Modern Measurements of Market Liquidity

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Runs and a new approach to Liquidity

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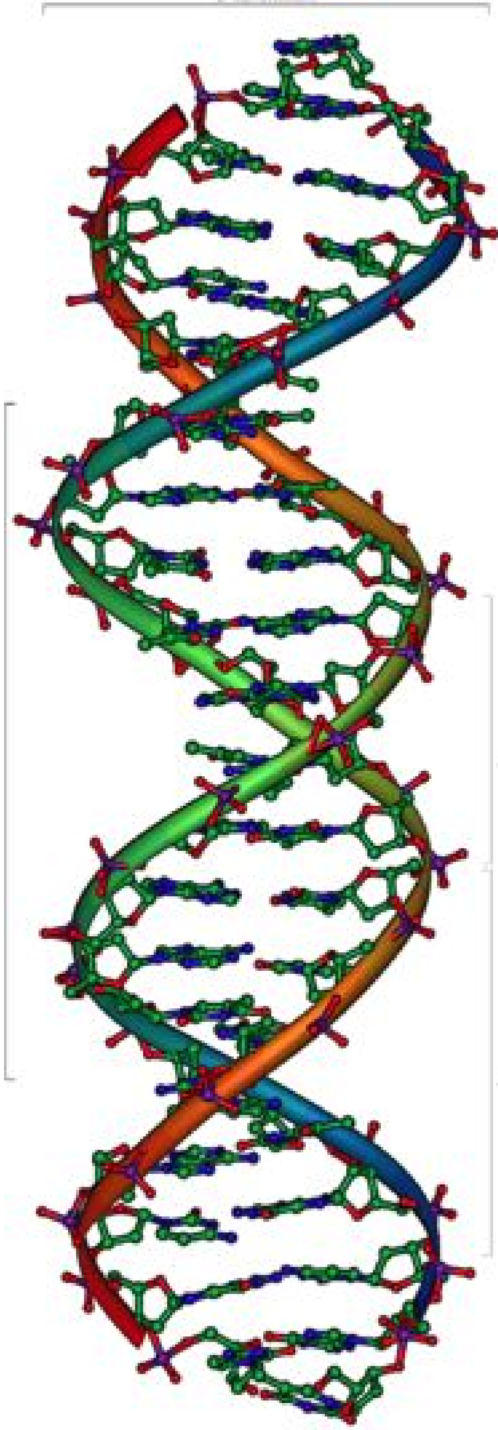
Liquidity

1. How much trading activity is there in a stock? Trading volume, turnover.
2. How much can you trade at any point in time without moving the price too much?

Run Length: the DNA of illiquidity:



Model

Stock returns are assumed to follow a random walk. We define h to be the inter-arrival time between trades. Thus, we examine an i.i.d. process for stock return innovations, conditional on h, i.e.

$$R|h = \mu h + \sigma \epsilon \sqrt{h}, \quad \epsilon \sim N(0,1), \tag{1}$$

which implies a return per trade interval with mean μh and variance $\sigma^2 h$. The variables μ and σ are the annual return and standard deviation of return (per chosen unit time interval) respectively.

The probability of a negative return for trade interval h is

$$p(h) = \Pr[R < 0] = \Pr[\mu h + \sigma \epsilon \sqrt{h} < 0] = \Phi\left(\frac{-\mu\sqrt{h}}{\sigma}\right) \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Note here that as $h \to 0$, this probability tends to one-half, because the variance swamps the mean.

Run Length

A run is an series of positive (negative) moves in the stock price, until interrupted by a negative (positive) move.

Conditional on being in a positive run, the average length of such runs will depend on the probability that a negative return does *not* occur. To begin, say we are interested in the survival probability of a positive run. Define L^+ as the length of a positive run. Then,

$$\Pr[L^+ = n] = [1 - p(h)]^{n-1}\, p(h). \tag{3}$$

For example, if $n = 1$, the run is comprised of one positive return, and is terminated. The probability of this happening is $p(h)$.

What is the average run length of a random walk?

Average Run Length

The average run length for positive runs is then given by the following infinite sum:

$$\begin{aligned}\bar{L}^{+} &= \sum_{n=1}^{\infty}\left\{n \times \Pr[L^{+}=n]\right\} \\ &= \sum_{n=1}^{\infty}\left\{n \times [1-p(h)]^{n-1}\, p(h)\right\} = \frac{1}{p(h)}.\end{aligned}$$

By analogy, the average length of a negative run is

$$\bar{L}^{-} = \frac{1}{1-p(h)}.$$

Average RL over positive and negative runs

In the limit, since there are an equal number of positive and negative runs (over a long sequence of trades), the average run length will be the average of the averages of positive and negative run lengths, i.e.,

$$\begin{aligned}\bar{L} &= \frac{1}{2}[\bar{L}^{+} + \bar{L}^{-}] \\ &= \frac{1}{2}\left[\frac{1}{p(h)} + \frac{1}{1-p(h)}\right], \\ &= \frac{1}{2}\left[\frac{1}{\Phi\left(\frac{-\mu\sqrt{h}}{\sigma}\right)} + \frac{1}{\Phi\left(\frac{\mu\sqrt{h}}{\sigma}\right)}\right] \end{aligned} \tag{7}$$

where the second line follows from equations (5) and (6). The third line comes from the result in equation (2).

Example: if $\mu = 0$, then we have a symmetric random walk, and $p(h) = 0.5, \forall h$. In this case, $\bar{L} = 2$.

How run length varies with inputs

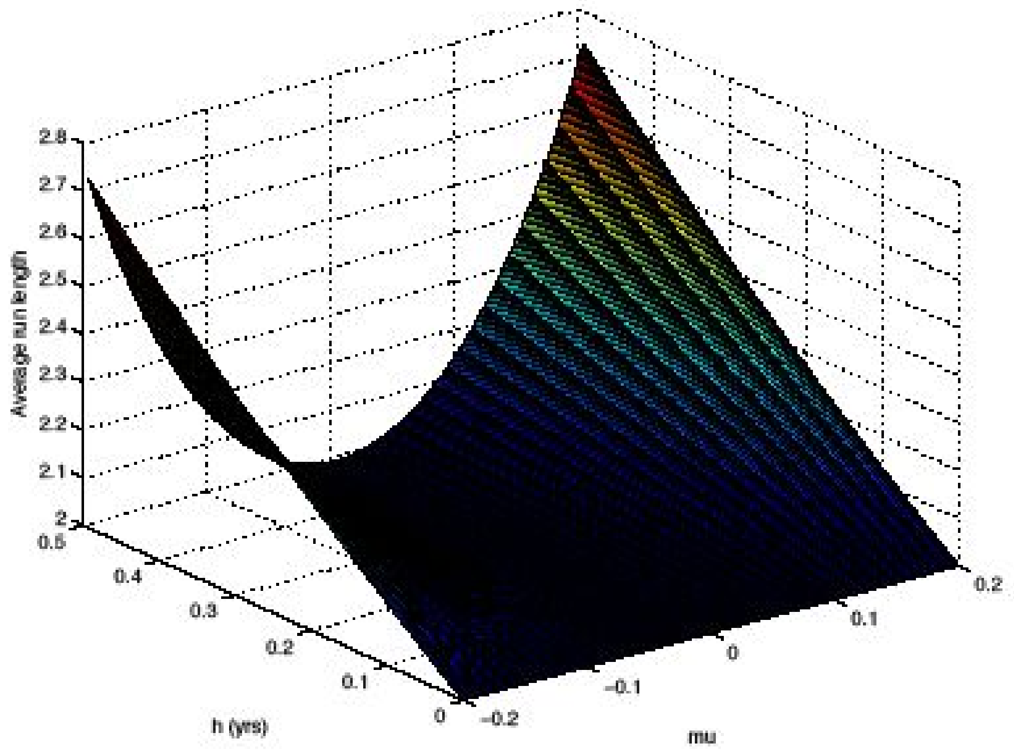


Figure 1: Variation in average run length with changing μ and h. ($\sigma = 0.2$). Think of h as a proxy for transaction volume, and μ as a proxy for price impact.

Runs and Trading Activity

Result 1: Trading activity. Run length is inversely related to trading activity (shorter trade inter-arrival times h), as may be seen from equation (7), where $\frac{\partial L}{\partial h} > 0$. This embodies the simple idea that when there is high liquidity, trading activity rises, which implies that the time interval between trades (h) shrinks. When $h \to 0$, i.e. as the frequency of sampling increases, the average run length tends to 2.

Corollary (Volatility): A known empirical regularity is that trading volume and volatility (σ) are positively related, see Karpoff (1987). Therefore, if volatility (related to trading volume) increases, equation (7) would indicate that the average run length would tend to decline ($\frac{\partial L}{\partial \sigma} < 0$), and in the limit would be 2.

Runs and Price Impact

Result 2: Price Impact. Trade price impact [see Glosten (1989), Chacko and Stafford (2004) for models of this] in equation (1) is a function of the absolute return per unit risk (the ratio $|\mu|/\sigma$ in equation 7) from each trade (keeping h fixed). The average run length $\bar{L}$ increases in $|\mu|/\sigma$. This is intuitive because the tendency to drift in one direction increases. Hence, increasing average run length corresponds to greater price impact from each trade, or lower liquidity.

Summary so far:

1. Average run length positively related to illiquidity
2. Trading activity negatively related to illiquidity
3. Price impact positively related to illiquidity

Extension to random trade arrivals

Random trade arrivals and sampling

Suppose h depends on a random arrival of trades. If the rate at which trades arrive is Poisson with parameter λ, then the average inter-arrival time between trades will be $1/\lambda$. The distribution of inter-arrival times will be exponential and the probability density of h will be $(1/\lambda)\exp[-h/\lambda]$. Simple extension of the previous calculations shows that the average run length will be given by

$$\bar{L} = \frac{1}{2}[\bar{L}^{+} + \bar{L}^{-}] = \frac{1}{2}\left[\frac{1}{p(\lambda)} + \frac{1}{1-p(\lambda)}\right], \tag{8}$$

$$p(\lambda) = \int_0^{\infty} \Phi\left(\frac{-\mu\sqrt{h}}{\sigma}\right)\frac{1}{\lambda}e^{-h/\lambda}\,dh. \tag{9}$$

The results have now been expressed as a function of the trade arrival rate λ as opposed to the trade interval h. It is easily checked that Results 1 and 2 above are unaltered for random arrivals. As λ increases, the average trade inter-arrival time decreases, and the run length falls as well. Therefore, random arrivals of trades do not impact the results we obtained from constant trade inter-arrival times. Finally, we note that the average number of runs per unit time will be $\lambda/\bar{L}$.

Sampling Frequency

Table 1: The effect of sampling interval on run lengths. We present the results of a simulation experiment to show that changes in sampling frequency do not change the ordering of stocks by run length. We assumed that trades arrive at Poisson frequency with mean λ trades per day for a total of 2600 days. We chose four different values of the trade arrival rate, $\lambda = \{0.25, 1, 10, 40\}$ trades per day, representing increasing frequency of trades (resulting in essentially four different stocks). We set $\mu = 0.1$ and $\sigma = 0.2$. We simulated price paths for all these stocks by generating inter-trade times (h) from an exponential distribution with parameter $1/\lambda$, and then returns using equation (1). Once the price series for all 2600 days is generated, we then sample the series at four different frequencies: (i) each trade, (ii) hourly, (iii) daily, (iv) weekly. The ordering of stocks by run length does not change with sampling frequency - note that the ordering of run length in each column has remained the same, even though the stock price is being sample less frequently as we go from left to right in the table. In fact, sampling less frequently exaggerates the difference in run lengths across the various trade frequencies. The numbers in columns 2 through 5 in the table below are the average run lengths for four stocks of different trade arrival rates (rows), sampled at four different frequencies (columns).

λ	Sampling frequency			
	Each trade	Hourly	Daily	Weekly
0.25	3.1000	3.4000	3.6122	12.0833
1.00	2.3648	2.2932	2.6023	5.3718
10.00	2.0167	2.0891	2.4351	4.7843
40.00	2.0100	2.0200	2.4235	4.2667

Relation of Runs to the Existing Literature

1. Conrad, Hameed, and Niden (1994): high transaction (or high volume) securities evidence greater reversals (lower run lengths).
2. The signed trading volume measure of Pastor and Stambaugh (2003) is consistent with the logic that increases in order flow result in shorter runs, as market-makers earn premia from injecting liquidity into the market.
3. Campbell, Grossman and Wang (1993) find that increased trading volume reduces serial dependence, analogous to shorter runs.
4. Hendershott and Seasholes (2007) show that liquidity injections through inventory build-ups by market makers is followed by reversals (shorter runs) that make it profitable for them to act as liquidity providers.
5. Run lengths are (in theory) positively correlated with the no-trade day count liquidity measure of Liu (2003).
6. Avramov, Chordia and Goyal (2006) show that liquidity infusions, after controlling for trading volume, result in price reversals (shorter runs).
7. Chordia, Roll and Subrahmanyam (2005) provide evidence that liquidity reduces autocorrelation in returns (shorter runs).
8. Momentum: Jegadeesh and Titman (1993), Carhart (1997). Long-run vs short-run phenomenon.

Data

Using the CRSP daily files we compute the yearly run length characteristics of each eligible stock on the NYSE, Amex, and Nasdaq exchanges from January 1962 to December 2005. Eligible stocks are common stocks with a year-end stock price between $5 and $1000, trading on the NYSE, AMEX and Nasdaq for at least 12 months. We took care to purge the daily file of equity-days where there was no trading volume to eliminate the possibility that a run is caused by stale data. We then proceed to identify for each stock the length of every run-up and run-down which we define as a period of uninterrupted rise (drop) in stock price by using the equity returns including all distributions calculated in CRSP. Days where there are zero returns for a given stock are assumed not to interrupt the current run (see the appendix which contains the algorithm). Run lengths are averaged for each security to create an average run length for the period of interest.

Descriptive Statistics

Period	Cross-Sectional Statistics	Run Length Mean	Run Length S Dev	Return (%) Mean	Return (%) S Dev
1962-1965	Mean	2.554	1.994	0.07462	1.65847
	Median	2.410	1.811	0.06406	1.50509
	N	3549	3549	3549	3549
1966-1970	Mean	2.479	1.908	0.04910	2.22603
	Median	2.404	1.804	0.03591	2.06242
	N	6650	6650	6650	6650
1971-1975	Mean	2.588	2.029	0.04425	2.27593
	Median	2.495	1.898	0.04381	2.16861
	N	4932	4932	4932	4932
1976-1980	Mean	2.531	1.953	0.10441	2.11741
	Median	2.471	1.863	0.08262	1.97729
	N	5868	5868	5868	5868
1981-1985	Mean	4.426	4.085	0.09026	2.19031
	Median	2.635	2.133	0.08847	2.05590
	N	10937	10937	10937	10937
1986-1990	Mean	3.948	3.440	0.04178	2.52132
	Median	2.505	1.972	0.04532	2.36671
	N	15190	15190	15190	15190
1991-1995	Mean	3.083	2.587	0.09645	2.77401
	Median	2.364	1.783	0.08514	2.53696
	N	18570	18570	18570	18570
1996-2000	Mean	2.509	2.029	0.09263	3.31840
	Median	2.230	1.641	0.07543	2.96127
	N	22185	22185	22185	22185
2001-2005	Mean	2.190	1.640	0.10244	2.75938
	Median	2.016	1.419	0.08293	2.40411
	N	17782	17782	17782	17782

Sorting Trading Volume by Factors

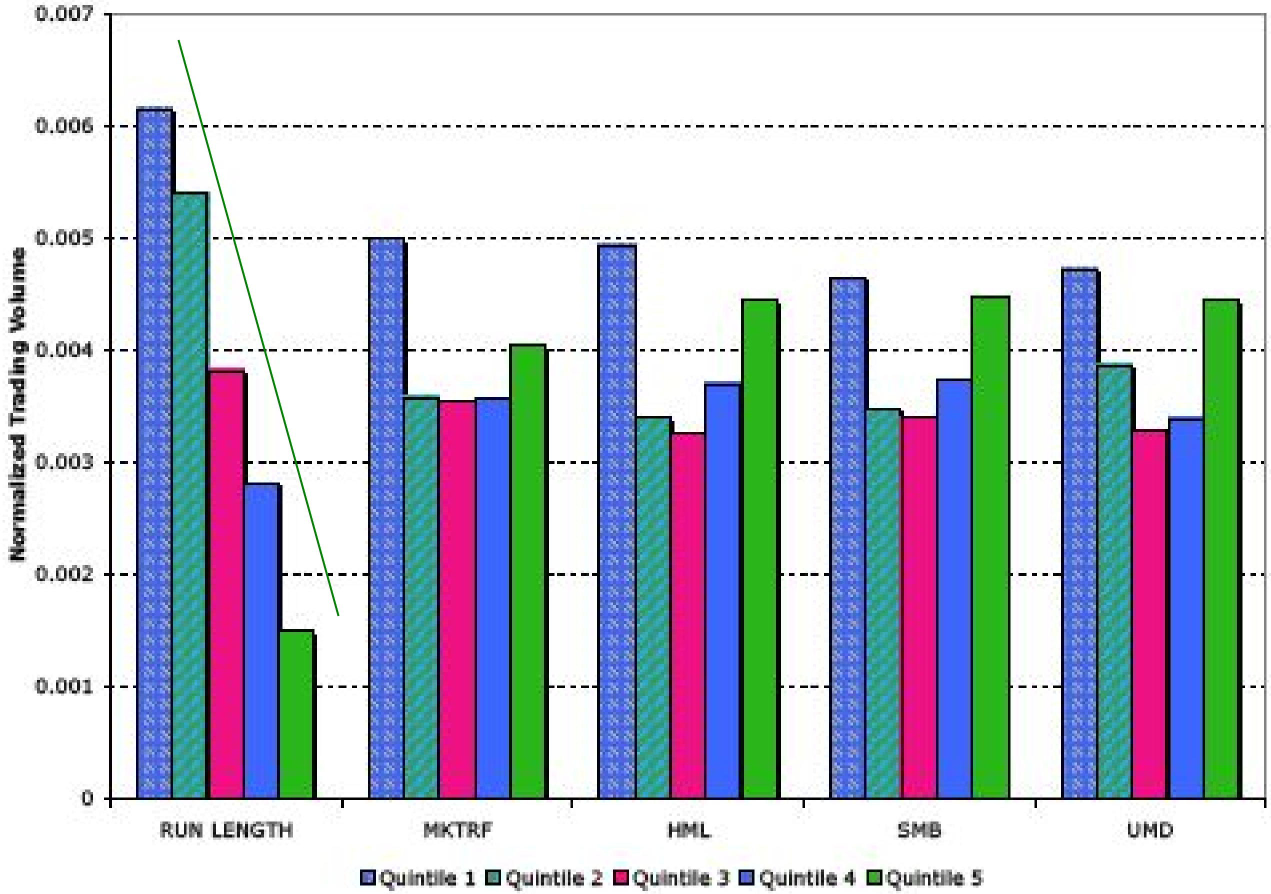


Figure 2: Sorts of normalized trading volume by quintiles formed on run length and Fama-French factors. We sorted all firms into quintiles based on the desired variable, i.e. run length, beta coefficients on the standard asset-pricing factors: Excess market return, HML, SMB, and UMD. For each stock we computed the normalized daily trading volume as the ratio of shares traded in a day to the outstanding shares in the firm. We then calculate the trading volume (equally weighted across firms) in each quintile. The only monotone pattern for trading volume is in the run length quintiles. We see that trading volume declines from the highest run length quintile to the lowest.

Turnover

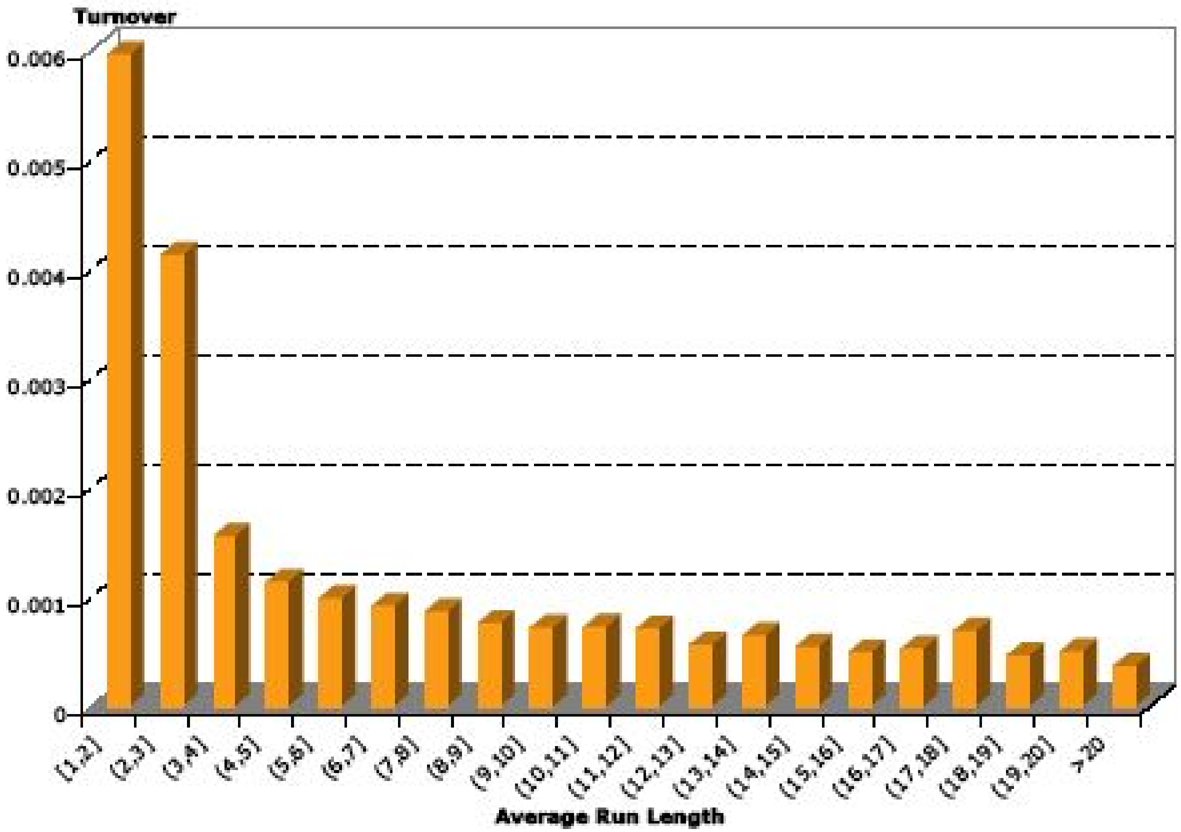


Figure 3: Run length and turnover. The plot shows the relationship between normalized trading volume (turnover) as run length increases. The declining turnover evidences the fact that run length may be used as a proxy for illiquidity.

Amihud Illiquidity

$$ILLIQ_{it} = \frac{1}{DAYS_{it}} \sum_{t=1}^{DAYS_{it}} \frac{|r_{it}|}{PRC_{it} \times VOL_{it}} \times 10^6$$

Table 3: Run lengths and the Amihud measure. This table presents average Amihud illiquidity for groups determined by sorting on run length. This measure is as follows for each stock i: $ILLIQ_{it} = \frac{1}{DAYS_{it}} \sum_{t=1}^{DAYS_{it}} \frac{|r_{it}|}{PRC_{it} \times VOL_{it}} \times 10^6$, where r_{it} is the ith stock's return for day t, PRC_{it} is closing price, and VOL_{it} is trading volume, which is the number of shares traded for a firm. $DAYS_{it}$ is the number of trading days for stock i in year t. Normalized trading volume (turnover) is defined as trading volume divided by shares outstanding. The quoted bid-ask spread is defined as the difference between the ask and the bid prices divided by the average of the two. All measures are equally-weighted.

Run Quintile:	Lowest	2	3	4	Highest
ILLIQ:	0.2634	0.3470	0.4512	0.7144	3.1385
Turnover:	0.0062	0.0055	0.0039	0.0028	0.0015
Bid-Ask Spread:	0.0154	0.0209	0.0251	0.0286	0.0550

Pre-sorting by Return

Table 4: The relation of Amihud's illiquidity measure with our run length measure. We examine illiquidity after controlling for contemporaneous stock returns (Panel A) and the contemporaneous standard deviation of stock returns (Panel B). The table reports a two-way 5×5 sort in which we first sort stocks by return (standard deviation of returns) into quintiles, and then within quintiles, sort stocks by run length. In each of the 25 cells in Table 4, we report the average of Amihud's illiquidity measure. We can see that the sort by our run length illiquidity measure lines up in the same way as does Amihud's measure, even after pre-sorting by returns or by standard deviation of returns.

Panel A: Stock Returns

	Run Quintile				
	Lowest	2	3	4	Highest
Average Return Quintile					
1	0.280	0.333	0.430	0.652	3.187
2	0.234	0.302	0.378	0.697	3.373
3	0.203	0.312	0.380	0.657	2.918
4	0.238	0.332	0.436	0.647	2.956
5	0.389	0.469	0.637	0.992	3.353

Panel B: Standard Deviation of Stock Returns

	Run Quintile				
	Lowest	2	3	4	Highest
StdDev of Return Quintile					
1	0.178	0.251	0.312	0.793	3.651
2	0.212	0.253	0.355	0.576	3.040
3	0.276	0.323	0.433	0.614	2.896
4	0.287	0.410	0.488	0.732	2.723
5	0.397	0.474	0.653	1.034	3.484

Table 5 Explaining liquidity with run length. The table presents results of regressions of two measures of liquidity, bid-ask spreads (BIDASK) and the Amihud illiquidity (ILLIQ) measure. After controlling for trading volume (NORMVOL), auto-correlation (AUTOCORR) in returns, an interaction term (NORMVOL*AUTOCORR), and separately we include the number of zero-return days (ZERORET) as in Lesmond, Ogden and Trzcinka (1999). In all cases the role of run length (MEANRUNLEN) as a proxy for illiquidity remains strongly statistically significant. In order to account for firm fixed-effects we demean each variable by the firm average. T-statistics estimated using GMM with the Newey and West (1987) correction with 2 lags are reported below the parameter estimates.

	Dependent variable			
	BIDASK		ILLIQ	
MEANRUNLEN	0.2667	0.6271	0.0014	0.0086
	8.21	25.54	5.74	43.06
AUTOCORR	-3.2877	-4.53892	-0.03432	-0.05993
	-24.35	-37.39	-36.15	-58.8
NORMVOL	-20.8808	-40.5924	-0.2116	-0.50297
	-3.85	-7.14	-8.78	-10.38
NORMVOL*AUTOCORR	246.3357	300.1359	1.4296	2.4606
	8.76	10.17	10.54	10.71
ZERORET	0.0166		0.0003	
	25.22		60.73	
R^2	11.99%	9.80%	36.76%	26.47%
N	66,732		105,200	

Linkage to other Liquidity Measures

Table 9 Principal components analysis of liquidity measures. We explore the commonality across different liquidity measures using a principal component analysis (PCA). In the PCA we use 6 monthly time-series of liquidity: bid-ask spread (BASPREAD), turnover (TURN), the Amihud illiquidity measure (ILLIQ), and the number of zero return days (ZERORET) are averaged monthly across securities. We also include the Pastor and Stambaugh (2003) liquidity innovations (PSINNOV). The measure of mean run length (MEANRUNLEN) is averaged across firms each month. The three panels below present the correlation matrix of the liquidity measures, the eigenvalues of the correlation matrix, and the loadings or eigenvectors. In the correlation matrix, each cell contains two numbers. The number on top is the correlation, and the number below it is the p-value. The number of paired observations in each cell used to compute the correlation is the number of months of data, i.e. 276.

Panel A: Correlation Matrix

	MEANRUNLEN	TURN	BASPREAD	ZERORET	ILLIQ
TURN	-0.7909				
	.0001				
BASPREAD	0.3810	-0.5896			
	.0001	.0001			
ZERORET	0.8736	-0.8729			
	.0001	.0001			
ILLIQ	0.3909	-0.5477	0.8444	0.6074	
	.0001	.0001	.0001	.0001	
PSINNOV	0.0603	-0.0789	-0.0150	0.0883	0.0153
	0.3183	0.1913	0.8042	0.1437	0.8005

Panel B: Eigenvalues of the Correlation Matrix

	Eigenvalue	Proportion	Cumulative
1	3.6549	0.6091	0.6091
2	1.0461	0.1744	0.7835
3	0.8961	0.1494	0.9329
4	0.1967	0.0328	0.9656
5	0.1575	0.0262	0.9919
6	0.0487	0.0081	1.0000

Conclusions

- Run lengths match features of liquidity.
- Run lengths are easy to compute - require no volume data.
- Longer average run length stocks earn higher returns.
- We can enhance the CAPM with average run length as a characteristic factor.

A New Measure of Illiquidity

Think of liquidity as the "price impact of a trade."

Illiquidity may be measured by this price impact, which increases when the price of immediacy is high.

We may use the price change of an ETF relative to the price change of the underlying NAV to extract the illiquidity in the market.

Think of illiquidity as the value of an option to exchange the ETF for the NAV, i.e., the difference between the ETF and the NAV upon exercise of the option is the effective transaction cost, and therefore the value of this option is the value of liquidity.

$$Illiquidity = Call(ETF, NAV) + Put(ETF, NAV)$$

$$Illiquidity^{*} = Call\left(\frac{ETF}{NAV}, 1\right) + Put\left(\frac{ETF}{NAV}, 1\right)$$

"An Index-Based Measure of Liquidity" (George Chacko, Sanjiv Das, Rong Fan)

BILLIQ

$$
\begin{aligned}
BILLIQ &= -10,000 \times \log\left[\frac{1}{1 + Illiquidity^*}\right] \\
&= -10,000 \times \log\left[\frac{1}{1 + Call(\frac{ETF}{NAV}, 1) + Put(\frac{ETF}{NAV}, 1)}\right]
\end{aligned}
$$

$$
\begin{aligned}
Illiquidity^* &= Call\left(\frac{ETF}{NAV}, 1\right) + Put\left(\frac{ETF}{NAV}, 1\right) \\
&= \max\left[\frac{ETF}{NAV} - 1, 0\right] + \max\left[1 - \frac{ETF}{NAV}, 0\right] \\
&= \left|\frac{ETF}{NAV} - 1\right|
\end{aligned}
$$

$$
BILLIQ = -10,000 \times \log\left[\frac{1}{1 + \left|\frac{ETF}{NAV} - 1\right|}\right]
$$

$$
BILLIQ = -10,000 \times \log\left[\frac{NAV}{NAV + |ETF - NAV|}\right]
$$

Table 1: Data Description

This table provides a description of each of the ETFs used in the study to compute bond illiquidity measures. All the ETF time series run up to April 1, 2009.

Ticker	Full Title	Issuer/Industry	
LQD	iShares iBoxx Investment Grade Bond Fund	Corp/Pref-Inv Grade	
HYG	iShares iBoxx High Yield Corporate Bond Fund	Corp/Pref-High Yield	
CSJ	iShares Barclays 1-3 Year Credit Bond Fund	Government/Corporate	
CFT	iShares Barclays Credit Bond Fund	Government/Corporate	
CIU	iShares Barclays Intermediate Credit Bond Fund	Corp/Pref-Inv Grade	
AGG	iShares Barclays Aggregate Bond Fund	Government/Corporate	
GBF	iShares Barclays Government/Credit Bond Fund	Government/Corporate	
GVI	iShares Barclays Intermediate Government/Credit Bond Fund	Government/Corporate	
MBB	iShares Barclays MBS Bond Fund	Asset Backed Securities	
EMB	iShares JP Morgan USD Emerging Markets Bond Fund	Emerging Market-Debt	
IVV	iShares S&P500 Index (NYSE)	Equity	
Ticker	**Rating Focus**	**Maturity Focus**	**Start Date**
LQD	Investment Grade	Intermediate Term (3-10 yr)	7/26/02
HYG	Speculative Grade/High Yield	Intermediate Term (3-10 yr)	4/11/07
CSJ	Investment Grade	Short Term (1-3 yr)	1/11/07
CFT	Investment Grade	Short/Intermediate Term	1/11/07
CIU	Investment Grade	Intermediate Term (3-10 yr)	1/11/07
AGG	Investment Grade	No Restriction	9/26/03
GBF	Investment Grade	Short/Intermediate Term	1/11/07
GVI	Investment Grade	Short/Intermediate Term	1/11/07
MBB	Investment Grade	No Restriction	3/16/07
EMB	Mixed	Intermediate/Long Term	12/19/07
IVV	N/A	N/A	5/15/00

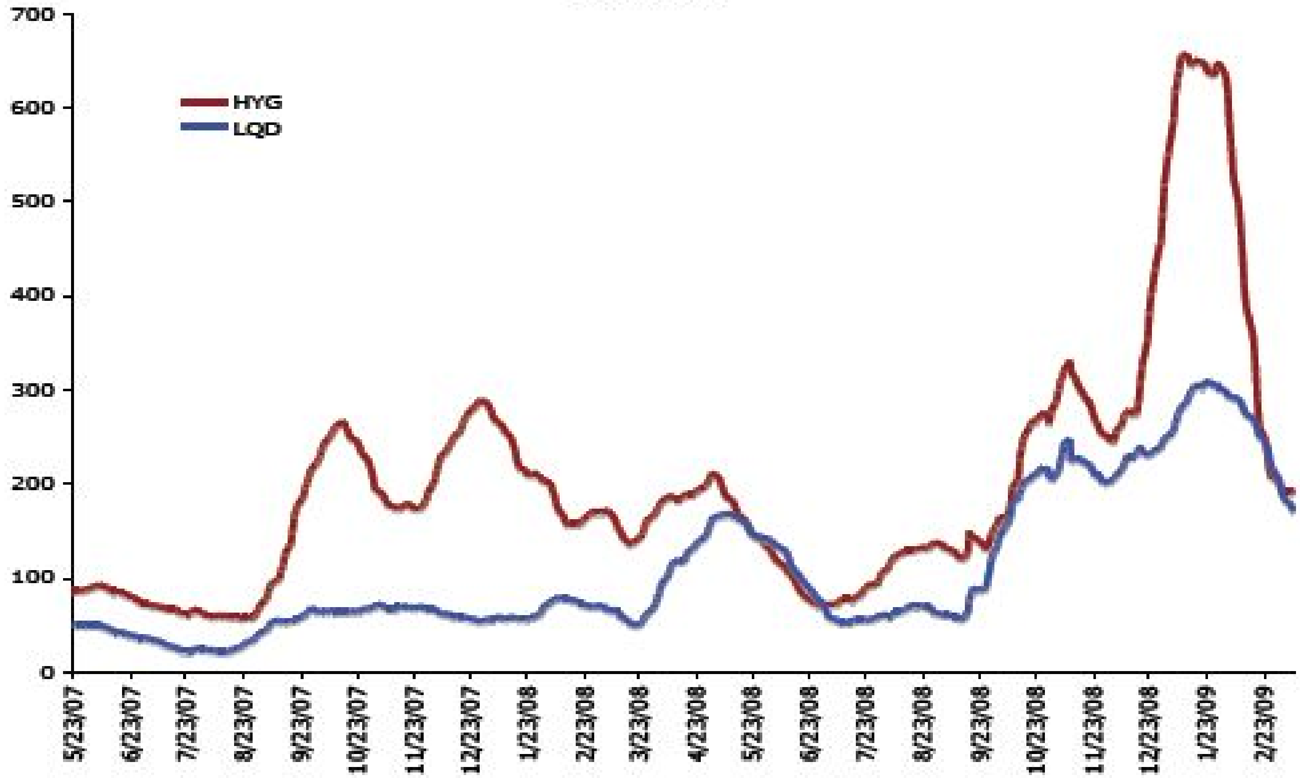


Figure 1: *BILLIQ* Time Series

This graph presents the time series of the various *BILLIQ* measures extracted from the ETFs. All series are presented in the upper graph. See Table 1 for a description of the different series. The lower graph shows the 30-day moving average of the illiquidity series for the investment grade sector (LQD) and high-yield sector (HYG) from May 2007 to March 2009. Illiquidity is expressed in basis points.

Table 2: Descriptive Statistics of ETF Series

The data is daily. The number of observations is the number of days for which it was possible to compute *BILLIQ* for each ticker. *BILLIQ* is represented in basis points. Price and NAV are in dollars. Volume is the number of units of the ETF that are traded per day. The second panel displays the correlations of returns amongst the ETFs. The lower triangle of the correlation matrix below comprises correlations computed for only the period over which all series had complete observations. In contrast, the upper triangle of the correlation matrix presents the pairwise correlations for all observations where complete data is available for the pair of ETFs. Hence, the upper triangle comprises correlations computed off more observations than the lower triangle. Bold font values are significant at the 5% level.

Ticker	Mean Price	Mean NAV	Mean Volume	Mean BILLIQ	No of Obs
LQD	106.62	106.13	259395	59.1259	1669
HYG	91.77	90.34	215750	198.7806	497
CSJ	100.71	99.80	55293	96.6358	557
CFT	97.67	96.74	13445	109.8060	552
CIU	98.78	97.84	30376	106.0145	553
AGG	100.74	100.43	307023	40.1194	1382
GBF	101.38	100.97	12096	48.3448	546
GVI	102.08	101.61	14064	51.8541	556
MBB	101.74	101.58	65428	18.9880	505
EMB	93.24	92.20	16637	160.3290	321
IVV	118.67				

	LQD	HYG	CSJ	CFT	CIU	AGG	GBF	GVI	MBB	EMB
LQD	1	**0.5261**	**0.4453**	0.0380	0.0249	0.9784	-0.0093	0.0305	0.0330	0.0420
HYG	**0.5599**	1	**0.3045**	0.0243	**0.2337**	**0.3334**	-0.0121	**0.0889**	-0.0032	0.0617
CSJ	**0.4365**	**0.3194**	1	0.0628	0.0294	0.5148	0.0054	0.0396	0.0231	0.0631
CFT	0.0648	0.0462	0.0762	1	-0.0042	0.0572	-0.0104	0.0001	0.0016	0.0062
CIU	**0.4307**	**0.2497**	**0.4479**	0.0608	1	0.0298	**0.3786**	-0.0016	0.0478	0.0667
AGG	**0.4349**	**0.3506**	**0.5101**	0.0697	**0.5379**	1	-0.0025	0.0428	0.0442	0.0695
GBF	**0.2706**	0.0227	**0.3615**	0.0795	**0.5170**	**0.4558**	1	-0.0053	-0.0063	0.0682
GVI	**0.3358**	0.0990	**0.4170**	0.0547	**0.3365**	**0.4563**	**0.5438**	1	0.0574	0.0344
MBB	**0.2762**	-0.0153	**0.2470**	0.0541	**0.4026**	**0.4337**	**0.4916**	**0.4873**	1	0.0146
EMB	0.0419	0.0617	0.0631	0.0062	0.0668	0.0697	0.0683	0.0345	0.0147	1

Table 3: Bond Liquidity Series Correlations

The lower triangle of the correlation matrix below comprises correlations computed for only the period over which all series had complete observations. In contrast, the upper triangle of the correlation matrix presents the pairwise correlations for all observations where complete data is available for the pair of ETFs. Hence, the upper triangle comprises correlations computed off more observations than the lower triangle. Only two correlations, the values in italics, are not significant at the 5% level; all other correlations are significant, mostly at the 1% level. The last line and column of the table shows the correlations of the various liquidity series with the Treasury-Eurodollar (TED) spread. Only two of these correlations are not significant (in italics) at the 5% level.

	LQD	HYG	CSJ	CFT	CIU	AGG	GBF	GVI	MBB	EMB	TED
LQD	1	0.6147	0.7061	0.7069	0.7224	0.4307	0.5859	0.6373	0.5832	0.3787	0.5820
HYG	0.6210	1	0.5989	0.5575	0.6088	0.4393	0.5124	0.6014	0.5618	0.5697	0.3588
CSJ	0.6158	0.6143	1	0.8240	0.8873	0.4129	0.7523	0.8485	0.5697	0.4954	0.3098
CFT	0.6182	0.5428	0.7473	1	0.8333	0.5942	0.7784	0.7993	0.5300	0.4165	0.4108
CIU	0.6325	0.6140	0.8351	0.7525	1	0.3621	0.7508	0.8414	0.5598	0.4415	0.2566
AGG	0.3532	0.4124	0.3332	0.5727	0.2635	1	0.5855	0.4898	0.3545	0.2975	0.4433
GBF	0.4773	0.4843	0.6773	0.7148	0.6691	0.5500	1	0.8052	0.5457	0.4373	0.3358
GVI	0.5300	0.6008	0.7923	0.7281	0.7777	*0.4288*	0.7527	1	0.6213	0.4889	0.3526
MBB	0.5607	0.5706	0.5326	0.4839	0.5180	*0.3152*	0.4943	0.5953	1	0.4152	0.4486
EMB	0.3779	0.5709	0.4942	0.4165	0.4406	0.2972	0.4368	0.4882	0.4141	1	0.2018
TED	0.3299	0.2435	*0.0969*	0.1824	*-0.0331*	0.4784	0.1555	0.1737	0.4474	0.2020	1

Table 4: Relationship between *BILLIQ*s and measures of price impact.

We report the correlation between our bond illiquidity time series and the illiquidity measure of Amihud (2002), as well as the absolute returns on an ETF of financial institutions (i.e., the XLF). The Amihud illiquidity measure is computed for the investment grade, high-yield, and combined bond returns. The measure is the absolute price return divided by trading volume for the day, and this is then averaged over the required period. Finally the measure is scaled by multiplying it by 100 to prevent the numbers from being too small. Return is calculated from daily price history where available, and is divided by the trading volume. Since price impact increases with illiquidity, we expect the correlations reported here to be statistically significant. The correlations that are reported in italics are the only ones that are not statistically significant at the 99% level. All correlations are positive.

	Correlation with			
Ticker	Amihud (All US Bonds)	Amihud (Inv Grade)	Amihud (High Yield)	Abs(XLF)
LQD	0.80	0.76	0.74	0.66
HYG	0.61	0.66	0.46	*0.21*
CSJ	0.72	0.72	0.63	0.60
CFT	0.76	0.78	0.63	0.68
CIU	0.61	0.63	0.52	0.54
AGG	0.72	0.81	0.52	0.61
GBF	0.66	0.69	0.55	0.61
GVI	0.70	0.73	0.58	0.53
MBB	0.77	0.81	0.61	0.34
EMB	0.69	0.75	0.51	*0.31*

Table 6: Relationship of $BILLIQ$ to Another Bond Illiquidity Measure

Results of regressing $BILLIQ$ for each ETF on the reciprocal of the Chacko illiquidity measure. We denote this measure $ILLIQFAC$. The regression equation is $BILLIQ = b_0 + b_1\ ILLIQFAC + \epsilon$. We report the coefficients, t-statistics, adjusted R-squareds, and the P-values of the F-statistics. The construction of $ILLIQFAC$ is described in Chacko (2009). The $ILLIQFAC$ series is monthly and so we have regressed the average $BILLIQ$ for each month on $ILLIQFAC$. The regressions have been adjusted for autocorrelation using the Cochrane-Orcutt correction. There are 81 observations of $ILLIQFAC$.

Ticker	b_0	b_1	b_0 tstat	b_1 tstat	R-sq	P-value	DW
LQD	-912.94	1,330.97	-3.22	3.46	41.4%	0.13	1.88
HYG	-1,173.92	1,877.45	-2.27	2.67	25.3%	0.01	1.94
CSJ	-1,319.32	1,945.24	-4.87	5.28	53.7%	0.00	1.17
CFT	-1,031.66	1,568.60	-5.58	6.23	61.8%	0.00	1.54
CIU	-1,091.06	1,644.77	-2.93	3.25	30.5%	0.00	1.13
AGG	-343.56	526.07	-5.52	6.17	37.3%	0.00	1.99
GBF	-497.94	749.06	-5.82	6.39	63.0%	0.00	1.75
GVI	-638.43	947.61	-5.85	6.35	62.7%	0.00	1.89
MBB	-134.83	210.29	-3.07	3.52	36.0%	0.00	1.73
EMB	-1,277.57	1,906.59	-2.55	2.87	38.8%	0.01	1.78

Table 9: Regression of Bond Index Returns on Pricing Factors.

PANEL A	Dependent Variables					
Independent Variables	USCorp Inv grade All	USCorp Inv grade Intermediate	USCorp Inv grade Long Term	USCorp High Yield All	USCorp High Yield Intermediate	USCorp High Yield Long Term
Intercept	-0.0243	-0.0186	-0.0393	-0.0168	-0.0166	-0.0213
T-stat	-1.79	-1.49	-2.01	-0.58	-0.58	-0.62
R_f	1.9945	1.7441	2.6515	2.2417	2.2368	2.4171
T-stat	1.89	1.79	1.74	0.99	1.01	0.90
$R_m - R_f$	-0.0061	-0.0110	0.0103	0.0330	0.0308	0.0548
T-stat	-1.77	-3.13	2.29	5.23	5.05	5.68
SMB	0.0005	0.0023	-0.0063	-0.0672	-0.0684	-0.0483
T-stat	0.07	0.30	-0.66	-5.07	-5.34	-2.38
HML	0.0140	0.0204	-0.0072	0.0012	-0.0028	0.0429
T-stat	1.69	2.46	-0.67	0.08	-0.19	1.88
$\Delta BILLIQ_{AGG}$	-0.0002	-0.0001	-0.0006	-0.0026	-0.0025	-0.0034
T-stat	-1.52	-0.60	-3.52	-10.55	-10.60	-9.03
Try LT-ST	56.1972	40.5482	103.5338	1.5489	-0.3170	15.4694
T-stat	67.91	48.60	95.63	1.02	-0.22	6.76
Adj R-sq	78.62%	65.73%	87.71%	12.07%	12.29%	11.35%
DW	2.09	2.05	2.14	2.19	2.20	2.10

Table 10: Bond Market Liquidity and Equity Market Liquidity

Results of regressing $BILLIQ$ for each ETF on equity market illiquidity as represented by $EILLIQ$. Equity market illiquidty is computed in the same way as $BILLIQ$, but the ETF used was the iShares S&P500 Equity Index (ticker IVV). The regression equation is $BILLIQ = b_0 + b_1\ EILLIQ + \epsilon$, and is run over 1682 observations. We report the coefficients, t-statistics, and adjusted R-squareds. These regression results are shown in Panel A. In Panel B, vector autoregressions are performed using $BILLIQ_{AGG}$ and $EILLIQ$ as endogenous variables. Coefficients are reported, with t-statistics shown below each respective coefficient. Results are reported after applying a Cochrane-Orcutt correction for autocorrelation.

Panel A: Regressions of $BILLIQ$ on $EILLIQ$

Ticker	b_0	b_1	b_0 tstat	b_1 tstat	Adj R-sq	DW
LQD	51.8396	0.6577	10.25	11.26	6.97%	2.44
HYG	189.4063	0.8798	6.96	3.34	2.00%	2.49
CSJ	96.0978	0.2451	4.62	2.41	0.86%	2.45
CFT	104.6084	0.5203	9.42	3.56	2.07%	2.38
CIU	105.1934	0.3984	4.57	4.19	2.92%	2.30
AGG	37.8260	0.2105	15.50	4.71	1.51%	2.22
GBF	45.9999	0.2178	7.44	3.04	1.49%	2.25
GVI	51.6460	0.0561	6.66	0.81	-0.06%	2.56
MBB	18.3094	0.0679	11.33	1.83	0.45%	2.46
EMB	151.8701	0.7588	5.79	2.18	1.16%	1.86

Table 11: Hedge Fund Returns and Liquidity Risk

Results of regressing different hedge fund strategies, represented by the CS Tremont Hedge Fund sub-indices, against the $BILLIQ_{AGG}$ measure, which captures aggregate bond market liquidity. The specific regression equations are of the form $Return_{Strategy} = b_0 + b_1\ BILLIQ_{AGG} + \epsilon$. The table contains the coefficients, t-statistics, adjusted R-squareds, F-statistics, and the P-values of the F-statistics. Number of observations is 67. Results are reported after applying a Cochrane-Orcutt correction for autocorrelation.

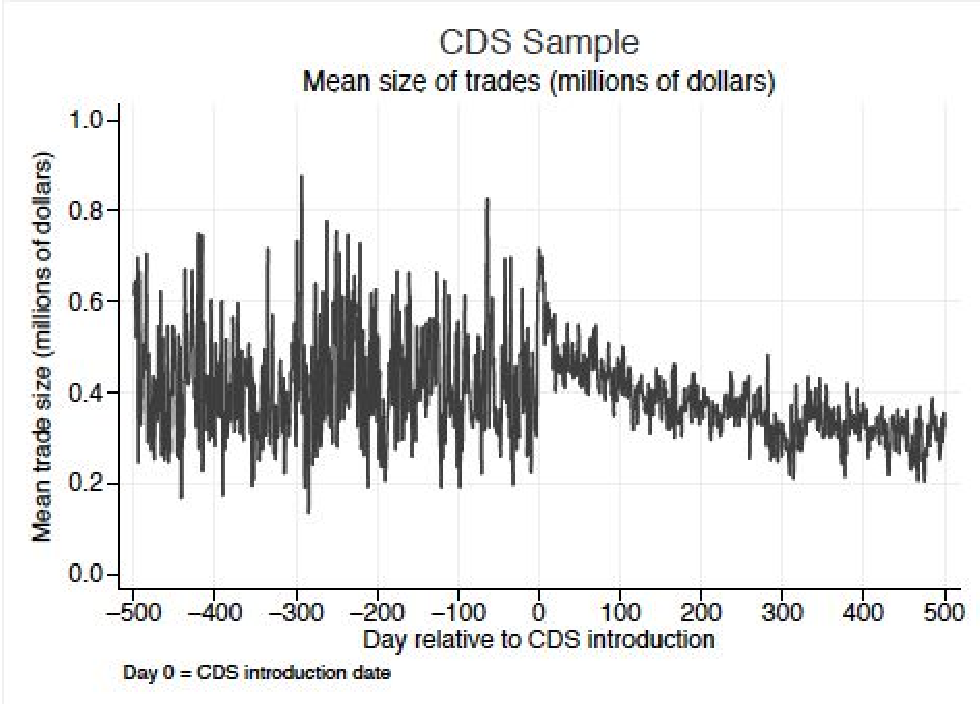
Strategy	b_0	b_1	b_0 tstat	b_1 tstat	R^2	F-stat	P-value	DW
Convertible Arbitrage	0.01	-0.00050	1.50	-2.92	11.73%	8.50	0.0049	1.7748
Dedicated Short Bias	-0.02	0.00041	-1.58	1.68	4.24%	2.83	0.0973	1.8871
Emerging Markets	0.04	-0.00092	4.21	-4.72	25.80%	22.25	0.0000	2.0456
Equity Market Neutral	0.03	-0.00083	6.81	-8.51	53.08%	72.41	0.0000	2.0664
Event Driven	0.00	0.00003	0.30	0.32	0.17%	0.10	0.7477	2.2218
Fixed Income Arbitrage	0.02	-0.00076	3.46	-6.98	43.25%	48.77	0.0000	1.8315
Global Macro	0.02	-0.00043	2.23	-3.72	17.99%	13.82	0.0004	1.6668
Long Short Equity	0.02	-0.00042	2.73	-3.08	12.93%	9.51	0.0030	1.9583
Managed Futures	-0.01	0.00035	-1.04	2.09	6.40%	4.37	0.0405	1.8122
Multi-Strategy	0.01	-0.00039	2.44	-3.71	17.73%	13.80	0.0004	2.0436
Credit Suisse / Tremont Blue Chip Index	0.01	-0.00032885	2.34	-3.48	15.90%	12.098	0.0009	1.9447

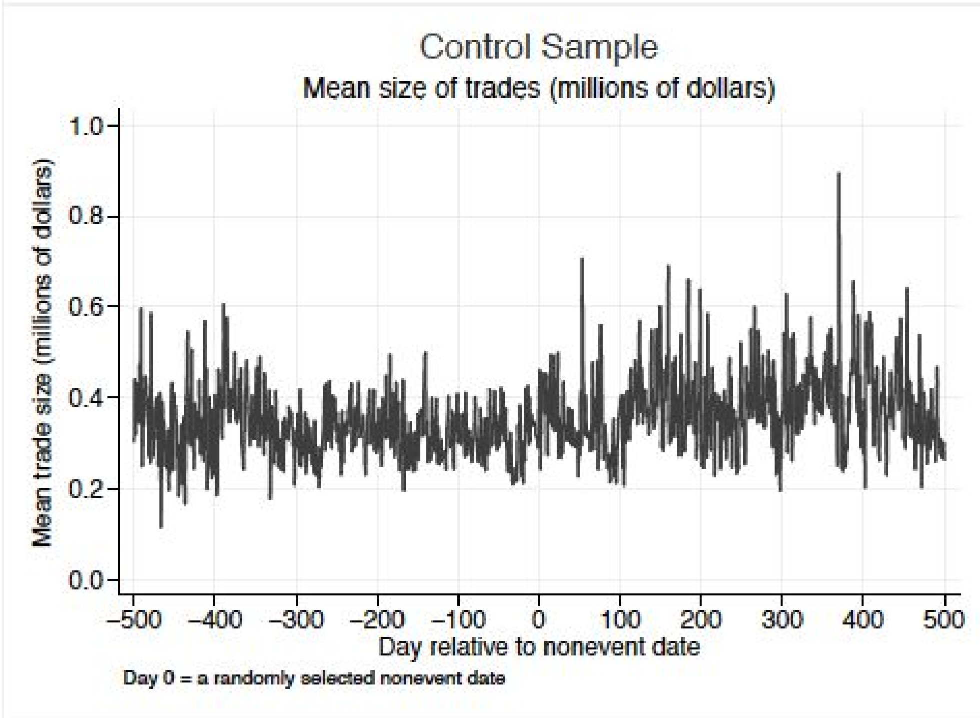
Liquidity Risk Metrics

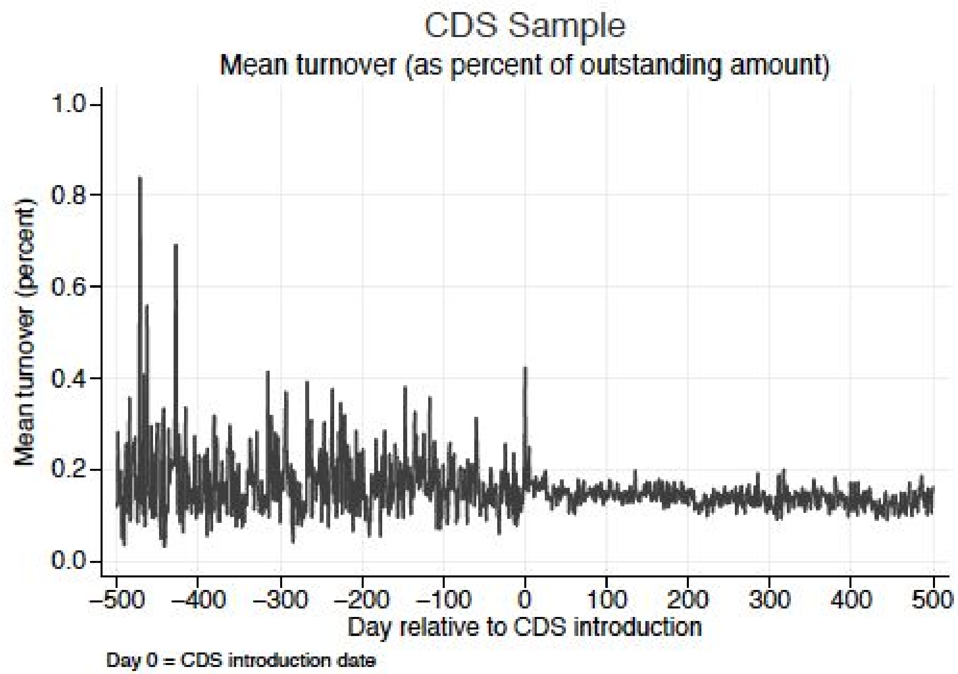
1. A simple count of the number of trades. We report the total number of trades over the entire pre- or post-CDS period, as well as the average number of trades per day (excluding and including zero trade days).

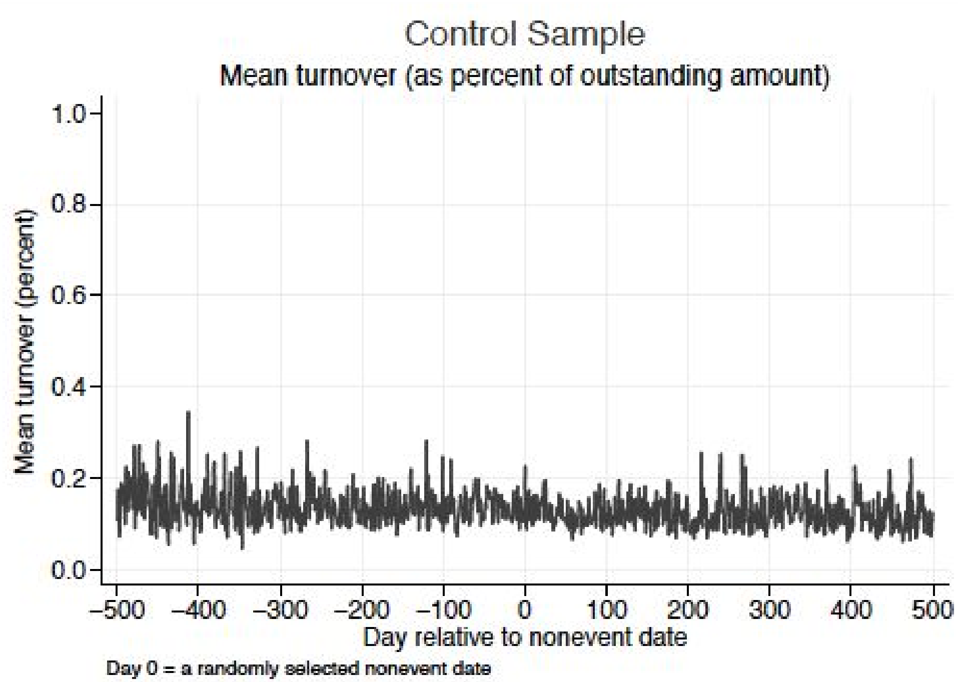
2. The dollar volume of trading, in millions of dollars. We compute the total volume over the entire period and the mean trade size per day and per transaction. Fig. 1 plots the trend of the mean size of each trade for the pooled sample of CDS issuers and the pooled control sample of CDS nonissuers.

3. Turnover, defined as the trading volume as a percentage of the outstanding amount of the bond issue. Again, we report the total (full period), mean daily, and mean per trade values. Fig. 2 plots the trend of mean turnover per transaction for the pooled samples of CDS issuers and nonissuers.[19]


CDS Sample
Mean size of trades (millions of dollars)
Mean trade size (millions of dollars)
1.0
0.8
0.6
0.4
0.2
0.0
−500
−400
−300
−200
−100
0
100
200
300
400
500
Day relative to CDS introduction
Day 0 = CDS introduction date


Control Sample
Mean size of trades (millions of dollars)
Mean trade size (millions of dollars)
1.0
0.8
0.6
0.4
0.2
0.0
−500
−400
−300
−200
−100
0
100
200
300
400
500
Day relative to nonevent date
Day 0 = a randomly selected nonevent date


CDS Sample
Mean turnover (as percent of outstanding amount)
Mean turnover (percent)
1.0
0.8
0.6
0.4
0.2
0.0
−500
−400
−300
−200
−100
0
100
200
300
400
500
Day relative to CDS introduction
Day 0 = CDS introduction date


Control Sample
Mean turnover (as percent of outstanding amount)
Mean turnover (percent)
1.0
0.8
0.6
0.4
0.2
0.0
−500
−400
−300
−200
−100
0
100
200
300
400
500
Day relative to nonevent date
Day 0 = a randomly selected nonevent date

4. The LOT measure of Lesmond, Ogden, and Trzcinka (1999). We use the Das and Hanouna (2010) adaptation of the LOT measure and compute three versions of this measure separately for pre- and post-CDS periods: (1) fraction of zero return trading days, (2) fraction of zero volume (i.e., no trade) trading days, and (3) fraction of zero return plus zero volume trading days. The total number of trading days in the entire pre- or post-CDS period constitutes the denominator of these fractions. Because nontrading days are included, the selection criteria for individual bonds is relaxed and the LOT measures reported in Panel B of Table 7 involve 257 pairs of individual bonds.

5. The Amihud (2002) illiquidity measure. It is computed as

$$\text{Amihud Illiquidity}_i = \frac{1}{DAYS_i} \sum_{t=1}^{DAYS_i} \frac{|bndret_{it}|}{\$VOL_{it}} \times 10^6, \tag{9}$$

where $bndret_{it}$ is the ith bond's return on day t, $\$VOL_{it}$ is the total daily trading volume in dollars, and $DAYS_i$ is the total number of trading days in the entire pre- or post-CDS period.

6. The Roll impact illiquidity measure. This is an extended Amihud proxy measure recommended by Goyenko, Holden, and Trzcinka (2009) for low frequency data. It is based on Roll (1984) spread illiquidity estimator and is computed as

$$\text{Roll Estimator}_i = \begin{cases} \sqrt{-(Cov(\Delta P_{it}, \Delta P_{i,t-1}))} & \text{if } Cov(\Delta P_{it}, \Delta P_{i,t-1}) < 0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

and

$$\text{Roll Impact}_i = \frac{10^6 \times \text{Roll Estimator}_i}{\left(\sum_{t=1}^{DAYS_i} \$VOL_{it}\right) / DAYS_i}, \tag{11}$$

where P_{it} is the daily mean bond price on day t, $\$VOL_{it}$ is the total daily trading volume in dollars, and $DAYS_i$ is the total number of trading days in the entire pre- or post-CDS period.

Summary of Liquidity Measures

1. Number of trades.
2. Trading volume in millions of dollars.
3. Turnover = trading volume as a percentage of outstanding amount.
4. LOT zeros measure: based on Das and Hanouna (2010) adaptation of Lesmond, Ogden, and Trzcinka (1999) measure; computed as frequency of zero return and zero volume trading days as a fraction of total number of trading days.
5. Amihud illiquidity measure: based on Amihud (2002).

$$\text{Amihud Illiquidity}_i = \frac{1}{DAYS_i} \sum_{t=1}^{DAYS_i} \frac{|bndret_{it}|}{\$VOL_{it}} \times 10^6.$$

6. Roll impact illiquidity measure: based on Roll (1984) and Goyenko, Holden, and Trzcinka (2009).

$$\text{Roll Estimator}_i = \begin{cases} \sqrt{-(Cov(\Delta P_{it}, \Delta P_{i,t-1}))} & \text{if } Cov(\Delta P_{it}, \Delta P_{i,t-1}) < 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\text{Roll Impact}_i = \frac{10^6 \times \text{Roll Estimator}_i}{\left(\sum_{t=1}^{DAYS_i} \$VOL_{it}\right) / DAYS_i}.$$

$bndret_{it}$ is the ith bond's return on day t, P_{it} is the daily mean bond price, $\$VOL_{it}$ is the total daily trading volume in dollars, and $DAYS_i$ is the total number of trading days in the entire pre- or post-CDS period.