Topology Homework 2
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1. Show that a space is simply connected if and only if all paths having the same endpoints are fixed endpoint homotopic.

   $\rightarrow$ Suppose X is simply connected. Consider two points x_0 and x_1, and two paths α and β connecting the points. We say $\bar{\beta}$ is $\beta(1-t)$. We can say that $[\alpha] \cong [\alpha] * [\bar{\beta} * \beta]$ because the path $[\bar{\beta} * \beta]$ is homotopic to a constant path at x_1. Then by associativity we have $[\alpha] \cong [\alpha * \bar{\beta}] * [\beta]$. Next, because $[\alpha * \bar{\beta}]$ is a loop based at x_0 we have $[\alpha] \cong [1] * [\beta]$. Finally we get $[\alpha] \cong [\beta]$ because $[1]$ is the identity. Thus α and β are fixed endpoint homotopic.

   $\leftarrow$ Suppose any two paths in X are fixed end point homotopic. Consider a loop that begins and ends at the point x_0 and a path that is constant at x_0. These paths have the same endpoints so they are homotopic. Thus any loop based at x_0 will be homotopic to the path that is constant at x_0, and X is by definition simply connected.

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