Rollover Risk and Capital Structure Covenants in Structured Finance Vehicles

Seoyoung Kim
Santa Clara University

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(joint work with Sanjiv Das)

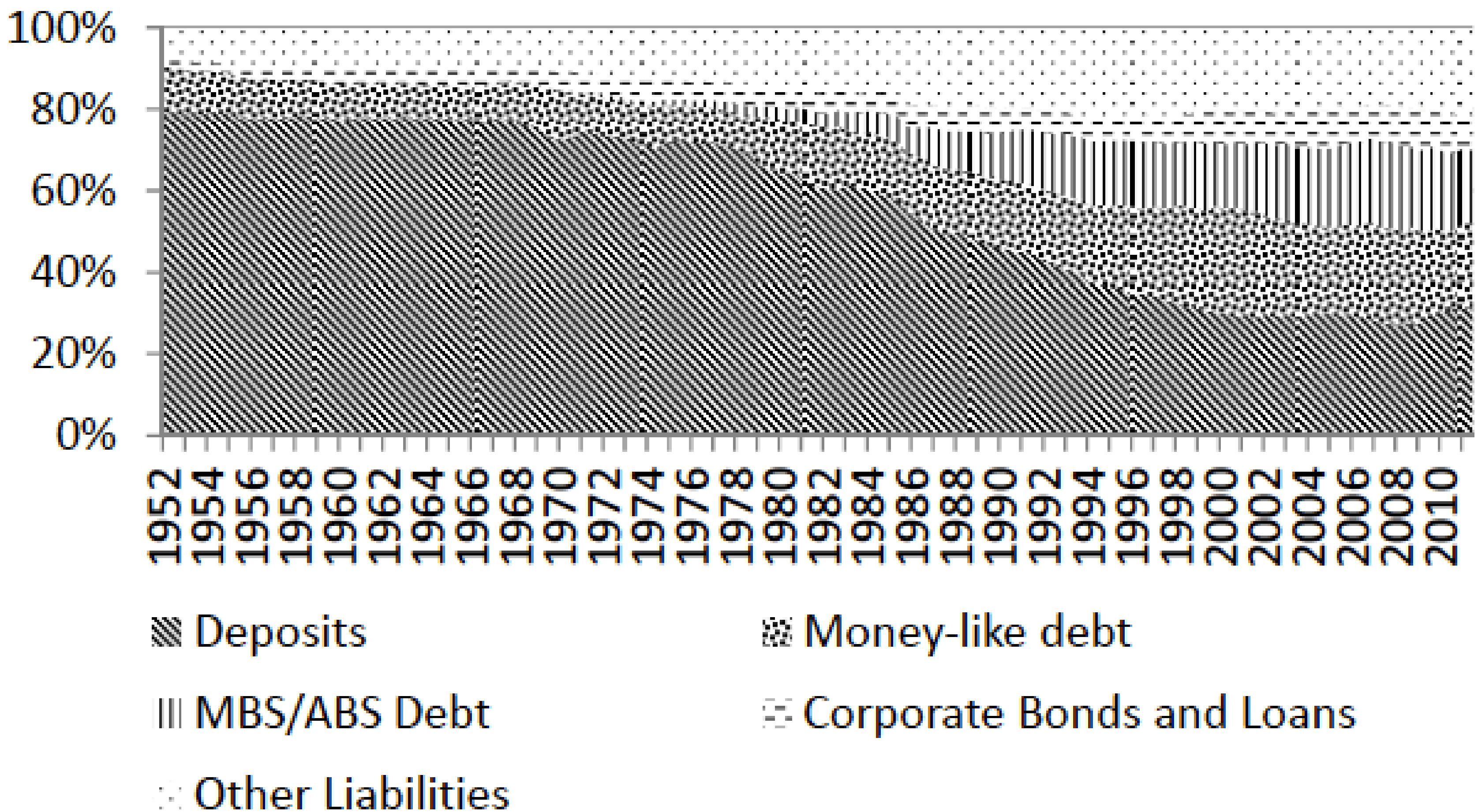
Big Picture

- The shadow banking system comprises leveraged vehicles characterized by high debt, with longer-term investments funded by shorter-term debt.
- These features interact to make funding risk and rollover risk particularly critical considerations in the risk management and design of a SPV.
- Our purpose is to study the inherent rollover risk in an SPV, and to explore its implications on the debt capacity and overall design of the vehicle.

What Is Structured Finance?

- A structured finance deal entails the pooling and tranching of assets into prioritized cash-flow claims
  - The asset pool may comprise a wide range of fixed income and credit assets, such as bonds, RMBS, CMBS, as well as other ABS collateralized by credit card loans, auto loans, home-equity loans, etc.
- This method allows a class of highly rated, low-risk claims, called the senior tranche, to be created from a pool of riskier assets (i.e., there is a subordinated equity tranche that bears first losses)
- In addition, the special purpose vehicle (SPV) outlines basic covenants pertaining to collateral quality, correlation, duration, liquidity, etc.
- Furthermore, roll dates of senior liabilities are staggered to prevent en-masse redemptions.

The Components of Safe Financial Debt



Excerpt from: Gorton, Lewellen, Metrick (2012), “The Safe-Asset Share”

Related Work

- Given their complexity and widespread importance, there is a growing body of work exploring the securitization and design of special purpose vehicles
  - DeMarzo and Duffie (1999): examine the role of information and liquidity costs inherent in selling tranches of a structured finance deal
  - DeMarzo (2005): demonstrates liquidity efficiencies to creating low-risk senior notes from the pooling and tranching of asset-backed securities
  - Coval, Jurek, and Stafford (2009a): argue that senior tranches are akin to economic catastrophe bonds, and offer lower compensation than investors should require
  - Coval, Jurek, and Stafford (2009b): demonstrate that small errors in correlation estimates of the collateral pool results in a large variation in actual riskiness of the senior tranches
  - Gennaioli, Shleifer, and Vishy (2013): argue that the shadow banking system can be welfare improving, but is vulnerable when investors ignore tail risks
- We explore a very different aspect of structured vehicle design: specifically, the adversarial game that arises among investors within the senior debt tranche.

Our Purpose

- Our purpose is to study the increasing risk that arises from staggering senior note holders across varying roll dates, and to explore its implications on the debt capacity and overall design of the SPV.

- We also intend to provide normative prescriptions as to the management of rollover risk in special purpose vehicles / structured deals.

Preview of Results

- Increasing the number of time tranches within the senior-debt class causes investors to adopt stricter rollover policies.
  - That is, investors increase the minimum acceptable asset-to-debt ratio at which they will roll over their capital as the number of time tranches increases.
  - As a result debt capacity of the SPV is reduced.

- A pre-designated leverage threshold on the SPV mitigates the increasing rollover risk arising from the varying roll dates within the senior-debt tranche.
  - This leverage threshold acts as a stop loss for investors, forcing partial liquidation of the SPV with pari passu distribution across all senior debt holders.
  - Thus, the SPV can remain diversified across roll dates while maintaining greater debt capacity.

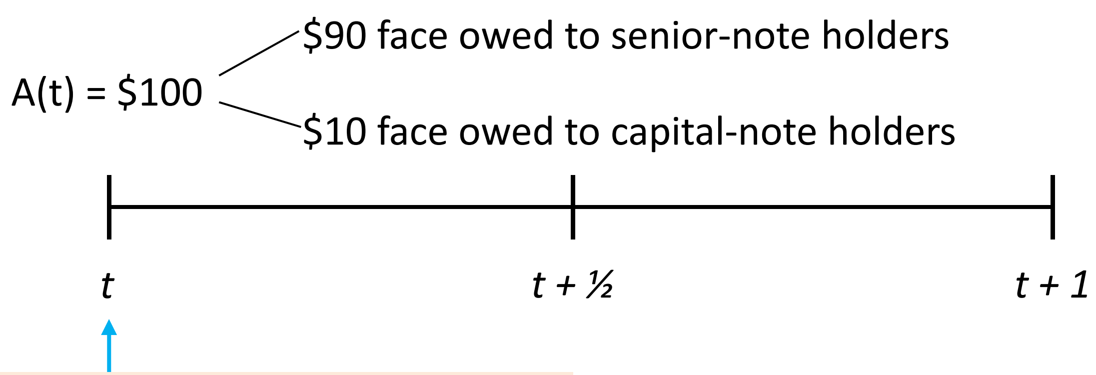
Notation and Setting

- Value of the underlying asset pool is $A(t)$, supported by senior debt $B(t)$
- Initial size of the senior tranche is $B(0) = D_B$
- Senior debt is comprised of m time tranches
  - The time tranches are of equal size (D_B/m) and have the same time to maturity (T) at issuance.
  - However, the time tranches have staggered maturity dates, with rollovers occurring at evenly spaced intervals of T/m.
- Asset/Fire-sale discounts are denoted by δ
- H_m denotes a maturing senior-debt investor's rollover decision rule under m time tranches
  - i.e., the minimum A(t)/B(t) ratio he requires to continue to roll over his investment

Notation and Setting

- Suppose $A(t) = \$100$, supported by senior debt $B(t) = D_B = \$90$ with time to maturity of $T = 1$ year at issuance.

- To mitigate funding risk (e.g., a run on the bank), the SPV has divided its senior debt capital into $m = 2$ time tranches, with staggered rollover dates occurring at time $t + ½$ and time $t + 1$, respectively.



Investor in time tranche 1 decides whether to roll over his \$45 investment capital based on whether \$100 / \$90 ≥ H_2

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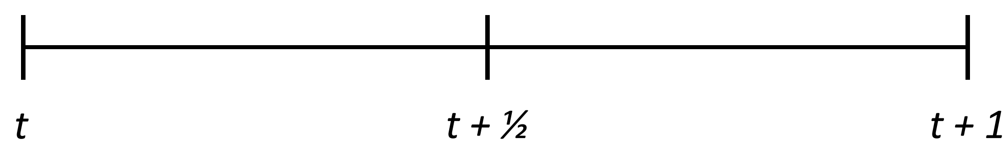
- To mitigate funding risk (e.g., a run on the bank), the SPV has divided its senior debt capital into $m = 2$ time tranches, with staggered rollover dates occurring at time $t + ½$ and time $t + 1$, respectively.

$B(t+½) =$ \$90 if time tranche 1 re-invested (i.e., m=2)
$B(t+½) =$ \$45 if time tranche 1 withdrew (i.e., m=1)



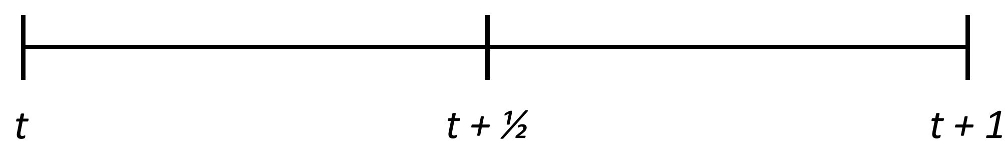
Investor in time tranche 2 decides whether to roll over his \$45 investment capital based on whether $A(t+½) / B(t+½) \geq H_m$

Notation and Setting

- Suppose $A(t) = \$100$, supported by senior debt $B(t) = D_B = \$90$ with time to maturity of $T = 1$ year at issuance.

- To mitigate funding risk (e.g., a run on the bank), the SPV has divided its senior debt capital into $m = 2$ time tranches, with staggered rollover dates occurring at time $t + ½$ and time $t + 1$, respectively.

$$B(t+½) = \begin{cases} \$90 \text{ if time tranche 1 re-invested (i.e., m=2)} \\ \$45 \text{ if time tranche 1 withdrew (i.e., m=1)} \end{cases}$$



Investor in time tranche 1 decides whether to roll over his \$45 investment capital based on whether $A(t+1) / B(t+1) \geq H_m$

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche; $H_1 = 1.05$; $\delta = 5\%$
- Asset values may increase or decrease by a factor of 0.025 each period

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche; $H_1 = 1.05$; $\delta = 5\%$
- Asset values may increase or decrease by a factor of 0.025 each period

$A(t) = 100$

$B(t) = 95$

$t = 0$

Example: The Cost of Rollover Risk in a Two-Period Framework

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche; H_1 = 1.05; δ = 5%
- Asset values may increase or decrease by a factor of 0.025 each period

A(t) = 100
B(t) = 95
A/B = 1.0526

A/B > H_1. Thus, investor decides to roll over his capital

t = 0

Example: The Cost of Rollover Risk in a Two-Period Framework

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- Asset values may increase or decrease by a factor of 0.025 each period

A = 102.50
B = 95.00

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 97.50
B = 95.00

$A/B > H_1$. Thus, investor decides to roll over his capital

t = 0

t = T/2 = ½

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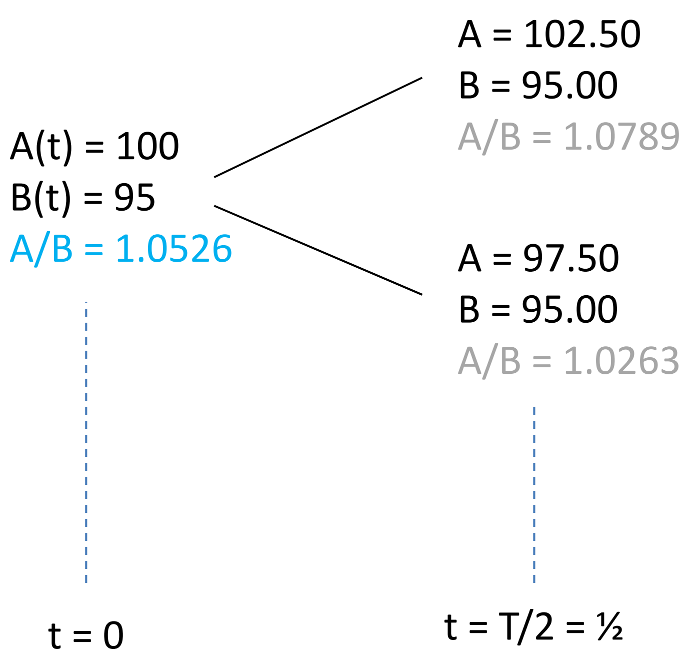

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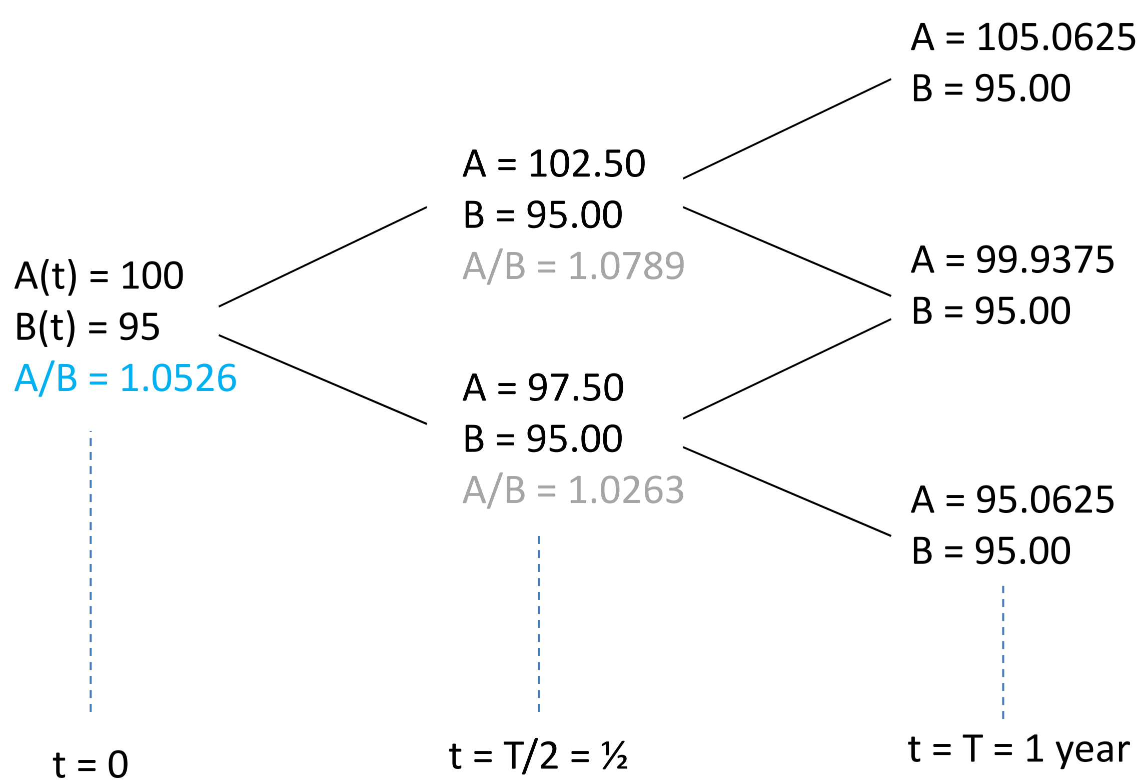

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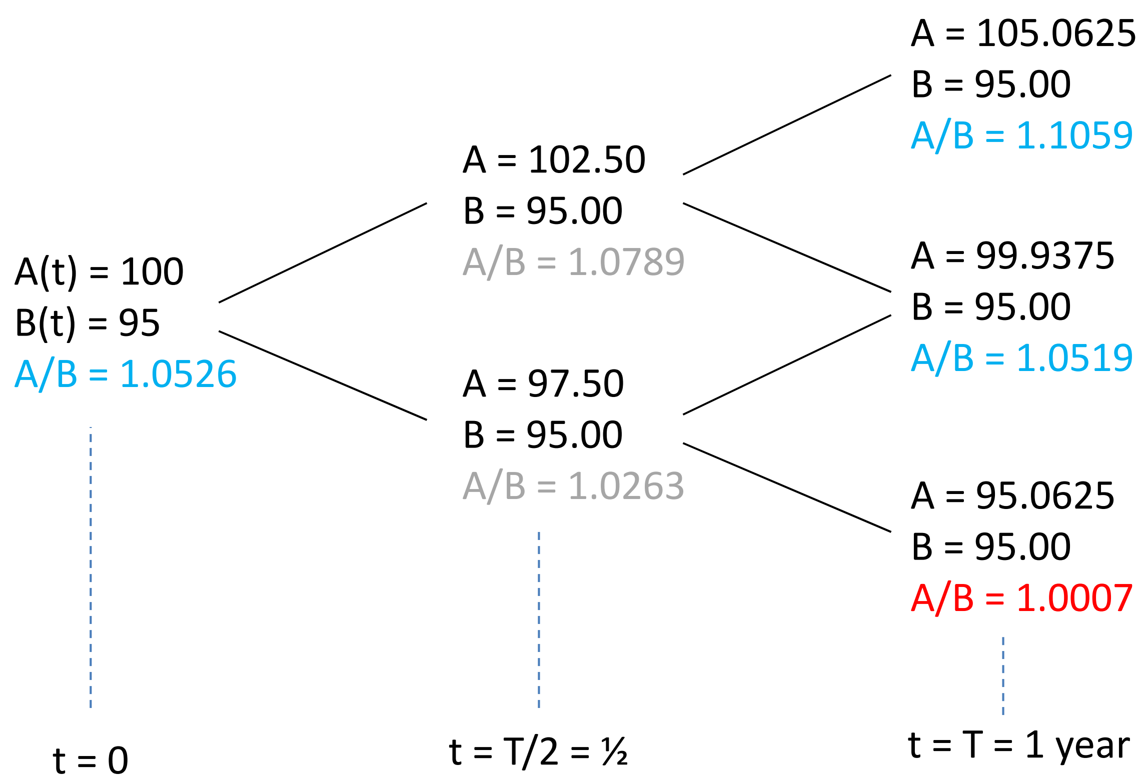

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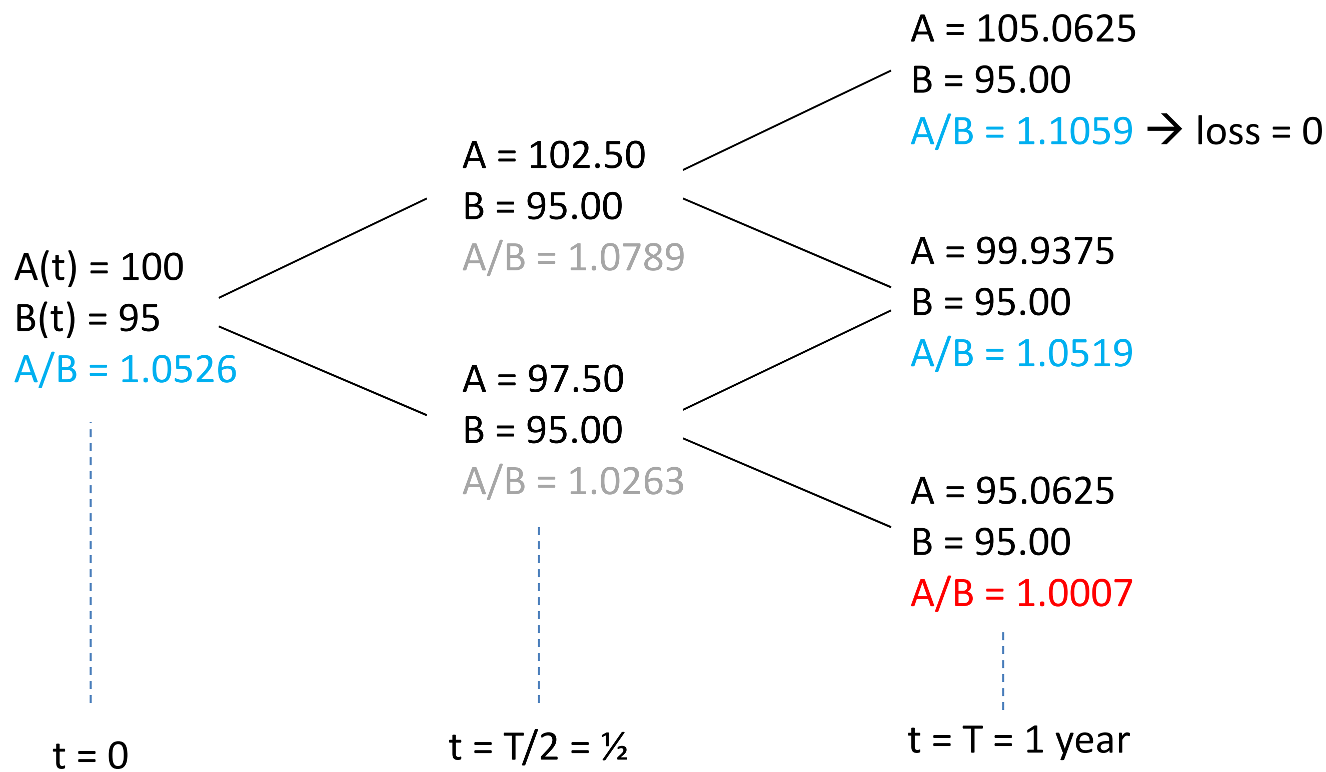

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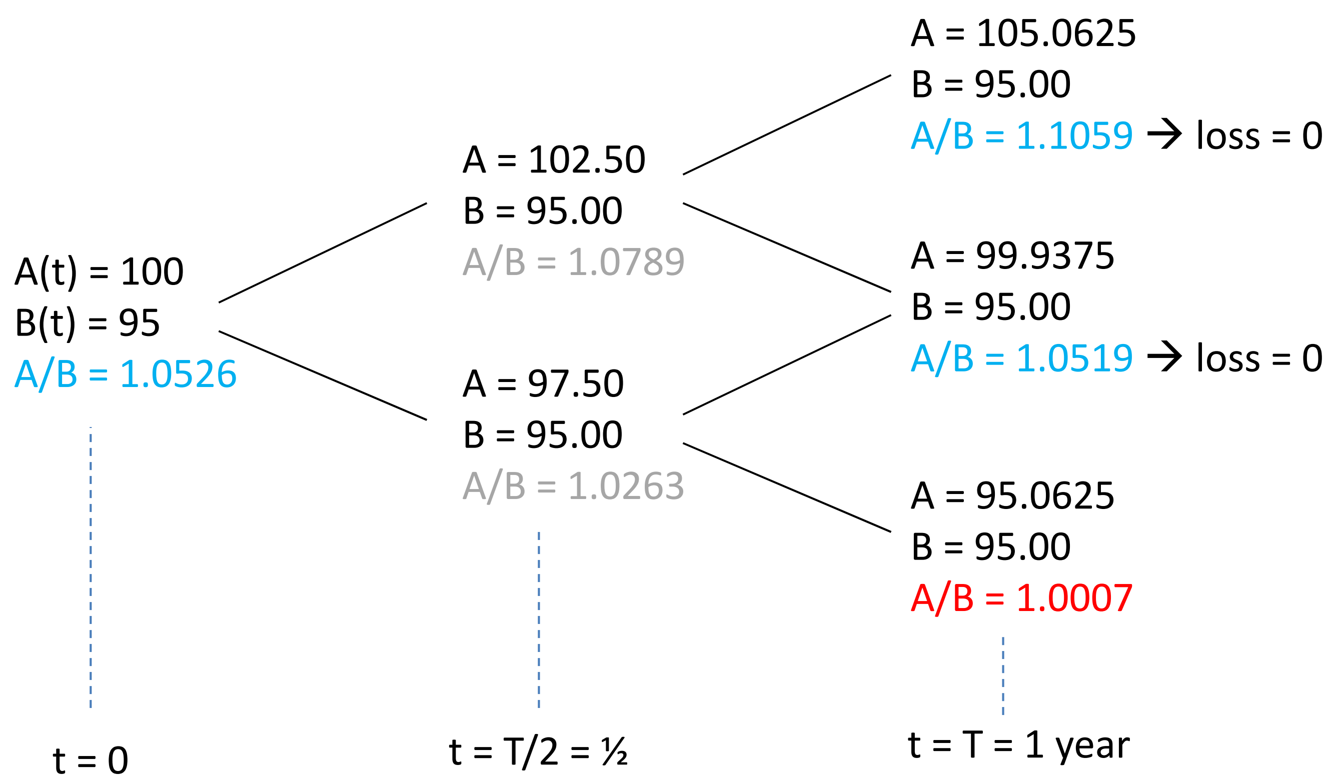

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- Asset values may increase or decrease by a factor of 0.025 each period

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 102.50
B = 95.00
A/B = 1.0789

A = 97.50
B = 95.00
A/B = 1.0263

A = 105.0625
B = 95.00
A/B = 1.1059 → loss = 0

A = 99.9375
B = 95.00
A/B = 1.0519 → loss = 0

A = ~~95.0625~~ sold for A×0.95 < $95
B = ~~95.00~~ repaid $90.3094
A/B = 1.0007 → loss = $4.6906

t = 0

t = T/2 = ½

t = T = 1 year

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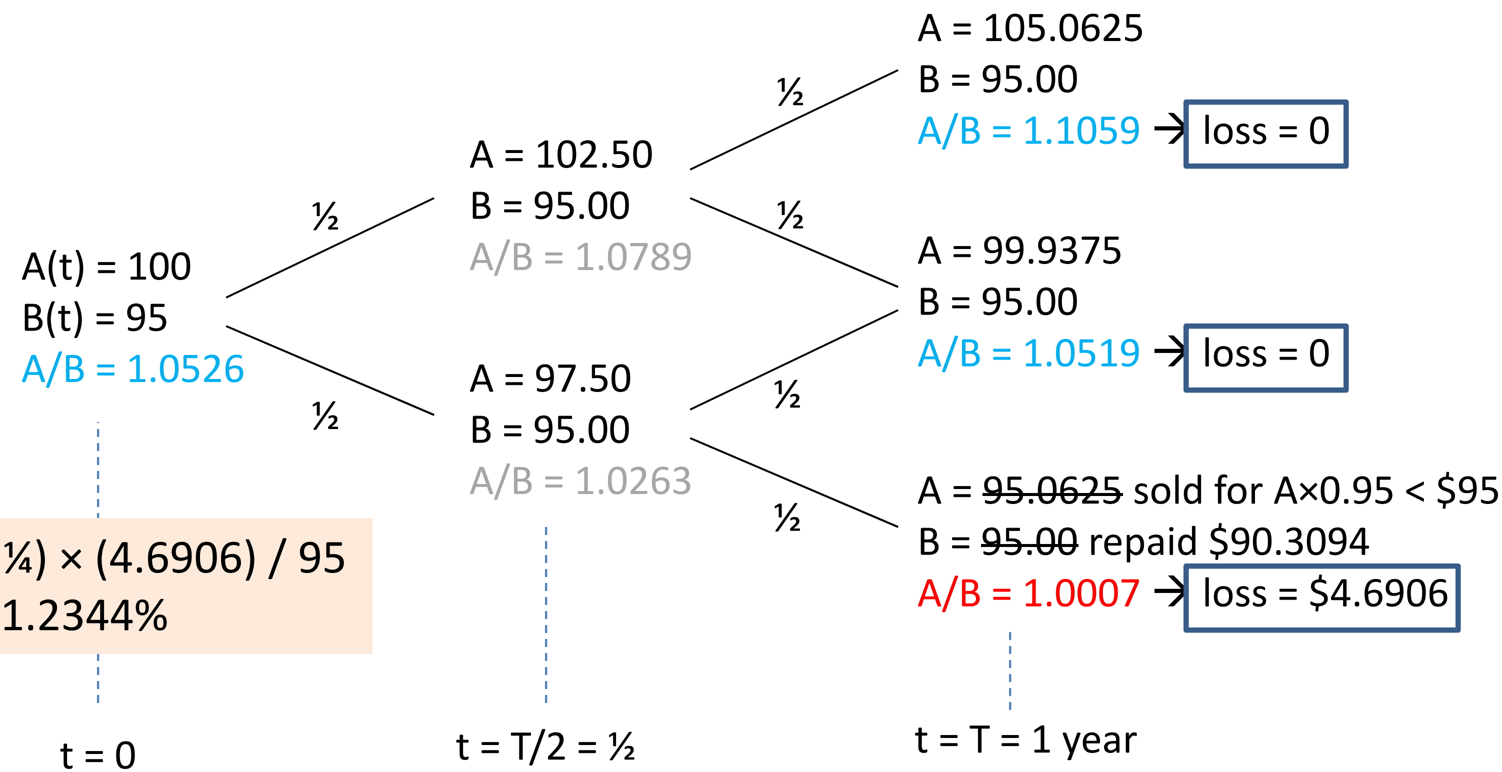

Thus, E(% loss) = (¼) × (4.6906) / 95
= 1.2344%

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

$A(t) = 100$
$B(t) = [47.50, 47.50]$

$t = 0$

Example: The Cost of Rollover Risk in a Two-Period Framework

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A(t) = 100
B(t) = [47.50, 47.50]
A/B = 1.0526

A/B > H_2. Thus, investor in time tranche 1 decides to roll over his capital

t = 0

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
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$A = 102.50$
$B = [47.50, 47.50]$

$A(t) = 100$
$B(t) = [47.50, 47.50]$
$A/B = 1.0526$

$A = 97.50$
$B = [47.50, 47.50]$

$A/B > H_2$. Thus, investor in time tranche 1 decides to roll over his capital

$t = 0$

$t = T/2 = ½$

Example: The Cost of Rollover Risk in a Two-Period Framework

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- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A(t) = 100
B(t) = [47.50, 47.50]
A/B = 1.0526

A = 97.50
B = [47.50, 47.50]
A/B = 1.0263

t = 0

t = T/2 = ½

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A(t) = 100
B(t) = [47.50, 47.50]
A/B = 1.0526

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

\$47.50 / 0.95 = \$50 worth of assets must be liquidated to pay exiting time tranche 2.

t = 0

t = T/2 = ½

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

$A(t) = 100$
$B(t) = [47.50, 47.50]$
$A/B = 1.0526$

$A = 102.50$
$B = [47.50, 47.50]$
$A/B = 1.0789$

$A = 105.0625$
$B = [47.50, 47.50]$

$A = 99.9375$
$B = [47.50, 47.50]$

$A = \cancel{97.50}\ 47.50$
$B = [47.50, \cancel{47.50}]$
$A/B = 1.0263$

$t = 0$

$t = T/2 = ½$

$t = T = 1$ year

Example: The Cost of Rollover Risk in a Two-Period Framework

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- $H_2 = H_1 = 1.05$; $\delta = 5\%$

t = 0

A(t) = 100
B(t) = [47.50, 47.50]
A/B = 1.0526

t = T/2 = ½

A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

t = T = 1 year

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059 → loss = 0

A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520 → loss = 0

A(t) = 100
B(t) = [47.50, 47.50]
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A = ~~97.50~~ 47.50
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A/B = 1.0263

t = 0

t = T/2 = ½

t = T = 1 year

Example: The Cost of Rollover Risk in a Two-Period Framework

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A(t) = 100
B(t) = [47.50, 47.50]
A/B = 1.0526

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B = [47.50, 47.50]
A/B = 1.0789

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059 → loss = 0

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520 → loss = 0

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

A = 48.6875
B = 47.50

A = 46.3125
B = 47.50

t = 0

t = T/2 = ½

t = T = 1 year

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A(t) = 100
B(t) = [47.50, 47.50]
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A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059 → loss = 0

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520 → loss = 0

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

A = 48.6875
B = 47.50
A/B = 1.025

A = 46.3125
B = 47.50
A/B = 0.975

t = 0

t = T/2 = ½

t = T = 1 year

Example: The Cost of Rollover Risk in a Two-Period Framework

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A(t) = 100
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A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059 → loss = 0

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520 → loss = 0

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

A = ~~48.6875~~ sold for A×0.95 < \$47.50
B = ~~47.50~~ repaid \$46.2531
A/B = 1.025 → loss = \$1.2469

A = 46.3125
B = 47.50
A/B = 0.975

t = 0

t = T/2 = ½

t = T = 1 year

Example 1: The Cost of Rollover Risk in a Two-Period Framework

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; $\delta = 5\%$

A(t) = 100
B(t) = [47.50, 47.50]
A/B = 1.0526

A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059 → loss = 0

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520 → loss = 0

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

A = ~~48.6875~~ sold for A×0.95 < \$47.50
B = ~~47.50~~ repaid \$46.2531
A/B = 1.025 → loss = \$1.2469

A = ~~46.3125~~ sold for A×0.95 < \$47.50
B = ~~47.50~~ repaid \$43.9969
A/B = 0.975 → loss = \$3.5031

t = 0

t = T/2 = ½

t = T = 1 year

Example: The Cost of Rollover Risk in a Two-Period Framework

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
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A(t) = 100
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A/B = 1.0526

½

A = 102.50
B = [47.50, 47.50]
A/B = 1.0789

½

A = 105.0625
B = [47.50, 47.50]
A/B = 1.1059 → $\boxed{\text{loss = 0}}$

½

A = 99.9375
B = [47.50, 47.50]
A/B = 1.0520 → $\boxed{\text{loss = 0}}$

½

A = ~~97.50~~ 47.50
B = [47.50, ~~47.50~~]
A/B = 1.0263

½

A = ~~48.6875~~ sold for A×0.95 < \$47.50
B = ~~47.50~~ repaid \$46.2531
A/B = 1.025 → $\boxed{\text{loss = \$1.2469}}$

½

A = ~~46.3125~~ sold for A×0.95 < \$47.50
B = ~~47.50~~ repaid \$43.9969
A/B = 0.975 → $\boxed{\text{loss = \$3.5031}}$

Thus, E(loss) = (¼) × (1.2469) + (¼) × (3.5031) = 1.1875
→ **E(% loss)** = 1.1875 / 45 = **2.5%** > 1.2344% (under m=1)

t = 0 t = T/2 = ½ t = T = 1 year

Analyses

- Thus we see the adversarial game that arises among senior debt investors in varying time tranches. Overall, the rollover decision rule (i.e., required A/B threshold) depends crucially on:
  - The credit quality of the underlying asset pool: σ
  - The time to maturity: T
  - The number of time tranches: m
- Similar results hold under a finer four-period asset tree (Section 2.2; Figures 4, 5, and 6)
- Let us now proceed to the generalized model using simulations for a continuous-time asset process →

Mean-Reverting Asset Process

- Since most SPVs were structured as pools of fixed-income securities, we employ a mean-reverting asset price process:

$$dA(t) = k[\theta - A(t)]\, dt + \sigma\, dW(t)$$

- Thus, $A(t+h) = A(t)e^{-kh} + \theta\left(1 - e^{-kh}\right) + \sigma \int_0^h e^{-k(h-s)} dW(s)$

- In our simulations, we set: $k = 0.5$; $\Theta = A(0) = 100$; and σ = {5, 10}

- The number of simulation paths is 10,000, with time step h = 1 month

Rollover Decision Rule: H_m

- H_m is the minimum possible $A(0)/D_B$ ratio such that expected (%) losses to senior debtholders do not exceed $L_0 = 0.01$.

- We solve for H_m using a sequential, recursive approach, as was demonstrated earlier in the simple tree examples.

- We begin by solving for H_1, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_1 = \int_{-\infty}^{D_B \cdot H_1} max\{0, D_B - (1-\delta)A(T)\} \cdot f[A(T)|A(0)]\, dA \leq 0.01$$

Rollover Decision Rule: H_m

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- We solve for H_m using a sequential, recursive approach, as was demonstrated earlier in the simple tree examples.

- We begin by solving for H_1, finding the minimum $A(0)/D_B$ ratio such that:

Loss upon failure to roll over

$$EL_1 = \int_{-\infty}^{D_B \cdot H_1} \underbrace{max\{0, D_B - (1-\delta)A(T)\}}_{\text{Loss upon failure to roll over}} \cdot f[A(T)|A(0)]\, dA \le 0.01$$

Investors decline to roll over if $A(T) < D_B \times H_1$

Rollover Decision Rule: H_m

- Using H_1 we proceed to solve for H_2, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_2 = \int_{D_B \cdot H_2}^{\infty} EL_1\left[H_2;\ A\left(\tfrac{T}{2}\right);\ \tfrac{D_B}{2};\ \theta;\ \sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) \middle| A(0)\right] dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1\left[H_1;\ A\left(\tfrac{T}{2}\right) - \tfrac{D_B/2}{1-\delta};\ \tfrac{D_B}{2};\ \gamma\theta;\ \gamma\sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) \middle| A(0)\right] dA \ \leq \ 0.01$$

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- Using H_1 we proceed to solve for H_2, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_2 = \int_{D_B \cdot H_2}^{\infty} EL_1\left[H_2;\ A\left(\tfrac{T}{2}\right);\ \tfrac{D_B}{2};\ \theta;\ \sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) \middle| A(0)\right] dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1\left[H_1;\ A\left(\tfrac{T}{2}\right) - \tfrac{D_B/2}{1-\delta};\ \tfrac{D_B}{2};\ \gamma\theta;\ \gamma\sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) \middle| A(0)\right] dA \ \leq\ 0.01$$

Investors in time tranche 2 proceed to roll over at time $t=T/2$ if $A(t) \geq D_B \times H_2$

Rollover Decision Rule: H_m

- Using H_1 we proceed to solve for H_2, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_2 = \int_{D_B \cdot H_2}^{\infty} EL_1\left[H_2;\ A\left(\tfrac{T}{2}\right);\ \tfrac{D_B}{2};\ \theta;\ \sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right)|A(0)\right] dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1\left[H_1;\ A\left(\tfrac{T}{2}\right) - \tfrac{D_B/2}{1-\delta};\ \tfrac{D_B}{2};\ \gamma\theta;\ \gamma\sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right)|A(0)\right] dA \ \leq\ 0.01$$

Investors in time tranche 1 continue to use decision rule H_2, with T/2 time left before their roll date

Rollover Decision Rule: H_m

- Using H_1 we proceed to solve for H_2, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_2 = \int_{D_B \cdot H_2}^{\infty} EL_1\left[H_2;\ A\left(\tfrac{T}{2}\right);\ \tfrac{D_B}{2};\ \theta;\ \sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) | A(0)\right] dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1\left[H_1;\ A\left(\tfrac{T}{2}\right) - \tfrac{D_B/2}{1-\delta};\ \tfrac{D_B}{2};\ \gamma\theta;\ \gamma\sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) | A(0)\right] dA \ \leq\ 0.01$$

Investors in time tranche 2 decline to roll over at time t=T/2 if $A(t) < D_B \times H_2$
Here, partial liquidation is undertaken to repay this exiting tranche.

Rollover Decision Rule: H_m

- Using H_1 we proceed to solve for H_2, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_2 = \int_{D_B \cdot H_2}^{\infty} EL_1\left[H_2;\ A\left(\tfrac{T}{2}\right);\ \tfrac{D_B}{2};\ \theta;\ \sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) \middle| A(0)\right] dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1\left[H_1;\ A\left(\tfrac{T}{2}\right) - \tfrac{D_B/2}{1-\delta};\ \tfrac{D_B}{2};\ \gamma\theta;\ \gamma\sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) \middle| A(0)\right] dA \ \leq \ 0.01$$

Now, investors in time tranche 1 use decision rule H_1, under a reduced portfolio size with asset-process parameters rescaled accordingly

$$\gamma = \frac{A\left(\frac{T}{2}\right) - \frac{B(0)/2}{1-\delta}}{A\left(\frac{T}{2}\right)}$$

Rollover Decision Rule: H_m

- Using H_1 we proceed to solve for H_2, finding the minimum $A(0)/D_B$ ratio such that:

$$EL_2 = \int_{D_B \cdot H_2}^{\infty} EL_1\left[H_2;\ A\left(\tfrac{T}{2}\right);\ \tfrac{D_B}{2};\ \theta;\ \sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) | A(0)\right] dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1\left[H_1;\ A\left(\tfrac{T}{2}\right) - \tfrac{D_B/2}{1-\delta};\ \tfrac{D_B}{2};\ \gamma\theta;\ \gamma\sigma;\ \tfrac{T}{2}\right] \cdot f\left[A\left(\tfrac{T}{2}\right) | A(0)\right] dA \ \leq\ 0.01$$

- Then, using $\{H_1, H_2\}$, we proceed to solve for H_3, and so on…

Table 1: SPV Structures and Expected Losses

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$ with $A(0) = \$100$ and $T = 1$ year
- The number of simulation paths is 10,000, with time step $h = 1$ month

Recov = 95%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0680	93.6360	30.94%	1.38%
	3	1.0680	93.6360	27.87%	1.75%
	4	1.0680	93.6360	24.93%	2.05%
	6	1.0680	93.6360	21.81%	2.64%

Recov = 98%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0353	96.5939	36.26%	1.00%
	2	1.0353	96.5939	30.97%	1.38%
	3	1.0353	96.5939	44.24%	1.75%
	4	1.0353	96.5939	24.93%	2.05%
	6	1.0353	96.5939	21.83%	2.64%

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	3	1.0680	93.6360	27.87%	1.75%
	4	1.0680	93.6360	24.93%	2.05%
	6	1.0680	93.6360	21.81%	2.64%

Recov = 98%	m	Let $H_m = H_1$	[illegible]	[illegible]	[illegible]
	1	1.0353	[illegible]	[illegible]	[illegible]
	2	1.0353	[illegible]	[illegible]	[illegible]
	3	1.0353	96.5939	44.24%	1.75%
	4	1.0353	96.5939	24.93%	2.05%
	6	1.0353	96.5939	21.83%	2.64%

As *m* increases, expected losses also increase if the rollover decision rule remains fixed.

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	4	1.0680	93.6360	24.93%	2.05%
	6	1.0680	93.6360	21.81%	2.64%

Recov = 98%	m	Let $H_m = H_1$
	1	1.0353
	2	1.0353
	3	1.0353
	4	1.0353
	6	1.0353

As *m* increases, expected losses also increase if the rollover decision rule remains fixed.

Thus, H_m must increase with *m*, and the debt capacity of the SPV decreases in *m*.

Table 2: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset-Liquidation Discounts

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$ with A(0) = $100 and T = 1 year
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0756	92.9704	24.52%	1.00%
	3	1.0797	92.6228	18.91%	1.00%
	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%

Recov = 98%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0353	96.5939	36.26%	1.00%
	2	1.0427	95.9067	24.53%	1.00%
	3	1.0466	95.5449	18.91%	1.00%
	4	1.0489	95.3361	23.47%	1.00%
	6	1.0527	94.9974	10.78%	1.00%

Table 2: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset-Liquidation Discounts

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	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%

Recov = 98%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0353	96.5939	36.26%	1.00%
	[illegible]	[illegible]	95.9067	24.53%	1.00%
	[illegible]	[illegible]	95.5449	18.91%	1.00%
	[illegible]	[illegible]	95.3361	23.47%	1.00%
	6	1.0527	94.9974	10.78%	1.00%

To maintain the same ex-ante expected loss, investors must require greater A/B ratios to roll over their capital as *m* increases.

Table 2: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset-Liquidation Discounts

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$ with $A(0)$ = \$100 and T = 1 year
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	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%

Recov = 98%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0353	96.5	[illegible]	[illegible]
	2	1.0427	95.9	[illegible]	[illegible]
	3	1.0466	95.5	[illegible]	[illegible]
	4	1.0489	95.3361	23.47%	1.00%
	6	1.0527	94.9974	10.78%	1.00%

Accordingly, the debt capacity of the SPV decreases in the number of time tranches

Table 2: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset-Liquidation Discounts

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	6	1.0859	92.0915	19.06%	1.00%

Recov = 98%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0353	96.5939	36.26%	1.00%
	2	1.0427	95.9067	24.53%	1.00%
	3	1.0466	95.5449	[illegible]	[illegible]
	4	1.0489	95.3361	[illegible]	[illegible]
	6	1.0527	94.9974	[illegible]	[illegible]

As expected, greater recovery rates allow for greater debt capacity, all else equal

Table 3: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset Volatilities

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$ with A(0) = $100 and T = 1 year
- The number of simulation paths is 10,000, with time step h = 1 month

Recovery = 95%					
$\sigma = 5$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0756	92.9704	24.52%	1.00%
	3	1.0797	92.6228	18.91%	1.00%
	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1265	88.7741	20.66%	1.00%
	2	1.1315	88.3755	14.36%	1.00%
	3	1.1371	87.9468	10.73%	1.00%
	4	1.1399	87.7269	16.89%	1.00%
	6	1.1467	87.2086	6.32%	1.00%

Table 3: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset Volatilities

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$ with $A(0) = \$100$ and $T = 1$ year
- The number of simulation paths is 10,000, with time step $h = 1$ month

Recovery = 95%

$\sigma = 5$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0756	92.9704	24.52%	1.00%
	3	1.0797	92.6228	18.91%	1.00%
	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1265	88.7741	20.66%	1.00%
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	3	1.1371	87.9468	10.[illegible]	[illegible]
	4	1.1399	87.7269	16.[illegible]	[illegible]
	6	1.1467	87.2086	6.32%	1.00%

Similarly, lower price volatility allows for greater debt capacity, all else equal

Table 3: SPV Structures and Maximum Senior-Debt Capacity Under Varying Asset Volatilities

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$ with A(0) = \$100 and T = 1 year
- The number of simulation paths is 10,000, with time step h = 1 month

Recovery = 97%

$\sigma = 5$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0459	95.6074	36.26%	1.00%
	2	1.0534	94.9269	24.52%	1.00%
	3	1.0574	94.5705	35.05%	1.00%
	4	1.0597	94.3656	27.51%	1.00%
	6	1.0635	94.0314	34.74%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1032	90.6432	20.67%	1.00%
	2	1.1082	90.2375	14.37%	1.00%
	3	1.1136	89.7968	10.[illegible]	[illegible]
	4	1.1164	89.5735	8.[illegible]	[illegible]
	6	1.1230	89.0455	29.05%	1.00%

Similarly, lower price volatility allows for greater debt capacity, all else equal

Mitigating Rollover Risk

- As we have now seen, losses to earlier investors compound as inherent losses due to asset risk interact with fire-sale losses incurred when later investors decline to roll over.

- This, in turn, prompts investors to require even higher rollover threholders as the number of time tranches increases.

- To mitigate this risk, we propose an ex-ante leverage threshold on the SPV. That is, we propose a defined trigger whereby assets are partially liquidated and all senior investors are (partially) repaid in unison once the trigger is breached.

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period

Assets are liquidated to repay senior debt holders if at any time A/B < K

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; m = 1 time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

$A(t) = 100$
$B(t) = 95$

$t = 0$

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = 95
A/B = 1.0526

t = 0

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 101.25
B = 95.00

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 98.75
B = 95

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 101.25
B = 95.00
A/B = 1.0658

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 98.75
B = 95

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 102.5156
B = 95.00

A = 101.25
B = 95.00
A/B = 1.0658

A = 99.9844
B = 95.00

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 98.75
B = 95

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 102.5156
B = 95.00

A = 101.25
B = 95.00
A/B = 1.0658

A = 99.9844
B = 95.00

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 98.75
B = 95
A/B = 1.0395

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 102.5156
B = 95.00

A = 101.25
B = 95.00
A/B = 1.0658

A = 99.9844
B = 95.00

A(t) = 100
B(t) = 95
A/B = 1.0526

A = ~~98.75~~
B = ~~95~~ repaid 93.8125
A/B = 1.0395 → loss = 1.1875

A/B < K, thereby triggering liquidation of the SPV, with fire-sale discount of $\delta = 5\%$

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 1$ time tranche
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 101.25
B = 95.00
A/B = 1.0658

A = 102.5156
B = 95.00
A/B = 1.0791

A = 99.9844
B = 95.00
A/B = 1.0525

A = ~~98.75~~
B = ~~95~~ repaid 93.8125
A/B = 1.0395 → loss = 1.1875

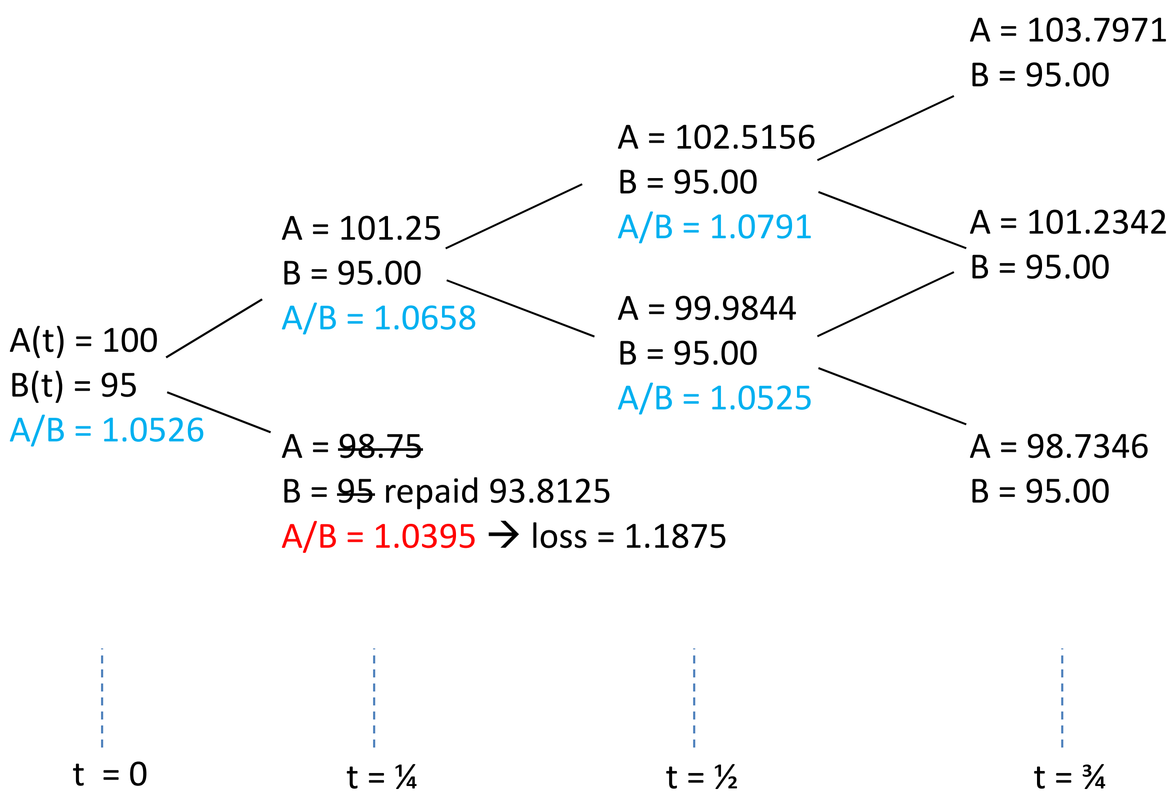
t = 0

t = ¼

t = ½

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0)$ = \$100; $B(0)$ = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period



Example: Mitigating Rollover Risk via Capital Structure Covenants

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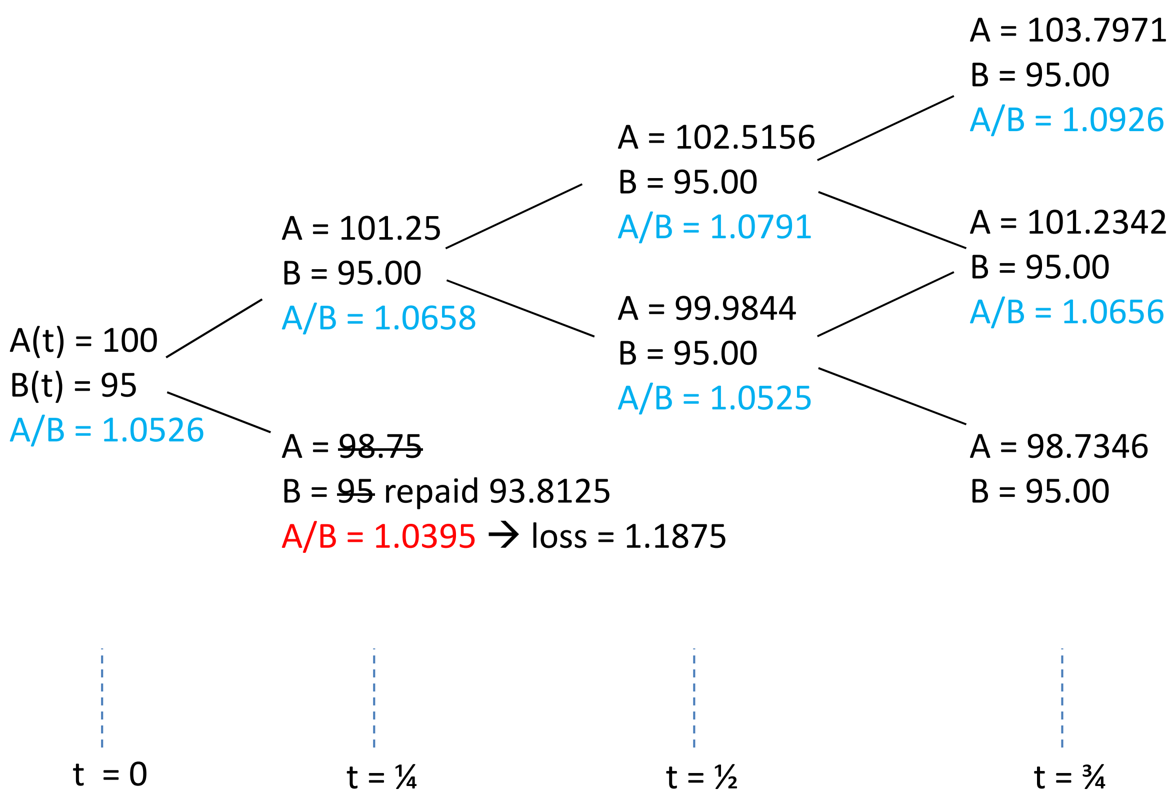
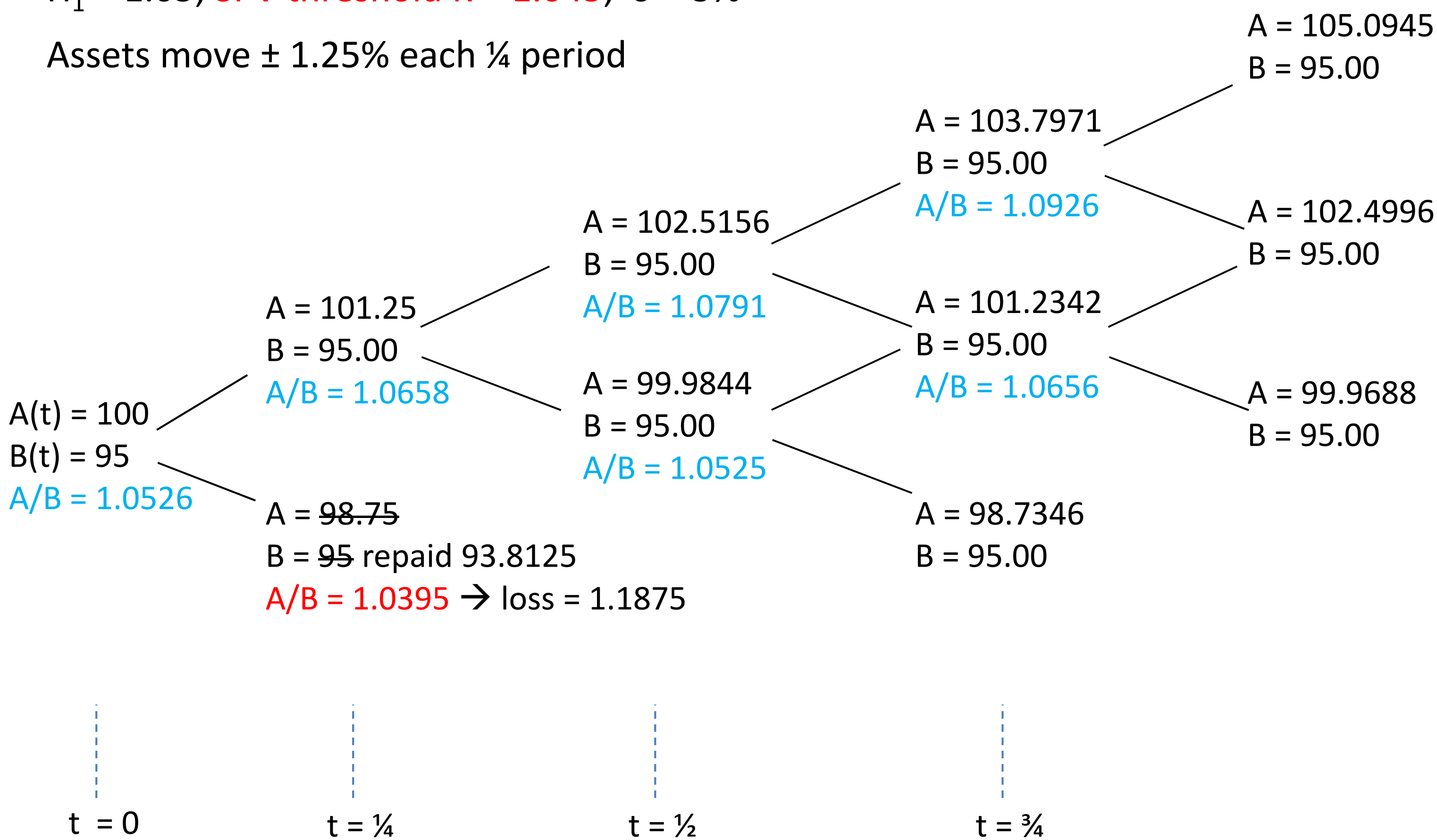
- $H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period



Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period



Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period

t = 0

A(t) = 100
B(t) = 95
A/B = 1.0526

t = ¼

A = 101.25
B = 95.00
A/B = 1.0658

A = ~~98.75~~
B = ~~95~~ repaid 93.8125
A/B = 1.0395 → loss = 1.1875

t = ½

A = 102.5156
B = 95.00
A/B = 1.0791

A = 99.9844
B = 95.00
A/B = 1.0525

t = ¾

A = 103.7971
B = 95.00
A/B = 1.0926

A = 101.2342
B = 95.00
A/B = 1.0656

A = 98.7346
B = 95.00
A/B = 1.0393

A = 105.0945
B = 95.00

A = 102.4996
B = 95.00

A = 99.9688
B = 95.00

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; m = 1 time tranche
- $H_1 = 1.05$; SPV threshold K = 1.045; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 101.25
B = 95.00
A/B = 1.0658

A = ~~98.75~~
B = ~~95~~ repaid 93.8125
A/B = 1.0395 → loss = 1.1875

A = 102.5156
B = 95.00
A/B = 1.0791

A = 99.9844
B = 95.00
A/B = 1.0525

A = 103.7971
B = 95.00
A/B = 1.0926

A = 101.2342
B = 95.00
A/B = 1.0656

A = ~~98.7346~~
B = ~~95.00~~ repaid 93.7979
A/B = 1.0393 → loss = 1.2022

A = 105.0945
B = 95.00

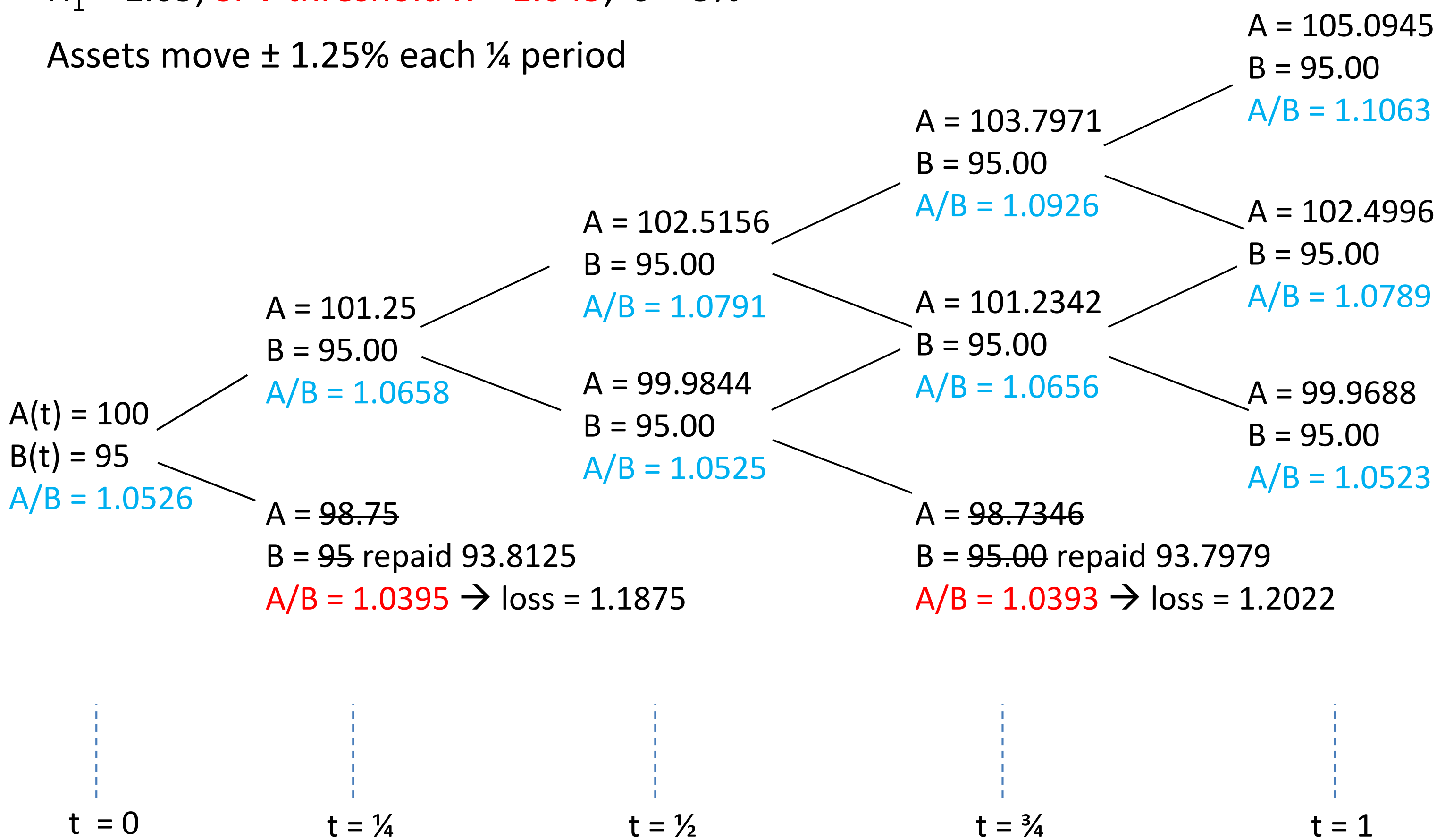
A = 102.4996
B = 95.00

A = 99.9688
B = 95.00

t = 0 | t = ¼ | t = ½ | t = ¾

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0)$ = \$100; $B(0)$ = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period



Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = 95
A/B = 1.0526

A = 101.25
B = 95.00
A/B = 1.0658

A = ~~98.75~~
B = ~~95~~ repaid 93.8125
A/B = 1.0395 → loss = 1.1875

A = 102.5156
B = 95.00
A/B = 1.0791

A = 99.9844
B = 95.00
A/B = 1.0525

A = 103.7971
B = 95.00
A/B = 1.0926

A = 101.2342
B = 95.00
A/B = 1.0656

A = ~~98.7346~~
B = ~~95.00~~ repaid 93.7979
A/B = 1.0393 → loss = 1.2022

A = 105.0945
B = 95.00
A/B = 1.1063 → loss = 0

A = 102.4996
B = 95.00
A/B = 1.0789 → loss = 0

A = 99.9688
B = 95.00
A/B = 1.0523 → loss = 0

t = 0 | t = ¼ | t = ½ | t = ¾ | t = 1

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 1 time tranche
- H_1 = 1.05; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period

t = 0

A(t) = 100
B(t) = 95
A/B = 1.0526

½ ½

t = ¼

A = 101.25
B = 95.00
A/B = 1.0658

A = ~~98.75~~
B = ~~95~~ repaid 93.8125
A/B = 1.0395 → $\boxed{\text{loss} = 1.1875}$

½ ½

t = ½

A = 102.5156
B = 95.00
A/B = 1.0791

A = 99.9844
B = 95.00
A/B = 1.0525

½ ½ ½ ½

t = ¾

A = 103.7971
B = 95.00
A/B = 1.0926

A = 101.2342
B = 95.00
A/B = 1.0656

A = ~~98.7346~~
B = ~~95.00~~ repaid 93.7979
A/B = 1.0393 → $\boxed{\text{loss} = 1.2022}$

½ ½ ½ ½

t = 1

A = 105.0945
B = 95.00
A/B = 1.1063 → $\boxed{\text{loss} = 0}$

A = 102.4996
B = 95.00
A/B = 1.0789 → $\boxed{\text{loss} = 0}$

A = 99.9688
B = 95.00
A/B = 1.0523 → $\boxed{\text{loss} = 0}$

Thus, $E(\text{loss}) = (\frac{1}{2}) \times (1.1875) + (\frac{1}{8}) \times (1.2022) = 0.7440$

→ **E(% loss)** = 0.7440/ 95 = **0.78%** < 0.93% (under m=1, no K)

Table 4: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 1 Time Tranche

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_1$ with $A(0) = \$100$
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0680	93.6360	1.00%
	1.040	1.0679	93.6468	1.00%
	1.045	1.0620	94.1591	1.00%
	1.050	1.0532	94.9515	1.00%

Recov = 98%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0353	96.5939	1.00%
	1.010	1.0335	96.7591	1.00%
	1.015	1.0260	97.4638	1.00%
	1.019	1.0190	98.1354	1.00%

Table 4: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 1 Time Tranche

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_1$ with A(0) = $100
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0680	93.6360	1.00%
	1.040	1.0679	93.6468	1.00%
	1.045	1.0620	94.1591	1.00%
	1.050	1.0532	94.9515	1.00%

When there is a leverage threshold in place, debtholders can relax their rollover decision rule.

	hold (K)	Min H_1	Max D_B	% E(loss)
	ne	1.0353	96.5939	1.00%
	0	1.0335	96.7591	1.00%
	1.015	1.0260	97.4638	1.00%
	1.019	1.0190	98.1354	1.00%

Table 4: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 1 Time Tranche

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_1$ with $A(0) = \$100$
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0680	$\boxed{93.6360}$	1.00%
	1.040	1.0679	93.6468	1.00%
	1.045	1.0620	94.1591	1.00%
	[illegible]	0532	$\boxed{94.9515}$	1.00%

Accordingly the debt capacity of the SPV increases.

Recov = 98%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0353	96.5939	1.00%
	1.010	1.0335	96.7591	1.00%
	1.015	1.0260	97.4638	1.00%
	1.019	1.0190	98.1354	1.00%

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

Triggers partial liquidation of assets to repay $B/m = \$47.50$, distributed equally across all senior debt holders, if at any time $A/B < K$
-- i.e., each time tranche is re-paid \$23.75

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

$A(t) = 100$
$B(t) = [47.5, 47.5]$

$t = 0$

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; SPV threshold K = 1.045; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

$A/B > H_2$. Thus, investor in time tranche 1 decides to roll over his capital

t = 0

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 101.25
B = [47.5, 47.5]

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

A/B > H_2. Thus, investor in time tranche 1 decides to roll over his capital

A = 98.75
B = [47.5, 47.5]

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 101.25
B = [47.5, 47.5]
A/B = 1.0658

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

A/B > K. SPV continues as is.

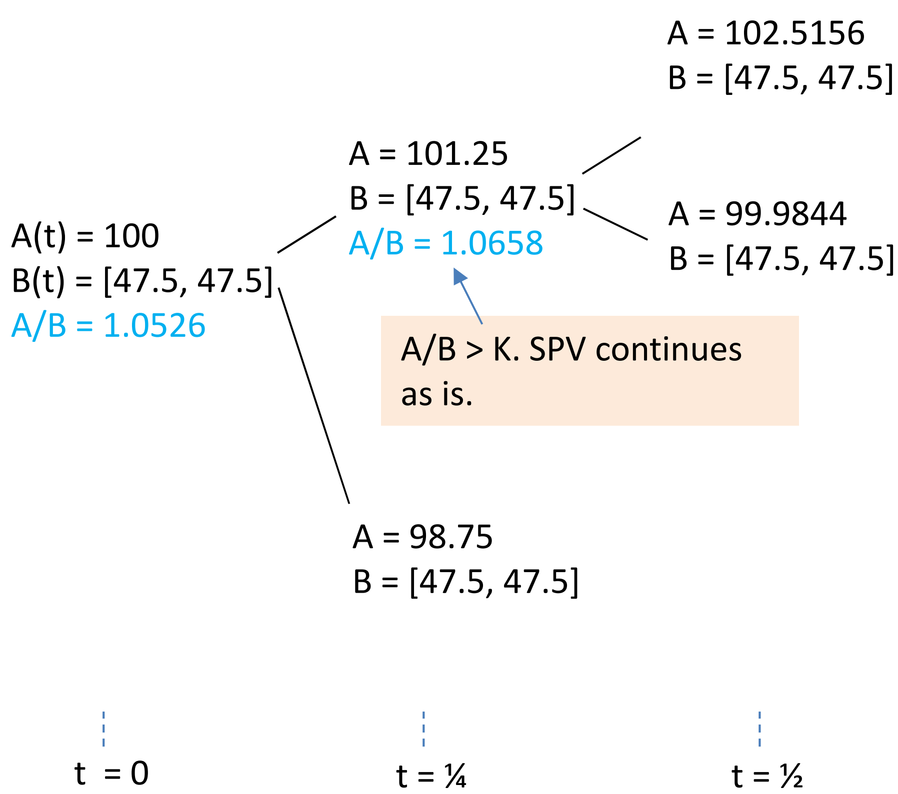
A = 98.75
B = [47.5, 47.5]

t = 0

t = ¼

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; SPV threshold K = 1.045; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period



Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A = 102.5156
B = [47.5, 47.5]

A = 101.25
B = [47.5, 47.5]
A/B = 1.0658

A = 99.9844
B = [47.5, 47.5]

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

A/B < K. Must liquidate \$47.50/0.95 = \$50 of assets to partially repay each time tranche \$22.75

A = 98.75
B = [47.5, 47.5]

A/B = 1.0395

t = 0 t = ¼ t = ½

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; $m = 2$ time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

A = 101.25
B = [47.5, 47.5]
A/B = 1.0658

A = 102.5156
B = [47.5, 47.5]

A = 99.9844
B = [47.5, 47.5]

A/B < K. Must liquidate $47.50/0.95 = $50 of assets to partially repay each time tranche $22.75

A = ~~98.75~~ 48.75
B = ~~[47.5, 47.5]~~
[23.75, 23.75]
A/B = 1.0395

t = 0 t = ¼ t = ½

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; SPV threshold K = 1.045; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

A = 101.25
B = [47.5, 47.5]
A/B = 1.0658

A = 102.5156
B = [47.5, 47.5]

A = 99.9844
B = [47.5, 47.5]

A/B < K. Must liquidate \$47.50/0.95 = \$50 of assets to partially repay each time tranche \$22.75

A = ~~98.75~~ 48.75
B = ~~[47.5, 47.5]~~
[23.75, 23.75]
A/B = 1.0395

A = 49.3594
B = [23.75, 23.75]

A = 48.1406
B = [23.75, 23.75]

t = 0 | t = ¼ | t = ½

Example: Mitigating Rollover Risk via Capital Structure Covenants

- A(0) = \$100; B(0) = \$95; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; SPV threshold K = 1.045; δ = 5%
- Assets move ± 1.25% each ¼ period

t = 0

A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

t = ¼

A = 101.25
B = [47.5, 47.5]
A/B = 1.0658

A = ~~98.75~~ 48.75
B = ~~[47.5, 47.5]~~
[23.75, 23.75]
A/B = 1.0395

t = ½

A = 102.5156
B = [47.5, 47.5]
A/B = 1.0791

A = 99.9844
B = [47.5, 47.5]
A/B = 1.0525

A = 49.3594
B = [23.75, 23.75]
A/B = 1.0391

A = 48.1406
B = [23.75, 23.75]
A/B = 1.0135

t = ¾

A = 103.7971
B = [47.5, 47.5]
A/B = 1.0926

A = 101.2342
B = [47.5, 47.5]
A/B = 1.0656

A = ~~98.7346~~
48.7346
B = ~~[47.5, 47.5]~~
[23.75, 23.75]
A/B = 1.0393

t = 1

A = 105.0945
B = [47.5, 47.5]
A/B = 1.1063

A = 102.4996
B = [47.5, 47.5]
A/B = 1.0789

A = 99.9688
B = [47.5, 47.5]
A/B = 1.0523

A = 49.3438
B = [23.75, 23.75]
A/B = 1.0388

A = 97.5004
B = [23.75, 23.75]
A/B = 1.0132

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; $T = 1$ year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; SPV threshold $K = 1.045$; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

t = 0

- A(t) = 100
  B(t) = [47.5, 47.5]
  A/B = 1.0526

t = ¼

- A = 101.25
  B = [47.5, 47.5]
  A/B = 1.0658
- A = ~~98.75~~ 48.75
  B = ~~[47.5, 47.5]~~
  [23.75, 23.75]
  A/B = 1.0395

t = ½

- A = 102.5156
  B = [47.5, 47.5]
  A/B = 1.0791
- A = 99.9844
  B = [47.5, 47.5]
  A/B = 1.0525
- A = ~~49.3594~~
  B = [~~23.75~~, ~~23.75~~]
  A/B = 1.0391 → loss = 0.6086
- A = ~~48.1406~~
  B = [~~23.75~~, ~~23.75~~]
  A/B = 1.0135 → loss = 1.7664

t = ¾

- A = 103.7971
  B = [47.5, 47.5]
  A/B = 1.0926
- A = 101.2342
  B = [47.5, 47.5]
  A/B = 1.0656
- A = ~~98.7346~~
  48.7346
  B = ~~[47.5, 47.5]~~
  [23.75, 23.75]
  A/B = 1.0393

t = 1

- A = 105.0945
  B = [47.5, 47.5]
  A/B = 1.1063 → loss = 0
- A = 102.4996
  B = [47.5, 47.5]
  A/B = 1.0789 → loss = 0
- A = 99.9688
  B = [47.5, 47.5]
  A/B = 1.0523 → loss = 0
- A = ~~49.3438~~
  B = [~~23.75~~, ~~23.75~~]
  A/B = 1.0388 → loss = 0.6234
- A = ~~97.5004~~
  B = [~~23.75~~, ~~23.75~~]
  A/B = 1.0132 → loss = 1.7809

Example: Mitigating Rollover Risk via Capital Structure Covenants

- $A(0) = \$100$; $B(0) = \$95$; T = 1 year; m = 2 time tranches
- $H_2 = H_1 = 1.05$; SPV threshold K = 1.045; $\delta = 5\%$
- Assets move ± 1.25% each ¼ period

Thus, $E(\text{loss}) = (\frac{1}{4}) \times (0.6086) + (\frac{1}{4}) \times (1.7664) + (\frac{1}{16}) \times (0.6234) + (\frac{1}{16}) \times (1.7809)$

→ **E(% loss)** = 0.7440/ 95 = **0.78%** (same as under m=1, with same K=1.045)

t = 0:
A(t) = 100
B(t) = [47.5, 47.5]
A/B = 1.0526

t = ¼ (½ up):
B = [47.5, 47.5]
A/B = 1.0658

t = ¼ (½ down):
A = ~~98.75~~ 48.75
B = ~~[47.5, 47.5]~~ [23.75, 23.75]
A/B = 1.0395

t = ½:
A = 102.5156

A = 99.9844
B = [47.5, 47.5]
A/B = 1.0525

A = ~~49.3594~~
B = [~~23.75~~, ~~23.75~~]
A/B = 1.0391 → loss = 0.6086

A = ~~48.1406~~
B = [~~23.75~~, ~~23.75~~]
A/B = 1.0135 → loss = 1.7664

t = ¾:
A = 103.7971
B = [47.5, 47.5]
A/B = 1.0926

A/B = 1.0656

A = ~~98.7346~~ 48.7346
B = ~~[47.5, 47.5]~~ [23.75, 23.75]
A/B = 1.0393

t = 1:
A = 105.0945
B = [47.5, 47.5]
A/B = 1.1063 → loss = 0

A = 102.4996
B = [47.5, 47.5]
A/B = ...789 → loss = 0

...38
B = [47.5, 47.5]
A/B = 1.0523 → loss = 0

A = ~~49.3438~~
B = [~~23.75~~, ~~23.75~~]
A/B = 1.0388 → loss = 0.6234

A = ~~97.5004~~
B = [~~23.75~~, ~~23.75~~]
A/B = 1.0132 → loss = 1.7809

(each branch probability ½)

t = 0 | t = ¼ | t = ½ | t = ¾ | t = 1

Table 5: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 2 Time Tranches

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_2$ with $A(0)$ = \$100 and T = 1 year
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0680	1.0756	92.9704	1.00%
	1.000	1.0680	1.0756	92.9704	0.51%
	1.025	1.0680	1.0756	92.9704	0.29%
	1.000	1.0680	1.0680	93.6356	0.62%

Recov = 98%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0353	1.0427	95.9067	1.00%
	1.000	1.0353	1.0427	95.9067	0.22%
	1.025	1.0353	1.0427	95.9067	0.05%
	1.000	1.0353	1.0353	96.5941	0.23%

Table 5: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 2 Time Tranches

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_2$ with A(0) = $100 and T = 1 year
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0680	1.0756	92.9704	1.00%
	1.000	1.0680	1.0756	92.9704	0.51%
	1.025	1.0680	1.0756	92.9704	0.29%
	1.000	1.0680	1.0680	93.6356	0.62%

With a leverage threshold in place, investors under m = 2 time tranches can relax their rollover decision rule.

	d (K)	H_1	H_2	Initial D_B	% E(loss)
		1.0353	1.0427	95.9067	1.00%
		1.0353	1.0427	95.9067	0.22%
	1.025	1.0353	1.0427	95.9067	0.05%
	1.000	1.0353	1.0353	96.5941	0.23%

Table 5: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 2 Time Tranches

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_2$ with $A(0) = \$100$ and $T = 1$ year
- The number of simulation paths is 10,000, with time step $h = 1$ month

Recov = 95%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0680	1.0756	$\boxed{92.9704}$	1.00%
	1.000	1.0680	1.0756	92.9704	0.51%
	1.025	1.0680	1.0756	92.9704	0.29%
	1.000	1.0680	1.0680	$\boxed{93.6356}$	0.62%

Recov = 98%	SPV			Initial D_B	% E(loss)
				95.9067	1.00%
	1.000	1.0353	1.0427	95.9067	0.22%
	1.025	1.0353	1.0427	95.9067	0.05%
	1.000	1.0353	1.0353	96.5941	0.23%

Accordingly the debt capacity of the SPV increases.

Table 6: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of m = 3 Time Tranches

- Initial senior debt is set to maximum capacity at $D_B = A(0)/H_3$ with A(0) = $100 and T = 1 year
- The number of simulation paths is 10,000, with time step h = 1 month

Recov = 95%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0680	1.0756	1.0797	92.6228	1.00%
	1.000	1.0680	1.0756	1.0797	92.6228	0.82%
	1.025	1.0680	1.0756	1.0797	92.6228	0.57%
	1.000	1.0680	1.0680	1.0680	93.6356	0.41%

Recov = 98%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0353	1.0427	1.0466	95.5449	1.00%
	1.000	1.0353	1.0427	1.0466	95.5449	0.11%
	1.025	1.0353	1.0427	1.0466	95.5449	0.03%
	1.000	1.0353	1.0353	1.0353	96.5941	0.13%

We make similar observations when senior debt is divided across *m = 3* time tranches.

Concluding Remarks

- Overall, we provide normative prescriptions as to the management of rollover risk in special purpose vehicles / structured deals.
- Interestingly, instead of providing safety, staggered roll dates across senior investors can accelerate the chances of failure and increase the expected losses to senior-note holders.
- Furthermore, expected losses and the rollover decision rule are sensitive to the collateral pool volatility and anticipated recovery rates.
- We recommend a method that, when carefully calibrated, can mitigate the rollover risk inherent in a structure with staggered roll dates across senior debt investors.

Thank you.