Computer Lab 1

General Notes

1. You are expected to do your own work in these computer labs.
2. Save all m–files. Some of them will come in handy later in the course.
3. Be sure that you hand in all the required results in the correct format before leaving the lab. Use the checklist provided to make sure you have everything.
4. Do not ignore Hints and Notes. The Hints will help you work efficiently. The Notes will help understanding the material.
5. Practice good programming techniques
   (a) Avoid infinite loops
   (b) For complicated programs, make a logic flowchart in advance
   (c) Read and understand any error messages
   (d) Use the (excellent) Matlab help menu as needed
   (e) Prepare professional plots:
      i. label all axes clearly
      ii. make sure multiple curves on the same plot are clearly labeled
      iii. use different colors and/or plotting symbols and styles to distinguish different curves.
   (f) Test your code by trying simple validation cases
   (g) Be sure to clear all arrays at the start of every mfile (except function m-files)

Problem 1.

We are going to solve the problem of flow in a channel numerically using the shooting method. For reference, the BVP for $u(y)$ is

$$\frac{d^2u}{dy^2} = G, \qquad u(-b) = 0 \quad \text{and} \quad u(b) = V. \tag{1}$$

The general approach is to start the integration at $x = -b$ (at which $u = 0$) by guessing the value of $u'(-b)$ and integrating to $x = +b$. If your current guess of $u'(-b)$ is correct, then $u(+b)$ will equal V (to a prescribed tolerance). Otherwise, one has to iterate on $u'(-b)$ as explained in lecture.

a. Write an m-file to integrate the ODE as an initial value problem, with a relative error of 10^-3. Use `ode45`. ***Hint***: It will be necessary to write the second order ode as a system of first order odes.

Using the values $b = 1$, $G = -2$, and $V = 0$, test your program by integrating once with $u'(-b) = 2$ (the exact answer for plane Poisseiulle flow). You should get the well-known parabolic velocity profile $u(y) = (1 - y^2)$. Do not attempt to do the rest of the lab until you complete this validation study. If you don't get the analytical solution, you're doing something wrong and things will only get worse.

b. For the values $b = 1$, $G = -2$, and $V = 1$, solve the problem for a moving plate by the shooting method, using the method of false position to generate the next guess for the slope at $-b$. Converge the solution to four significant figures in the value of the slope. ***Note***: This is different from requiring four significant figures in `ode45`, but the two are related in an obvious way: the value of the slope cannot be found to an accuracy greater than the accuracy of the integration routine. To be on the safe side, what accuracy should you prescribe for the integration?

For initial guesses use $u'(0) = 1$ and $u'(0) = 4$.

Hint: If you have done everything correctly, the first application of false position will give the correct slope (to within the combined truncation and roundoff error of the computation). This is because the problem is linear - see Problem 2b of the Assignment 1. But it will take at least two iterations to show that your answer has the required number of significant figures. Why?

Hand in

- ⋆ Your m–files
- ⋆ A plot of the numerical solution from part **a.**, together with the analytical solution. Since one expects the two curves to be indistinguishable, use symbols for the numerical solution and a smooth curve for the analytical solution.
- ⋆ Your numerical value for $u'(-1)$ for $V = 1$.
- ⋆ A plot of the numerical solution for $V = 1$, together with the analytical solution. See note above about plotting styles with two nearly indistinguishable curves.

Problem 2.

Consider the following nonlinear BVP:

$$\frac{d^2y}{dx^2} = -4y\frac{dy}{dx}, \qquad y(0) = 0 \quad \text{and} \quad y(1) = 1.5 \tag{2}$$

Solve this problem using the shooting method. Use ode45 and the method of false position or the secant method. Find the solution to three significant figures. ***Note***:

a. This is a nonlinear ODE. So the iteration will not necessarily converge as quickly as in **Problem 1.**.

b. As with any second order equation, you will have to write it as a system of two first order equations.

c. Adapt the m–file from **Problem 1.** to this problem.

d. In contrast to **Problem 1.**, you are not given any initial guesses for the slope $y'(0)$. You will have to do some numerical experimentation in order to find suitable values. If you find two that bracket the root, then false position is guaranteed to converge. If you start with two that don't bracket the root and use the secant method, you may or may not achieve convergence.

Hand in

- ⋆ Your m–files
- ⋆ A short discussion of how you found your starting values for the iteration
- ⋆ A single plot of $y'(0)$ and $y(1)$ vs the iteration number
- ⋆ A single plot of the successive relative error in both $y'(0)$ and $y(1)$ vs. the iteration number
- ⋆ A plot of the numerical solution
- ⋆ Your numerical value for $y'(0)$