1. A compact set $E \subset \mathbb{R}^n$ is bounded.
   *Proof.* By contrapositive, suppose $E$ is not bounded. This means that $E$ is not covered by any ball of finite radius. However, all of $\mathbb{R}^n$, and thus $E$, will be covered by the sequence of open balls
$$B_n := \{x : |x| < n\}.$$
   Any union of a finite subset of these open balls will be an open ball of finite radius, and thus will not cover $E$. Therefore $E$ is not compact.$\square$

2. A compact set $E \subset \mathbb{R}^n$ is closed.
   *Proof.* We will prove that the complement of $E$ is open. Consider a point $x \in \backslash E$ and a collection of sets
$$G_k := \{y : |y - x| > \frac{1}{k}\}.$$
   We can see that $\cup_{k=1}^{\infty} G_k = \mathbb{R}^n / x$. Since $x \notin E$, $E \subset \cup G_k$, which means $G_k$ is an open cover $E$. We know that $E$ is compact so it can be covered by finitely many of the $G_k$. The $G_k$ are ordered by containment ($G_k \subset G_{k+1}$) so $E$ can be covered by exactly one of the $G_k$, call this set $G_l$. Now, all of the $G_k$ with indices strictly greater than $l$ have complements that are disjoint from $E$ ($\backslash G_{k+1} \subset \backslash G_k$). The interiors of the complements of the $G_k$ are the open balls $B(x, \frac{1}{k})$. This sequence of open balls for all $k > l$ contains the point $x$ and does not intersect $E$. The point $x$ was arbitrarily chosen in $\backslash E$. Therefore $\backslash E$ is open, and $E$ by definition is closed. $\square$

3. Let $[a_j, b_j]$ be a nested sequence of nonempty closed intervals in $\mathbb{R} : [a_j, b_j] \supset [a_{j+1}, b_{j+1}]$ for all $j$. Then $\cap_{j=1}^{\infty} [a_j, b_j]$ is non-empty.
   *Proof.*

4. Let $[\mathbf{a_j}, \mathbf{b_j}]$ be a nested sequence of nonempty closed n-cells in $\mathbb{R}^{\ltimes} : [\mathbf{a_j}, \mathbf{b_j}] \supset [\mathbf{a_{j+1}}, \mathbf{b_{j+1}}]$ for all $j$. Then $\cap_{j=1}^{\infty} [\mathbf{a_j}, \mathbf{b_j}]$ is nonempty.
   *Proof.*

5. Any closed n-cell is compact.
   *Proof.*

6. A closed subset $F$ of a compact set $K$ in $\mathbb{R}^n$ is compact.
   *Proof.*