2. SHOW THAT THE MINIMIZER FOR L_2 REGULARIZATION is $\vec{\omega} = (\lambda I + \Phi^T \Phi)^{-1} \Phi^T t$

ERROR IS GIVEN BY:

$$\tilde{E}(\omega) = \frac{1}{2} \sum_{n=1}^{N} \{y_n(x_n, \omega) - t_n\}^2 + \frac{\lambda}{2} \omega^T \omega$$

OR $$\tilde{E}(\omega) = \left[\frac{1}{2} \sum_{n=1}^{N} (t_n - \bar{\omega}^T \phi(\bar{x}_n))^2 \right] + \left[\frac{\lambda}{2} \omega^T \omega \right]$$

→ TAKE $\nabla \tilde{E}$ AND SOLVE WHERE $\nabla \tilde{E} = 0$

$$\vec{0}^T = \nabla(\hat{E}(\omega)) = \left[\frac{\partial}{\partial \omega_0} (\hat{E}(\omega)), \frac{\partial}{\partial \omega_1} (\hat{E}(\omega)), \frac{\partial}{\partial \omega_2} (\hat{E}(\omega)) \ldots \right]$$

$$\vec{0}^T = \frac{\cancel{2}}{\cancel{2}} \sum_{n=1}^{N} (t_n - \bar{\omega}^T \phi(x_n))(-\phi(x_n)) + \lambda \omega^T$$

$$\vec{0}^T = \sum_{n=1}^{N} \left[(t_n - \bar{\omega}^T \phi(x_n))(-\phi(x_n)) \right] + \lambda \omega^T$$

DEFINE: (AS IN THE NOTES)

$$\Phi = \begin{bmatrix} \phi_0(\bar{x}_1) & \phi_1(\bar{x}_1) & \ldots & \phi_{M-1}(\bar{x}_1) \\ \phi_0(\bar{x}_2) & \phi_1(x_2) & \ldots & \phi_{M-1}(\bar{x}_2) \\ \vdots & \vdots & & \vdots \\ \phi_0(\bar{x}_n) & \phi_1(\bar{x}_n) & \ldots & \phi_{M-1}(\bar{x}_n) \end{bmatrix}$$

$$\vec{0}^T = -t^T \Phi + \omega^T \Phi^t \Phi + \lambda \omega^T$$

$$\boxed{\vec{\omega} = (\Phi^t \Phi + \lambda I)^{-1} \Phi t^T}$$