Managing Rollover Risk with Capital Structure Covenants in Structured Finance Vehicles

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Abstract

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The shadow banking system comprises special purpose vehicles (SPVs) characterized by high debt, illiquid long-maturity assets funded predominantly by short-maturity debt, and tranched liabilities also known as the capital structure of the SPV. These three features lead to an adversarial game among senior-note holders, who solve for an optimal rollover policy based on the other senior tranches with varying rollover dates. This rollover policy is, in turn, taken into account by capital-note holders (i.e., investors in the equity tranche) when choosing the capital structure (i.e., the assets-to-debt ratio) of the SPV. Rollover risk increases in the number of time tranches, resulting in a lower equilibrium level of debt and higher cost of debt. The expected life of the SPV may also be shortened. We propose a covenant-based capital structure that mitigates these problems and is Pareto-improving for equity and debt holders in the SPV.

Keywords: special purpose vehicle; structured finance; rollover risk; leverage; capital structure; covenants.

1 Introduction

Structured finance deals emerged as an increasingly important means of risk sharing and obtaining access to capital prior to the financial crisis of 2008. The demand for safe, money-like debt has been on the rise, particularly since the rise and burst of the NASDAQ/tech bubble (Caballero and Krishnamurthy (2009); Krishnamurthy and Vissing-Jorgensen (2012)). This, coupled with market segmentation, has resulted in a marked shift in the suppliers of safe debt from the commercial banking system to the shadow banking system (Gennaioli, Shleifer, and Vishny (2013)).

Through special purpose vehicles (SPVs), the shadow banking system has provided an increasing share of "safe" assets, overtaking many traditional sources of safe debt (Gorton, Lewellen, and Metrick (2012)). At their peak in 2007, Pozsar, Adrian, Ashcraft, and Boesky (2012) estimate that shadow banking liabilities had grown to nearly $22 trillion. Given its scope and magnitude, it is crucial to understand this relatively new and highly complex asset class. For example, before the financial crisis, twenty-nine structured investment vehicles (SIVs), a special sub-class of SPVs, held an estimated $400 billion in assets.[1] Despite their AAA rating, senior notes issued by these SPVs experienced an average 50% loss, and subordinated notes experienced near total loss.[2]

Overall, the devastating effects of over-leveraging, coupled with rollover risk and liquidity risk, became starkly apparent in the financial crisis. Although studies in the aftermath have extensively explored the collateral quality and over-leveraging of SPVs, the role of funding/rollover risk and, in particular, the adversarial game arising among investors *within* the senior-debt tranche, remain unattended. Our purpose is to analyze the additional risks

[1] http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/5769361/400bn-SIV-market-sold-off-in-two-years.html

[2] http://www.risk.net/risk-magazine/news/1517514/almost-siv-assets-sold-fitch

borne by SPVs given funding diversity and varying rollover dates across senior note holders, thereby arriving at the maximal safe level of debt (i.e., senior tranche size) for the SPV accounting for these factors.

A structured finance deal is engineered to tranche investments in an asset pool into prioritized cash-flow claims. The resulting liability structure comprises two broad sources of capital for SPVs: the so-called equity portion, which comprises equity or backstop notes commonly denoted as "capital" notes, and the de facto debt portion of the capital structure, known as "senior" notes, which are supported by the subordinated capital/equity notes. Senior debt forms the primary source of financing, often accounting for more than 90% of the liabilities. In general, structured finance deals are designed with the intent to secure a AAA rating for the senior notes, which pay slightly above the risk free rate of interest, making them attractive as money-market instruments. Senior notes have maturities shorter than that of the investment assets, and are typically rolled over at maturity, provided lenders are satisfied with a sufficiently low credit risk profile.

These senior notes are typically issued in tiered tranches, mostly identical in their maturities at issuance but with different rollover dates, resulting in sequential rollover decisions where the exit/re-investment opportunity is presented to each time tranche in a staggered manner. This tiering of senior-note rollover dates creates an adversarial relation among the varying time-tranche holders, who risk rolling over their investments only to have the next maturing tier(s) exit at their expense. That is, although the extent of tiering/time-tranching decreases funding risk since it guarantees that all investors cannot exit en masse, it also risks altering investor behavior by increasing the safety threshold at which successive senior note holders are willing to roll over their investments. In this paper, we analyze rollover risk in the face of this adversarial problem among senior-note holders of varying rollover dates, and we explore remedies to mitigate this issue.

To this end, we develop a discrete-time model to examine how the leverage threshold (i.e., asset to senior-debt ratio) at which an investor is willing to roll over his investment is affected by the number of time tranches themselves. We then explore how this adversarial game among investors of varying rollover dates is further exacerbated by illiquid asset-sale discounts, whereby investment assets must be liquidated at a discount to pay off an exiting tranche. In doing so, we demonstrate how a small drop in asset values can result in a rapid shrinking of debt capacity as senior note holders decline to roll over their investments.

We also propose a remedy to mitigate rollover risk in maturing senior notes, and we show that this remedy decreases ex-ante expected losses to investors in the SPV. Specifically, we suggest a covenant to mitigate rollover risk by committing to a partial liquidation of assets once a pre-set leverage threshold is breached, with pari passu distribution of proceeds across all senior note holders without regard to their respective rollover dates. Intuitively, this covenant acts as a stop loss mechanism, allowing a maturing tranche holder to roll over his investment with greater confidence that his reinvested capital is not simply allowing subsequent time tranches a riskless exit at his expense.

Our paper complements the work of Acharya, Gale, and Yorulmazer (2011) and He and Xiong (2012), who also explore the adversarial position across debt tranches in a continuous-time model. We extend the work of He and Xiong (2012) by providing a remedy to the adversarial problem that exacerbates rollover risk, and in contrast to Acharya, Gale, and Yorulmazer (2011), our model does not rely on information-theoretic arguments. Furthermore, we are especially interested in the capital structure of a structured finance vehicle in the presence of rollover risk. Thus, we also derive the maximal possible debt (i.e., equilibrium senior tranche size) of the SPV given senior note holders' risk preferences, while accounting for the rollover decisions of the various senior tranche holders.

The main features of this paper are as follows. First, we consider the capital structure decision of the SPV, i.e., we solve an optimization problem subject to rollover risk. The goal of sponsors / equity holders in the SPV is to maximize the amount of debt issued (which maximizes their return on equity), subject to a cap on the ex-ante expected percentage loss on debt to ensure a sufficiently high quality rating so that the SPV is marketable. Since the expected percentage loss depends on the inherent rollover risk, our model captures this feature in the optimization problem.

Second, we model the optimal rollover decisions of senior note holders. Debt holders specify a rule where they decline to roll over debt when assets have fallen below a specified level and the leverage in the SPV (ratio of assets to debt) has become unsustainable with respect to their acceptable expected loss levels. When a maturing tranche holder takes his turn in deciding whether or not to roll over his investment, he must account for the fact that if he decides to reinvest, he becomes vulnerable to the risk that subsequent time tranches may withdraw prior to his next rollover date. Thus, in equilibrium, all time tranches will withdraw sooner, i.e., at higher asset-to-debt ratios, and this problem is exacerbated when there are more time tranches / tiers of maturity dates.

Overall, the SPV becomes more susceptible to defeasance (i.e., a wind down via asset sales), where value is further lost through asset-liquidation discounts. These effects, in turn, reduce the amount of senior debt that can be issued by the SPV, impacting the return to equity note holders. Thus, solving for the capital structure decision of the SPV not only depends on the underlying investment assets and acceptable expected loss levels to investors, but also requires the solution of the optimal rollover decision of the debt holders. We are able to solve these interlocking problems, demonstrating how rollover risk arises, the ensuing reactions of debt holders, and the ultimate design of the SPV in the presence of these interactions.

Third, our model shows that once tranches begin to withdraw, the presence of asset-liquidation discounts increases the likelihood that, at the next rollover date, another tranche will also withdraw, leading to a death spiral for the SPV. The failures of SPVs in the recent subprime financial crisis followed this same pattern, as asset-sale discounts reached increasingly high levels,[3] triggering further increases in leverage and credit risk.

Fourth, we suggest a pre-determined remedy for this phenomenon that (i) results in orderly deleveraging and (ii) mitigates the rollover risk arising from the adversarial game among senior note holders of differing rollover dates. That is, we propose an additional covenant that not only imposes a leverage constraint on the SPV, but also requires partial liquidation of assets with an equal distribution of the proceeds among all senior note holders. Specifically, if the asset-to-debt ratio drops below a pre-set threshold, the SPV must repay the amount of debt equivalent to one time tranche (i.e., a partial deleveraging), but the payment is split *pari passu* across all time tranches, irrespective of their individual rollover dates.

As a result, a partial deleveraging is undertaken at an earlier stage, and because all time tranches are partially repaid in an equal manner, we mitigate each tranche holder's concern that subsequent time tranches will exit in succession, forcing asset liquidations at a discount and leaving insufficient funds to repay the latest investor to roll over. Thus, at each rollover date, the likelihood of funding withdrawal is reduced, and the end result is that ex-ante expected losses are reduced not only for the senior note holders, but also for

[3] For instance, Cheyne Finance recovered 44% of par value in initial liquidation rounds, and Sigma Finance recovered 15%. See http://www.risk.net/risk-magazine/news/1504163/cheyne-assets-disappoint-in-rescue-auction; http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/5769361/400bn-SIV-market-sold-off-in-two-years.html.

the equity note holders, who are the residual claimants. In turn, the SPV can sustain higher levels of senior debt and has greater longevity than it would in absence of this covenant. We present numerical examples based on calibrated simulations of our model that demonstrate these intuitions. The analyses in this paper should inform investment banks, rating agencies, and regulators concerned with the design and structure of highly-leveraged special purpose vehicles.

In the end, designing a deal that is safe for senior note holders is inherently difficult, with risks arising from a variety of sources. For instance, DeMarzo and Duffie (1999) examine the role of information and liquidity costs inherent in a structured finance deal, highlighting the lemons problem in the issuance of asset backed securities, and Hanson and Sunderam (2013) argue that pooling and tranching into prioritized cash-flow claims creates "safe" senior tranches owned by the majority of investors, which leads to a dearth of informed investors in good times and results in insufficient risk controls. Coval, Jurek, and Stafford (2009a) discuss the features of securitized pools, where senior notes are akin to economic catastrophe bonds, failing under extreme situations, but offering lower compensation than investors should require. In related work, Coval, Jurek, and Stafford (2009b) argue that small errors in parameter estimates of the collateral pool result in a large variation in the riskiness of the senior notes. Complementing this asset-side result, we find that finer details of the liability side also matter: irrespective of the risk parameters of the collateral pool, funding diversity via varying rollover dates exacerbates rollover risk, and can have a material impact on the riskiness and value of the senior tranches.

Overall, our paper explores a very different aspect of structured finance design than has been considered in the literature so far, and is aimed at determining the equilibrium design of a SPV, i.e., the capital structure and risk controls, in the face of rollover risk. Many results are not obvious from the onset, suggesting that a naive approach to SPV design might exacerbate risk rather than mitigate it.

The rest of the paper proceeds as follows. In Section 2, we present simple examples to demonstrate the intuition for the results. In Section 3, we present the comprehensive model where asset values follow a continuous-time mean-reverting process. We also present simulation-based outcomes to the senior-note and capital-note holders' problems, and we demonstrate the effects of the remedial covenants designed to mitigate rollover risk. In Section 4, we discuss and conclude.

2 Model

We consider a SPV with assets $A(t)$ supported by senior debt $B(t)$ at time t. These two quantities define a leverage ratio $A(t)/B(t)$. For a solvent SPV, this ratio is greater than 1. Initial senior debt in the SPV is denoted $D_B = B(0)$.

The senior debt in the SPV comprises m equal-sized time tranches, each of which has a different maturity date, but with the same rollover horizon, T, at issuance. At the maturity date of a tranche, the lenders in that time tranche have the option to roll over their debt for another period of length T. Each rollover occurs at evenly spaced intervals of T/m. For example, T may be equal to one year, and we may have $m = 4$ time tranches each of size $D_B/4$. Hence, every quarter, one of the time tranches is faced with whether to withdraw their capital or to roll over the debt for another year. The interval at which these rollover decisions occur is shorter than the original maturity of each time tranche's debt.

This tiered structure creates additional rollover risk, as each time tranche factors in the possibility that if they choose to re-invest their capital, subsequent maturing time tranches may instead decide to withdraw, leaving losses to be borne by the latest investors. The staggered debt structure we utilize is common in practice, and our paper focuses on SPV risk and design in this setting, in contrast to other work that assumes a single tier of debt within prioritized tranches (i.e., a single rollover date across all investors in the same class of debt). In related work, He and Xiong (2012) examine rollover risk in a model of debt runs, and assess how this risk varies with asset volatility, debt maturity, and the presence of credit lines. We employ a different representation of the rollover risk problem in discrete time, with a recursive solution, and use it to assess the drivers of rollover risk and how it affects ex-ante SPV design. In extending and complementing the extant literature, this paper further explores how risk management covenants can resolve the rollover risk game among time tranches.

Intuitively, when a SPV's assets decline in value and its debt-to-equity ratio increases beyond a certain threshold, senior debt holders are unwilling to roll over their investments because their expected losses become exceedingly high. That is, senior debt holders have a threshold level of leverage H_m (dependent on the number of tranches m) below which they are unwilling to continue to fund the SPV. Insofar as $A(t)/B(t) \geq H_m$, these debt holders are satisfied with the expected percentage loss of face value, denoted L_0. However, if $A(t)/B(t) < H_m$ at the time their tranche matures, then they will decline to rollover. Thus, we can think of H_m as the strike level at which senior debt holders call back their debt.

This threshold H_m depends not only on the riskiness of collateral and investors' level of risk aversion, but also on the number of time tranches as well as the anticipated asset-liquidation discount. When assets must be sold to repay an exiting time tranche, a fire-sale discount, δ, is incurred. Ex-post, this loss is borne by the latest time tranches to invest; thus, sequentially rational investors will factor this risk into their ex-ante rollover decision.

The end result is that each maturing time tranche will require an even greater $A(t)/B(t)$ to re-invest their capital, i.e., the equilibrium level of H_m is raised to reduce exposure to rollover risk. This, in turn, increases expected losses from fire sale discounts, making all time tranches worse off, ex-ante. Later, we will determine risk management covenants that will remedy this problem. A summary of variables and definitions is as follows.

Variable	Definition
$A(t)$	Total asset value
$B(t)$	Total face value of senior debt
$A(t)/B(t)$	(Inverse) leverage ratio of the SPV at time t
D_B	Initial face value of total senior debt (i.e., $D_B = B(0)$)
m	Number of senior-debt time tranches, each with face value $B(0)/m$ and rollover horizon T
T	Time to maturity at issuance (i.e., rollover horizon)
δ	Percentage loss incurred on fire sale of assets when rollover is declined, i.e., recovery rate is $(1-\delta)$
H_m	Leverage threshold, based on a total of m time tranches, below which the maturing time tranche declines to roll over debt (i.e., the maturing time-tranche holder will re-invest as long as $A(t)/B(t) \geq H_m$). H_m will vary with investors' acceptable level of expected losses, L_0.

In order to set intuition, we present two simple tree-based examples that (a) illustrate the effects of rollover risk; (b) explain the recursive solution procedure; and (c) demonstrate how our proposed risk management covenants are pareto-improving. Without loss of generality, we assume that the risk free rate is zero, and that asset returns are sufficient to pay interim coupons on the debt. By abstracting away from these cash flows, including dividends on equity, we highlight our main focus: the impact of rollover risk on ex-ante expected losses on the face value of senior debt in the SPV.

Later, when we move to our comprehensive continuous-time stochastic model and simulation design, any appreciation in the assets on a rollover date is paid as a residual to the equity notes. Therefore, at each rollover date, the vehicle either fails to roll over, or is reset to its original state ($A(t) = A(0)$), resulting in a repeated scenario. This design-imposed stationarity of these structured finance vehicles means that our solution for one rollover cycle is sufficient to capture all dynamics across repeated rollover periods, specifically because (unlike the simple examples below), we will deal with vehicles that begin at maximal debt capacity (i.e., vehicles that are not overcollateralized).

2.1 Example 1: The cost of rollover risk in a two-period framework

We begin with a simple two-period example where $T = 1$ year, i.e., each period is a half year. We assume that $A(0) = 100$ and $B(0) = 95$. Asset values may move up and down with equal probability by an up-factor of 1.025 per period, and a corresponding down factor of 0.975. We also assume an asset-liquidation discount of $\delta = 5\%$. As mentioned previously, coupon rates and the dividend rate are zero. The asset price tree for this process is depicted in **Figure 1**.

In this framework, each time tranche of senior debt rolls over at the end of two periods from inception. Because the SPV continues indefinitely, provided it remains solvent and debt holders continue to roll over their investments, the tree repeats itself in perpetuity but may scale up or down depending on the initial portfolio value, $A(0)$. We now proceed to calculate ex-ante expected losses to senior debt holders under a rollover leverage threshold of $H = 1.05$.

2.1.1 A single senior-debt time tranche

First, we calculate the expected loss to senior debt holders when there is just a single time tranche (i.e., $m = 1$). The process is as follows. Given a self-imposed leverage cut-off of $H_1 = 1.05$, the initial leverage ratio of $A(0)/B(0) = 100/95 = 1.0526$ satisfies this threshold and the debt holder decides to roll over his investment. At time $t = 1/2$, whether assets have risen or fallen, the debt holder has no option but to continue, given his original time to maturity of $T = 1$ year at issuance. Finally, at time $t = T = 1$, asset values fall in $A(1) = \{105.0625, 99.9375, 95.0625\}$, all of which satisfy the debt holder's leverage threshold except for the bottom-most value, which results in a leverage ratio of $95.0625/95 = 1.0007 < 1.05$. The resulting tree with debt amounts, (inverse) leverage ratios, and attendant losses for each node is shown in **Figure 2**.

In the first two scenarios, the senior debt holder will roll over his investment, sustaining zero losses, and the SPV will continue. However, in the final scenario, when $A/B = 1.0007 < H_1$, the senior debt holder will decline to re-invest, forcing the SPV into defeasance. Given a fire-sale discount of $\delta = 5\%$, the senior debt holder will be repaid $95.0625 \times 0.95 = 90.3094$, thereby sustaining a loss of 4.6906 (i.e., 4.94% of par value). Because this state occurs with probability 1/4, the ex-ante expected loss as a percentage of par is $(1/4) \times (4.6906)/95 = 1.2344\%$.

We note that H_1 does not depend on A or B. It is the level of leverage that provides a decision rule, stipulating the minimum asset-to-debt ratio that is acceptable to the lender in a SPV where the senior debt is comprised of a single time tranche. In other words, if senior debt holders are comfortable with an expected loss of 1.2344%, then they will select $H_1 = 1.05$ as their decision criteria with regard to whether or not they will roll over their investment when it comes due. On the other hand, if senior debt holders desire ex-ante expected losses no greater than $L_0 = 1.00\%$, then they will choose a greater H_1 to manage their expected losses accordingly (provided it is feasible).

2.1.2 Two time tranches of senior debt

Next, we calculate the expected loss to senior debt holders when there are two time tranches / tiers of senior debt (i.e., $m = 2$), whereby the rollover dates are now staggered over time. In this case we have two rollover thresholds, H_2 and H_1. Each tranche applies the rollover rule H_2 when both tranches exist, but when one tranche refuses to rollover, leaving a single tranche, the remaining tranche applies rule H_1 when deciding whether or not to rollover. As we demonstrate shortly, it is not necessarily the case that $H_2 = H_1$.

To start, we calculate expected losses to senior debt holders under the same decision rule: $H_2 = H_1 = 1.05$, and we will demonstrate that expected losses will now be greater than the 1.2344% expected loss we derived earlier under $m = 1$ time tranche. It will then follow that investors' rollover threshold under two time tranches must be greater than that under a single time tranche (i.e., $H_2 > H_1$) if the latest investors wish to keep expected losses from rising. We now proceed to outline this case, which we depict graphically in **Figure 3**.

As before, each time tranche has a maturity of $T = 1$ year, the tree has two periods, initial assets are $A(0) = 100$, and initial total senior debt is $B(0) = 95$, though $B(0)$ now comprises two staggered time tranches of 47.50 each. At the initial time $t = 0$, investors in the first time tranche of senior debt (time tranche 1) must decide whether to roll over their investment. Since the leverage ratio $A(0)/B(0) = 1.0526$ is greater than the investors' rollover threshold of $H_2 = 1.05$, the investors continue to roll over their investment, the total senior debt remains unchanged at $B(0) = D_B = 95$, and the number of time tranches remains at $m = 2$.

In the following period at time $t = 1/2$, investors in the second time tranche of debt (time tranche 2) can now decide whether to roll over their investment. If assets have appreciated in value to $A(\frac{1}{2}) = 102.50$, then once again, the threshold $H_2 = 1.05$ is not violated and investors in time tranche 2 also continue to roll over their investment, keeping total senior debt at $B(\frac{1}{2}) = D_B = 95$ and the number of time tranches at $m = 2$.

However, if assets have depreciated in value to $A(\frac{1}{2}) = 97.50$, now $A(\frac{1}{2})/B(\frac{1}{2}) = 97.50/95 = 1.0263$, which is less than the rollover threshold $H_2 = 1.05$. In this scenario, investors will decline to roll over and a partial liquidation of assets must be undertaken to repay this exiting time tranche of investors. Specifically, based on the fire-sale discount of $\delta = 5\%$, the SPV must sell $47.50/0.95 = 50$ worth of assets in order repay the exiting investors their total face value of 47.50. Thus, the SPV now has $A(\frac{1}{2}) = 97.50 - 50 = 47.50$ and total senior debt of $B(\frac{1}{2}) = 95 - 47.50 = 47.50$, with just $m = 1$ time tranche remaining (see time $t = 1/2$ in **Figure 3**). We note that, now, $A(\frac{1}{2})/B(\frac{1}{2}) = 47.50/47.50 = 1$, which is less than $H_1 = 1.05$. However, investors of time tranche 1 cannot yet choose to exit, because their rollover date is still 1/2 year away.

In the final period at time $t = T = 1$, investors in time tranche 1 can now decide whether to withdraw or to roll over their investment. In the upper branch of the tree, asset values are sufficient in both scenarios to meet the $H_2 = 1.05$ rollover threshold. Thus, investors in time tranche 1 will yet again continue to roll over their capital, and the SPV will continue operations with a total senior debt of $B(1) = 95$ and $m = 2$ time tranches.

In the lower (half) branch of the tree, asset values are insufficient in both scenarios to meet the $H_1 = 1.05$ rollover threshold. Thus, investors will decide to withdraw their capital, and asset liquidations must take place to repay these investors their $B(1) = 47.50$ face value of debt. After accounting for the asset-liquidation discount of $\delta = 5\%$, we see that remaining assets are insufficient to repay these investors in full (i.e., $48.6875 \times 0.95 = 46.2531$ and $46.3125 \times 0.95 = 43.9969$). Thus, investors sustain losses of $47.50 - 46.2531 = 1.2469$ (i.e., 2.625%) and $47.50 - 43.9969 = 3.5031$ (i.e., 7.375%) in each respective scenario of the lower branch. Ultimately, the SPV ends in complete defeasance, with $A(1) = 0$, total senior debt $B(1) = 0$, and $m = 0$ time tranches remaining.

Thus, we see that with $m = 2$ time tranches, the ex-ante expected loss to investors in time tranche 1 (under a rollover decision rule of $H_2 = H_1 = 1.05$) is $(1/2)(0) + (1/4)(1.2469) + (1/4)(3.5031) = 1.1875$, which is equivalent to 2.5% of the \$47.50 face value of debt. This expected loss is substantially greater than the 1.2344% expected loss we found earlier under $m = 1$ time tranche, suggesting that investors must increase their rollover threshold, H_m, to achieve the same level of ex-ante expected losses as the number of time tranches grows (i.e., H_m is increasing in m). Intuitively, the trigger level of $H_2 = 1.05$ is too low, and the investors in time tranche 1 do not exit in time, effectively providing the capital for the investors in time tranche 2 to subsequently exit with full payment. Here, the investors in time tranche 1 should have set for themselves a greater rollover threshold $H_2 > 1.05$, opting to withdraw their capital at $t = 0$.

Overall, in the presence of rollover risk with multiple time tranches of senior debt, ex-ante expected losses are greater because of the risk that subsequent time tranches may choose to exit just after the latest investors have re-committed. In order to mitigate this, we can see that trigger levels must be set such that $H_2 > H_1$, which further increases rollover risk and also forces the SPV to operate at lower debt-to-equity ratios, thereby decreasing expected returns to capital-note holders in the equity tranche of the SPV.

2.2 Example 2: A finer, four-period tree

In this section, we extend our tree to four periods over the same $T = 1$ rollover horizon as in the previous section, and we repeat the earlier examples. While the specific numbers change, the qualitative results are unaffected. Eventually, we implement the model using a continuous-time stochastic process, and the results remain robust. Therefore, there are no qualitative modeling differences in discrete versus continuous time.

Because each step now represents four three-month quarters (rather than two six-month semi-annual periods), we adjust the step sizes accordingly to increase by a factor of 1.0125 or to decrease by a factor of 0.9875 (i.e., $\pm 1.25\%$), with equal probability. This four-period tree is shown in **Figure 4**.

To begin, we calculate the ex-ante expected losses to senior debt holders when there is just a single time tranche (i.e., $m = 1$). As before, the initial leverage ratio of $A(0)/B(0) = 1.0526$ satisfies the leverage threshold of $H_1 = 1.05$, and the senior debt holder decides to roll over his investment. Because his lockup period is for $T = 1$ year, he has no option to withdraw until the end of the fourth quarter/period. On that date, the five possible values of assets, as shown in **Figure 4** are:

$$A(1) = \{105.0945, 102.4996, 99.9688, 97.5004, 95.0930\},$$

which correspond to leverage ratios of:

$$A(1)/B(1) = \{1.1063, 1.0789, 1.0523, 1.0263, 1.0010\}.$$

Thus, in the first three scenarios, the leverage threshold of $H_1 = 1.05$ is satisfied and the senior debt holders continue to roll over their investment. However, $A(1)/B(1)$ does not satisfy the leverage threshold in the fourth and fifth cases, and investors will opt to withdraw their capital, sustaining losses of $95 - 97.5004 \times 0.95 = 2.3746$ and $95 - 95.0930 \times 0.95 = 4.6617$, respectively, given a fire-sale discount of $\delta = 5\%$. Since these states occur with probability 1/4 and 1/16, respectively, the ex-ante expected loss as a percentage of par value is $[(1/4)(2.3746) + (1/16)(4.6617)]/95 = 0.009316$ (i.e., 0.9316%). The populated tree with debt amounts, leverage ratios, and losses sustained is shown in **Figure 5**.

Next, we calculate the expected loss to senior debt holders when there are two time tranches / tiers of debt (i.e., $m = 2$), whereby the rollover dates are staggered and occur every $T/m = 1/2$ years. Thus, the total senior debt of $B(0) = 95$ is now comprised of two time tranches of 47.50 each; i.e., $B(0) = \{47.50, 47.50\}$. This new tree is depicted in **Figure 6**. As before, we calculate these expected losses under the same decision rule: $H_2 = H_1 = 1.05$, and we analyze the sequence of decisions as follows:

1. At time $t = 0$, investors in the first time tranche of debt (time tranche 1) must decide whether to roll over their investment. Since the leverage ratio $A(0)/B(0)$ is greater than $H_2 = 1.05$, investors in time tranche 1 continue to re-invest their capital and the total face value of senior debt remains unchanged at $B(0) = D_B = 95$ and the number of time tranches remains at $m = 2$.

2. At time $t = 1/4$, neither tranche is maturing. Thus, investors have no choice but to continue, regardless of whether asset values have increased or fallen.

3. At time $t = 1/2$, investors in the second time tranche of debt (time tranche 2) can now decide whether to re-invest or withdraw their capital. There are three possible scenarios at this node, with $A(\frac{1}{2}) = \{105.5156, 99.9844, 97.5158\}$, which correspond to leverage ratios of $A(\frac{1}{2})/B(\frac{1}{2}) = \{1.0791, 1.0525, 1.0265\}$, respectively.

   In the first two scenarios, investors in time tranche 2 will continue to roll over their investment, since their leverage threshold of $H_2 = 1.05$ is satisfied. In these cases, total senior debt remains at $B(\frac{1}{2}) = D_B = 95$ and the number of time tranches remains at $m = 2$.

However, in the third scenario, these investors will opt to withdraw their capital, and a partial liquidation of assets must occur to repay this exiting time tranche of investors. Specifically, based on a fire-sale discount of $\delta = 5\%$, the SPV must sell $47.50/0.95 = 50$ worth of assets to repay the exiting investors' total face value of 47.50. In this case, the SPV will now have $A(\frac{1}{2}) = 97.5158 - 50 = 47.5158$ and senior debt of $B(\frac{1}{2}) = 95 - 47.50 = 47.50$, with just $m = 1$ time tranche remaining. The tree continuing from this node is shown in the lower graph of **Figure 6**.

4. At time $t = 3/4$, neither tranche is maturing. Again, any remaining investors have no choice but to continue.

5. Finally, at time $t = T = 1$, investors in time tranche 1 can now decide whether to re-invest or withdraw their capital. The losses realized in each of the seven possible scenarios are depicted on the terminal nodes in **Figure 6**, and aggregate to an ex-ante percentage expected loss of 1.2422%. That is:

$$[(1/16)(1.2247) + (2/16)(2.3673) + (1/16)(3.4816)]/47.50 = 0.012422$$

Thus, as in Example 1, the ex-ante expected loss under $m = 2$ time tranches (1.2422%) exceeds that under $m = 1$ time tranche (0.9316%) due to the increased rollover risk as the number of time tranches increases. Again, this increased risk suggests that new investors must impose a greater rollover threshold H_m as m increases if they wish to maintain the same level of ex-ante expected losses derived under smaller m.

2.3 Mitigating rollover risk via capital structure covenants

Because rollover risk results in a non-cooperative game among the varying time tranches of senior-note holders, the ex-ante expected loss to each time tranche is higher as the number of time tranches increases. As we have seen in the prior examples, losses compound as the inherent losses due to asset risk interact with fire-sale losses incurred when a subsequent time tranche declines to roll over, prompting cautious investors to require ever higher rollover thresholds as the number of time tranches increases. In order to mitigate this risk, we propose that the SPV implement a leverage trigger at which point all time tranches are partially repaid in equal amounts via a partial deleveraging of the SPV. This SPV threshold, K, is lower than H_m (else the SPV might be unnecessarily liquidated when investors are otherwise satisfied with the extant risks), and applies even when no time tranche is due for rollover.

To illustrate, we consider a SPV leverage threshold of $K = 1.045$ on our four-period tree under $m = 1$ time tranche. That is, if $A/B \leq 1.045$ at any time t, assets are liquidated to repay senior debt holders, regardless of whether or not time t is a rollover date. The resultant tree of asset values and losses incurred is shown in **Figure 7**. We note that there are two nodes on the tree where the leverage trigger is breached, i.e., in the lowest nodes at time $t = 1/4$ and time $t = 3/4$. For instance, possible asset values at time $t = 1/4$ are $A(\frac{1}{4}) = \{101.25, 98.75\}$, with leverage ratios $A(\frac{1}{4})/B(\frac{1}{4}) = \{1.0658, 1.0395\}$. Thus, the SPV threshold is breached in the bottom node, and assets are liquidated to repay the senior debt holders, who incur a loss of $95 - 98.75 \times 0.95 = 1.1875$. Similarly, the SPV threshold is breached in the bottom node at time t=3/4, and senior debt holders incur a loss of $95 - 98.7346 \times 0.95 = 1.2022$. No losses are incurred in all other nodes of the tree.

Overall, the ex-ante expected percentage loss is $[(1/2)(1.1875) + (1/8)(1.2022)]/95 = 0.7832\%$. We see that this expected loss is lower than that in the case of one time tranche when the SPV threshold covenant, K, is not applied, as previously shown in **Figure 5** (i.e., under $K = 0$, the expected loss was 0.9316%). Equivalently, the threshold covenant allows investors to impose a lower rollover threshold when deciding whether to reinvest or withdraw their capital. Thus, by imposing a capital structure covenant on the SPV, we effectively mitigate rollover risk and the increasing costs borne by investors due to rollover risk.

We now proceed to demonstrate the expected losses when employing this remedy under $m = 2$ time tranches, which we show graphically in **Figure 8**. As before, investors in time tranche 1 decide to reinvest their capital at time $t = 0$, since $A(0)/B(0) > 1.05$. In the following period at time $t = 1/4$, possible asset values are $A(\frac{1}{4}) = \{101.25, 98.75\}$, with leverage ratios $A(\frac{1}{4})/B(\frac{1}{4}) = \{1.0658, 1.0395\}$. Because the SPV threshold is breached in the bottom node, the SPV must undergo partial deleveraging, repaying the equivalent of one time tranche of senior debt to all senior-debt investors pari passu. That is, $47.50/0.95 = 50$ worth of assets must be liquidated to repay 47.50 worth of debt. This 47.50 will be disbursed equally across investors of both time tranches, regardless of their respective rollover dates. Thus, the SPV continues with $m = 2$ time tranches of senior debt, but with $B(\frac{1}{4}) = \{23.75, 23.75\}$, and $A(\frac{1}{4}) = 98.75 - 50 = 48.75$. Despite this partial deleveraging, we see that in continuing the tree from this node, further deleveraging is subsequently required and the SPV enters total defeasance.

We populate the entire tree following this procedure (as depicted in **Figure 8**), and we see that ultimately, ex-ante percentage expected losses are 0.7832% (i.e., identical to that under the $m = 1$ tranche case). That is:

$$[(1/16)(0.6234) + (1/16)(1.7809) + (1/4)(0.6086) + (1/4)(1.7664)]/95 = 0.7832\%$$

Thus, in employing the proper SPV threshold, the two time-tranche case does not entail any additional expected losses from rollover risk created by the adversarial game among the senior debt holders in time tranches of varying rollover dates. These examples show that this capital-structure covenant reduces the expected losses to senior debt holders, and dominates the setting when there are no SPV covenants. In the following section we generalize the discrete-time tree model to a continuous time stochastic process. Given that the SPV covenant reduces expected losses for senior-note holders, expected losses are also reduced for equity-note holders, who are the residual claimants, resulting in a Pareto-improving SPV structure.

3 Comprehensive Model

In this section, we generalize the model outlined in the preceding section using simulations with continuous-time stochastic processes. Since most SPVs were structured as pools of mortgage debt securities, we employ a mean-reverting asset price process, rather than a process such as the geometric Brownian motion, which is used for equity portfolios. In addition, we solve the equity-note holders' problem, whose objective is to maximize the extent of senior-debt funding (thereby maximizing their expected returns) while keeping expected losses at a level that is acceptable to senior debt holders so as to ensure funding can be acquired. This analysis will deliver the equilibrium capital structure for the SPV, accounting for rollover risk in the face of multiple time tranches of senior debt with varying rollover dates. We will then show how this structure is improved when our proposed capital-structure covenant is included in the ex-ante design of the SPV.

3.1 Mean-reverting asset process

The assets in the SPV evolve according to the following stochastic differential equation (SDE):

$$dA(t) = k[\theta - A(t)]\, dt + \sigma\, dW(t) \tag{1}$$

Here k is the rate of mean reversion, θ is the long run mean of the asset process, with volatility σ. Given time interval h, the solution to this SDE is:

$$A(t+h) = A(t)e^{-kh} + \theta(1 - e^{-kh}) + \sigma \int_0^h e^{-k(h-s)}\, dW(s) \tag{2}$$

At time t, conditional on $A(0)$, $A(t)$ is normally distributed with mean and variance as follows:

$$\begin{aligned} \text{Mean: } \alpha(t) &= E[A(t)|A(0)] = A(0)e^{-kt} + \theta(1 - e^{-kt}) & (3) \\ \text{Variance: } \beta(t) &= Var[A(t)|A(0)] = \frac{\sigma^2}{2k}(1 - e^{-2kt}) & (4) \end{aligned}$$

Based on these parameters, the probability density function of asset values at time t is:

$$f[A(t)|A(0)] = \frac{1}{\sqrt{2\pi\beta(t)}} \exp\left[-\frac{(A(t) - \mu(t))^2}{2\beta(t)}\right] \tag{5}$$

By setting $\theta > A(0)$, we can inject an upward drift to capture expected returns from the assets. However, in general, θ is set to par value, so that assets are pulled to par as is the case with fixed-income securities.

3.2 Characterizing the decision to roll over a time tranche of senior debt

We specify the number of time tranches to be m. In our model, we simulate asset prices over time across many paths, and as each time tranche matures, investors in that time tranche decide to roll over their capital only if the assets to total senior debt ratio is greater than H_m, noting also that $H_m > 1$. H_m is determined as the minimum possible rollover decision rule such that ex-ante expected (%) losses do not exceed an acceptable level, L_0. In doing so, investors must account for the risk that other time tranches may fail to roll over, the risk of additional losses from asset-liquidation discounts (δ), and the risk inherent in the asset pool itself.

We solve for H_m using a sequential, recursive approach, as was done in the simple tree examples in the preceding section. We begin by solving for H_1 under the $m = 1$ time tranche case. That is, we seek to find the minimum threshold rule that provides investors with an ex-ante percentage expected loss that is no greater than L_0, where the ex-ante expected loss is defined as:

$$EL_1 = \int_{-\infty}^{D_B \cdot H_1} \max[0, D_B - (1-\delta)A(T)] \cdot f[A(T)|A(0)] \, dA \tag{6}$$

In the equation above, losses occur at the terminal date when the single time tranche does not rollover, i.e., when $A(T)/B(0) < H_1$. Thus, we integrate over the range $A(T) \leq B(0) \cdot H_1$. We note that the fire-sale discount, δ, is taken into account as well. The percentage expected loss is then defined as

$$PL_1 = \frac{EL_1}{D_B} \leq L_0 \tag{7}$$

The *objective function* of the SPV designer (i.e., the equity-note holders) is to maximize the amount of initial senior debt that can be raised, $B(0) = D_B$, subject to the expected percentage loss on the senior debt being capped at a given level L_0, so that the senior debt capital can be placed with investors. Thus, we search over a range of potential H_1 values, from 1.0 and onward, and we set the initial debt $D_B = A(0)/H_1$, using the above formula to calculate the ex-ante expected loss to senior debt holders under this leverage threshold and initial capital structure. At very high levels of debt, i.e., low levels of H_1, expected percentage loss is likely to exceed L_0, resulting in a non-viable structure. We then increase H_1 by small increments, thereby reducing the extent of senior-debt funding and decreasing default risk, until we achieve $PL_1 = L_0$.

Moreover, any appreciation in the assets at the end of the rollover cycle is paid as a residual to the equity note holders, resulting in a repeated scenario. That is, at the end of the rollover cycle, the senior debt holders either: (1) decline to roll over, in which case $A(t)/D_B < H_1$ (i.e., $A(t) < A(0)$); or, (2) they decide to roll over their capital, in which case $A(t)/D_B \geq H_1$ (i.e., $A(t) \geq A(0)$), and the vehicle is reset to its original state ($A(t) = A(0)$), with the equity holders receiving the residual $A(t) - A(0)$. Thus, our solution for one rollover cycle is sufficient to capture all dynamics across repeated rollover periods, specifically because, by construction, vehicles begin at maximal debt capacity (i.e., vehicles are not overcollateralized).

Having computed H_1, the second step is to solve for H_2, investors' rollover-decision leverage threshold under $m = 2$ time tranches, again searching over a range of potential H_2 values until we find the minimum threshold that results in an ex-ante expected loss of L_0. For the SPV with $m = 2$ time tranches of senior debt, we assume that each time tranche at inception has a rollover horizon of T years, with each maturing $T/m = T/2$ years apart (i.e., rollover dates occur at $t = 0, T/2, T, ...$). At time $t = 0$, investors in time tranche 1 must decide whether to roll over their capital, which they base on whether $A(0)/B(0) > H_2$. We assume, without loss of generality, that we begin with sufficient equity capital at $t = 0$ to meet this threshold. Then, at time $t = T/2$, investors in time tranche 2 are faced with whether to roll over their capital, which they base on whether $A(\frac{T}{2})/B(\frac{T}{2}) > H_2$. This decision entails two possible outcomes:

1. If $A(\frac{T}{2})/B(\frac{T}{2}) > H_2$, then investors in time tranche 2 decide to re-invest at time $t = T/2$. Because $A(\frac{T}{2}) > A(0)$, the vehicle is reset to $A(\frac{T}{2}) = A(0)$, with the residual paid to the equity holders. Then the SPV continues its operations until time $t = T = 1$, at which point investors in time tranche 1 are again faced with the decision to re-invest or withdraw based on whether $A(T)/B(T) > H_2$.

2. If $A(\frac{T}{2})/B(\frac{T}{2}) \leq H_2$, then investors in time tranche 2 opt to withdraw their capital at time $t = T/2$, and they must be repaid $B(\frac{T}{2})/2$, whereby the SPV must liquidate $\frac{B(\frac{T}{2})/2}{1-\delta}$ worth of assets to make these payments. From this point on, the remaining assets evolve under the same process, but there is only $m = 1$ time tranche remaining and the face value of senior liabilities is reduced to $B(\frac{T}{2}) - B(\frac{T}{2})/2 = B(\frac{T}{2})/2$.

   Then, at time $t = T = 1$, investors in time tranche 1 are again faced with the decision to re-invest or withdraw based on whether $A(T)/B(T) > H_1$. Note that the investors now revert to decision rule H_1 because there is only $m = 1$ time tranche remaining at this point. In this case, losses to investors in time tranche 1 are higher, because they have been subjected to funding withdrawal by the investors in time tranche 2, which is the cost of the adversarial game that arises in the presence of rollover risk under multiple time tranches of varying rollover dates.

The ex-ante expected loss to investors (in time tranche 1) under $m = 2$ time tranches may be expressed as:

$$EL_2 \quad = \quad \int_{D_B \cdot H_2}^{\infty} EL_1 \left[H_2; A(0); D_B/2; \gamma_1 \cdot \theta; \gamma_1 \cdot \sigma; T/2\right] \cdot f[A(\frac{T}{2})|A(0)] \; dA$$

$$+ \int_{-\infty}^{D_B \cdot H_2} EL_1 \left[H_1; A(\frac{T}{2}) - \frac{D_B/2}{1-\delta}; D_B/2; \gamma_2 \cdot \theta; \gamma_2 \cdot \sigma; T/2 \right] \times$$

$$f[A(\frac{T}{2})|A(0)] \, dA \tag{8}$$

where $\gamma_1 = \frac{A(\frac{T}{2})-A(0)}{A(\frac{T}{2})}$ and $\gamma_2 = \frac{A(\frac{T}{2})-\frac{B(0)/2}{1-\delta}}{A(\frac{T}{2})}$ to re-scale the asset-process parameters, θ and σ, accounting for partial liquidation along the way.

The two lines of the equation above correspond to the two cases outlined above. If rollover occurs at time $t = T/2$, then the function EL_1 is called with parameter H_2 (and a re-set asset pool, $A(\frac{T}{2}) = A(0)$), whereas if investors in time tranche 2 decline to rollover, then EL_1 is called with parameter H_1 (and reduced asset pool, $A(\frac{T}{2}) - \frac{B(0)/2}{1-\delta}$), as only one tranche remains. Since the computation of EL_2 requires calling the function for EL_1, the recursive nature of the approach is revealed.

Furthermore, the first line of the equation emphasizes the source of the adversarial relation between investors in time tranche 1 and those in time tranche 2. Specifically, in the case that investors in time tranche 2 decide to roll over their investment, we see that the expected loss function to investors in time tranche 1 depends on the original pool of assets from the onset of their investment horizon (since $A(\frac{1}{2})$ is reset to $A(0)$), though, their stake comprises only half of the debt pool and the time to maturity is now only $T/2$. That is, at each roll over date, investors in the maturing time tranche risk the possibility that their re-invested capital is facilitating a clean exit for investors in the subsequent time tranche.

In our implementation, which we present in the following section, we implement the integrals above from simulated paths, which allows easy tracking of remaining time tranches and any losses sustained. We also note that in the function EL_2, values are scaled for one time tranche by dividing appropriately by $m = 2$, the number of time tranches. Then, the ex-ante expected percentage loss is expressed as:

$$PL_2 = \frac{EL_2}{B(0)/2} \leq L_0 \tag{9}$$

Having determined H_1 and H_2, we proceed to find H_3, and so on, in a similar fashion. As m increases through values $1, 2, 3, \ldots$, the equations involve additional nested integrals, and tractability is achieved by computing expected losses under simulated paths. We use the `R` programming language for the simulations, and even over 10,000 simulated paths, the root finder in `R` is extremely efficient in finding the values of $H_1, H_2, \ldots, H_m$, that ensure that ex-ante percentage expected losses are capped at L_0.[4] In the next subsection, we demonstrate several numerical examples to solidify intuition for the model, and we discuss additional results and insights.

3.3 Suboptimal rollover trigger levels with multiple time tranches

We now implement the full simulation model to examine the costs of rollover risk and the acceptable extent of senior-debt funding in a SPV, accounting for the adversarial game among senior investors across multiple time tranches. We first explore, under varying asset-liquidation discounts $\delta = \{10\%,\ 5\%,\ 3\%,\ 2\%,\ 0\%\}$, how the ex-ante expected losses to senior-debt investors are affected as we increase the number of time tranches, m (while keeping the rollover-decision, $H_m = H_1$, fixed). For demonstrative purposes, we first find the H_1 that results in an expected loss of 1%. Then, we fix all subsequent H_m equal to this H_1, and we calculate the expected losses to investors as m grows (but H_m does not).

We assume initial assets of $A(0) = \theta = 100$, which follows the asset process outlined

[4] The root finding function we use is `uniroot`, included in the base distribution of `R`.

earlier in Section 3.1, with mean-reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$. We assume that the initial time-to-maturity at issuance is $T = 1$ year, and we execute our simulations in incremental time steps of $h = 1$ month. Thus, we generate 10,000 simulated paths, each comprising 12-month asset processes, and we take the average loss across all 10,000 simulations to determine expected losses under each scenario. For all $m > 2$, we assume that all senior debt has the same maturity ($T = 1$ year) at issuance, but that each of the m time tranches mature at equally spaced intervals of T/m; i.e., for $m = 2$, the time tranches mature six months apart, and for $m = 3$, the time tranches mature four months apart.

The results, which we present in **Table 1**, show that ex-ante expected losses increase with the number of time tranches, m, when the rollover-decision leverage threshold, H_m remains fixed. For instance, under an asset-liquidation discount of $\delta = 5\%$, a rollover-decision threshold of $H_1 = 1.0677$ yields an expected loss of 1% to the senior debt investors in a SPV with initial senior debt of $D_B = A(0)/H_1 = 93.6563$ comprising just one time tranche. That is, if investors decide to roll over their investment in this SPV as long as the asset-to-debt ratio is greater than $H_1 = 1.0677$, their ex-ante expected loss is capped at 1%.

However, we see that if investors continue to use this same decision rule as the number of time tranches (m) grows, then expected losses also grow. For instance, under $m = 2$ and $m = 3$, the expected losses to investors increase to 1.39% and 1.75%, respectively. By $m = 6$ time tranches, using $H_6 = H_1 = 1.0677$, the expected loss to investors reaches $2.63\% > 1\%$, indicating that investors must increase their rollover leverage thresholds (i.e., require lower levels of senior debt in the SPV) to maintain their original, acceptable level of expected losses. We observe similar results across the varying fire-sale discounts, δ.

Looking across the different asset-liquidation discounts, we also see that as discounts decrease (i.e., as recovery rates rise), the H_1 required to achieve a 1% expected loss decreases, and equivalently, the maximum feasible amount of senior debt increases. That is, as asset-liquidation discounts decrease, the SPV can sustain greater amounts of senior debt since the deadweight cost of rollover risk (and the losses on ensuing asset liquidations) declines.

Overall, these results demonstrate that, interestingly, diversifying debt funding sources across time tranches increases rollover risk and expected losses to senior debt holders, and may thereby increase the cost of senior debt financing. Collectively, these outcomes also suggest that the SPV must assume a lower amount of senior debt to remain viable if its senior debt funding is diversified across many time tranches with varying rollover dates.

3.4 Optimal rollover decisions with multiple time tranches

As we demonstrated earlier with discrete tree examples in Section 2.1 and Section 2.2, the investors' decision to roll over is determined by H_1 when there is $m = 1$ tranche in the SPV, and $H_2 > H_1$ when there are $m = 2$ tranches. We now generalize this result, implementing the full simulation model to examine the appropriate H_m that maintains the same expected loss to investors, $L_0 = 1\%$, even as m increases.

We assume the same asset process and initial parameters as before (in Section 3.3), and again, we explore the implications under varying asset-liquidation discounts, δ. We also report the probability of experiencing a loss, which differs from the expected (%) loss in that it represents the percentage of occurrences, across our simulations, in which any amount of loss is incurred. As before, we take the average loss across all 10,000 simulated paths to obtain the expected loss to investors under each scenario.

The results, which we present in **Table 2**, show that to keep their expected losses fixed, investors demand an ever-increasing asset-to-debt ratio, H_m, to roll over their investments as the number of time tranches, m, in the SPV grows. For example, under a fire-sale discount of $\delta = 5\%$, $H_1 = 1.0677$ represents the minimum rollover threshold that yields an expected loss of 1% to senior-debt holders when there is just a single time tranche in a SPV with initial senior debt maxed out to $D_B = A(0)/H_1 = 93.6563$. However, as the number of time tranches (m) grows, investors must increase their minimum rollover threshold to achieve the same 1% expected loss, and equivalently, the SPV's maximum senior-debt capacity is reduced.

Under $m = 2$ and $m = 3$, investors demand thresholds of $H_2 = 1.0754$ and $H_3 = 1.0795$, respectively, to roll over their investments, which correspond to maximum senior-debt capacities of $D_B = \{92.9908, 92.6391\}$, respectively, for the SPV. By $m = 6$ time tranches, investors demand an even greater threshold of $H_6 = 1.0854$, requiring that the asset-to-debt ratio be greater than 1.0854 to roll over their investment and thereby reducing the SPV's maximum senior-debt capacity to $D_B = 92.1282$. We also see that as m and H_m increase, the higher thresholds are associated with a lower probability of loss. Therefore, increasing the number of tranches lowers the total senior debt that may be issued, but correspondingly lowers the probability of losses.

In a related analysis, we examine how varying asset volatility, $\sigma = \{5, 10\}$, impacts rollover risk and the maximum senior-debt capacity of the SPV. The results, which we present in **Table 3**, show that for a fixed asset-liquidation discount, investors demand a greater asset-to-debt ratio, H_m, to roll over their investments when the underlying asset pool has greater price volatility. For instance, under a liquidation discount of $\delta = 5\%$ and under $m = 1$ time tranche, investors require a threshold of $H_1 = 1.0677$ to roll over their capital when asset volatility is $\sigma = 5$, but they require a threshold of $H_1 = 1.1253$ when asset volatility is $\sigma = 10$. These translate to maximum senior-debt capacities of

$D_B = \{93.6563, 88.8644\}$, respectively, for the SPV. Similarly, under $m = 6$ time tranches, $H_6 = 1.0854$ when $\sigma = 5$, but $H_6 = 1.1453$ when $\sigma = 10$, which translates to maximum senior-debt capacities of $D_B = \{92.1282, 87.3130\}$, respectively.

Overall, we observe a similar pattern of H_m increasing in m across the varying fire-sale discounts, δ. That is, rollover risk increases as the number of time tranches increases, thereby promoting an adversarial game among senior-debt holders, who require ever greater asset-to-debt ratios to continue to roll over their investments. Furthermore, the general level of H_m is lower and the maximum senior-debt capacity of the SPV is greater as asset volatility and fire-sale discounts decrease. That is, investors accept a lower rollover threshold, since the losses incurred due to the adversarial relation among investors are reduced when asset liquidations are less likely and are not as costly.

3.5 Covenants for managing rollover risk

Given the escalating rollover risk and expected losses as the number of time tranches increases, a natural question arises as to whether we can design an ex-ante mechanism to mitigate this adversarial game among investors. Because the additional risk arises from the probability that an investor's capital will be used to fund the exit of another time tranche, we propose a de-leveraging solution that repays, in part, all senior investors equally irrespective of their specific time tranche/rollover date. Specifically, we propose that the SPV implement an ex-ante capital-structure covenant, defining a leverage trigger that, once breached, requires all time tranches to be partially repaid in equal amounts (as was outlined earlier in Section 2.3), thereby protecting all time tranches equally regardless of their varying rollover dates. Following this remedy, we now implement our full simulation model to examine the effects of having this ex-ante covenant in place.

Using the same asset process and initial parameters as before, we begin with the case of $m = 1$ time tranche. The results, which we present in **Table 4**, show that with the implementation of a SPV threshold covenant, the rollover threshold, H_1, demanded by investors is reduced. For instance, under $\delta = 5\%$, without a SPV threshold covenant in place (i.e., $K = 0$), investors demand a rollover threshold of $H_1 = 1.0677$, which allows them to satisfy their expected loss requirement of 1%. However, with a SPV threshold covenant of $K = 1.05$ (i.e., regardless of whether the trigger is breached on a rollover date, the SPV will liquidate and repay investors if assets-to-debt ratio drops below $K = 1.05$), investors' threshold requirement drops to $H_1 = 1.0532$, allowing the SPV to maintain a higher senior-debt capacity. Intuitively, the SPV threshold covenant acts as a stop loss for investors during the investment lock-up period, allowing them to roll over their investments at lower asset-to-debt ratios with greater confidence.

To generalize this result, we also examine the effects of implementing a SPV threshold covenant under $m = 2$ time tranches (presented in **Table 5**) and under $m = 3$ time tranches (presented in **Table 6**). We find that with a SPV threshold covenant in place, we can effectively mitigate the additional rollover risk and expected losses that arise as we increase the number of time tranches. For instance, for $m = 2$ time tranches (and under a fire-sale discount of $\delta = 10\%$), we see that without the SPV threshold in place (i.e., $K = 0$), investors' demand a rollover threshold of $H_2 = 1.1351 > H_1$ to achieve the acceptable expected loss of $L_0 = 1\%$. However, with a SPV threshold covenant of $K = 1.025$, the expected loss to investors still falls within the acceptable $L_0 = 1\%$, even when investors relax their demanded threshold to $H_2 = 1.1271 = H_1$. We make similar observations under $m = 3$ time tranches.

Overall, with the introduction of an effective, ex-ante SPV threshold covenant K, senior-debt holders no longer demand an ever-increasing H_m to roll over their capital. Thus, the maximum senior-debt capacity of the SPV can be maintained, even as it increases the number of time tranches for greater funding diversity.

4 Concluding Comments

Special purpose vehicles (SPVs) are characterized by high leverage, illiquid assets that incur asset-sale discounts, and staggered liabilities that are susceptible to rollover risk. In the recent financial crisis, SPVs experienced triple-witching: falling asset values leading to unsustainable leverage, sharp increases in asset illiquidity and greater fire-sale discounts, and withdrawal of capital leading to a death spiral as SPVs defeased concurrently.

We present a model that incorporates these features, and we suggest a covenant-based remedy that results in timely deleveraging and mitigates rollover risk, thereby decreasing the ex-ante expected losses for both equity-note holders and senior-debt holders. We do this in an adversarial model where the varying time tranches of senior debt compete with one other in making rollover decisions, and equity-note holders attempt to maximize the senior debt in the SPV capital structure, accounting for the game among senior-debt holders, and the optimal rollover decisions of varying time-tranche holders.

Overall, designing a viable SPV is a complex problem, requiring decisions not only along the suitable portfolio composition, leverage (asset-to-debt ratio), and asset-liability gap of the SPV, but also along finer details of the funding sources with regard to the diversity of sources and rollover dates. As we have seen, diversifying financing sources by having many time tranches of senior debt exacerbates the adversarial game among senior-debt holders with otherwise equal priority, makes debt more costly, and, thus, may not be optimal for either senior-debt holders or equity-note holders in the SPV. However, these problems may be mitigated via our suggested covenant structure. Further research will no doubt uncover other solutions to the optimal SPV design problem, and some, such as contingent capital are particularly ripe for exploration.

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Table 1: SPV Structures and Expected Losses. This table shows the expected losses to investors as the number of time tranches, m, increases (but their rollover threshold rule, H_m, does not). H_m represents the minimum asset-to-debt ratio required for investors in an m-tranche SPV to continue to roll over their investment. Expected (%) losses are calculated as the average loss incurred across all simulation paths, and the probability of loss is simply the percentage of simulated occurrences in which any loss occurs. We report these expected losses under varying recovery rates of $\{90\%, 95\%, 97\%, 98\%, 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.1271	88.7271	36.43%	1.00%
	2	1.1271	88.7271	31.20%	1.39%
	3	1.1271	88.7271	27.95%	1.75%
	4	1.1271	88.7271	25.08%	2.05%
	6	1.1271	88.7078	21.94%	2.63%
Recov = 95%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0677	93.6563	36.43%	1.00%
	2	1.0677	93.6563	31.20%	1.39%
	3	1.0677	93.6563	27.95%	1.75%
	4	1.0677	93.6563	33.23%	2.05%
	6	1.0677	93.6563	21.94%	2.63%
Recov = 97%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0457	95.6282	36.43%	1.00%
	2	1.0457	95.6282	31.20%	1.39%
	3	1.0457	95.6282	44.37%	1.75%
	4	1.0457	95.6282	25.08%	2.05%
	6	1.0457	95.6282	49.52%	2.63%
Recov = 98%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0351	96.6139	36.43%	1.00%
	2	1.0351	96.6139	31.20%	1.39%
	3	1.0351	96.6139	27.95%	1.75%
	4	1.0351	96.6139	39.08%	2.05%
	6	1.0351	96.6139	32.67%	2.63%
Recov = 100%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0144	96.6139	36.43%	1.00%
	2	1.0144	98.6139	31.20%	1.39%
	3	1.0144	98.6139	27.95%	1.75%
	4	1.0144	98.6139	39.08%	2.05%
	6	1.0144	98.6139	32.67%	2.63%

Table 2: SPV Structures and Maximum Senior-Debt Capacity, Under Varying Asset-Liquidation Discounts. This table shows the maximum senior-debt capacity, D_B, of the SPV as the number of time tranches, m, increases, and investors uniformly demand expected losses to be capped at $L_0 = 1\%$. Min H_m represents the minimum rollover threshold (asset-to-debt ratio) required for investors in an m-tranche SPV to continue to roll over their investment, given their level of acceptable expected losses. Max D_B is then the corresponding maximum senior-debt capacity of the SPV. Expected (%) losses are calculated as the average loss incurred across all simulation paths, and the probability of loss is simply the percentage of simulated occurrences in which any loss occurs. We report these statistics under varying recovery rates of $\{90\%, 95\%, 97\%, 98\%, 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1271	88.7271	36.43%	1.00%
	2	1.1351	88.0966	24.69%	1.00%
	3	1.1394	87.7633	35.16%	1.00%
	4	1.1421	87.5581	15.41%	1.00%
	6	1.1458	87.2784	11.10%	1.00%
Recov = 95%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0677	93.6563	36.43%	1.00%
	2	1.0754	92.9908	24.69%	1.00%
	3	1.0795	92.6391	35.16%	1.00%
	4	1.0820	92.4223	15.41%	1.00%
	6	1.0854	92.1282	11.10%	1.00%
Recov = 97%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0457	95.6282	36.43%	1.00%
	2	1.0532	94.9479	24.68%	1.00%
	3	1.0572	94.5870	19.05%	1.00%
	4	1.0597	94.3680	23.48%	1.00%
	6	1.0631	94.0676	11.10%	1.00%
Recov = 98%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0351	96.6139	36.43%	1.00%
	2	1.0424	95.9287	24.70%	1.00%
	3	1.0464	95.5617	19.05%	1.00%
	4	1.0489	95.3392	15.40%	1.00%
	6	1.0523	95.0341	11.03%	1.00%
Recov = 100%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0144	98.5852	36.43%	1.00%
	2	1.0216	97.8859	24.69%	1.00%
	3	1.0255	97.5149	19.07%	1.00%
	4	1.0279	97.2873	27.45%	1.00%
	6	1.0312	96.9733	11.03%	1.00%

Table 3: SPV Structures and Maximum Senior-Debt Capacity, Under Varying Asset-Volatilities. This table shows the maximum senior-debt capacity, D_B, of the SPV as the number of time tranches, m, increases, and investors uniformly demand expected losses to be capped at $L_0 = 1\%$. Min H_m represents the minimum rollover threshold (asset-to-debt ratio) required for investors in an m-tranche SPV to continue to roll over their investment, given their level of acceptable expected losses. Max D_B is then the corresponding maximum senior-debt capacity of the SPV. Expected (%) losses are calculated as the average loss incurred across all simulation paths, and the probability of loss is simply the percentage of simulated occurrences in which any loss occurs. We report these statistics under varying asset volatilities of $\sigma = \{5, 10\}$ and varying recovery rates of $\{95\%, 97\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = \{5, 10\}$; $T = 1$ year; and time step $h = 1$ month. Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recovery = 95%					
$\sigma = 5$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0677	93.6563	36.43%	1.00%
	2	1.0754	92.9908	24.69%	1.00%
	3	1.0795	92.6391	35.16%	1.00%
	4	1.0820	92.4223	15.41%	1.00%
	6	1.0854	92.1282	11.10%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1253	88.8644	21.08%	1.00%
	2	1.1314	88.3844	14.41%	1.00%
	3	1.1360	88.0261	29.95%	1.00%
	4	1.1400	87.7213	8.67%	1.00%
	6	1.1453	87.3130	6.55%	1.00%

Recovery = 97%					
Recov = 97%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0457	95.6282	36.43%	1.00%
	2	1.0532	94.9479	24.68%	1.00%
	3	1.0572	94.5870	19.05%	1.00%
	4	1.0597	94.3680	23.48%	1.00%
	6	1.0631	94.0676	11.10%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1021	90.7355	21.08%	1.00%
	2	1.1081	90.2469	14.41%	1.00%
	3	1.1126	89.8811	10.98%	1.00%
	4	1.1165	89.5688	16.86%	1.00%
	6	1.1217	89.1541	6.55%	1.00%

Table 4: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of $m = 1$ Time Tranche. This table shows the maximum senior-debt capacity, D_B, of a one-tranche SPV, now with a threshold covenant K in place, where investors uniformly demand expected losses to be capped at $L_0 = 1\%$. The SPV Threshold, K, represents the asset-to-debt ratio that, once breached, requires asset liquidation to repay investors (regardless of whether the breach occurred on a rollover date). Min H_1 represents the minimum rollover threshold (asset-to-debt ratio) required for investors in the one-tranche SPV to continue to roll over their investment, given their level of acceptable expected losses $L_0 = 1\%$. Max D_B is then the corresponding maximum senior-debt capacity of the SPV. Expected (%) losses are calculated as the average loss incurred across all simulation paths. We report these statistics under varying recovery rates of $\{95\%, 97\%, 98\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 95%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0677	93.6563	1.00%
	1.040	1.0679	93.6397	1.00%
	1.045	1.0625	94.1142	1.00%
	1.050	1.0532	94.9470	1.00%
Recov = 97%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0457	95.6282	1.00%
	1.020	1.0447	95.7297	1.00%
	1.025	1.0375	96.3397	1.00%
	1.029	1.0303	97.0286	1.00%
Recov = 98%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0351	96.6139	1.00%
	1.010	1.0336	96.7466	1.00%
	1.015	1.0264	97.4258	1.00%
	1.019	1.0194	98.0985	1.00%

Table 5: Enhancing Senior Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of $m = 2$ Time Tranches. This table shows the expected losses and senior-debt capacity, D_B, of a two-tranche SPV, now with an ex-ante SPV threshold covenant, K, in place. The SPV Threshold, K, represents the asset-to-debt ratio that, once breached, requires partial asset liquidation, with the proceeds split equally across all senior-debt investors regardless of their respective rollover dates (see Section 3.5 for details). H_m represents the minimum asset-to-debt ratio required for investors in an m-tranche SPV to continue to roll over their investment. Expected (%) losses are calculated as the average loss incurred across all simulation paths. We report these statistics under varying recovery rates of $\{90\%, 95\%, 97\%, 98\% \ 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.1271	1.1351	88.0966	1.00%
	1.000	1.1271	1.1351	88.0966	0.77%
	1.025	1.1271	1.1351	88.0966	0.58%
	1.000	1.1271	1.1271	88.7271	1.02%
	1.025	1.1271	1.1271	88.7271	0.74%
Recov = 95%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0677	1.0754	92.9908	1.00%
	1.000	1.0677	1.0754	92.9908	0.32%
	1.025	1.0677	1.0754	92.9908	0.11%
	1.000	1.0677	1.0677	93.6563	0.36%
	1.025	1.0677	1.0677	93.6563	0.12%
Recov = 97%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0457	1.0532	94.9479	1.00%
	1.000	1.0457	1.0532	94.9479	0.14%
	1.025	1.0457	1.0532	94.9479	0.03%
	1.000	1.0457	1.0457	95.6282	0.15%
	1.025	1.0457	1.0457	95.6282	0.04%
Recov = 98%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0351	1.0424	95.9287	1.00%
	1.000	1.0351	1.0424	95.9287	0.08%
	1.025	1.0351	1.0424	95.9287	0.01%
	1.000	1.0351	1.0351	96.6139	0.09%
	1.025	1.0351	1.0351	96.6139	0.02%
Recov = 100%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0144	1.0216	97.8859	1.00%
	1.000	1.0144	1.0216	97.8859	0.02%
	1.000	1.0144	1.0144	98.5852	0.03%

Table 6: Enhancing Senior-Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of $m = 3$ Time Tranches. This table shows the expected losses and senior-debt capacity, D_B, of a three-tranche SPV, now with an ex-ante SPV threshold covenant, K, in place. The SPV Threshold, K, represents the asset-to-debt ratio that, once breached, requires partial asset liquidation, with the proceeds split equally across all senior-debt investors regardless of their respective rollover dates (see Section 3.5 for details). H_m represents the minimum asset-to-debt ratio required for investors in an m-tranche SPV to continue to roll over their investment. Expected (%) losses are calculated as the average loss incurred across all simulation paths. We report these statistics under varying recovery rates of $\{90\%, 95\%, 97\%, 98\% \; 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial senior debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.1271	1.1351	1.1394	87.7633	1.00%
	1.000	1.1271	1.1351	1.1394	87.7633	0.61%
	1.025	1.1271	1.1351	1.1394	87.7633	0.46%
	1.000	1.1271	1.1271	1.1271	88.7271	0.91%
	1.025	1.1271	1.1271	1.1271	88.7271	0.63%
Recov = 95%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0677	1.0754	1.0795	92.6391	1.00%
	1.000	1.0677	1.0754	1.0795	92.6391	0.27%
	1.025	1.0677	1.0754	1.0795	92.6391	0.15%
	1.000	1.0677	1.0677	1.0677	93.6563	0.38%
	1.025	1.0677	1.0677	1.0677	93.6563	0.26%
Recov = 97%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0457	1.0532	1.0572	94.5870	1.00%
	1.000	1.0457	1.0532	1.0572	94.5870	0.17%
	1.025	1.0457	1.0532	1.0572	94.5870	0.11%
	1.000	1.0457	1.0457	1.0457	95.6282	0.28%
	1.025	1.0457	1.0457	1.0457	95.6282	0.22%
Recov = 98%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0351	1.0424	1.0464	95.5617	1.00%
	1.000	1.0351	1.0424	1.0464	95.5617	0.14%
	1.025	1.0351	1.0424	1.0464	95.5617	0.09%
	1.000	1.0351	1.0351	1.0351	96.6139	0.25%
	1.025	1.0351	1.0351	1.0351	96.6139	0.15%
Recov = 100%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0144	1.0216	1.0255	97.5149	1.00%
	1.000	1.0144	1.0216	1.0255	97.5149	0.10%
	1.000	1.0144	1.0144	1.0144	98.5852	0.19%

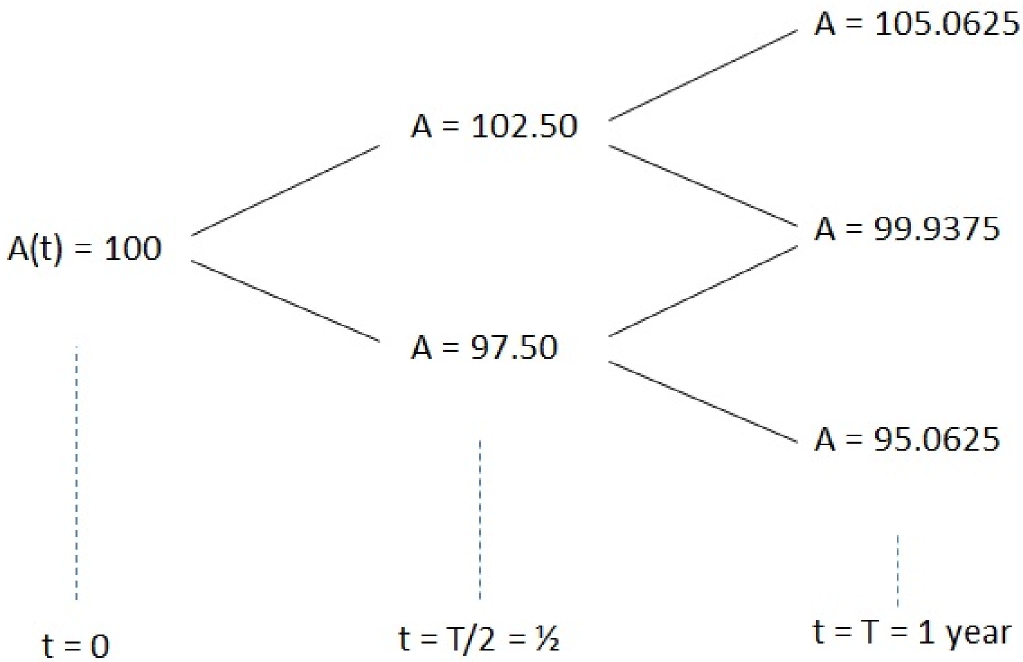

Figure 1: Two-period asset tree, $T = 1$ year. The up and down moves each period are based on a $\pm 2.5\%$ range.

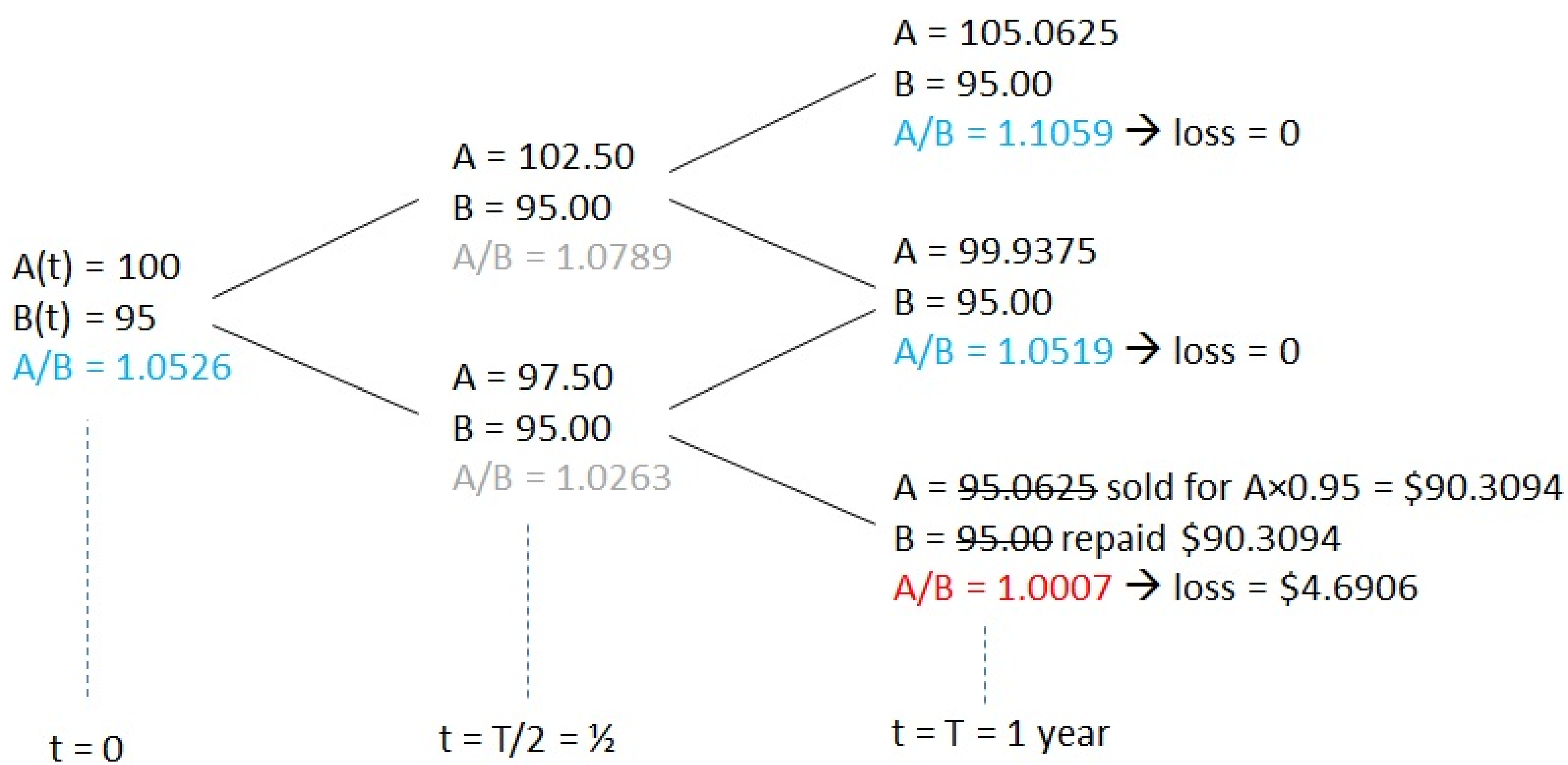

Figure 2: Two-period asset tree, $T = 1$ year, $m = 1$ time tranche, $H_1 = 1.05$. Senior-debt face values outstanding are shown below asset values for each node along the tree. Losses are shown as incurred. In this case, since just one time tranche of debt is considered, the only possible loss occurs at the end of two periods, when the senior debt comes due for rollover.

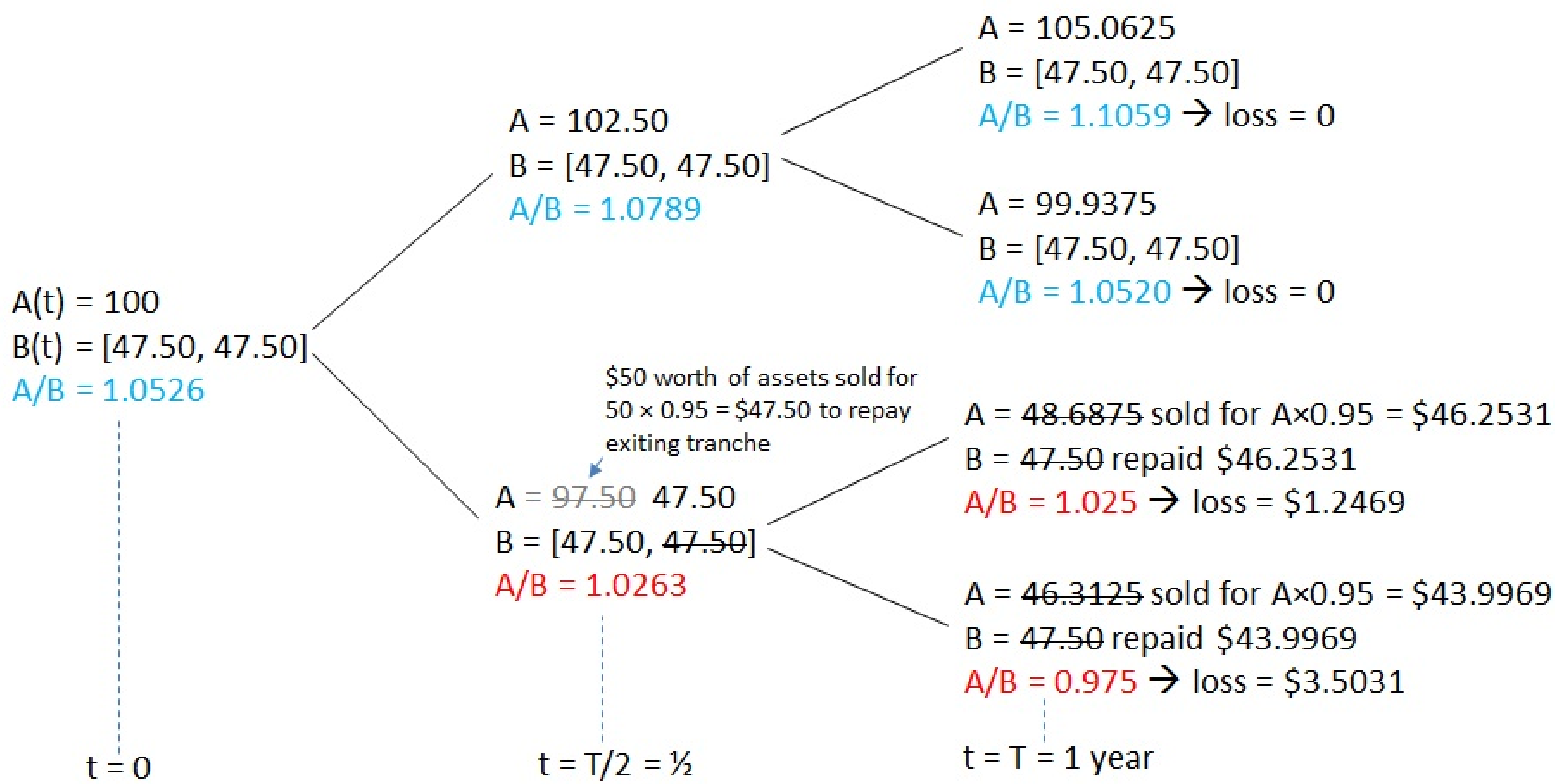

Figure 3: Two-period asset tree, $T = 1$ year, $m = 2$ time tranches, $H_2 = H_1 = 1.05$. Senior-debt face values are shown below asset values, partitioned by time tranche. Losses are shown when incurred, as are reductions in debt.

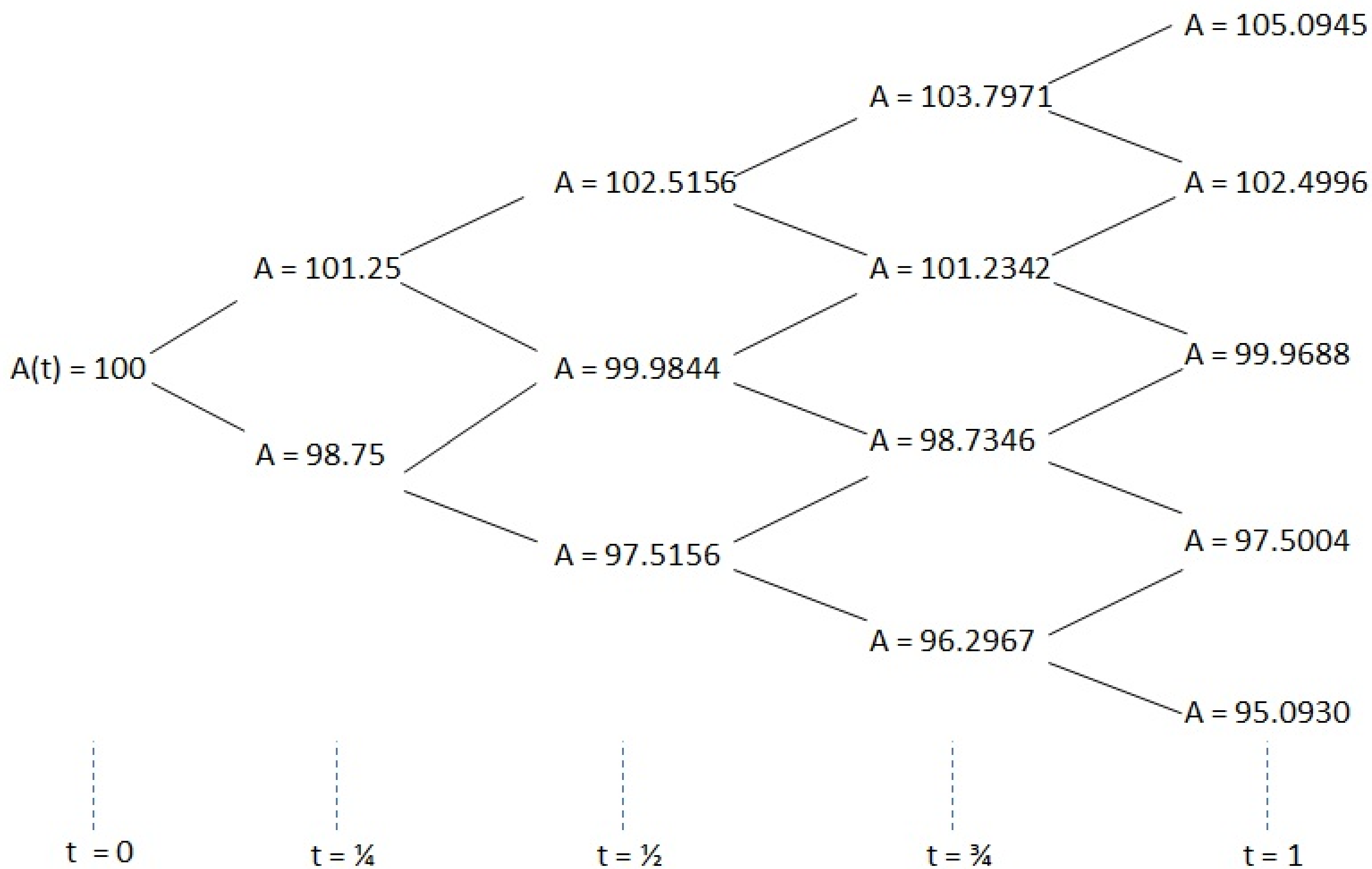

Figure 4: Four-period asset tree, $T = 1$ year. The up and down moves each period are based on a $\pm 1.25\%$ range, half that in the case of the two-period tree in the preceding examples.

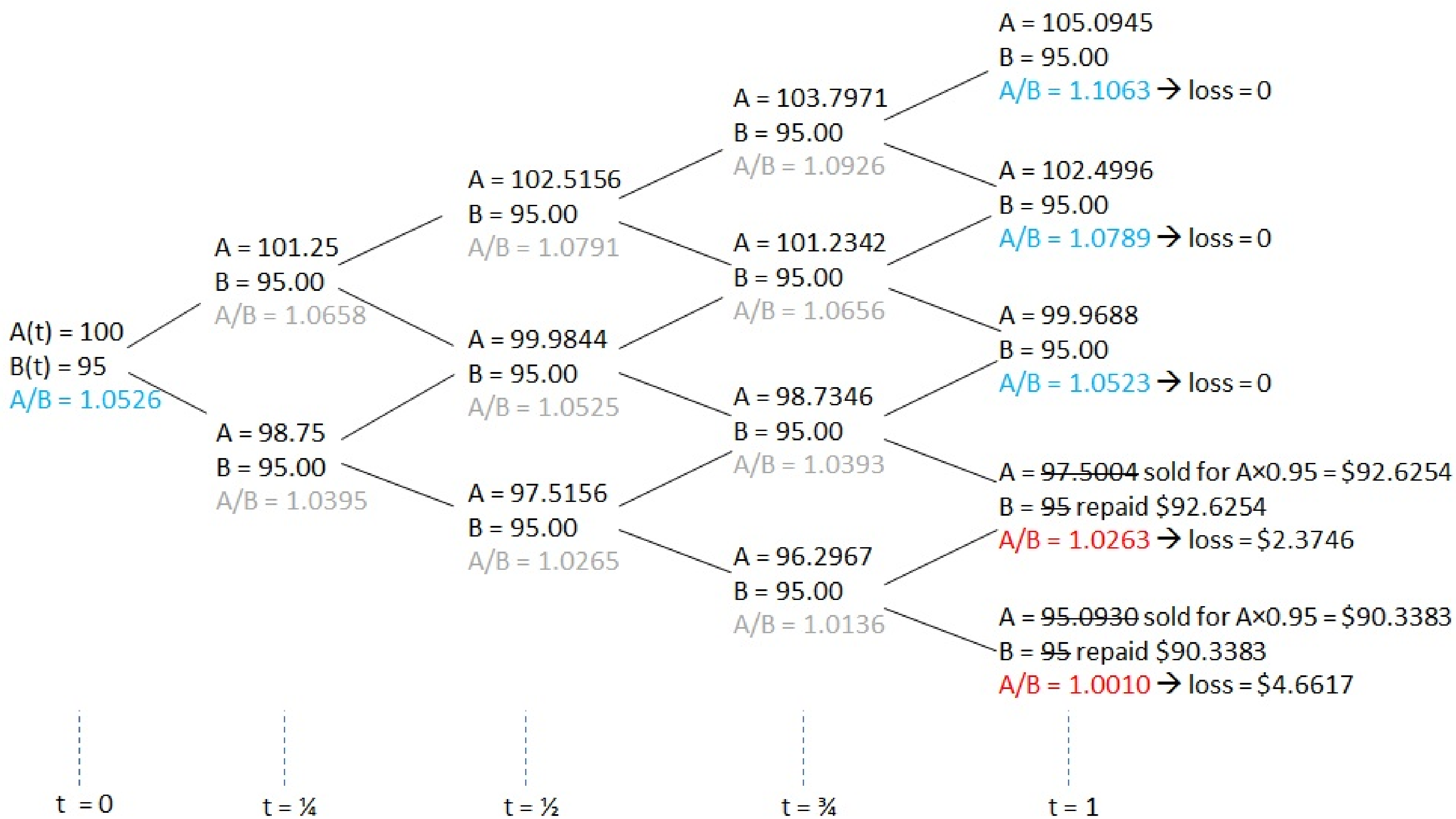

Figure 5: Four-period asset tree, $T = 1$ year, $m = 1$ time tranche, $H_1 = 1.05$. Senior-debt face values are shown below asset values. Losses are shown as incurred. Since just one time tranche of debt is considered, the only possible loss occurs at the end of four periods, when the senior debt comes due for rollover.

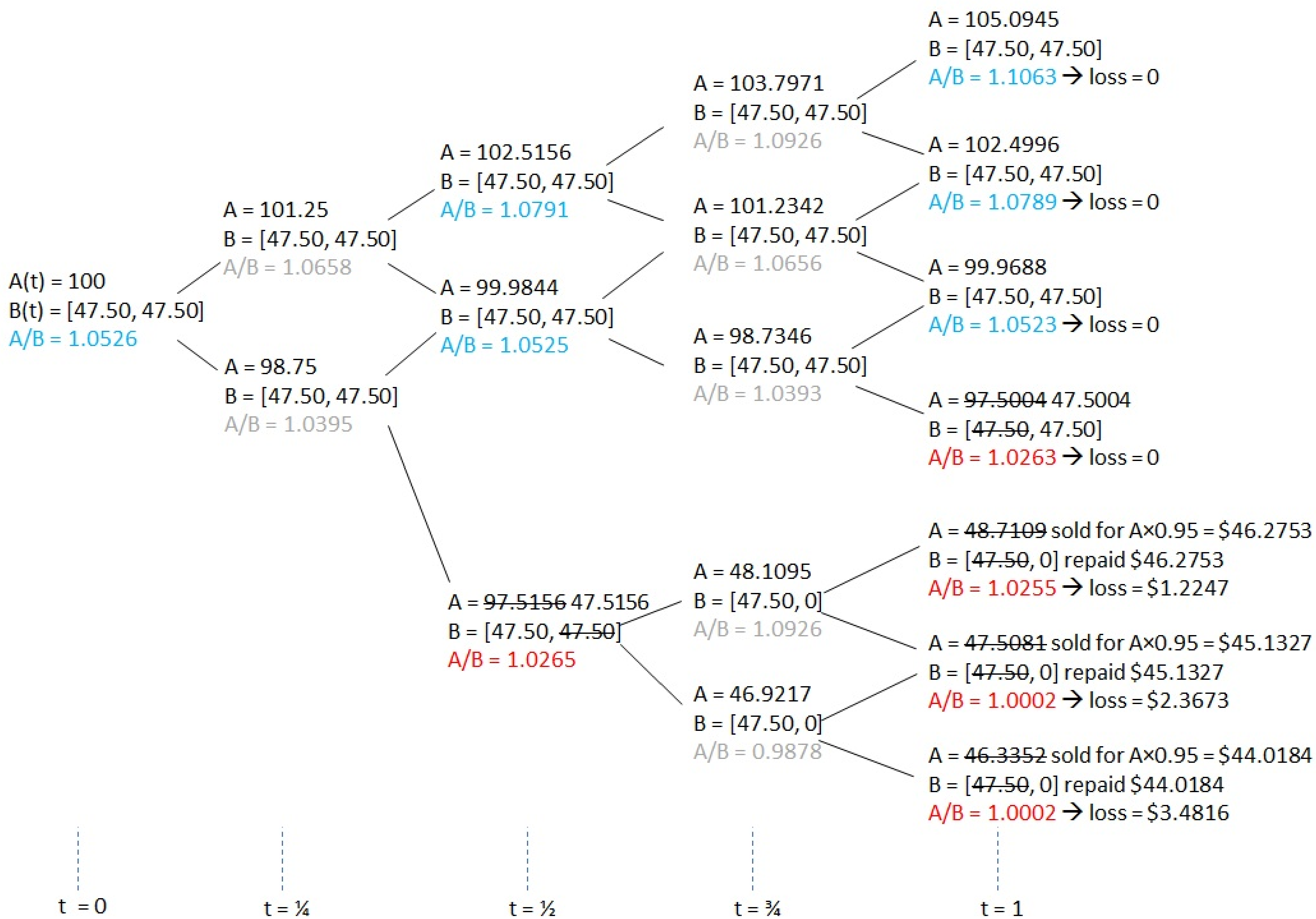

Figure 6: Four-period asset tree, $T = 1$ year, $m = 2$ time tranches, $H_2 = H_1 = 1.05$. Senior-debt face values are shown below asset values, partitioned by time tranche, and losses as well as senior debt reductions are shown as incurred. When investors in a time tranche decline to roll over, the debt value for that time tranche is zeroed out, and asset values are shown before and after repayment of that particular time tranche. The graph above has two sections, and the lower section continues the tree from the lowest node after two periods.

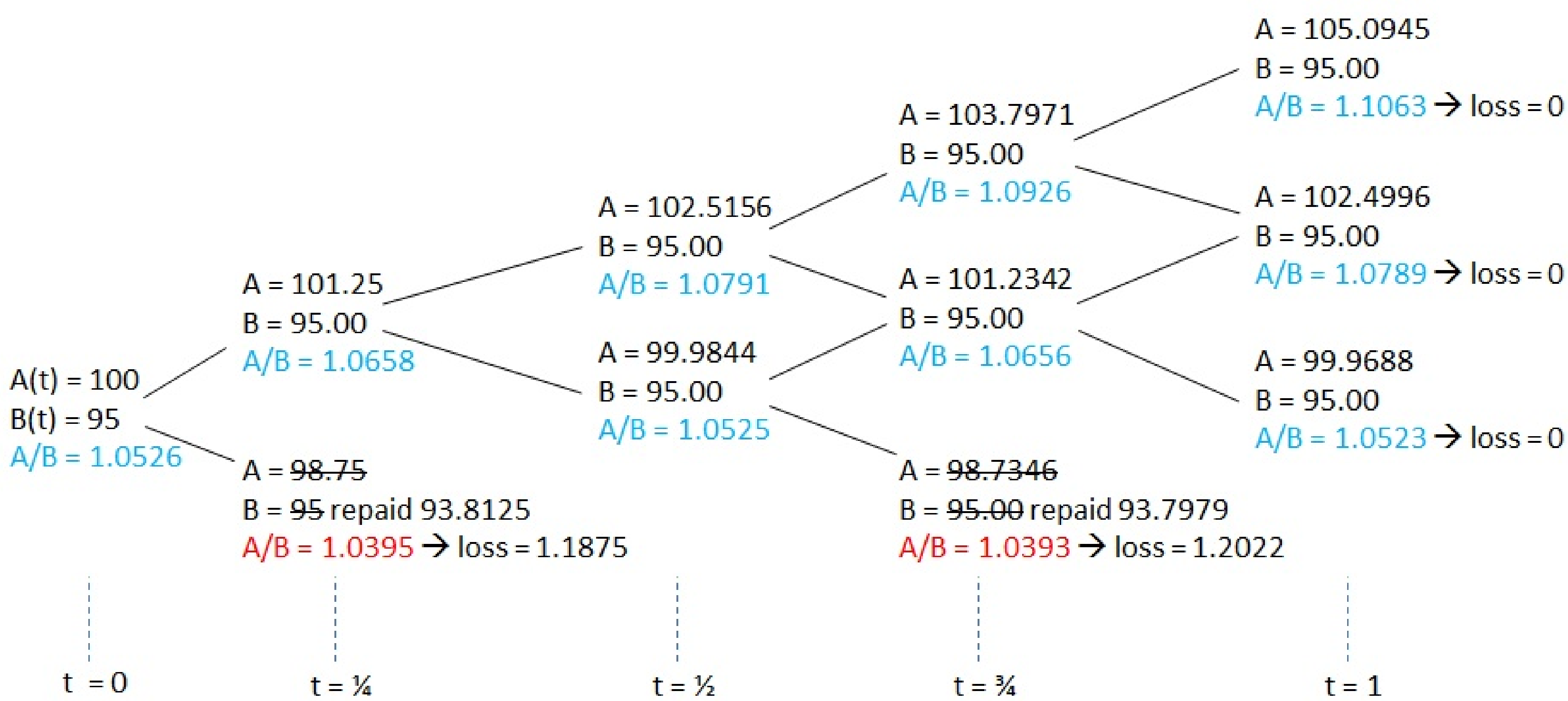

Figure 7: Four-period asset tree, $T = 1$ year, $m = 1$ time tranche, $H_1 = 1.05$, SPV leverage threshold $K = 1.045$. Senior-debt face values and leverage ratios are shown below asset values, and losses are shown as incurred. The SPV leverage trigger, $K = 1.045$, specifies that assets are liquidated to repay senior-debt holders if at any time $A/B \leq K$. The expected loss here is 0.78%, in contrast to the case without the covenant, where the loss is 0.93% (as was shown in **Figure 5**).

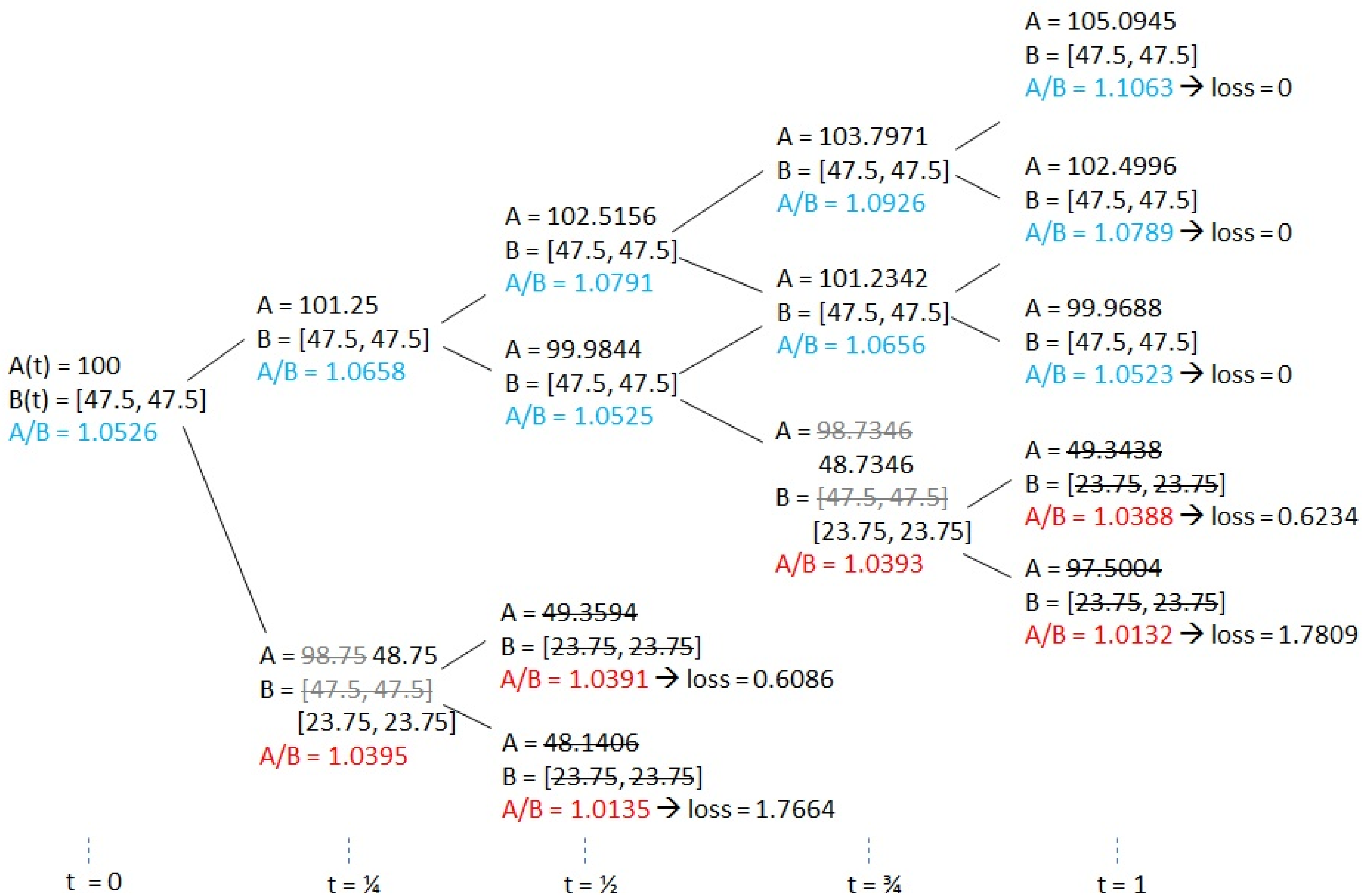

Figure 8: Four-period asset tree, $T = 1$ year, $m = 2$ time tranches, $H_2 = H_1 = 1.05$, SPV leverage threshold $K = 1.045$. Senior-debt face values are shown below asset values, partitioned by time tranche, and losses are shown as incurred. The SPV leverage trigger, $K = 1.045$, specifies a partial liquidation of assets equal to the total amount of one time tranche if at any time $A/B \leq K$ with pari-passu distribution of proceeds across all senior-debt holders without regard to their respective rollover dates. Ex-ante expected losses here are 0.78%, which is the same as that under $m = 1$ time tranche.