# Computer Lab 2

## General Notes

1. You are expected to do your own work in these computer labs.
2. Save all m–files. Some of them will come in handy later in the course.
3. Be sure that you hand in all the required results in the correct format. Use the checklist provided to make sure you have everything.
4. Do not ignore Hints and Notes. The Hints will help you work efficiently. The Notes will help understanding the material.
5. Practice good programming techniques
   (a) Avoid infinite loops
   (b) For complicated programs, make a logic flowchart in advance
   (c) Read and understand any error messages
   (d) Use the (excellent) Matlab help menu as needed
   (e) Prepare professional plots:
      i. label all axes clearly
      ii. make sure multiple curves on the same plot are clearly labeled
      iii. use different colors and/or plotting symbols and styles to distinguish different curves.
   (f) Test your code by trying simple validation cases
   (g) Be sure to clear all arrays at the start of every mfile (except function m-files)

## Problem 1. A problem in fluid mechanics

***Objective***: *Solve a complicated nonlinear boundary value problem from fluid mechanics using shooting.*

As you will learn in fluids, high Reynolds number flow over solid objects creates boundary layers, which are very thin regions near the no-slip surface where viscous forces and inertial effects are of equal importance. You do not need to know the details in order to work this lab, but the following description will provide the context for the problem.

There is a set of boundary layer assumptions that result in significant simplifications of the Navier-Stokes equations. Consider flow past a flat plate at zero angle of incidence as sketched. The boundary layer is the thin region where the velocity profile varies with both the downstream ($x$) and cross-stream ($y$) directions.

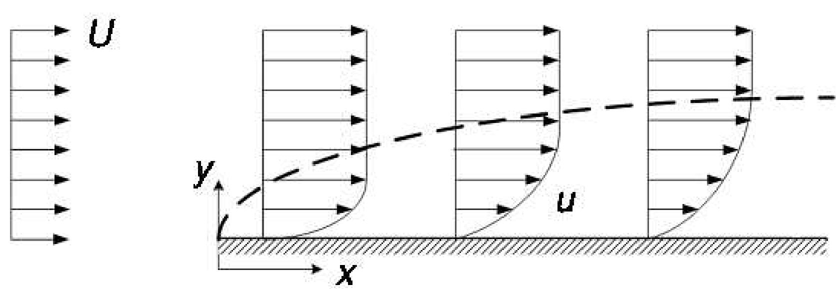


Figure 1: Boundary layer on a flat plate

The $x$–component of velocity, $u(x, y)$, is a function of a similarity variable, $\eta = y\sqrt{U/\nu\, x}$. In this definition, $\nu$ is the kinematic viscosity of the fluid and $U$ is the free stream velocity. The problem for $u(x, y)$ reduces to finding a function $f(\eta)$ defined such that $u(x, y)/U = f'(\eta)$. Under suitable assumptions (and lots of development), $f(\eta)$ satisfies the following third order non-linear ODE (called the Blasius equation):

$$f''' + \frac{1}{2}f''f = 0, \quad f(0) = f'(0) = 0 \quad \text{and} \quad f'(\infty) = 1 \tag{1}$$

where primes on $f$ denote derivatives with respect to $\eta$. The 'exact' solution is tabulated in most fluid mechanics text books and is available on Vista (file name BlasiusExactSoln.mat).

Solve this problem using shooting methods in the following steps.

**a.** Write the third order ODE as a system of three first order ODEs.
**b.** Integrate the equation as an Initial Value Problem by guessing the value of $f''(0)$
**c.** Integrate the system to 'infinity' to check if $f'(\infty) = 1$ to some desired accuracy.
**d.** If not, then iterate; if so, then store the value of $f''(0)$ and $f'(\eta)$

***Note***: BVPs on infinite domains require a suitable choice of 'infinity', $\eta_{max}$ say, at which to apply the boundary condition. In this case, we are going to see how the result depends on this choice.

Solve the Blasius equation to 3 significant figures of $f'(\eta)$ for the choices of $\eta_{max} = 5.0,\ 6.0,\ 7.0$ and 10. Use the method of false position to generate the next guess for $f''(0)$. For the initial guesses use $f''(0) = 0$ and $f''(0) = 1$.

## Hand in

- ⋆ Your m–files
- ⋆ A table of $\eta_{max}$, your values of $f''(0)$ for each given $\eta_{max}$ and relative error in $f''(0)$ compared to the exact value.
- ⋆ A short discussion of which of your values is the most accurate. Explain why.
- ⋆ Plotting $f'$ vs. $\eta$ gives a function. Plotting $\eta$ vs. $f'$ gives a 'picture' of the velocity profile. For your most accurate solution, provide a graph of $f'$ vs. $\eta$ and $\eta$ vs. $f'$. In each case, plot your numerical solution as a continuous curve and the 'exact' solution available on Vista (file name: BlasiusExactSoln.mat) using points.

**Problem 2. A convective heat transfer problem**

***Objective***: *To solve a two point boundary value problem using finite differences and compare against an exact solution.*

Consider the problem of thermal convection near a porous plate:

$$\frac{dT}{dx} + \frac{1}{Pe}\frac{d^2T}{dx^2}, \quad T(0) = 1 \quad \text{and} \quad T(\infty) = 0 \tag{2}$$

Solve this problem by finite difference techniques in the following steps:

**a.** Convert the ODE to algebraic equations using second order central difference formula for both the first and second derivatives.
**b.** Write an m–file that follows the following logical progression

   **i.** Clear all arrays.

**ii.** Set parameters such as mesh spacing, $h$, number of mesh points, $N$, and domain length, $L$ (not all these are independent).

**iii.** Form the coefficient matrix that represents the differential equation in all grid points.

**iv.** Form the right hand side vector of the algebraic equations.

**v.** Solve the algebraic equations for the choice $h = 1$. Use $Pe = 1$ and a mesh between $x = 0$ and $x = 10$. (***Note***: see discussion above about problems in infinite domains).

**c.** Plot the solution and compare against the analytical solution.

## Hand in

- ⋆ Your m–files
- ⋆ A graph of your numerical solution compared with the exact solution

**Problem 3. A short study in truncation error**

***Objective***: *Learn more about 'big Oh' and study truncation error and study the dependence of the truncation error on mesh size.*

Solve the convective heat transfer problem above for $Pe = 1$ on a domain $0 < x < 10$ for four values of the mesh size, $h$ : $h = 1,\ 0.5,\ 0.1$ and $0.05$. Evaluate the numerical solution at $x = 1$ and compare it against the exact value, $T(1) = e^{-1} = 0.3678794$.

## Hand in

- ⋆ A log-log plot of the relative error in $T(1)$ vs. the mesh size.
- ⋆ Your numerical value for the power of $h$ by which relative error in $T(1)$ decreases as $h$ gets smaller.
- ⋆ Since you have used second order differences, the error should go to zero as $h^2$. Provide a short discussion of how your error varies as a function of $h$ in light of this expectation.

**Problem 4. A short study in Fourier series**

***Objective***: *To observe the convergence of a simple Fourier series.*

The solution of one dimensional transient conduction developed in lecture involved the following Fourier series:

$$1 = 4\sum_{n=1}^{\infty}\frac{\sin((2n-1)\pi x)}{(2n-1)\pi} = \frac{4}{\pi}\left[\sin(\pi x) + \frac{1}{3}\sin 3\pi x + \ldots\right] \tag{3}$$

Using a loop and plot functions, evaluate and plot the partial sums $S_N(x)$, i.e. the first $N$ terms of this series, for $N = 1$, 10, 30 and 100.

$$S_N = 4\sum_{n=1}^{\lceil N/2\rceil}\frac{\sin((2n-1)\pi x)}{(2n-1)\pi}, \quad \lceil N/2\rceil = \begin{cases} N/2 & \text{if } N \text{ is even} \\ (N+1)/2 & \text{if } N \text{ is odd} \end{cases} \tag{4}$$

Evaluate the value of the series at the mid-point $x = 1/2$, and compute the relative error and the number of significant figures there for the above values of $N$. ***Hint***: Evaluate the Fourier series at $x = 1/2$ and compute its partial sums in the same loop.

## Hand in

- ⋆ A single plot of the partial sums for the given values of $N$.
- ⋆ A table of $S_N(1/2)$ and the number of significant figures for the given values of $N$.