
- **Patience** One explanation for the conventional wisdom in political economy, and the phenomenon where crises sometimes enable radical reforms.
- **Future studies** Explore pie-division with no disposal setting to address inefficiency; endogenous stochastic states with implemented policies; further generalize the utility function etc.

Takeaways

- **Policy stability** can be achieved regardless of future uncertainty, provided that legislators are sufficiently patient.
- **Uncertainty** Greater patience among legislators is required as uncertainty increases.
- **Waste** Policy persistence may result in ongoing inefficiencies.

Bottomline

Thanks for your attention.

Q&A

The Game

Voting rules

- Voting rule \mathcal{D} , chosen exogenously.
- \mathcal{D} is *proper*, i.e., every pair of decisive coalitions has nonempty intersection: $C, C' \in \mathcal{D}$ implies $C \cap C' \neq \emptyset$.
- \mathcal{D} is *monotonic*, i.e., any superset of a decisive coalition is itself decisive: $C \in \mathcal{D}$ and $C' \subseteq C$ imply $C' \in \mathcal{D}$.
- \mathcal{D} is *noncollegial*, i.e., no player has a veto: $N \setminus \{i\} \subseteq \mathcal{D}$ for all $i \in N$.
- Allowing for any quota rule defined by $\mathcal{D} \equiv \{C : |C| \geq q\}$, with the only restriction on the quota q being $\frac{n}{2} < q < n$.

Back: Setup.



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The Game

Utility

- Discount factor of player i : $\delta_i \in [0, 1)$.
- Stage utility $u_i(s^t, x_i^t)$ of player i from policy $x_i^t \in x^t$ in period t when the state is s^t .
- $u_i(s^t, x_i^t) : \mathbf{S} \times [0, 1] \rightarrow \mathbb{R}$ is strictly increasing in both s^t and x_i^t , twice continuously differentiable with respect to both s^t and x_i^t , concave in x_i^t , and $\frac{\partial^2 u_i}{\partial s^t \partial x_i^t} > 0$.
- Inputs are essential, i.e., $u_i(s^t, 0) = u_i(0, x_i^t) = 0$.
- Given $\{x^t\} \in \mathbf{X}^\infty$ and realized states $\{s^t\} \in \mathbf{S}^\infty$, player i 's discounted utility is $(1 - \delta_i) \sum_{t=0}^{\infty} \delta_i^t \cdot u_i(s^t, x_i^t)$.

Back: Setup.



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- **Stationary Markov perfect equilibrium** (SMPE), a subgame perfect Nash equilibrium (SPNE) in which all players use stationary Markov strategies.
- Stationary Markov strategy $\sigma_i = (\pi_i, \alpha_i)$, $i \in N$ consists of
 - a proposal strategy $\pi_i : \mathbf{S} \times \mathbf{X} \rightarrow \mathbf{X}$, where $\pi_i(S, x)$ is the proposal made by player i when the current status quo and state is x and S respectively (conditional on her being selected to propose);
 - a voting strategy $\alpha_j : \mathbf{S} \times \mathbf{X}^2 \rightarrow \{0, 1\}$, where $\alpha_j(S, x, y)$ is the (degenerate) probability that player $j \in N$ votes to accept a proposal y when the current status quo is x at state S .
- Strategy profile $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$.
- Focusing on SMPEs in stage-undominated voting strategies.

Back: The Game.

Absorbing Points and No-delay Strategies

- Given σ , defines a transition function $P^\sigma : \mathbf{S} \times \mathbf{X}^2 \rightarrow [0, 1]$, $P^\sigma(S, x, y)$ is the probability the alternative implemented in the next period is y , when current status quo is x , at state S .
- $x \in \mathbf{X}$ is an *absorbing point* of σ if and only if $P^\sigma(S, x, x) = 1$ for all states $S \in \mathbf{S}$.
- Set of absorbing points $A(\sigma) \equiv \{x \in \mathbf{X} : P^\sigma(S, x, x) = 1\}$.
- σ is *no-delay* if and only if $A(\sigma)$ is non-empty and for all $x \in \mathbf{X}$ and $S \in \mathbf{S}$ there is $y \in A(\sigma)$ such that $P^\sigma(S, x, y) = 1$.

Back: The Game.

- A class of pure strategy no-delay SMPEs, in which each player $j \in N$ is offered only two different shares of the pie: a "high" offer $x_j > 0$, and a "low" offer $y_j < x_j$, after any history, and each player receives a low offer from at least one proposer.
- **Definition 1.** Let $\mathcal{C} \equiv \{C_i\}_{i \in N} \subseteq \mathcal{D}$ be a class of coalitions such that, for each $i \in N$, $i \in C_i$ and $i \notin C_j$ for some $j \in N \setminus \{i\}$. Let $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ be two vectors in $[0, 1]^n$, for all $i \in N$:

$$\begin{aligned} \sum_{j \in C_i} x_j + \sum_{j \notin C_i} y_j &\leq 1 \\ x_i &> y_i \end{aligned} \tag{3}$$

- The simple solution induced by (\mathcal{C}, x, y) is the set of policies $X \equiv \{x^{C_i}\}_{i \in N}$, where

$$x^{C_i} \equiv \begin{cases} x_j & \text{if } j \in C_i \\ y_j & \text{if } j \notin C_i \end{cases} \text{ for all } i, j \in N \quad (4)$$

- A set of policies $X \subseteq \mathbf{X}$ is a simple solution if there exists a triplet (\mathcal{C}, x, y) such that X is a simple solution induced by (\mathcal{C}, x, y) .

Back: The Game.

- **Corollary 1** For voting rules where \mathcal{D} is proper, monotonic, and noncollegial, and for a pie with size drawn independently and identically from a distribution function with an upper bound $\bar{s} < \infty$. The following statements hold: there exists a threshold $\bar{\delta} \in (0, 1)$, such that each statement is true whenever $\min_{i \in N} \delta_i > \bar{\delta}$.
- ① There exist multiple no-delay stationary bargaining equilibria, for any realization of the pie size;
- ② Any policy that allocates a positive share to some players within a winning coalition is an absorbing point in some no-delay stationary bargaining equilibrium;
- ③ A proposal strategy that allocates the entire pie to a single player is not in a no-delay stationary bargaining equilibrium;

- ④ There are no-delay stationary bargaining equilibria which fail the size principle;
- ⑤ For any $\varepsilon \in (0, 1)$, there is no-delay stationary bargaining equilibrium σ such that $P^\sigma(\mathcal{S}, x, x) = 1$ for all $x \in \mathbf{X}_\varepsilon$;
- ⑥ There exist no-delay stationary bargaining equilibria in which the agreement wastes a portion of the pie and violates the size principle.

Back: Results.