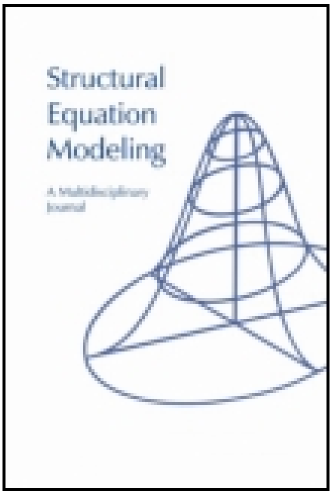

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Recent Changes Leading to Subsequent Changes: Extensions of Multivariate Latent Difference Score Models

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Latent difference score models (e.g., McArdle & Hamagami, 2001) are extended to include effects from prior changes to subsequent changes. This extension of latent difference scores allows for testing hypotheses where recent changes, as opposed to recent levels, are a primary predictor of subsequent changes. These models are applied to bivariate longitudinal data collected as part of the Baltimore Longitudinal Study of Aging on memory performance, measured by the California Verbal Learning Test, and lateral ventricle size, measured by structural MRIs. Results indicate that recent increases in the lateral ventricle size were a leading indicator of subsequent declines in memory performance from age 60 to 90.

Keywords: latent difference score, dynamic, longitudinal, growth, memory, brain

Understanding the dynamic interplay between two (or more) constantly changing constructs is a central goal of developmental science. Take, for example and presented in Figures 1a and 1b, changes in memory performance, as measured by the California Verbal Learning Test (CVLT), and changes in the size of the brain, as indexed inversely by the relative size of the lateral ventricle, measured from participants in the Baltimore Longitudinal Study of Aging (BLSA) between ages 60 and 90. Overall, memory performance appears to decline over this

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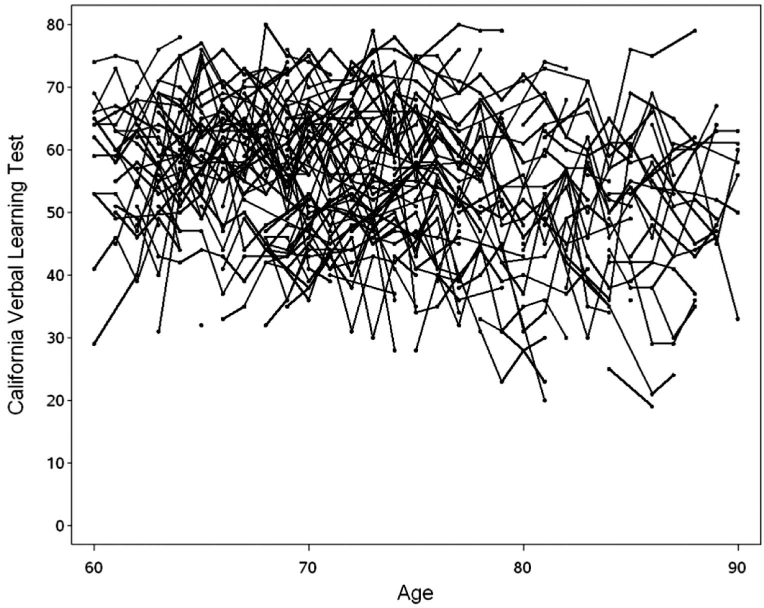
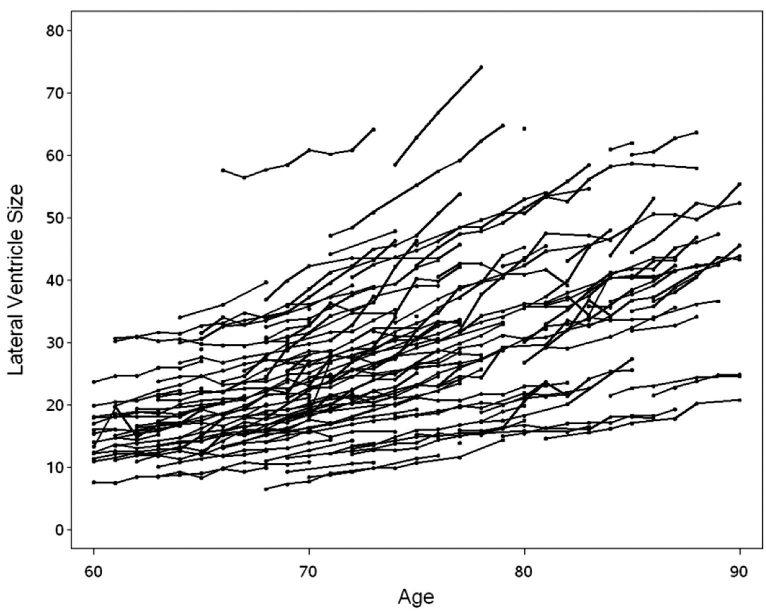


(a)



(b)

FIGURE 1 Longitudinal plot of (a) California Verbal Learning Test scores (sum of 5 immediate recall trials) and (b) lateral ventricle size against age of measurement for Baltimore Longitudinal Study of Aging participants.

age period, whereas the size of the lateral ventricle increases. It is expected that memory and the size of certain brain regions are related cross-sectionally and that changes in memory and changes in the brain are somehow linked longitudinally. Although not specific to Alzheimer's disease, increases in lateral ventricle size (LVS) reflect decreases in brain tissue volume and are associated with cognitive impairment and dementia (Jack et al., 2005; Jack et al., 2008). Investigators often assume that changes in the brain precede memory change but tests of this directionality remain limited (e.g., Jack et al., 2010).

The link between changes in memory and lateral ventricle volume could be static or dynamic. A static relationship between changes would suggest that changes in memory over time are simply associated with changes in brain size, such that people who show greater declines in memory performance tend to show greater increases in the size of the lateral ventricle over time. This type of relationship is often indexed by the correlation between slopes in a bivariate growth model (sometimes referred to as a parallel process model; McArdle, 1988). Key features of this type of relationship are that directionality is not determined and the association is between persons. A dynamic relationship between changes involves time such that changes in the first construct temporally precede changes in the second. This type of relationship is often described as a lead–lag relationship where changes in the first construct lead and changes in the second construct lag behind those of the first. In our example data, there are two possible dynamic change relationships. The first is if changes in ventricular volume are subsequently followed by changes in memory; the second would involve changes in memory subsequently followed by changes in ventricular volume. Key features of this type of relationship are that directionality can be studied and the association is within person (although sometimes tested with a combination of within- and between-person associations).

Models able to examine dynamic change relationships of different forms include the classic autoregressive cross-lag (ACL) model (Jöreskog, 1970, 1974), latent difference score (LDS) models (McArdle, 2001, 2009), autoregressive latent trajectory (ALT) models (Bollen & Curran, 2004), and latent differential models (Boker, Neale, & Rausch, 2004; see Ferrer & McArdle, 2003; Grimm, 2007 for comparisons of specific models). Each model specifies the dynamic relationship in different ways, which leads to different expected longitudinal trajectories. In this article we propose extensions of LDS models to examine different types of dynamic relationships. We concentrate on LDS models because of their flexibility for modeling time-sequential changes in multiple variables over time, common use, ability to model both mean changes and time-sequential dynamic relationships, and because such models can only model time in discrete form and this is the form that time usually takes in change models fit in the structural equation modeling framework. We begin by reviewing univariate and bivariate LDS models and introduce extensions of these models allowing for a different type of time-related dynamics, specifically related to prior changes leading to subsequent changes. Finally, we illustrate the application of traditional LDS models as well as the proposed extensions to examine lead–lag associations between changes in the size of the lateral ventricle and changes in memory in the BLSA.

Extensions of LDS models are proposed because common specifications are limited in terms of expected trajectories, determinants of change, and theories that can be tested. Specifically, common specifications are limited to exponential change trajectories and change determined by prior levels of constructs. Thus, such specifications are limited to testing theories that conform to these circumstances. We propose extensions based on the idea that prior changes can lead (predict) to subsequent changes and discuss how such models might be a step forward in linear dynamic modeling with latent difference scores.

LATENT DIFFERENCE SCORE MODELS

LDS modeling (Hamagami & McArdle, 2001; McArdle, 2001; McArdle & Hamagami, 2001; McArdle & Nesselroade, 1994) is a framework for studying longitudinal change that combines features of ACL (e.g., Jöreskog, 1970, 1974) and latent curve models (e.g., McArdle & Epstein, 1987; Meredith & Tisak, 1990) for panel data. We refer to LDS modeling as a *framework* because models typically applied to longitudinal panel data (e.g., repeated measures data) can be respecified to involve latent differences. Such models include the ACL, latent curve models, and the time-varying covariate model—these models can be subsumed within the LDS framework. However, common specifications of LDS models do not show the breadth of models that can be fit within this framework for studying and understanding time-related change.

Univariate Models

In this section we describe commonly specified univariate LDS models. Starting with ideas from classical test theory, observed scores of the same variable measured at time t $(Y[t]_n)$ for subject n are composed of theoretical true scores $(y[t]_n)$ and unique scores at time t $(u[t]_n)$, which can be written as

$$Y[t]_n = y[t]_n + u[t]_n. \tag{1}$$

True scores covary over time, whereas the unique scores do not. Often the variance of the unique score is held constant across time ($\text{var}(u[t]_n) = \sigma_u^2$), but this constraint is not necessary in many applications (Grimm & Widaman, 2010). In LDS modeling, latent difference scores are created not by taking the difference between observed scores, but by fixed structural relationships between consecutive true scores. To do this, true scores follow a fixed unit autoregressive process such that the true score at time t $(y[t]_n)$ is a function of the true score at time $t-1$ $(y[t-1]_n)$ plus change in the true score from time $t-1$ to t $(\Delta y[t]_n)$. This can be written as

$$y[t]_n = y[t-1]_n + \Delta y[t]_n. \tag{2}$$

Rearranging terms, we could write the change in the true score from time $t-1$ to t is the simple difference between true scores at time t and $t-1$ or

$$\Delta y[t]_n = y[t]_n - y[t-1]_n. \tag{3}$$

The trajectory equation in LDS models is different from typical latent curve models because the focus is placed on the latent difference scores, as opposed to latent true scores. The trajectory equation for LDS models begins with an initial true level of the attribute from which a series of true changes occur. These changes are summed over time and can be specified as

$$y[t]_n = y[0]_n + \sum_{r=1}^{r=t}(\Delta y[r]_n), \tag{4}$$

where $y[0]_n$ is the initial true score for subject n, which can be described by its mean (μ_{y_0}) and variance ($\sigma^2_{y_0}$). Following Equation 4, the true score of variable Y at time t is equal to subject n's initial true state ($y[0]_n$) plus an accumulation or sum of subject n's changes that have occurred up to that point in time, $r = t$.

Next, an equation for the latent difference scores is specified to describe how its determinants combine to predict time-sequential change. These change equations control the structure of change over time (e.g., linear, exponential, etc.). In the univariate case, there are three commonly specified models for the latent difference scores. These include the (a) constant change model where $\Delta y[t]_n = \alpha \cdot s_n$, where α is a fixed parameter (often equal to 1) and s_n is the constant change component for subject n, which can be described by a mean (μ_s) and variance (σ^2_s); (b) the proportional change model where $\Delta y[t]_n = \beta \cdot y[t-1]_n$, where β is an estimated parameter and not allowed to vary over subjects, thus making time-sequential change proportional to its previous true state; and (c) the dual change model where $\Delta y[t]_n = \alpha \cdot s_n + \beta \cdot y[t-1]_n$, where subsequent changes have a constant component and are also based on its previous true state. In the proportional and dual change models, β is often invariant with respect to time; however, this constraint is not often needed for identification and is testable (e.g., Grimm, 2006). One reason β is often specified to be time invariant is the idea of constant dynamics—regardless of when observation occurs, the dynamics of the system are constant.

Figure 2 is a path diagram of the dual change score model with five observed repeated measurements coded $Y[0]$ to $Y[4]$. Observed scores are composed of latent true scores ($y[0]$ to $y[4]$) and unique scores ($u[0]$ to $u[4]$) following Equation 1. Unique scores have zero means, a time-invariant variance (σ^2_u), and are not allowed to covary over time. True scores after the initial true score have inputs from the prior true score and the latent difference score with fixed regression weights equal to 1, which makes each true score the sum of the prior true score and latent difference score as in Equation 2. Focusing on the latent difference scores ($\Delta y[1]$ to $\Delta y[4]$), each latent difference score has two inputs—the prior true score with weight β and the constant change component (s) with weight α following the specification of the change equation for the dual change score model. The constant change component and the initial true score ($y[0]$) have means (μ_{y0} & μ_s), standard deviations (σ_{y0} & σ_s), and a correlation ($\rho_{y0,s}$). The latent variables $y[0]^*$ and s^* are standardized forms of the initial true score and constant change component, which allow for the estimation of the correlation, as opposed to the covariance, between the two.

The univariate LDS models discussed are functionally equivalent to more commonly specified latent curve models. Specifically, the constant change model, where there is a constant amount of change for a one-unit change in time, is functionally equivalent to a linear growth model. This can be seen by examining the trajectory equations. The trajectory equation for the linear model can be specified as $y[t]_n = y[0]_n + s_n \cdot t$, where $y[0]_n$ is the true score at $t = 0$ (i.e., intercept) and s_n is a linear slope for individual n. The constant change model can be written as $y[t]_n = y[0]_n + \sum_{r=1}^{r=t} s_n$ and adding s_n at each measurement occasion after the initial measurement occasion up to time t is equal to $s_n \cdot t$ (see also McArdle & Grimm, 2010). A second way to see the equivalence is by taking the first derivative of the linear model with respect to time. The first derivative, which represents the rate of change, is $y'[t]_n = s_n$, which is the continuous form of the latent change equation for the constant change model (assuming $\alpha = 1$). The proportional change model is equivalent to an exponential growth or decay model with a positive true score at time $t = 0$. Figure 3a is a plot of two growth trajectories based

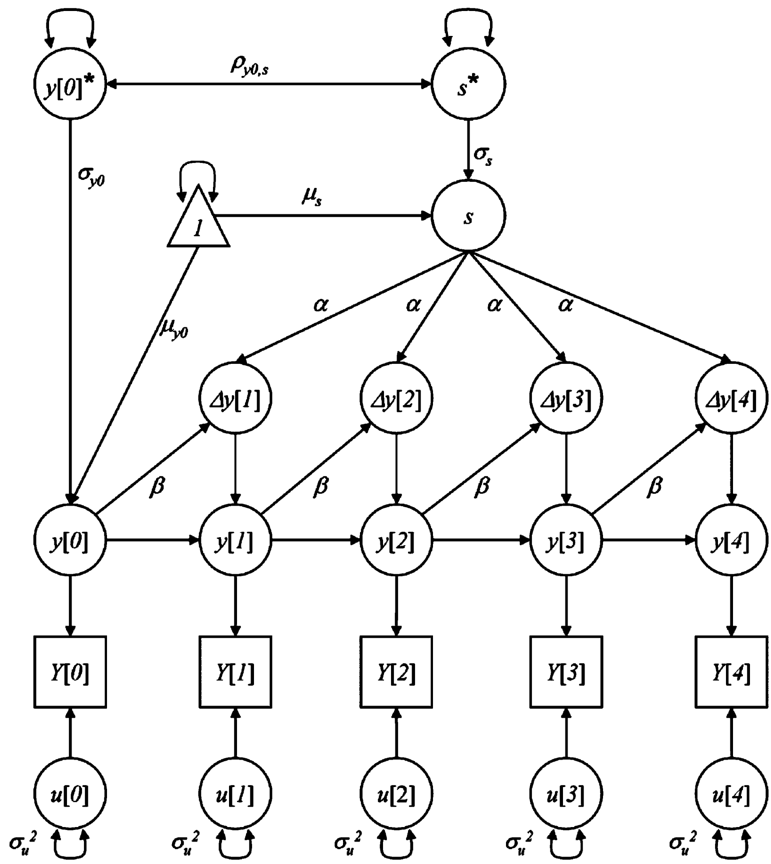
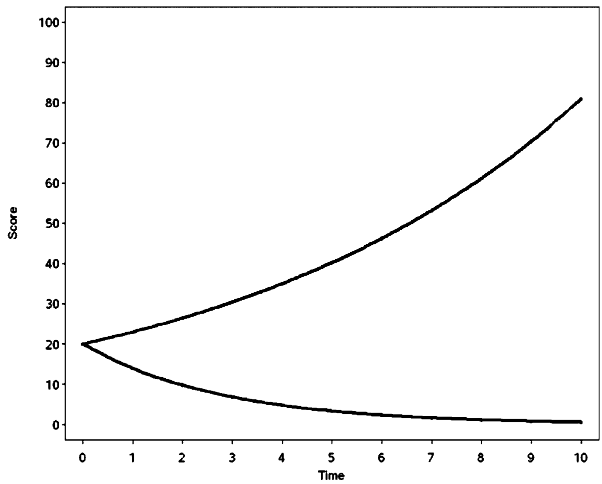


FIGURE 2 Path diagram of the dual change score model with five measurement occasions. Note: Unlabeled paths are fixed equal to 1.

on the proportional change model—the first represents exponential growth ($\beta = .15$) and the second describes exponential decay ($\beta = -.30$). The proportional change parameter, β, controls whether the function increases or decreases with time. The dual change model is equivalent to an exponential function. The sign and magnitude of the constant change and proportional change parameter in conjunction with the sign and magnitude of the true score at $t = 0$ determine whether the function is (a) positively accelerating (exponential growth), (b) positively decelerating, (c) negatively accelerating, and (d) negative decelerating (exponential decay). Trajectories for these four types of exponential functions are displayed in Figures 3b through 3e, respectively. In these plots $\beta = .2$ and $\mu_s = 20$, $\beta = -.2$ and $\mu_s = 20$, $\beta = -.2$ and $\mu_s = -20$, and $\beta = .2$ and $\mu_s = -20$, respectively. If the change process does not follow one of these expected trajectories, then LDS models, as often implemented, will not adequately capture the change process or the dynamics (i.e., will not fit the data). Thus, different change equations are necessary—one reason why extensions, such as those proposed here, are needed.



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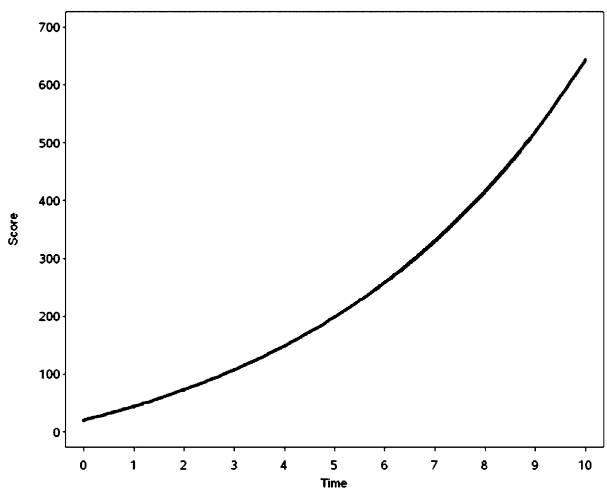
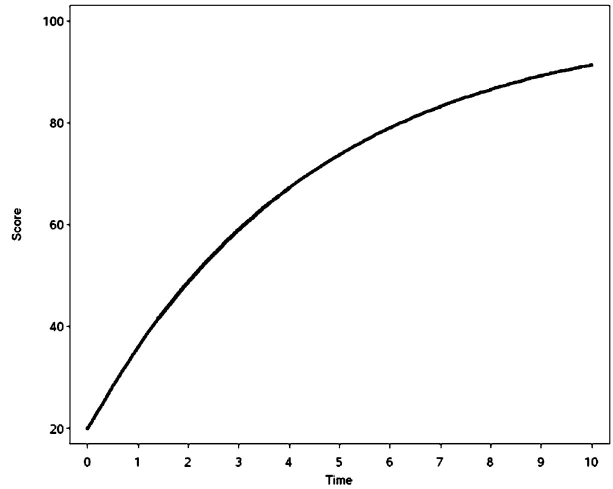


(b)

FIGURE 3 Expected trajectories based on the (a) proportional change model and (b–e) dual change model.
(*continued*)

Bivariate Models

LDS models were specifically developed to study multivariate change processes and time dependencies between two or more processes while accounting for individual change. In the bivariate case, the change process for the second variable (X) is set up in a similar manner with



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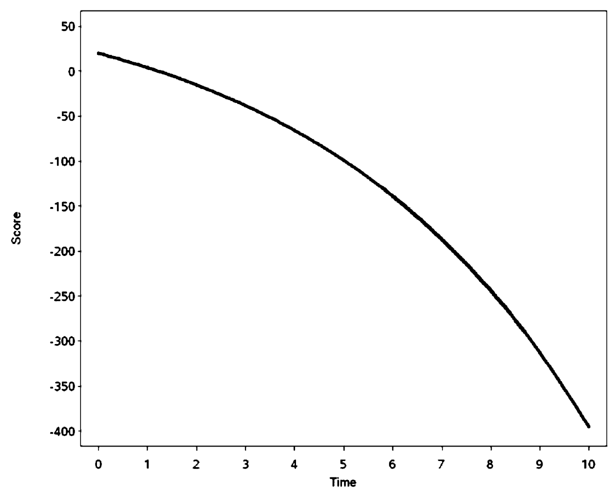
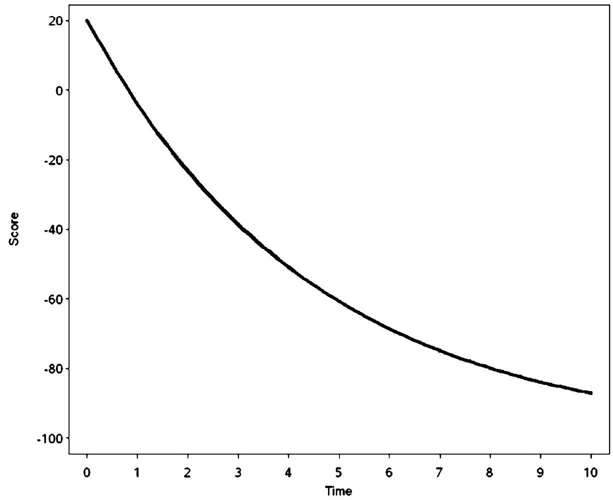


(d)

FIGURE 3 (*Continued*).

the distinction between true and unique scores (Equation 1) and the trajectory as a function of an initial state plus the accumulated changes (Equation 4). Similarly, the latent difference scores are specified to have similar determinants (i.e., constant change component and proportional change parameter). Bivariate LDS models are useful for examining whether changes in one process are also determined by the previous state of the second process, and vice versa. Commonly



(e)

FIGURE 3 (*Continued*).

implemented equations for latent difference scores for two processes (Y & X) are written as

$$\begin{aligned}\Delta y[t]_n &= \alpha_y \cdot s_{yn} + \beta_y \cdot y[t-1]_n + \gamma_{yx} \cdot x[t-1]_n \\ \Delta x[t]_n &= \alpha_x \cdot s_{xn} + \beta_x \cdot x[t-1]_n + \gamma_{xy} \cdot y[t-1]_n,\end{aligned} \tag{5}$$

where α_y and α_x, s_{yn} and s_{xn}, and β_y and β_x are as previously defined, but are now subscripted by y or x to denote the process, γ_{yx} is the regression coefficient for the effect of x on subsequent changes in y, and γ_{xy} is the regression coefficient for the effect of y on subsequent changes in x. These coefficients are termed *coupling* parameters and are useful for examining a specific type of lead–lag dynamic relationships in multivariate repeated measures-namely how levels predict subsequent changes. Figure 4 is a path diagram of the bivariate latent difference score model with change equations from Equation 5 for five repeated measures (coded 0–4). In Figure 4, the latent difference scores for Y and X have three predictors following Equation 5. Additionally, initial true scores and constant change components are allowed to covary across processes. Finally, unique scores for Y and X at each measurement occasion are allowed to covary within time and this covariance is often specified to be invariant with respect to time.

The expected trajectories that can be modeled based on the bivariate change equations of Equation 5 are more varied than in the univariate case due to the additional effect (coupling) from the second process. If coupling parameters equal 0, then the expected trajectories are limited to those described in the univariate models. However, if coupling parameters are nonzero, then the exponential trends have deflections from their usual trajectory. Suppose, for example, the parameters of the univariate systems predict positively decelerating trends for both change processes (e.g., $\beta = -.4$ & $\mu_s = 20$). A positive coupling parameter would lead

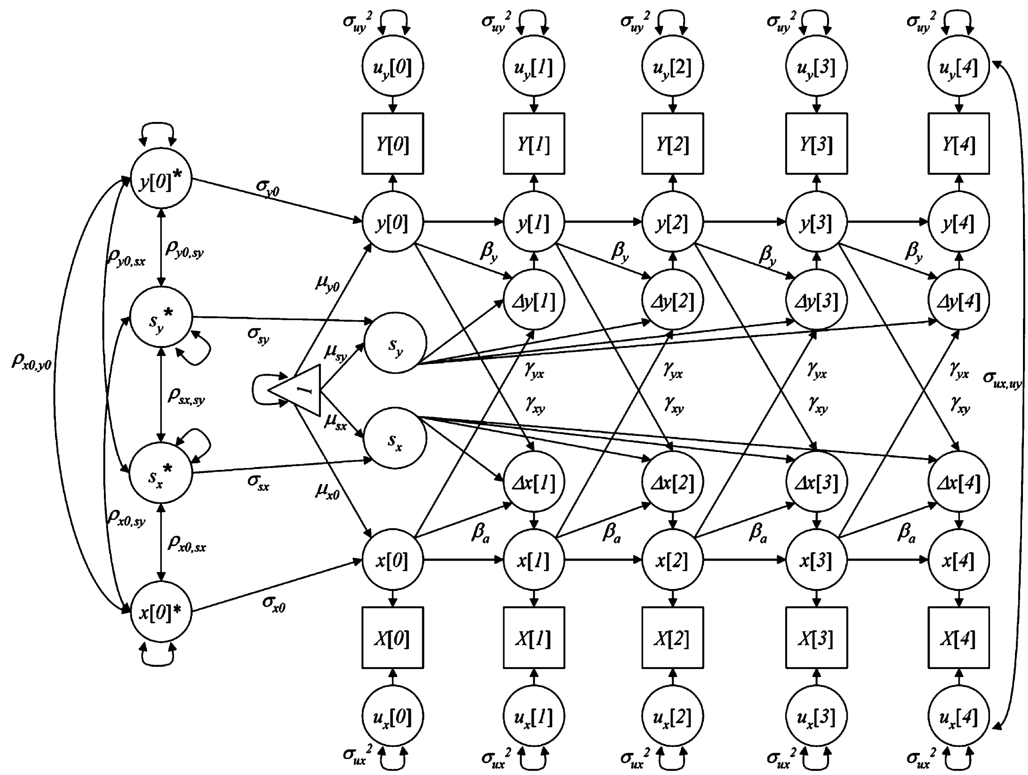


FIGURE 4 Path diagram of a bivariate latent difference score model with five measurement occasions. Note: Unlabeled paths are fixed equal to 1.

to a similar exponential trend, but with a positive deflection and further increasing, whereas a negative coupling parameter would lead to a negative deflection and decline as time increases. Figure 5 shows these types of deflections from the simple exponential trend. In Figure 5 the coupling parameter (γ) from the second process is .1, 0, or −.15.

EXTENSIONS

Univariate and multivariate LDS models can and have been extended in a variety of ways to represent different expectations regarding the change processes and their dynamic relationships. The change equations of Equation 5 can be modified to include additional change components and test their necessity. For example, Hamagami and McArdle (2007) expanded traditional specifications of univariate and bivariate LDS models in two different ways. In the first extension, termed the parallel process change score model, two latent change scores representing rise and fall of the process, load onto the latent changes. This extension enabled the fitting of trajectories

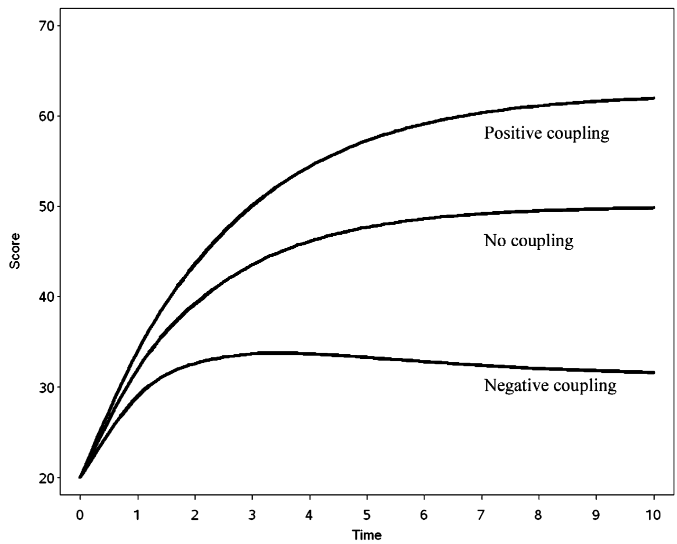


FIGURE 5 Example expected trajectories from a bivariate dual change model with positive, zero, and negative coupling.

that followed the rise and fall of the dual exponential growth curve (see McArdle, Ferrer-Caja, Hamagami, & Woodcock, 2002). The second extension, termed the latent acceleration model, included second-order latent difference scores representing differences in the first-order latent difference scores. Thus, in this formulation, first-order difference scores represent velocity of the trajectory, whereas the second-order differences represent acceleration of the trajectory. The second-order latent differences, or latent acceleration scores, can be determined by the prior status or level of the attribute or the previous first-order latent difference score. These determinants, combined with the constant change component, can account for a variety of nonlinear trends, including trajectories that can follow waveforms that rise and fall multiple times (i.e., oscillations). In a similar vein, Grimm (2006) examined the influences of a linear change component and linearly changing dynamic (i.e., proportional change and coupling) parameters in the trajectories of LDS models. In this work, the addition of a linear change component (in addition to the constant change component) led to quadratic trends modified by the proportional change parameter. Linearly changing dynamics were based on the idea, common in developmental theories (e.g., Cattell's investment hypothesis; Cattell, 1971), that the influence one variable has on the subsequent changes of the other variable can wax or wane over time.

Changes to Changes Extension

In this project we propose a new extension that enables researchers to examine how prior changes relate to subsequent changes. Recalling our initial example of changes in memory and in the size of the lateral ventricle, a model including prior changes as a predictor of subsequent changes would additionally be able to examine whether recent changes in the size of the lateral ventricle precede changes in memory (and vice versa). Thus, in the univariate case, change from time $t-2$ to $t-1$ is included as a predictor of change from time $t-1$ to t. Based on this notion, the univariate change equation for Y can be written as

$$\Delta y[t]_n = \alpha \cdot s_n + \beta \cdot y[t-1]_n + \phi \cdot \Delta y[t-1]_n, \tag{6}$$

where α, s_n, and β are as previously defined, $\Delta y[t-1]_n$ is the latent difference score from time $t-2$ to $t-1$, and ϕ is the regression coefficient describing the effect of prior changes on subsequent changes. The addition of the ϕ parameter can have a substantial effect on the expected trajectories. For example, Figure 6 is a plot of two trajectories following Equation 6 with nonzero ϕ parameters ($\mu_s = 20$, $\beta = -.4$, $\phi = -.6$ or .6). As seen in Figure 6, one curve appears to follow a step function, where the second curve initially rises and subsequently falls.

The univariate change equation of Equation 6 can, of course, be extended to the bivariate case when attempting to jointly understand two change processes and their dynamics. Bivariate change equations, based on this idea, can be written as

$$\begin{aligned} \Delta y[t]_n &= \alpha_y \cdot s_{yn} + \beta_y \cdot y[t-1]_n + \gamma_{yx} \cdot x[t-1]_n + \phi_y \cdot \Delta y[t-1]_n + \xi_{yx} \cdot \Delta x[t-1]_n \\ \Delta x[t]_n &= \alpha_x \cdot s_{xn} + \beta_x \cdot x[t-1]_n + \gamma_{xy} \cdot y[t-1]_n + \phi_x \cdot \Delta x[t-1]_n + \xi_{xy} \cdot \Delta y[t-1]_n, \end{aligned} \tag{7}$$

where ϕ_y and ϕ_x are parameters describing how the changes from time $t-1$ to time t are determined by the changes from time $t-2$ to time $t-1$ within each process and ξ_{yx} and

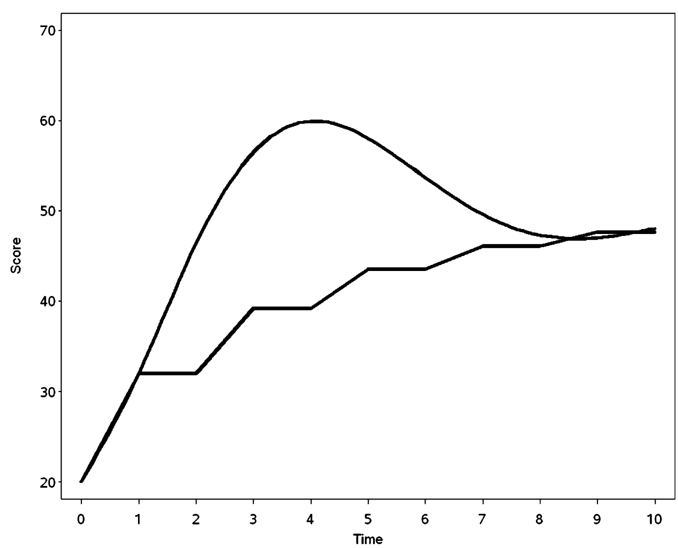


FIGURE 6 Example expected trajectories from a univariate changes to changes latent difference score model.

ξ_{xy} are parameters describing how the changes from time $t-1$ to time t are determined by the changes from time $t-2$ to time $t-1$ across processes. A path diagram of this model is contained in Figure 7 with five measurement occasions.

The traditional bivariate dual change model (Equation 5) posits that changes in Y are dependent on a constant change component that differs across persons, the previous true state of Y, and the previous true state of X. Thus, if the coupling parameter is significantly different from zero, then the level of X at time $t-1$ predicts subsequent change in Y from time $t-1$ to t. The bivariate extension of Equation 7 allows for a more complicated dynamic system, such that recent changes can also influence subsequent changes. It might be theoretically important and statistically useful to additionally examine whether the changes in X from time $t-2$ to $t-1$ predict changes in Y from time $t-1$ to t. Theoretically, previous states might not be important determinants of change, but how each process has recently changed might be key determinants. One advantage of this bivariate extension is that we are able to statistically test whether previous levels and previous changes are leading indicators of subsequent changes within and across variables.

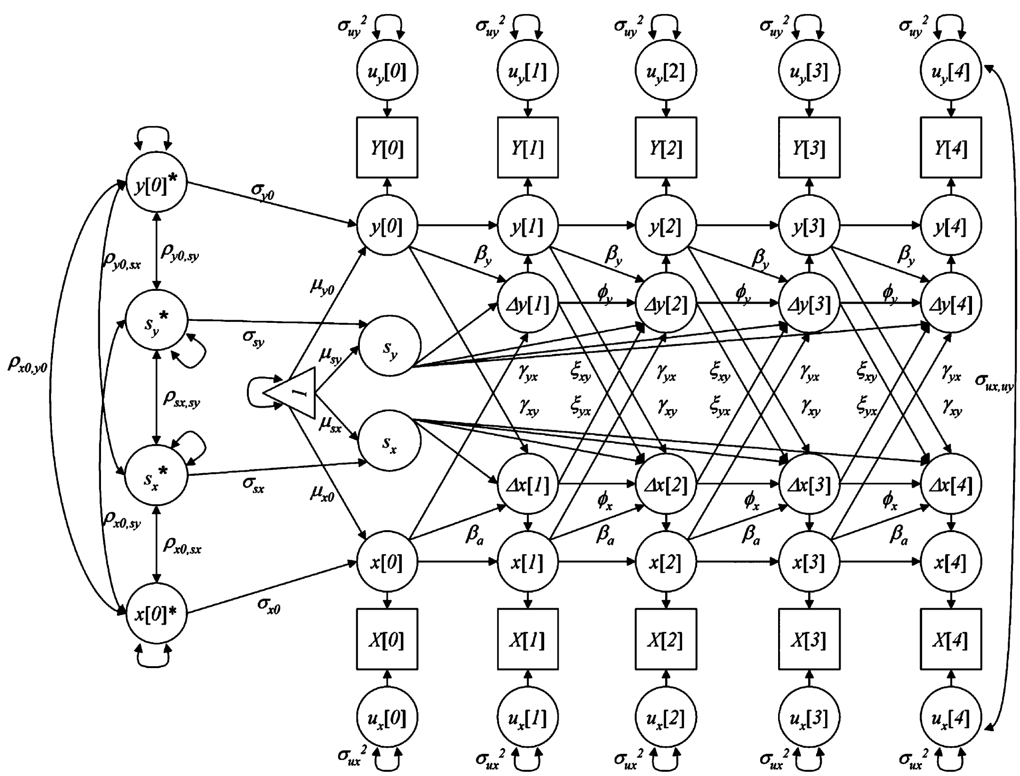


FIGURE 7 Path diagram of the changes to changes extension of the bivariate latent difference score model with five measurement occasions. Note: Unlabeled paths are fixed equal to 1.

ILLUSTRATIVE EXAMPLE

To illustrate the proposed LDS extensions described here, we fit commonly specified univariate and bivariate LDS models as well as the proposed extensions to longitudinal data from the BLSA (Shock et al., 1984) on short-term memory and the relative size of the lateral ventricle.

Participants

The sample includes 149 participants[1] (62 women) from the neuroimaging substudy (Resnick et al., 2000) of the BLSA. The neuroimaging study began in 1994, and participants returned approximately every year for biomedical and psychological evaluations. These analyses focus on up to 10 years of follow-up with an average of 6.8 ($SD = 2.7$) measurement occasions. Participants were mostly White (90%) and well educated ($M = 16.3$, $SD = 2.8$ years of education). At the first measurement occasion participants ranged in age from 56 to 86 years ($M = 69.9$, $SD = 7.9$ years). A total of 958 observations were used in the analyses after we excluded data points after diagnosis for some subjects who later developed dementia and restricted the age range from 60 to 90 years.

All participants were generally in good health and free of dementia at entrance with exclusionary criteria encompassing central nervous system disease, severe cardiovascular disease, severe pulmonary disease, or metastatic cancer. Analyses were restricted to data from participants who were not depressed at the time of data collection. The study was approved by the local institutional review boards, and all participants gave written informed consent prior to each assessment.

Measures

California Verbal Learning Test. The CVLT is a measure of verbal memory (Delis, Kramer, Kaplan, & Ober, 1987). The test is composed of two 16-item shopping lists. Each list contains four items (e.g., grapes) from four semantic categories (e.g., fruits). On the learning trial, 16 items are read aloud to the participant, who is asked to immediately recall as many words as possible. Five consecutive learning trials are performed with list A. The total correct over the five learning trials is a measure of new learning and short-term memory and was used for analyses in this study. A longitudinal plot of CVLT total immediate recall performance against age is shown in Figure 1a. As seen in Figure 1a, CVLT scores appear to gradually decline from age 60 to 90 and there appears to be sizeable interindividual differences in average performance and changes in performance.

Lateral ventricle size. Brain structures were assessed using MRI scans. Procedures for imaging processing were previously described in detail (Davatzikos, Genc, Xu, & Resnick, 2001; Driscoll et al., 2009; Goldszal et al., 1998; Resnick, Pham, Kraut, Zonderman, &

[1]This sample size is generally small for LDS modeling; however, the measures generally have good measurement properties and there are several assessments. Determining an appropriate sample size for this type of modeling depends on several issues including the size of measurement error, effect size, attrition, number, and spacing of measurement occasions.

Davatzikos, 2003). In brief, images were corrected for head tilt and rotation, and reformatted parallel to the anterior-posterior commissure plane. Extracranial tissue was removed using a semiautomated procedure followed by manual editing. Images were then segmented into white matter (WM), gray matter (GM), and cerebrospinal fluid (CSF). The final step involved stereotaxic normalization and tissue quantitation for specific regions of interest using a template-based deformation approach; all images were normalized individually to the same template.

In this analysis, we selected the LVS as a measure of ventricle volume and a broad indicator of brain atrophy or tissue loss. A longitudinal plot of the LVS by age is contained in Figure 1b. As seen in Figure 1b, the size of the lateral ventricle appears to gradually increase from age 60 to 90, indicating increasing brain atrophy with age. Additionally, changes in ventricle volume appear to accelerate with increased age. As with CVLT, there appears to be sizeable interindividual differences in ventricle volume at any given age as well as interindividual differences in the rate of change.

Analytic Techniques

Traditional univariate and bivariate latent difference score models (i.e., Equation 5) as well as the extensions proposed here (i.e., Equation 7) were fit to the repeated measurements of the CVLT and LVS with age as the timing metric in an exploratory nature (all models were fit and compared). Age was rounded to the nearest year and models were fit from age 60 to 90 in a structural modeling framework using the M*plus* program (Muthén & Muthén, 1998–2007). Rounding age to the nearest year leads to a loss of information; however, LDS models cannot be directly fit to data with individually varying time metrics without some accommodation such as this (Δt needs to be constant at the latent level). Models were compared using likelihood ratio tests and information criteria due to data sparseness. Likelihood ratio tests (change in -2 log-likelihood [-2LL] with respect to the change in the number of estimated parameters) could be conducted for the majority of model comparisons as all models discussed are nested, with the exception of the constant and proportional change models. Information criteria included Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC). The AIC and BIC are based on the -2LL plus a penalty function based on the number of estimated parameters. The penalty for the AIC is $2p$ where p is the number of parameters and the penalty for the BIC is $ln(N)p$ where ln is the natural log and N is the sample size. For the information criteria, lower values indicate better fit. The full information maximum likelihood estimator was used to account for the incomplete data and analyze all available data. Input and output scripts from M*plus* are available on the Web (see http://psychology.ucdavis.edu/labs/Grimm/personal/downloads.html).

Models. Four univariate LDS models were fit to the repeated measures of the CVLT and LVS separately. The models included the (a) proportional change model, where yearly changes are proportional to the level at the previous year; (b) constant change model, where a constant amount of change occurs each year; (c) dual change model where yearly changes have a constant influence and are proportional to the level at the previous year; and (d) changes to changes model (Equation 6) where yearly changes have a constant influence, are proportional to the level at the previous year, and are proportional to the changes in the previous year.

Bivariate models were fit in two series. The first series included four traditional latent difference score models—no coupling (M1a; $\gamma_{xy} = \gamma_{yx} = 0$), level of memory leading to changes in LVS (M2a; $\gamma_{xy} = 0$), LVS leading to changes in memory (M3a; $\gamma_{yx} = 0$), and bidirectional coupling model that includes both coupling parameters (M4a; Equation 5). Building on this series of models, the second series of models included parameters examining how prior changes related to subsequent changes. In this series, the within-construct model included the constant change component, proportional change parameter, and changes to changes parameter as described in Equation 6. The four models, in this bivariate series, mimicked Models M1a through M4a such that models only differed in terms of their cross-construct relations. The first model (M1b) included all parameters of the bidirectional coupling model (M4a) and added the within-construct changes to changes parameters (ϕ_x & ϕ_y)—only ξ_{xy} and ξ_{yx} were not estimated (fixed to 0). Thus, changes in memory were determined by the constant change component, previous level of memory, previous ventricle volume, and previous changes in memory. Similarly, changes in ventricle volume were determined by the constant change component, previous ventricle volume, previous level of memory, and previous changes in ventricle volume. In Models M2b and M3b, one cross-construct changes to changes parameter was estimated. M2b included changes in memory as a predictor of changes in ventricle volume ($\xi_{xy} = 0$), whereas M3b included changes in ventricle volume as a predictor of changes in memory ($\xi_{yx} = 0$). Finally, M4b included the estimation of both cross-construct changes to changes parameters (Equation 7).

RESULTS

Univariate Models

Four LDS models were fit to the repeated measures of the CVLT and LVS and fit statistics for these models are shown in Table 1. Changes in memory performance, measured by the CVLT, from age 60 to 90 were best represented by the constant change model ($-2LL = 6{,}210$, parameters = 6). Thus, the changes in CVLT were not dependent on the previous level of CVLT or how CVLT previously changed. Parameter estimates along with standard errors for the constant change model for CVLT are contained in Table 2. Based on the constant change model, participants had an average CVLT score of 56.883 at age 60 and there was a nonsignificant yearly mean decrease ($\mu_s = -.153$) in scores. However, there was significant variation in both the level of performance at age 60 ($\sigma^2_{y_0} = 107.751$) and yearly changes ($\sigma^2_s = .586$). Furthermore, participants with a higher level of performance at age 60 tended to show greater yearly declines ($\sigma_{y_0,s} = -4.590$).

Changes in the LVS from age 60 to 90 were best captured by the dual change score model ($-2LL = 3{,}804$, parameters = 7). Thus, yearly changes in the LVS were dependent on the previous LVS and the constant change component, but not how the LVS recently changed. Parameter estimates along with standard errors for the dual change model for LVS are also contained in Table 2. On average, participants had a ventricle volume of 16.961 at age 60. Yearly changes in ventricle volume had two predictors—the constant change component, which had a nonsignificant mean ($\mu_s = -.115$), and the proportional change parameter, which was significant and estimated to be .041. Thus, average yearly increases in ventricle volume were

TABLE 1
Fit Statistics for Univariate Latent Difference Score Models Fit to California Verbal Learning Test and Lateral Ventricle Size

	Proportional Change Model	*Constant Change Model*[a]	*Dual Change Model*	*Changes to Changes Model*
(A) California Verbal Learning Test				
−2LL	6,262	6,210	6,209	6,207
Parameters	4	6	7	8
AIC	6,270	6,222	6,223	6,223
BIC	6,282	6,240	6,244	6,247
ABIC	6,269	6,221	6,221	6,222

	Proportional Change Model	*Constant Change Model*	*Dual Change Model*[a]	*Changes to Changes Model*
(B) Lateral Ventricle Size				
−2LL	4,315	3,864	3,804	3,804
Parameters	4	6	7	8
AIC	4,323	3,876	3,819	3,820
BIC	4,335	3,894	3,840	3,844
ABIC	4,322	3,875	3,817	3,819

Note. −2LL = −2 log-likelihood; AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion; ABIC = sample size adjusted Bayesian Information Criterion.
[a]Selected model.

TABLE 2
Parameter Estimates for Chosen Univariate Latent Difference Score Models Fit to the Longitudinal California Verbal Learning Test and Lateral Ventricle Size Data

	California Verbal Learning Test		*Lateral Ventricle Size*	
	Constant Change Model		*Dual Change Model*	
	Parameter Estimate	*SE*	*Parameter Estimate*	*SE*
Fixed effects				
μ_{y0}	56.883	1.238	16.961	.850
μ_s	−.153	.092	−.115	.157
β	—		.041	.006
Random effects				
σ^2_{y0}	107.750	26.163	77.596	10.014
σ^2_s	.586	.145	.241	.039
$\sigma_{y0,s}$	−4.590	1.751	−3.257	.655
σ^2_u	30.047	1.692	.812	.044

Note. — indicates parameter was not estimated.

4% of the previous year's ventricle volume plus a constant of −.115. There was significant variation in both the size of the lateral ventricle at age 60 ($\sigma^2_{y_0}$ = 77.596) and constant change component (σ^2_s = .241). Finally, there was a negative covariance between ventricle volume at age 60 and the constant change component ($\sigma_{y_0,s}$ = −3.257). Thus, participants with a greater ventricle volume at age 60 tended to have a more negative constant change component.

Bivariate Models

Fit statistics for bivariate models are contained in Table 3. First, we focus on Models M1a through M4a, which are based on commonly specified bivariate LDS models. Based on the likelihood fit statistics, Model M4a (−2LL = 9,973, parameters = 21) was considered the best representation of the dynamics between memory and ventricle volume. This model fit significantly better than the no coupling model (Δ−2LL = 28, Δparameters = 2) and both unidirectional coupling models (M4a vs. M2a: Δ−2LL = 7, Δparameters = 1; M4a vs. M3a: Δ−2LL = 24, Δparameters = 1). Parameter estimates along with standard errors from M4a are contained in Table 4. Focusing on the dynamic parameters (β & γ) as these parameters

TABLE 3
Fit Statistics for Traditional Bivariate Latent Difference Score Models and Extensions of Bivariate Latent Difference Score Models Jointly Fit to Data on the California Verbal Learning Test and Lateral Ventricle Size

(A) Traditional Bivariate Latent Difference Score Models				
	M1a No Coupling	*M2a CVLT[t − 1] → ΔLVS[t]*	*M3a LVS[t − 1] → ΔCVLT[t]*	*M4a Bidirectional Coupling*[a]
−2LL	10,001	9,980	9,997	9,973
Parameters	19	20	20	21
AIC	10,039	10,020	10,037	10,016
BIC	10,096	10,080	10,096	10,079
ABIC	10,035	10,017	10,033	10,012
(B) Extensions of Bivariate Latent Difference Score Models				
	M1b No Changes to Changes Coupling	*M2b CVLT[t − 1] → ΔLVS[t]*	*M3b LVS[t − 1] → ΔCVLT[t]*	*M4b Full Model*[a]
−2LL	9,939	9,939	9,931	9,927
Parameters	23	24	24	25
AIC	9,985	9,986	9,979	9,977
BIC	10,053	10,058	10,050	10,052
ABIC	9,981	9,982	9,974	9,973

Note. CVLT = California Verbal Learning Test; LVS = lateral ventricle size; −2LL = −2 log-likelihood; AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion; ABIC = sample size adjusted Bayesian Information Criterion.

[a]Selected model.

TABLE 4
Parameter Estimates for Chosen Bivariate Latent Difference Score Models Fit to the Longitudinal California Verbal Learning Test and Lateral Ventricle Size Data

	M4a: Bidirectional Coupling Model				*M4b: Full Model*			
	CVLT		*LVS*		*CVLT*		*LVS*	
	Parameter Estimate	*SE*	*Parameter Estimate*	*SE*	*Parameter Estimate*	*SE*	*Parameter Estimate*	*SE*
Fixed effects								
μ_{y0}	57.671	1.071	17.374	.857	53.953	1.768	17.781	.820
μ_s	−9.226	2.111	8.029	2.646	1.238	3.513	.793	.512
β	.139	.034	.011	.014	−.019	.058	−.001	.006
γ	.055	.014	−.135	.044	.054	.029	−.009	.009
ϕ	—		—		−.620	.208	.716	.168
ξ	—		—		−1.969	.645	−.282	.153
Random effects								
Univariate								
σ^2_{y0}	84.161	11.399	70.530	10.163	118.215	29.771	73.736	10.017
σ^2_s	1.709	.775	1.771	1.036	1.895	.925	.058	.045
$\sigma_{y0,s}$	−11.104	3.029	−2.918	1.327	−4.958	4.018	−.112	.525
σ^2_u	32.853	1.889	.713	.042	29.869	1.706	.723	.042
Bivariate								
$\sigma_{y0,y0}$	−14.516		7.758		−14.497		10.575	
$\sigma_{y0,s}$	11.814		4.031		−.487		.858	
$\sigma_{s,y0}$	−2.009		1.432		−4.669		2.782	
$\sigma_{s,s}$	−1.538		.816		.299		.139	
$\sigma_{u,u}$	−.140		.212		−.229		.191	

Note. — indicates parameter was not estimated. CVLT = California Verbal Learning Test; LVS = lateral ventricle size.

describe the dynamic interplay between memory and ventricle volume,[2] changes in ventricle volume were negatively impacted by the level of memory performance and changes in memory performance were positively impacted by the prior size of the lateral ventricle. Thus, based on M4a, it appears that yearly changes in the size of the lateral ventricle increased at a slower pace if the individual had higher memory scores the previous year and that yearly changes in memory performance decreased more rapidly if the individual had a smaller lateral ventricle the previous year.

The second series of bivariate models (e.g., M1b–M4b) that included prior changes as predictors of subsequent changes were fit and, generally, were an improvement over the first series (e.g., M1a–M4a). Starting from model M4a, the traditional bivariate LDS model with bidirectional coupling and best fitting bivariate model from the first series of bivariate models,

[2]When dynamic parameters are estimated in univariate and bivariate latent difference score models, the sign and magnitude of the mean of the constant change parameter varies as a function of magnitude and sign of the scale (of raw scores) because this estimated parameter functions as an intercept in the change equations. For example, adding 10 to all observed scores results in a new estimate for the mean of the constant change component; however, the estimates for all dynamic parameters are identical. Thus, interpreting the mean of the constant change component can be difficult.

Model M1b showed better fit ($\Delta{-}2LL = 34$, Δparameters $= 2$). In this model, yearly changes in memory performance were strongly negatively predicted by previous yearly changes in memory performance and yearly changes in ventricle volume were strongly positively related to previous yearly changes in ventricle volume. Thus, if previous changes in memory performance were large, then subsequent changes in memory were expected to be smaller, whereas if previous changes in ventricle volume were large, then subsequent changes in ventricle volume were expected to be larger—an acceleration of changes. Next, the parameter from prior changes in memory to subsequent changes in ventricle volume was estimated (M2b). The addition of this parameter did not significantly improve model fit ($\Delta{-}2LL = 0$, Δparameters $= 1$). However, when the parameter from prior changes in ventricle volume to subsequent changes in memory was estimated (M3b) there was a significant improvement in fit ($\Delta{-}2LL = 8$, Δparameters $= 1$). Finally, the full model (M4b), which includes all changes to changes parameters, fit significantly better than all previous models ($-2LL = 9{,}927$, parameters $= 25$; M4b vs. M1b: $\Delta{-}2LL = 12$, Δparameters $= 2$; M4b vs. M2b: $\Delta{-}2LL = 12$, Δparameters $= 1$; M4b vs. M3b: $\Delta{-}2LL = 4$, Δparameters $= 1$). Parameter estimates along with standard errors from M4b are contained in Table 4. The change equations for this final model were estimated as

$$\begin{aligned}
\Delta mem[t]_n &= \mathbf{1.238} - .019 \cdot mem[t-1]_n + .054 \cdot lvs[t-1]_n \\
&\quad - \mathbf{.620} \cdot \Delta mem[t-1]_n - \mathbf{1.969} \cdot \Delta lvs[t-1]_n \\
\Delta lvs[t]_n &= \mathbf{.793} - .001 \cdot lvs[t-1]_n - .009 \cdot mem[t-1]_n \\
&\quad + \mathbf{.716} \cdot \Delta lvs[t-1]_n - .282 \cdot \Delta mem[t-1]_n,
\end{aligned} \tag{8}$$

where *mem* represents memory performance as measured by the CVLT, Δmem represents changes in memory performance, *lvs* represents LVS, and Δlvs represents changes in LVS. Parameters shown in bold are significantly different from 0 with *p* values less than .05. Again, we focus on the dynamic parameters (β, γ, ϕ, & ξ). The change equations describe a dynamic system such that yearly changes in ventricle volume were significantly determined by prior yearly changes in ventricle volume ($\phi = .716$) and yearly changes in memory performance were significantly determined by prior yearly changes in memory performance ($\phi = -.620$) and prior yearly changes in ventricle volume ($\xi = -1.969$). Thus, if the size of the lateral ventricle recently increased, these equations predict larger subsequent yearly declines in memory performance and larger yearly increases in the size of the lateral ventricle, whereas if memory performance recently decreased, then subsequent yearly declines in memory performance occurred at a slower rate. For example, three predicted trajectories for scores on the CVLT are contained in Figure 8. The parameters that generated these predicted trajectories are identical, with the exception of the size of the constant change component for the LVS. For the upper dotted curve, the constant change component for the LVS was $\frac{1}{2}$ *SD* below the mean (smaller increases in the LVS); for the middle solid line, the constant change component for the LVS was at the mean; and for the lower dotted line, the constant change component for the LVS was $\frac{1}{2}$ *SD* above the mean (larger increases in the LVS). Thus, if increases in the LVS were larger, there was a predicted sharp decline in memory performance, whereas if increases in the LVS were smaller, there was a predicted increase in memory performance over time.

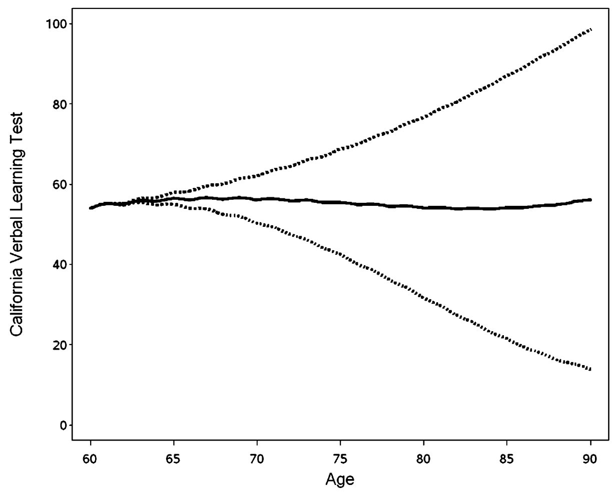


FIGURE 8 Predicted trajectories for the California Verbal Learning Test as a function of variation in the lateral ventricle size change component.

This plot exemplified how variations in the size of the changes in LVS affect the trajectories for the CVLT.

DISCUSSION

LDS modeling is a framework for studying and examining growth and decline and time dependency in multivariate repeated measures that combines aspects of latent growth models and ACL (time-series) models. Traditionally, a specific series of models is fit to determine leading and lagging processes and to understand their joint trajectory. This traditional series of models is limited in terms of expected trajectories, determinants of change, and theories that can be tested. Specifically, univariate trajectories can follow linear and exponential trajectories and bivariate models enable deflections from these univariate trajectories. In terms of time dependency, traditional LDS models only test whether changes in one construct can be determined by the previous *status* of itself and the second construct. However, some developmental theories do not posit this type of time dependency and, therefore, should not be tested with traditional LDS models. Here, we adapted the LDS framework to test and examine developmental theories that posit an alternative type of time dependency—one where recent changes affect subsequent changes.

Memory and Brain Structure Dynamics

The proposed models were fit to longitudinal data to examine the dynamics between brain and behavioral changes. Results indicated a specific type of dynamic relation, such that changes in ventricle volume precede changes in memory performance; however, the opposite effect was not significant. Longitudinal changes and the dynamics between brain structures and memory performance were also studied by McArdle et al. (2004) using data from the Normative Aging Study (NAS). McArdle et al. (2004) fit LDS models that were identical to the first series of bivariate models presented. McArdle et al. (2004) found that ventricle volume in the previous year was predictive of yearly subsequent declines in memory performance, but memory in the previous year was not predictive of subsequent yearly increases in ventricle volume. Thus, results from McArdle et al. (2004) do not align with results reported here because coupling between memory and ventricle volume were found to be bidirectional in the BLSA. Furthermore, the direction of the effect from the LVS to subsequent changes in memory was opposite to the effect reported in McArdle et al. (2004). There are several potential sources of differences. One potential source is the age span—models were fit from age 60 to 90 in the BLSA, whereas models were fit from age 30 to 90 in the NAS. However, McArdle et al. (2004) did not examine how previous changes related to subsequent changes and examining this type of dynamic might change their conclusions. In both studies, the size of the ventricle volume or changes in the ventricle volume appeared to precede changes occurring in memory.

Latent Difference Score Modeling and Extensions

LDS models have been utilized to study a variety of developmental issues. Such issues include the developmental sequencing between basic cognitive skills and achievement (e.g., Ferrer & McArdle, 2003), behavior and achievement (Grimm, 2007; McArdle & Hamagami, 2001), anger, internalizing, and externalizing behaviors (Kim & Deater-Deckard, 2011), and sleep and mood disturbances (Sbarra & Allen, 2009). All of these examples utilized common specifications where prior levels were the predictors of subsequent changes. If testing for additional impacts from prior changes, results and conclusions might change (as found here with memory and ventricle volume), leading to a more nuanced dynamic system and understanding of time-related change.

Theoretically, traditional specifications of LDS models that place the focus on prior levels are a continuous version of a threshold model, where a certain level of attribute is needed to obtain certain changes. In many cases, this type of model is very reasonable. For example, changes in reading behavior might require a certain amount of basic cognitive skills and without that level of basic cognitive skill, subsequent changes in reading are negatively impacted. The extensions proposed here add prior changes as a possible predictor of subsequent changes. Thus, the proposed models are not solely threshold-type models as recent occurrences can impact subsequent changes. For example, changes in reading might require a certain amount of basic cognitive skills, but such changes could be accelerated when cognitive skills recently increased. Therefore, the proposed models are able to take more information into account—not just where you are, but where you've recently been. This extension is similar to the idea of adding additional lags allowing for prediction to not only take into account the recent past ($t-1$), but earlier points in time (e.g., $t-2$, $t-3$)—this is common in time-series models, such as the dynamic factor model (e.g., Ram et al., 2005). Incorporating lag-2 effects (i.e., from $t-2$) in LDS models is possible and might be theoretically important; however, this is statistically different from the models proposed here.

The proposed extensions are not without their limitations. The inclusion of predictions from prior changes can result in trajectories that are not smooth (i.e., step functions, short-term waves within a longer term trajectory) and such trajectories might be theoretically unrealistic for many constructs. Additionally, including prior changes in the dynamic leads to a more complicated dynamic system in which interpretation becomes more difficult.

Finally, it's important to note several limitations of the BLSA for understanding time-related dynamics. Issues such as attrition, retest effects (specifically for CVLT) and cohort differences impact the estimation of the dynamic parameters. Additionally, changes were modeled based on age; however, a combination of time bases (age, measurement occasion, retest interval) might be needed. At this time, examining changes with LDS models requires the same time basis (e.g., age) and the most appropriate time-basis might be different for each construct.

Concluding Remarks

LDS modeling is a framework for understanding determinants of change. Theoretically, this framework has advantages over traditional latent curve modeling as all latent curve models could be respecified within the LDS modeling framework and time-sequencing in multivariate data can also be studied. Empirically, latent difference scores have been specified in limited ways. Most often, latent changes are specified as outcomes of previous states and a constant interindividual differences factor. These models are appropriate for studying developmental and neurodegenerative theories that posit this type of dynamic; however, change equations associated with the traditional specification should not be forced on data generated to test theories that posit other types of dynamic relationships within and between processes. Change equations should be specified that match the developmental or neurodegenerative theory being evaluated and the models proposed here are a step in this direction.

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