H-Infinity Static Output-Feedback Control for Rotorcraft

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The problem of stabilization of an autonomous rotorcraft platform in a hover configuration subject to external disturbances is addressed. Necessary and sufficient conditions are presented for static output-feedback control of linear time-invariant systems using the H_∞ approach. Simplified conditions are derived which only require the solution of two coupled matrix design equations. It is shown that the static output-feedback H_∞ solution does not generally yield a well-defined saddle point for the zero sum differential game; conditions are given under which it does. This paper also proposes a numerically efficient solution algorithm for the coupled design equations to determine the output-feedback gain. A major contribution is that an initial stabilizing gain is not needed. The efficacy of the control law and the disturbance accommodation properties are shown on a rotorcraft design example. The rotorcraft model is first loop shaped to achieve desired characteristics about the hover operating condition.

Nomenclature

A	=	system or plant matrix
B	=	control input matrix
C	=	output or measurement matrix
D	=	disturbance matrix
$x(t)$	=	internal state
$y(t)$	=	measured output
$u(t)$	=	control input
$z(t)$	=	performance output
$d(t)$	=	disturbance
Q	=	state weighting matrix
R	=	control weighting matrix
K	=	*static* output feedback gain matrix
γ	=	system L_2 gain
G	=	nominal plant
G_s	=	control input matrix
U	=	body frame x-axis velocity
V	=	body frame y-axis velocity

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P	=	roll angular rate
Q	=	pitch angular rate
ϕ	=	roll angle
θ	=	pitch angle
a_s	=	longitudinal blade flapping angle
b_s	=	lateral blade flapping angle
W	=	body frame z-axis velocity
r	=	yaw angular rate
r_{fb}	=	yaw rate feedback

I. Introduction

THIS paper investigates the application of a novel restricted-measurement static output-feedback (OPFB) control methodology to control an unmanned rotorcraft. In the past few years, there has been a significant interest in using unmanned aerial vehicles for applications such as search and rescue, surveillance and remote inspection. Rotorcrafts (especially helicopters) have several significant advantages over conventional fixed wing platforms in conducting several of these tasks. The advantages are exemplified by certain unique capabilities of the rotorcraft e.g., they can hover and can take-off and land in very limited spaces. Moreover, helicopters are highly maneuverable making them preferable for such tasks.

Several linear as well as non-linear control strategies have been proposed for control of helicopters and can be found in Refs. 1-3 and references within. The methodologies used in Refs. 1 & 2 use a adaptive feedback linearization approach where in a neural network approximates the uncertainties and the network weights are updated adaptively based on the trajectory tracking errors. While the controllers are efficient, one introduces additional dynamics to synthesize the controller. Further this makes the controller of very high order and there is no optimality guaranteed against the specific classes of disturbances/uncertainties considered. Alternately, we propose a static output feedback control structure based on H_∞ theory. We specifically focus on the problem of control in a hover configuration which in general is an unstable configuration. Further, in the presence of disturbances, the helicopter exhibits deviations in the dynamical states which complicate the control problem as the helicopter dynamical states are very tightly coupled. For example, in hover, pitch motion almost always is accompanied by forward and vertical motion and all three states need to be controlled simultaneously (See Ref. 4).

The static output-feedback problem is one of the most researched problems in systems and control theory. The use of output feedback allows flexibility and simplicity of implementation. Moreover, in practical applications, full state measurements are not usually possible. The restricted-measurement static output-feedback problem is of extreme importance in practical controller design applications including flight control in Ref. 5, manufacturing robotics in Ref. 6, and elsewhere where it is desired that the controller have certain pre-specified desirable structure, e.g., unity gain outer tracking loop and feedback only from certain available sensors. A survey of OPFB design results is presented in Ref. 7. Finally, though many theoretical conditions have been offered for the existence of OPFB, there are few good solution algorithms. Most existing algorithms require the determination of an *initial stabilizing gain*, which can be extremely difficult.

It is well known that the OPFB optimal control solution can be prescribed in terms of three coupled matrix equations,[8] namely two associated Riccati equations and a spectral radius coupling equation. A sequential numerical algorithm to solve these equations is presented in Ref. 9. OPFB stabilizability conditions that only require the solution of two coupled matrix equations are given in Refs. 10-12. Some recent LMI approaches for OPFB design are presented in Ref. 13-15. These allow the design of OPFB controllers using numerically efficient software, e.g., the MATLAB LMI toolbox[16]. However several problems are still open. Most of the solution algorithms are hard to implement, are difficult to solve for higher order systems, may impose numerical problems and may have restricted solution procedures such as the initial stabilizing gain requirements.

H_∞ design has played an important role in the study and analysis of control theory since its original formulation in an input-output setting in Ref. 17. It is well known that, through conservative, they provide better response in the presence of disturbance than H_2 optimal techniques. State-space H_∞ solutions were rigorously derived for the linear time-invariant case that required solving several associated Riccati equations in Ref. 18. Later, more insight into the problem was given after the H_∞ linear control problem was posed as a zero-sum two-player differential game[19]. A thorough treatment of H_∞ design is given in Ref. 20, which also considers the case of OPFB using *dynamic* feedback. An excellent treatment of H2 and H_∞ is given in Ref. 21.

Static OPFB design, as opposed to dynamic output feedback with a regulator, is suitable for the design of aircraft controllers of prescribed structure. H_∞ design has been considered for static OPFB, Hol and Scherer[22] addressed the applicability of matrix-valued sum-of-squares (sos) techniques for the computations of LMI lower bounds. Prempain and Postlethwaite[23] presented conditions for a static output loop shaping controller in terms of two coupled matrix inequalities. Recently, the application of loop shaping procedure in Helicopter control has lead to several improvements[24].

The aim of this paper is to demonstrate that high performance low order controllers can be easily and efficiently computed using H_∞ Static Static Output Feedback Techniques given in Refs. 25-26. In this paper, we show that the H_∞ approach can be used for static OPFB design to yield a simplified solution procedure that only requires the solution of *one* associated Riccati equation and a coupled gain matrix condition. This explains and illuminates the results in Ref. 11. That is, H_∞ design provides more straightforward design equations than optimal control, which requires solving three coupled equations. We have two objectives. First, we give necessary and sufficient conditions for OPFB with H_∞ design. Second, we suggest a less restrictive numerical solution algorithm with *no initial stabilizing gain requirement*. The design synthesis procedure is applied to the robust stabilization of an autonomous rotorcraft.

The paper is organized as follows. Section II details the formulation of Necessary and Sufficient Condition for H-Infinity OPFB Control. A solution Algorithm is proposed in Section III. Section IV illustrates the Unmanned Aerial Vehicle (UAV) model; controller structure, H-Infinity loop shaping design procedure, simulation results with disturbance effects.

II. Necessary and Sufficient Condition for H-Infinity OPFB Control

In this section we present a method for finding H_∞ static output feedback (OPFB) gains. It is seen that the H_∞ OPFB gain is computed in terms of only two coupled matrix equations. This is a simpler problem to solve than the optimal OPFB problem given in terms of three coupled equations [Ref. 5]. Moreover, a numerical algorithm is given to solve these equations that does not require an initial stabilizing OPFB gain.

A. System Description and Definitions

Consider the linear time-invariant system of Fig. 1 with control input *u(t)*, output *y(t)*, and disturbance *d(t)* given by

$$\dot{x} = Ax + Bu + Dd \ , \ y = Cx \ , \tag{1}$$

and a performance output *z(t)* that satisfies

$$\|z(t)\|^2 = x^T Q x + u^T R u \ . \tag{2}$$

A static output-feedback control is given by

$$u = -Ky = -KCx \ . \tag{3}$$

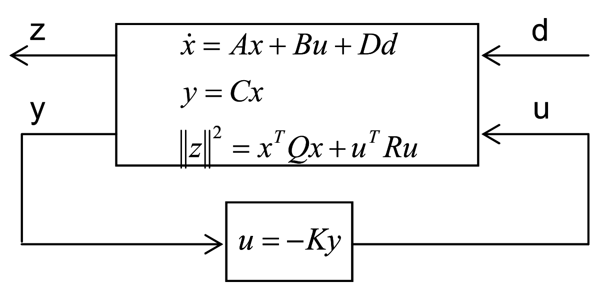

Fig. 1: System description

By definition the pair *(A, B)* is said to be *stabilizable* if there exists a real matrix K such that $A - BK$ is (asymptotically) stable. The pair *(A, C)* is said to be *detectable* if there exists a real matrix L such that $A - LC$ is stable. System (1) is said to be *output feedback stabilizable* if there exists a real matrix K such that $A-BKC$ is stable.

B. Bounded L_2 Gain Design Problem

The System L_2 gain is said to be *bounded or attenuated by* γ if

$$\frac{\int_0^\infty \|z(t)\|^2 dt}{\int_0^\infty \|d(t)\|^2 dt} = \frac{\int_0^\infty (x^T Qx + u^T Ru)dt}{\int_0^\infty (d^T d)dt} \le \gamma^2 \tag{4}$$

for any non-zero energy-bounded disturbance input d. Call γ^* the minimum gain for which this occurs. For linear systems, these are explicit formulae to compute γ^*, see e.g., Ref. 27. Throughout this paper we shall assume that γ is fixed and $\gamma > \gamma^*$. The case when $\gamma = \gamma^*$ is called H_∞ Control. It is desired to find a static OPFB gain K such that the system is stable and the L_2 gain is bounded by a prescribed value γ.

The next theorem gives necessary and sufficient conditions for existence of bounded L_2 gain static OPFB control, *c.f.* Ref. 26.

Theorem 1. Necessary and Sufficient Conditions for Bounded L_2 gain Static OPFB Control:
For a given $\gamma > \gamma^*$, there exists an OPFB gain such that $A_0 \equiv (A-BKC)$ is asymptotically stable with L_2 gain bounded by γ if and only if:

i. (A,C) is detectable

and there exist matrices L and $P = P^T \ge 0$ such that:

ii. $$KC = R^{-1}(B^T P + L). \tag{5}$$

iii. $$PA + A^T P + C^T C + \frac{1}{\gamma^2} PDD^T P - PBR^{-1}B^T P + L^T R^{-1} L = 0 \tag{6}$$

Proof:

Necessity:

Note that (*A-BKC*) stable implies (*A, C*) detectable. Now set $Q = C^T C$. Suppose that there exists an output-feedback gain K that stabilizes the closed loop system $A_c \equiv A - BKC$ and satisfies L_2 gain bounded by γ. Consider the equation

$$PA_c + A_c^T P + \frac{1}{\gamma^2} PDD^T P + Q + C^T K^T RKC = 0 \tag{7}$$

From Knobloch *et al.* (Ref. 20), Theorem 2.3.1, closed-loop stability and L_2 gain boundedness implies that (7) has a unique symmetric solution such that $P \ge 0$. Rearranging (7) and completing the square will yield

$$PA + A^T P + Q + \frac{1}{\gamma^2} PDD^T P - PBR^{-1}B^T P + (KC - R^{-1}B^T P)^T R(KC - R^{-1}B^T P) = 0. \tag{8}$$

Substituting the gain defined by (5) in (8) yields (6).

Sufficiency:

Set $Q = C^T C$ and define $\overline{Q} \equiv Q + C^T K^T RKC + \frac{1}{\gamma^2} PDD^T P$ and $\widetilde{Q} \equiv Q + C^T K^T RKC$. Suppose that (5) and (6) hold, then (7) follows, so that

$$PA_c + A_c^T P + \overline{Q} = 0\,. \tag{9}$$

We claim that detectability of $(A, \sqrt{Q})$ implies detectability of $(A_c, \sqrt{\overline{Q}})$. For, note that detectability of $(A, \sqrt{Q})$ implies that $A - \widetilde{L}\sqrt{Q}$ is stable for some $\widetilde{L}$. One can write

$$A - \widetilde{L}\sqrt{Q} = A_c - \overline{L}\sqrt{\overline{Q}} \tag{10}$$

for $\left(\sqrt{\overline{Q}}\right)^T \equiv \begin{bmatrix} \left(\sqrt{Q}\right)^T & (\sqrt{R}KC)^T & \left(\frac{1}{\gamma}\right)PD \end{bmatrix}$ and $\overline{L} \equiv \begin{bmatrix} \widetilde{L} & -BR^{-1/2} & 0 \end{bmatrix}$. It follows that if pair $(A, \sqrt{Q})$ is detectable then pair $(A_c, \sqrt{\overline{Q}})$ is detectable as well. Moreover $\overline{Q} \geq 0$, and $P \geq 0$. Therefore, using the Lyapunov stability criteria (Ref. 28, Lemma 12.2], (9) implies closed-loop stability. Finally for $z = [\sqrt{Q}x \quad \sqrt{R}\,u]^T$ one has $z^T z = x^T \widetilde{Q} x$, and

$$PA_c + A_c^T P + \frac{1}{\gamma^2} PDD^T P + \widetilde{Q} = 0\,, \tag{11}$$

and system L_2 gain boundedness then follows from Van der Schaft (Ref. 29, Theorem 2.)

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III. Solution Algorithm

Most existing iterative algorithms for OPFB design require the determination of an initial stabilizing gain, which can be very difficult for practical aerospace systems such as the stabilization of an autonomous rotorcraft in hover. The following algorithm is proposed to solve the two coupled design equations in Theorem 1. Note that *it does not require an initial stabilizing gain* since, in contrast to Kleinman's state feedback Algorithm,[30] and the OPFB algorithm of Moerder and Calise[9], it uses a Riccati equation solution, not a Lyapunov equation, at each step.

1. Initialize:
   Set $n=0$, $L_0 = 0$, and select γ, Q and R.
2. *n-th* iteration:
   solve for P_n in

$$P_n A + A^T P_n + Q + \frac{1}{\gamma^2} P_n DD^T P_n - P_n BR^{-1} B^T P_n + L_n^T R^{-1} L_n = 0 \tag{12}$$

   Evaluate gain and update L

$$K_{n+1} = R^{-1}(B^T P_n + L_n) C^T (CC^T)^{-1} \tag{13}$$

$$L_{n+1} = RK_{n+1}C - B^T P_n \tag{14}$$

   If K_{n+1} and K_n are close enough to each other, go to
   3 otherwise set $n = n+1$ and go to 2.
3. Terminate:
   Set K= K_{n+1}

■

Note that this algorithm uses well-developed techniques for solving Riccati equations available, for instance, in MATLAB. It generalizes the algorithm in Ref. 12 to the case of nonzero initial gain. It is described in section IV that this algorithm is also suitable to find static output-feedback gains for loop-shaped plants.

IV. Attitude Control Loop Design Example

A. System Description:

The controller design is based on an 11-state linear model of a "Raptor-90" helicopter shown in Fig.2.

Fig. 2: Raptor-90 helicopter.

The results are based on the model derived at National University of Singapore. A linearized model for hover operating point has been established. The model currently used is a state-space model which represents the helicopter as 6-degree-of-freedom (DOF) rigid body augmented with servo/rotor dynamics and artificial yaw damping dynamics[31]. The state vector physically shown in Fig. 3 contains eleven states and can be expressed as $x = [U \quad V \quad p \quad q \quad \phi \quad \theta \quad a_s \quad b_s \quad W \quad r \quad r_{fb}]^T$.

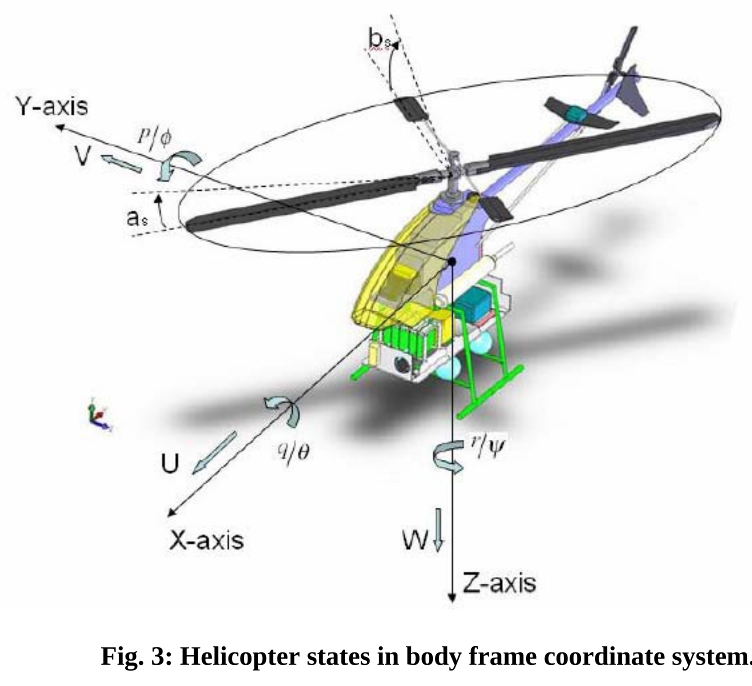

Fig. 3: Helicopter states in body frame coordinate system.

The input vector can be written as $u = [\delta_{lati} \quad \delta_{longi} \quad \delta_{ped}]^T$. Where δ_{lati} is the lateral channel input and affects roll motion, δ_{longi} is longitudinal channel input and affects pitch, δ_{ped} is pedal channel input of remote controller and affects yaw motion. In helicopters there is a high degree of coupling between lateral and longitudinal dynamics. In this paper collective channel, the fourth actuator which produces lift, is left to be controlled manually.

The primary variables to be controlled are the pitch angle and roll angle. Two extra rate gyros measuring pitch angular rate and roll angular rate will also be used for feedback purposes. Five system states constitute the output vector $y = [\phi \quad \theta \quad r \quad p \quad q]^T$. The rotorcraft equations mentioned were trimmed in a hover configuration to obtain the reference trim condition. The nonlinear equations then linearized for the hover configuration based on the reference values obtained[5]. The plant linear matrices are as below

$$A = \begin{bmatrix} -0.1778 & 0 & 0 & 0 & 0 & -9.7807 & -9.7807 & 0 & 0 & 0 & 0 \\ 0 & -0.3104 & 0 & 0 & 9.7807 & 0 & 0 & 9.7807 & 0 & 0 & 0 \\ -0.3326 & -0.5353 & 0 & 0 & 0 & 0 & 75.7640 & 343.8600 & 0 & 0 & 0 \\ 0.1903 & -0.2940 & 0 & 0 & 0 & 0 & 172.6200 & -59.9580 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -8.1222 & 4.6535 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -0.0921 & -8.1222 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17.1680 & 7.1018 & -0.6821 & -0.1070 & 0 \\ 0 & 0 & -0.2834 & 0 & 0 & 0 & 0 & 0 & -0.1446 & -5.5561 & -36.6740 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7492 & -11.1120 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.0632 & 3.3390 & 0 \\ 3.1739 & 0.2216 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -74.3640 \\ 0 & 0 & 0 \end{bmatrix}$$

B. Wind Turbulence Model

The disturbance vector d given in (15) has wind components along the $[x \quad y]^T$ fuselage axes, disturbance input matrix D defines dynamics involved with body frame x, and y velocities. For this example D is a 11×2 matrix and is constituted from first two columns of the plant matrix A.

$$d = [d_U \quad d_V]^T \tag{15}$$

In Hall and Bryson[32] the wind components along the fuselage axes are modeled by independently excited correlated Gauss-Markov processes

$$\begin{bmatrix} \dot{d}_U \\ \dot{d}_V \end{bmatrix} = \begin{bmatrix} -1/\tau_c & 0 \\ 0 & -1/\tau_c \end{bmatrix} \begin{bmatrix} d_U \\ d_V \end{bmatrix} + \rho * B_w \begin{bmatrix} q_U \\ q_V \end{bmatrix} \tag{16}$$

Equation (16) is called a "shaping filter" for the wind, where q_U, and q_V are independent with zero mean, $\tau_c = 3.2\,\mathrm{sec}$ is the correlation time of the wind, $\sigma_{q_U}, \sigma_{q_V} = 20\,ft/s$, B_w is the turbulence input identity matrix, and $\rho = \frac{1}{2}$ is the scalar weighting factor.

C. Controller Structure

The control structure shown in Fig. 4 is basically an attitude control loop; each input channel is augmented with a compensator. Precompensators $G_{lat}(s)$, $G_{long}(s)$, and $G_{ped}(s)$ shape the plant prior to closing the loop. Loop shaping procedure is explained in the next section.

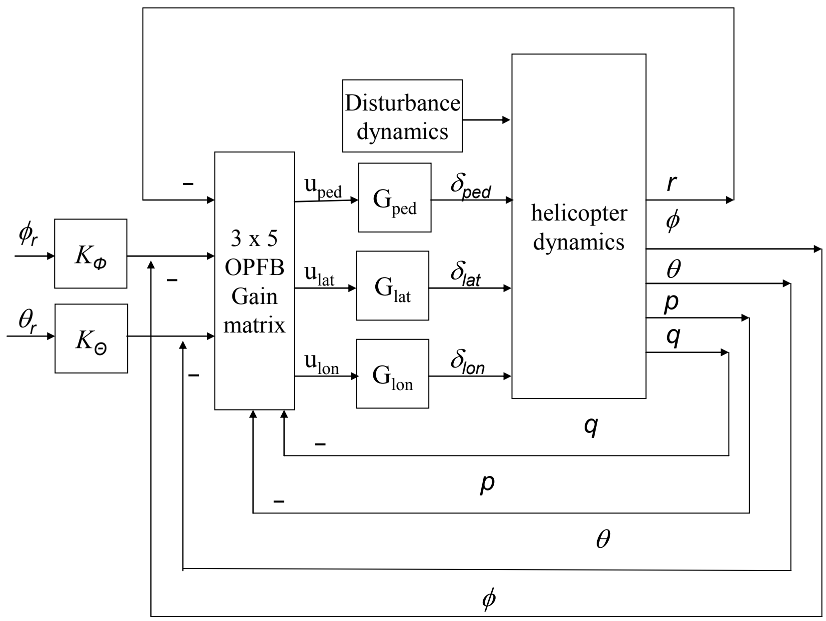

Fig. 4: Controller structure.

D. H-Infinity Loop Shaping Design Procedure

We will now formally state the design procedure. The objective of this approach is to balance the tradeoff between performance and robustness in loop shaping. The procedure couples loop shaping design with H-Infinity output-feedback control techniques.

- Using a precompensator W_1 and a postcompensator W_2, the singular values of the nominal plant are shaped to achieve a desired open-loop shape.
- The nominal plant G and the compensators are combined to form the shaped plant G_s. Let (A, B, C, D) be a realization of G_s.

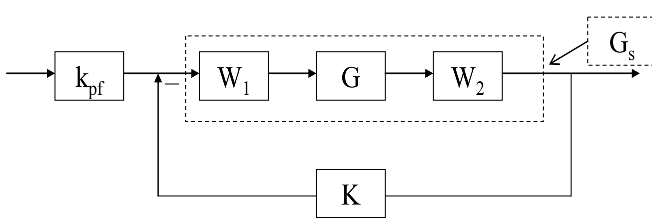

Fig. 5: Loop shaped plant with controller.

- Choose weighing matrices Q and R for G_s .
- Use H-Infinity static output feedback algorithm to find the static output-feedback gain. The algorithm is described in section III.
- Find prefilter gain K_{pf} for unity steady state gain between input and output pairs.

Loop Shaping

In this example precompensators $G_{lat}(s), G_{lon}(s), \text{and } G_{ped}(s)$ all are chosen as $G_{precomp}(s) = 2\left(\dfrac{s+0.5}{s(s+5)}\right)$ to shape the open loop plant. Additional dynamics in the Pre-Compensators is included to pull the cut-off to within the 2-5 rad/sec region, which is typical with aircraft and rotorcraft controllers. The design was effective using only the Pre-Compensators, so no Post-Compensators were chosen, i.e., the Post-Compensator weights was set to the identity matrix. The singular value plots of the original loop-gain and the shaped loop-gain are shown in Fig. 6. Also shown is the wind gust spectrum.

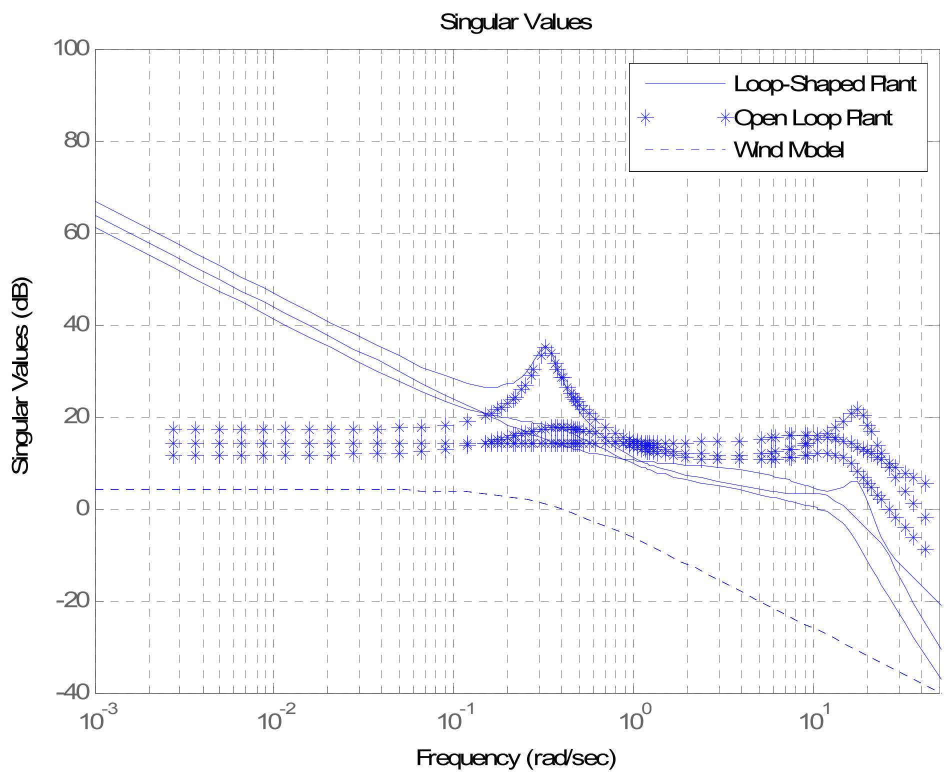

Fig. 6: Loop-gain singular value plots.

Weighting Matrices

In this example the weighting matrices are taken as

$$Q = \text{diag}\begin{bmatrix}0.25 & 0.25 & 0.01 & 0.01 & 100 & 100 & 1E\text{-}4 & 1E\text{-}4 & 0.25 & 0.01 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$R = \text{diag}\begin{bmatrix}169 & 169 & 0.78\end{bmatrix}.$$

The selection of Q and R is further discussed in the next subsection.

E. Simulation Results with Disturbance effects

The static output feedback solution derived in section III is applied to obtain an output feedback controller to stabilize the loop-shaped plant. The controller is then simulated subject to the wind disturbances to evaluate the efficacy of the proposed control law. The closed-loop system is shown in Fig. 7, where the exogenous disturbance input *d(t)* is a random variable, shown in Fig. 8, generated in the time domain to match statistical properties of the turbulence model, as discussed in Section B.

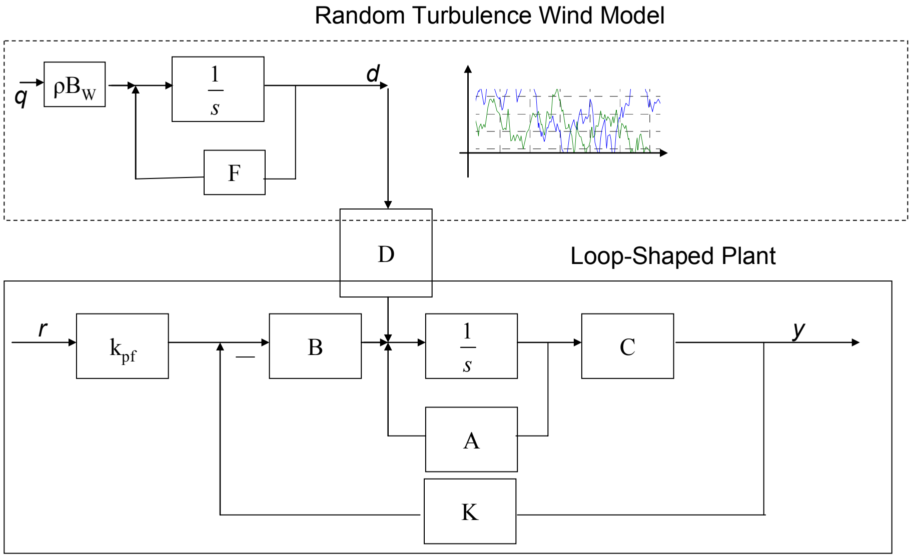

Fig. 7: Simulation with turbulence model.

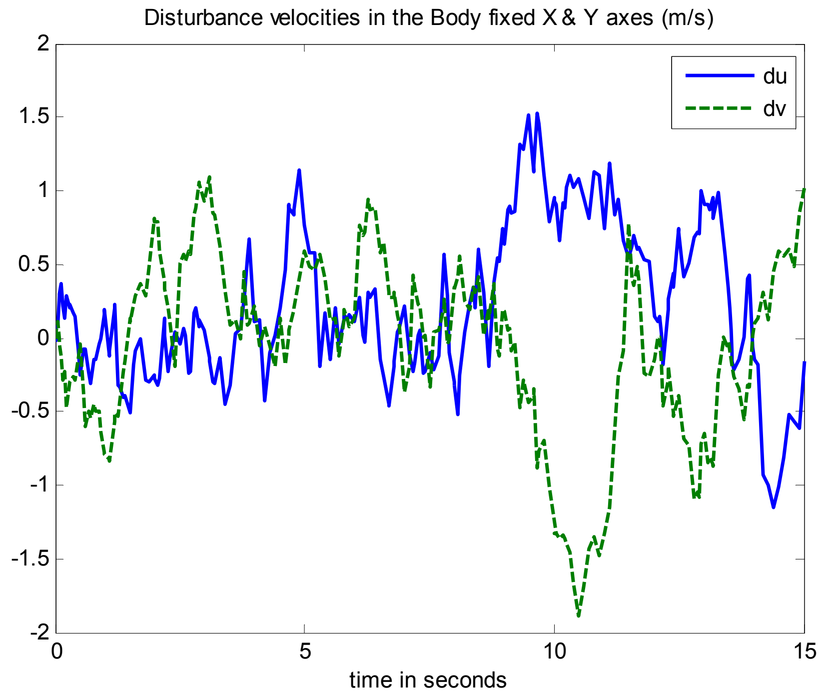

Fig. 8: Random velocity disturbance vector

For the computation of the output-feedback gain K it is necessary to select weighting matrices Q and R. A diagonal structure is used for Q and R. The diagonal entries are tuned iteratively. That is, for a given selection of Q and R, our algorithm was run to find the OPFB gain K. Then, the closed-loop system was simulated. If the results are not satisfactory, Q and R were modified and the procedure was repeated. Our algorithm makes it very fast and easy to perform this procedure., To avoid the excitation of un-modeled high frequency dynamics, the control input and velocity states are heavily penalized.

The gain parameter γ defines the desired L_2 gain bound. For the initial design, a fairly large γ is selected. If the algorithm converges, the parameter γ may be reduced. If γ is taken too small the algorithm will not converge since the Algebraic Riccati Equation has no positive semidefinite solution. After some design repetitions, which were performed very quickly using the algorithm; we found the smallest value of the gain to be 0.62.

Two particular cases were simulated to evaluate the closed loop system performance, namely bank angle command tracking i.e. $\phi_{command}$ and a pitch angle command tracking i.e. $\theta_{command}$

A) Bank angle command tracking ($\phi_{command}$)

The step responses of the lateral-directional states for a unit bank angle command (equivalent of 1 radian) are shown in Fig. 9. The inner loop simulation is begun at a hover configuration at an altitude of 50 m and was subjected to a turbulent wind disturbance with peak amplitude of 4.0 m/s. Considering, that the helicopter is initially in the hover configuration, this is a significant perturbation. It is seen that the bank angle settles to less than 5% of the steady state value within 5 seconds. The overshoot is 18.5%. The roll rate does not peak beyond 1 rad/s, which is within acceptable limits. We also note that the yaw rate activity is consistent with the build up in the lateral velocity. It is to be mentioned that throughout this inner loop control design, the collective pitch is not utilized. The consequence of this is a velocity build-up that causes the helicopter to drift from its current position. It was seen that without the collective pitch being active, the helicopter loses altitude very rapidly as the main rotor thrust vector is no longer aligned along the inertial Z-axis. The only way to increase the component of the thrust along the inertial Z-axis to balance the weight of the helicopter is to use the collective pitch.

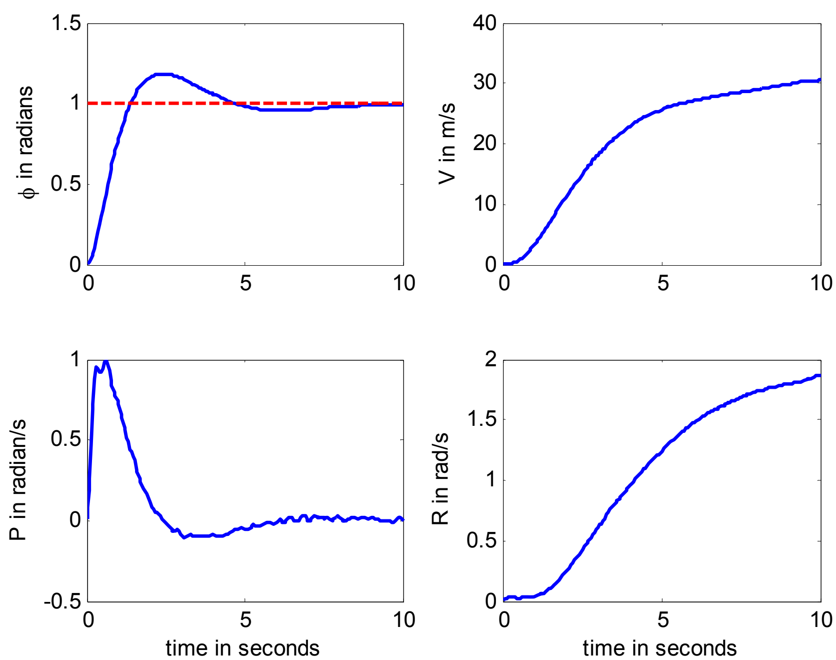

Fig. 9: Closed-loop lateral-directional state responses to a unity bank angle step demand.

Fig. 10 shows the longitudinal state responses for this case. It is seen that the states are all within acceptable limits. Note, the slight build up in the body axes U and W components of the velocities is attributed to the loss in lift due to the vectoring of the main rotor thrust to achieve the desired bank angle as well as the coupling between the longitudinal and lateral-directional dynamics. In addition, there is a velocity disturbance along the body X-axis due to turbulent wind.

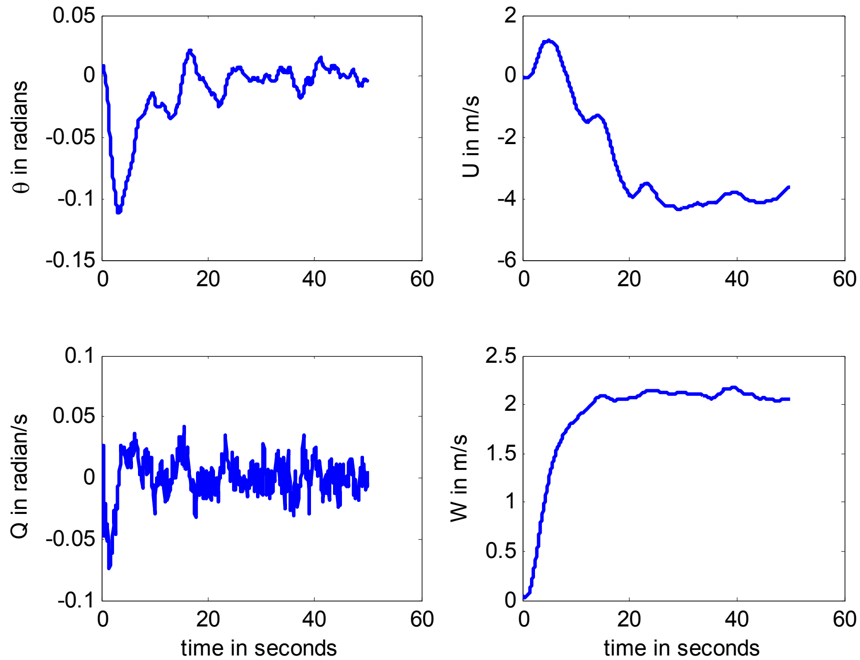

Fig. 10: Closed-loop longitudinal state responses to a unity bank angle step demand.

Fig. 11 shows the cyclic-pitch activity in the lateral as well as the longitudinal axes and the rudder pedal activity. As expected the activity in the rudder is minimal. The longitudinal cyclic-pitch responds to arrest the build up in the longitudinal states (pitch angle and pitch rate).

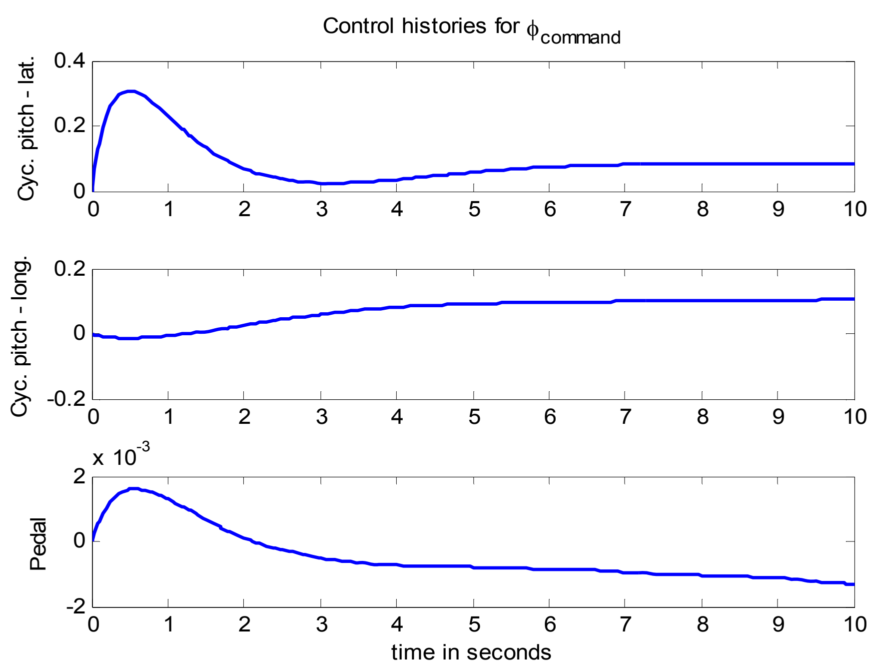

Fig. 11: Control history for a unity bank angle step demand.

B) **Pitch angle command tracking ($\theta_{command}$)**

The step responses of the longitudinal states for a unit pitch angle command (equivalent of 1 radian) are shown in Fig. 12. The helicopter configuration is identical to the earlier case, i.e. there is no collective pitch activity and similar turbulent wind disturbances are injected into the system. It is seen that the pitch angle settles to less than 6% of the steady state value within 5 seconds. The overshoot is < 18.5%. The slight oscillations within the 5% settling band are due to the external state disturbance (due to turbulent wind). The pitch rate does not peak beyond 1.5 rad/s, which is within acceptable limits (< 90 deg/s). While there is a significant change in the horizontal velocity there isn't as much change in the vertical velocity. The pitch angle demand is very aggressive, almost 60 degrees whose primary effect is to drastically slow down the helicopter. In the hover configuration, this would mean that the helicopter moves backwards while losing altitude. There is also a lateral shift in the inertial position due to the external disturbance activity and the weak coupling inherent in the vehicle dynamics. One way to arrest the build up in the translational velocities is to include an inner-loop for the translational dynamics (velocity loops) and use collective pitch.

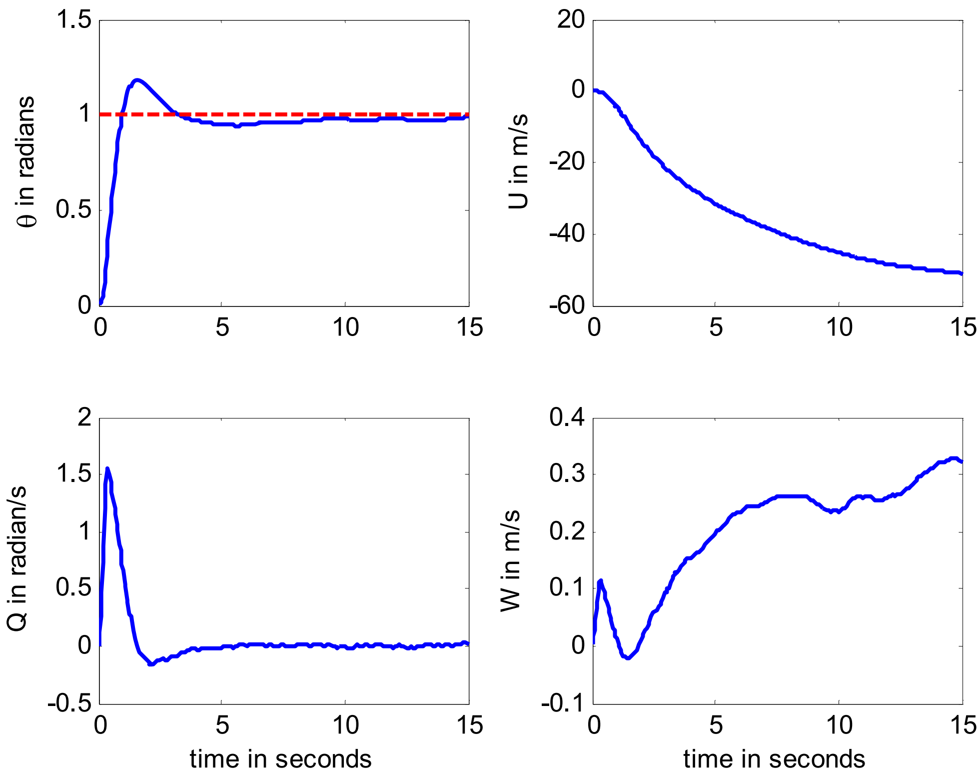

Fig. 12: Closed-loop longitudinal state responses to a unity pitch angle step demand.

Figs. 13 and 14 show the lateral-directional responses and the control activity for this maneuver (pitch angle command). As it is seen from the plots, the lateral-directional responses are within acceptable limits and the control histories are as expected.

We note from the plots for both the maneuvers the cross coupling between the longitudinal and lateral-directional modes is minimal. Additionally the roll rate and the pitch rate have low peaks for the respective maneuvers (< 90 deg/s). Our objective was to reduce these rate peaks as much as possible and also obtain good step responses in the attitude variables.

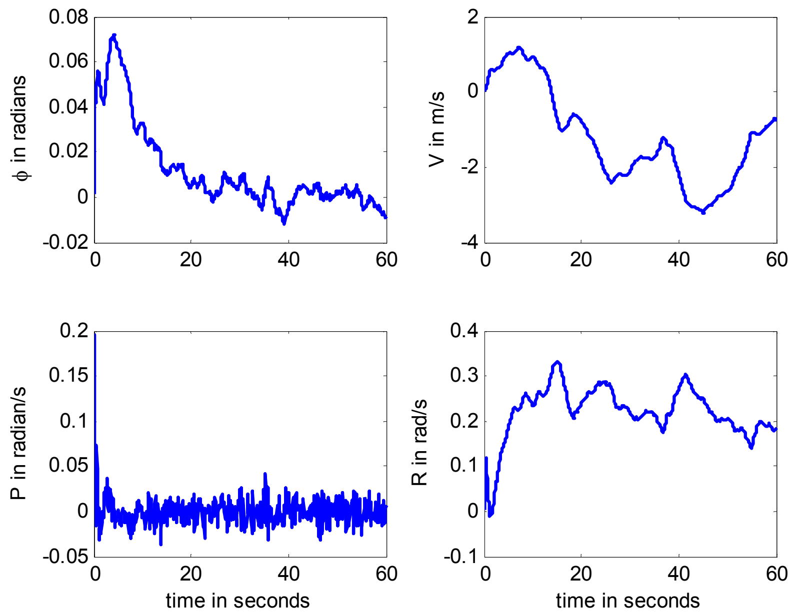

Fig. 13: Closed-loop lateral-directional state responses to a unity pitch angle step demand.

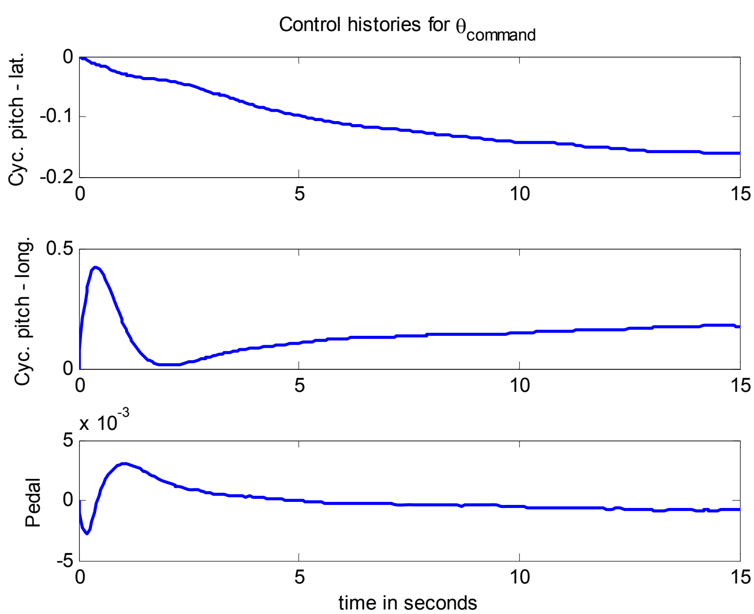

Fig. 14: Control history for a unity bank angle step demand.

V. Conclusion

The problem of disturbance attenuation with stability using static output-feedback for linear time-invariant systems has been studied. Necessary and sufficient conditions were developed, which yield two coupled matrix design equations to be solved for the OPFB gain. A computational algorithm to solve for the output-feedback gain that achieves pre-specified disturbance attenuation was developed. The algorithm requires no initial stabilizing gain, in contrast to other existing recursive OPFB solution algorithms. This procedure allows output-feedback control design with pre-specified controller structures and guaranteed performance. A robust controller for stabilizing an autonomous rotorcraft in hover was designed using the algorithm highlighted in the paper.

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References

[1]Nakwan Kim, Anthony J. Calise, J. Eric Corban, and J.V.R. Prasad. "Adaptive Output Feedback for Altitude Control of an Unmanned Helicopter Using Rotor RPM". In *AIAA Guidance, Navigation and Control Conference*, August 2004

[2]Eric N. Johnson and Suresh K. Kannan. "Adaptive Trajectory Control for Autonomous Helicopters". *AIAA Journal of Guidance, Control, and Dynamics*, 28(3), May-June 2005

[3]Linda Wills, Suresh K. Kannan, Sam Sander, Murat Guler, Bonnie Heck, J.V.R Prasad, Daniel P. Schrage, and George Vachtsevanos. An Open Platform for Reconfigurable Control. In Tariq Samad and Gary J. Balas, editors,*Software-Enabled Control: Information Technology for Dynamical Systems*. IEEE Press, 2001.

[4]Wayne Johnson, "Helicopter Theory", *Dover Publications*, Mineola, N. Y.

[5]Stevens, B. L., and Lewis, F. L., *Aircraft control and simulation*, 2nd ed , Wiley Interscience, New York, 2003, pp 403-419.

[6]Kim, Y. H., and Lewis F. L., *High-Level Feedback Control with Neural Networks*, World Scientific, Singapore, 1998, pp 55-75.

[7]Syrmos, V. L., Abdallah, C.; Dorato, P., "Static output feedback: a survey," *Proceedings of 33rd IEEE Conference on Decision and Control*, Orlando, FL, 1994, pp. 837-842

[8]Lewis, F. L. and Syrmos V. L., *Optimal Control*, 2nd ed , John Wiely and Sons, New York, 1995, pp. 359-375.

[9]Moerder, D. D., and Calise, A. J., "Convergence of a Numerical Algorithm for calculating Optimal output feedback gains," *IEEE Transactions on Automatic Control*, Vol. 30, No. 9, 1985, pp. 900, 903.

[10]Trofino-Neto, A., and Kucera, V., "Stabilization via static output feedback," *IEEE Transactions on Automatic Control*, Vol. 38, No. 5, 1993, pp. 764, 765.

[11]Kucera, V., and De Souza, C. E., "A necessary and sufficient condition for output feedback stabilizability," *IFAC Journal, AUTOMATICA*, Vol. 31, No. 9, 1995, pp. 1357, 1359.

[12]Geromel, J. C., and Peres, P. L. D., "Decentralized load-frequency control," *IEE Proceedings,* Vol. 132, Pt. D, No. 5, 1985, pp. 225, 230.

[13]El Ghaoui, L., Oustry, F., AitRami, M., "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Transactions on Automatic Control*, Vol. 42, No. 8, 1997, pp. 1171-1176.

[14]Geromel, J.C., de Souza, C.C.; and Skelton, R.E., "Static output feedback controllers: stability and convexity," *IEEE Transactions on Automatic Control*, Vol. 43, No. 1, 1998, pp. 120-125.

[15]Cao, Y., Lam, J., and Sun, Y., "Static Output Feedback Stabilization: An ILMI approach," *IFAC Journal, AUTOMATICA*, Vol. 34, No. 12, 1998, pp. 1641- 1645.

[16]Gahinet, P. , Nemirovski, A., Laub, A., and Chilali, M., LMI Control Tool box, MATLAB., Mathworks, Inc., Natick, MA, 1995.

[17]Zames, G., "Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximate inverses," *IEEE Transactions on Automatic Control*, Vol. 26, No. 2, 1981, pp. 301-320.

[18]Doyle, J. H, Glover, K., Khargonekar, P., and Francis, B.., "State state solutions to standard H_2 and H_∞ Control Problems," *IEEE Transactions on Automatic Control*, Vol. 34, No. 8, 1989, pp. 831-847.

[19]Basar ,T. P., and Bernard, P., *H_∞ Optimal Control and Related Minimax Design Problems*, Berlin, Germany, Birkhauser, 1991, pp 4-6, 33-48.

[20]Knobloch, H. W., Isidori, A., and Flockerzi, D., *Topics in control theory*, Birkhauser, Berlin, Germany, 1993, pp. 43-49, 58-67, 99-111.

[21]Colaneri, P., Geromel, J. C., and Locatelli, A., *Control theory and design, an RH_2 and RH_∞ viewpoint*, Academic Press Interscience, San Diego, 1997, pp. 87-261.

[22]Holl, C., and Scherer, C., "Computing Optimal Fixed Order H_∞-synthesis Values by matrix Sum of Squares Relaxation,"*43rd IEEE Conference on Decision and Control*, Atlantis, Paradise Island, Bahmas, 2004, pp. 3147-3153.

[23]Prempain, E., and Postlethwaite, I., "Static H_∞ loop Shaping Control of a Fly-by-wire Helicopter,"*43rd IEEE Conference on Decision and Control*, Atlantis, Paradise Island, Bahmas, 2004, pp. 1188-1195.

[24] Smerlas A.J., Walker D.J., Postlethwaite I., Strange M.E., Howitt J., Gubbels A.W., "Evaluating. H∞ controllers on the NRC Bell 205 fly-by-wire helicopter," *Control Engineering Practice,* 9(1), 2001, pp. 1-10.

[25]Gadewadikar J , Lewis F., Abu-Khalaf M., "Necessary and Sufficient Conditions for H-∞ Static Output-Feedback Control," *Journal of Guidance, Control, and Dynamics,*Vol. 29, No. 4, July–August 2006, pp. 915-920.

[26]Gadewadikar J, Lewis F., Xie L., Kucera V., Abu-Khalaf M. "Parameterization of All Stabilizing H∞ Static State-Feedback Gains: Application to Output-Feedback Design," *Automatica,* in review,

[27]Chen, B. M.,"*Robust and H_∞ Control*", Springer, Berlin, 2000.
[28]Wonham, W. M., "*Linear Multivariable Control*" Springer Verlag , Berlin, Germany, 1985.
[29]Van der schaft, A. J., "L_2 gain Analysis of Nonlinear Systems and Nonlinear State Feedback H_∞ Control," *IEEE Transactions on Automatic Control*, vol. 37, pp. 770-784, 1992.
[30]Klienman, D. L., "On an Iterative Technique for Riccati Equations Computations," *IEEE Transactions on Automatic Control*, Vol. 13, Issue 1. 1968, pp.114- 115.
[31]Chen B., "Fourth Half Yearly Progress Report for the TYIA 2003 Project on Nonlinear Control Methods for Flight Control Systems of Flying Vehicles,DSTA, NUS, and ST AEROSPACE, Nov. 2005.
[32]Hall Jr. W.E., and Bryson Jr. A.E., "The inclusion of rotor dynamics in controller design for helicopters," *J. Aircraft*, Vol 10, No 4., pp. 200-206.