Quantile Treatment Effects of Partially Linear Models

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Abstract

In this paper, we consider the quantile treatment effects estimations of partially linear models. We derive the asymptotics of our estimators. We also apply our methods to some empirical applications.

Keywords:

JEL Classification: C14, C33, C34.

1 Introduction

2 Nonparametric estimation

Consider the potential outcomes or counterfactuals as follows

$$
\begin{aligned}
y^*_{0i} &= m_0(x_{1i}) + x'_{2i}\beta + \varepsilon_{0i}, \\
y^*_{1i} &= m_1(x_{1i}) + x'_{2i}\beta + \varepsilon_{1i}, \\
W_i &= \mathbf{1}(z'_i\gamma + u_i \geq 0),
\end{aligned}
$$

where the conditional quantiles $Q_{\varepsilon_0|x}(\tau) = 0$ and $Q_{\varepsilon_1|x}(\tau) = 0$. Therefore, $Q_{y_0|x}(\tau) = m_0(x_1) + x'_2\beta$ and $Q_{y_1|x}(\tau) = m_1(x_1) + x'_2\beta$, where $x = (x_1, x_2)$.

The observed outcome is

$$y_i = y^*_{1i}W_i + y^*_{0i}(1 - W_i). \tag{2.1}$$

The equation (2.1) can be written as

$$y_i = m_0(x_{1i}) + x'_{2i}\beta + W_i(m_1(x_{1i}) - m_0(x_{1i})) + \varepsilon_{0i}(1 - W_i) + \varepsilon_{1i}W_i. \tag{2.2}$$

The conditional quantile treatment effect is given as $m_1(x_1) - m_0(x_1)$.

The model (2.2) is similar to the following model considered in Das (2005)

$$y_i = m(x_i, d_i) + \varepsilon_i,$$

where d_i is a univariate discrete endogenous regressor with finite support which characterize the treatment, x_i is an $l_1 \times 1$ vector of exogenous covariates, m is an unknown smooth function. Further, the model can be written as can be written as

$$
\begin{aligned}
y_i &= m(x_i, d_i) + \varepsilon_i \\
&= m_1(x_i)\mathbf{1}(d = k_1) + m_2(x_i)\mathbf{1}(d = k_2) + \cdots + m_J(x_i)[1 - \mathbf{1}(d = k_1) - \cdots - \mathbf{1}(d = k_{J-1}) + \varepsilon_i \\
&= \alpha_0(x_i) + \left(\sum_{j=1}^{J-1} m_j(x_i)\mathbf{1}(d = k_j)\right) + \varepsilon_i.
\end{aligned}
$$

Here, we only have d_i as a binary variable with values 0 and 1.

Das (2005) focused on the mean estimation of such a model, but we focus on the quantile treatment effect. We adopt the idea of the control variable approach proposed in Imbens and Newey (2009).

Following Imbens and Newey (2009), we assume there exist a control variable v_i satisfying the following conditions.

Assumption 1 W_i and $\varepsilon_{0i}(1 - W_i) + \varepsilon_{1i}W_i$ are independent conditional on (x_i, v_i).

Assumption 2 The support of v_i conditional on W_i is equal to the support of v_i.

From the equation (6) in Imbens and Newey (2009), if we define

$$G(y, x, w) = \int F_{y|x,w,v}(y|x, w, v)F_v(dv),$$

Then,

$$G(m_0(x_1) + x_2'\beta + w(m_1(x_1) - m_0(x_1)), x, w) = \tau.$$

Following the estimation in Imbens and Newey (2009), given the independent and identically distributed sample $(x_{1i}, x_{2i}, y_i, z_i)$ we can first estimate the control variable v_i as

$$\hat{v}_i = \hat{F}_{w|z}(w_i, z_i).$$

Then, we can get

$$\hat{m}_0(x_1) + x_2'\hat{\beta} + w(\hat{m}_1(x_1) - \hat{m}_0(x_1)) = \hat{G}^{-1}(y, x, w),$$

and

$$\hat{G}(y, x, w) = \frac{1}{n}\sum_{i=1}^{n} \hat{F}_{y|x,w,v}(y|x, w, \hat{v}_i).$$

Therefore,

$$\hat{m}_1(x_1) - \hat{m}_0(x_1) = \hat{G}^{-1}(y, x, 1) - \hat{G}^{-1}(y, x, 0).$$

3 Asymptotics

4 Empirical Applications

References

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