Rollover Risk and Capital Structure Covenants in Structured Finance Vehicles

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Abstract

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The shadow banking system comprises special purpose vehicles (SPVs) characterized by high debt, illiquid long-maturity assets funded predominantly by short-maturity debt, and tranched liabilities also known as the capital structure of the SPV. These three features lead to an adversarial game among senior-note holders, who solve for an optimal rollover policy based on the other senior tranches with varying rollover dates. This rollover policy is, in turn, taken into account by capital-note holders (i.e., investors in the equity tranche) when choosing the capital structure (i.e., the assets-to-debt ratio) of the SPV. Rollover risk increases in the number of tranches, resulting in a lower equilibrium (optimal) level of debt and higher cost of debt. The expected life of the SPV may also be shortened. We propose a covenant based capital structure that mitigates these problems and is Pareto-improving for equity and debt holders in the SPV.

Keywords: special purpose vehicle; structured finance; rollover risk; leverage; capital structure; covenants.

1 Introduction

Structured finance deals emerged as an increasingly important means of risk sharing and obtaining access to capital prior to the financial crisis of 2008. The demand for safe, money-like debt has been on the rise, particularly since the rise and burst of the NASDAQ/tech bubble (Caballero and Krishnamurthy (2009); Krishnamurthy and Vissing-Jorgensen (2012)). This, coupled with market segmentation, has resulted in a marked shift in the suppliers of safe debt from the commercial banking system to the shadow banking system (Gennaioli, Shleifer, and Vishny (2013)).

Through special purpose vehicles (SPVs), the shadow banking system has provided an increasing share of "safe" assets, overtaking many traditional sources of safe debt (Gorton, Lewellen, and Metrick (2012)). At their peak in 2007, Pozsar, Adrian, Ashcraft, and Boesky (2012) estimate that shadow banking liabilities had grown to nearly $22 trillion. Given their scope and magnitude, it is crucial to understand this relatively new and highly complex asset class. For example, before the financial crisis, twenty-nine structured investment vehicles (SIVs), a special sub-class of SPVs, held an estimated $400 billion in assets.[1] Despite their AAA rating, senior notes issued by these SPVs experienced an average 50% loss, and subordinated notes experienced near total loss.[2]

Overall, the devastating effects of over-leveraging, coupled with rollover risk and liquidity risk, became starkly apparent in the financial crisis. Although studies in the aftermath have extensively explored the collateral quality and over-leveraging of SPVs, the role of funding risk and rollover risk remain unattended. Our purpose is to analyze the additional risks borne by SPVs given funding diversity and varying rollover dates across senior note holders, thereby arriving at the maximal safe level of debt (i.e., senior tranche size) for the SPV accounting for these factors.

A structured finance deal is engineered to tranche investments in an asset pool into prioritized cash-flow claims. The resulting liability structure comprises two broad sources of capital for SPVs: the so-called equity portion, which comprises equity or backstop notes commonly denoted as "capital" notes, and the de facto debt portion of the capital structure, known as "senior" notes, which are supported by the subordinated capital/equity notes. Senior debt forms the primary source of financing, often accounting for more than 90% of the liabilities. In general, structured finance deals are designed with the intent to secure a AAA rating for the senior notes, which pay slightly above the risk free rate of interest,

[1] http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/5769361/400bn-SIV-market-sold-off-in-two-years.html

[2] http://www.risk.net/risk-magazine/news/1517514/almost-siv-assets-sold-fitch

making them attractive as money-market instruments. Senior notes have maturities shorter than that of the investment assets, and are rolled over at maturity, provided lenders are satisfied with a sufficiently low credit risk profile.

These notes are typically issued in tiered tranches, mostly identical in their maturities at issuance but with different rollover dates, resulting in sequential rollover decisions where the exit/re-investment opportunity is presented to each tranche in a staggered manner. This tiering of senior-note rollover dates creates an adversarial relation among tranche holders, who risk rolling over their investments only to have the next maturing tier(s) exit at their expense. That is, although the extent of tiering/tranching decreases funding risk since it guarantees that all investors cannot exit en masse, it also risks altering investor behavior by increasing the safety threshold at which successive senior note holders are willing to roll over their investments. In this paper, we analyze rollover risk in the face of this adversarial problem among senior-note holders of varying rollover dates, and we explore remedies to mitigate this issue.

To this end, we develop a discrete-time model to examine how the leverage threshold (i.e., asset to senior-debt ratio) at which an investor is willing to roll over his investment is affected by the number of tranches themselves. We then explore how this adversarial game among tranches is further exacerbated by illiquid asset-sale discounts, whereby investment assets must be liquidated at a discount to pay off an exiting tranche. In doing so, we demonstrate how a small drop in asset values can result in a rapid shrinking of debt capacity as tranche holders decline to roll over their investments.

We also propose a potential remedy to mitigate rollover risk in maturing senior notes, and we show that this remedy decreases ex-ante expected losses to both debt and equity investors of the SPV. Specifically, we suggest a covenant to mitigate rollover risk by committing to a partial liquidation of assets once a pre-set leverage threshold is breached, with pari passu distribution of proceeds across all senior note holders without regard to their respective rollover dates. Intuitively, this covenant acts as a stop loss mechanism, allowing a maturing tranche holder to roll over his investment with greater confidence that his reinvested capital is not simply allowing subsequent tranches a riskless exit at his expense.

Our paper complements the work of Acharya, Gale, and Yorulmazer (2011) and He and Xiong (2012), who also explore the adversarial position across debt tranches in a continuous-time model. We extend the work of He and Xiong (2012) by providing a remedy to the adversarial problem that exacerbates rollover risk, and in contrast to Acharya, Gale, and Yorulmazer (2011) our model does not rely on information-theoretic arguments. Furthermore, we are especially interested in the capital structure of a structured finance vehicle in the presence of rollover risk. Thus, we also derive the maximal possible debt (i.e., equilibrium senior tranche size) of the SPV given senior note holders' risk preferences, while accounting for the rollover decisions of the various senior tranche holders.

The main features of this paper are as follows. First, we consider the capital structure decision of the SPV, i.e., we solve an optimization problem subject to rollover risk. The goal of equity holders in the SPV is to maximize the amount of debt issued (which maximizes their return on equity), subject to a cap on the ex-ante expected percentage loss on debt to ensure a sufficiently high quality rating (i.e., so that the SPV is marketable). Since the expected percentage loss depends on the inherent rollover risk, our model captures this feature in the optimization problem.

Second, we model the optimal rollover decisions of debt tranche holders. Debt holders specify a rule where they decline to roll over debt when assets have fallen below a specified level and the leverage in the SPV (ratio of assets to debt) has become unsustainable with respect to their acceptable expected loss levels. When a tranche holder takes his turn in deciding whether or not to roll over his investment, he must account for the fact that if he decides to reinvest, he becomes vulnerable to the risk that subsequent tranches may withdraw prior to his next rollover date. Thus, in equilibrium, all tranches will withdraw sooner, i.e., at higher asset-to-debt ratios, and this problem is exacerbated when there are more tranches/tiers of maturity dates.

Overall, the SPV becomes more susceptible to defeasance (i.e., a wind down via asset sales), where value is further lost through asset-liquidation discounts. These effects, in turn, reduce the amount of debt issued by the SPV, impacting the return to equity holders. Thus, solving for the capital structure decision of the SPV not only depends on the underlying investment assets and acceptable expected loss levels, but also requires the solution of the optimal rollover decision of the debt holders. We are able to solve these interlocking problems, demonstrating how rollover risk arises, the ensuing reactions of debt holders, and the ultimate design of the SPV in the presence of these interactions.

Third, our model shows that once tranches begin to withdraw, the presence of asset-liquidation discounts increases the likelihood that, at the next rollover date, another tranche will also withdraw, leading to a death spiral for the SPV. The failures of SPVs in the recent subprime financial crisis followed this same pattern, as asset-sale discounts reached increasingly high levels,[3] triggering further increases in leverage and credit risk.

[3]For instance, Cheyne Finance recovered 44% of par value in initial liquidation rounds, and Sigma Finance recovered 15%. See http://www.risk.net/risk-magazine/news/1504163/cheyne-assets-disappoint-in-rescue-auction; http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/5769361/400bn-SIV-market-sold-off-in-two-years.html.

Fourth, we suggest a pre-determined remedy for this phenomenon that (i) results in orderly deleveraging and (ii) mitigates the rollover risk arising from the adversarial game among senior note holders of differing rollover dates. That is, we propose an additional covenant that not only imposes a leverage constraint on the SPV, but also requires partial liquidation of assets with an equal distribution of the proceeds among all senior noteholders. Specifically, if the asset-to-debt ratio drops below a pre-set threshold, the SPV must repay the amount of debt equivalent to one tranche size (i.e., a partial deleveraging), but the payment is made *pari passu* across all tranches, irrespective of their individual rollover dates.

As a result, deleveraging is undertaken at an earlier stage, and because all tranches are partially repaid in an equal manner, we mitigate each tranche holder's concern that subsequent tranches will exit in succession, forcing asset liquidations at a discount and leaving insufficient funds to repay the latest investor to roll over. Thus, at each rollover date, the likelihood of funding withdrawal is reduced, and the end result is that ex-ante expected losses are reduced not only for the debt holders, but also for the equity holders, who are residual claimants. In turn, the SPV can sustain higher levels of debt and has greater longevity than it would in absence of this covenant. We present numerical examples based on calibrated simulations of our model that demonstrate these intuitions. The analyses in this paper should inform investment banks, rating agencies, and regulators concerned with the design and structure of highly-leveraged special purpose vehicles.

In the end, designing a deal that is safe for senior note holders is inherently difficult, with risks arising from a variety of sources. For instance, DeMarzo and Duffie (1999) examine the role of information and liquidity costs inherent in selling tranches of a structured finance deal, highlighting the lemons problem in the issuance of asset backed securities, and Hanson and Sunderam (2013) argue that pooling and tranching creates "safe" senior tranches owned by the majority of investors, leading to a dearth of informed investors in good times and resulting in insufficient risk controls. Coval, Jurek, and Stafford (2009a) discuss the features of securitized pools, where senior tranches are akin to economic catastrophe bonds, failing under extreme situations, but offering lower compensation than investors should require. In related work, Coval, Jurek, and Stafford (2009b) argue that small errors in parameter estimates of the collateral pool result in a large variation in the riskiness of the senior tranches. Complementing this asset-side result, we find that finer details of the liability side also matter: irrespective of the risk parameters of the collateral pool, funding diversity via varying rollover dates exacerbates rollover risk, and can have a material impact on the riskiness and value of the senior tranches.

Overall, our paper explores a very different aspect of structured finance design than has been considered in the literature so far, and is aimed at determining the equilibrium design of a SPV, i.e., the capital structure and risk controls, in the face of rollover risk. Many results are not obvious from the onset, suggesting that a naive approach to SPV design might exacerbate risk rather than mitigate it.

The rest of the paper proceeds as follows. In Section 2, we present simple examples to demonstrate the intuition for the results. In Section 3, we present the comprehensive model where asset values follow a continuous-time mean-reverting process. We also present simulation-based outcomes to the equity holders' and debt holders' problems, and we demonstrate the effects of the remedial covenants designed to mitigate rollover risk. In Section 4, we discuss and conclude.

2 Model

We consider a SPV with assets $A(t)$ supported by debt $B(t)$ at time t. These two quantities define a leverage ratio $A(t)/B(t)$. For a solvent SPV, this ratio is greater than 1. Initial debt in the SPV is denoted $D_B = B(0)$.

The debt in the SPV comprises m equal-sized tranches, each of which has a different maturity date, but with the same rollover horizon, T, at issuance. At the maturity date of a tranche, the lenders in that tranche have the option to roll over their debt for another period of length T. Each rollover occurs at evenly spaced intervals of T/m. For example, T may be equal to one year, and we may have $m = 4$ tranches each of size $D_B/4$. Hence, every quarter, one of the tranches is faced with whether to withdraw their capital or to roll over the debt for another year. The interval at which these rollover decisions occur is shorter than the original maturity of each tranche's debt.

This tranche structure creates additional rollover risk, as each tranche factors in the possibility that if they choose to re-invest their capital, subsequent maturing tranches may instead decide to withdraw, leaving losses to be borne by the latest investors. The staggered debt structure we utilize is common in practice, and our paper focuses on SPV risk and design in this setting, in contrast to other work that assumes a single tier of debt within prioritized tranches (i.e., a single rollover date across all investors in the same class of debt). In related work, He and Xiong (2012) examine rollover risk in a model of debt runs, and assess how this risk varies with asset volatility, debt maturity, and the presence of credit lines. We employ a different representation of the rollover risk problem in discrete time, with a recursive solution, and use it to assess the drivers of rollover risk and how it affects SPV design. In extending and complementing the extant literature, this paper further explores how risk management covenants can resolve the rollover risk game between tranches.

Intuitively, when a SPV's assets decline in value and its debt-to-equity ratio increases beyond a certain threshold, debt holders are unwilling to roll over their investments because their expected losses become exceedingly high. That is, debt holders have a threshold level of leverage H_m (dependent on the number of tranches m) below which they are unwilling to continue to fund the SPV. Insofar as $A(t)/B(t) \geq H_m$, debt holders are satisfied with the expected percentage loss of face value, denoted L_0. However, if $A(t)/B(t) < H_m$ at the time their tranche matures, then they will decline to rollover. Thus, we can think of H_m as the strike level at which debt holders call back their debt.

This threshold H_m depends not only on the riskiness of collateral and investors' level of risk aversion, but also on the number of tranches/tiers as well as the anticipated asset-liquidation discount. When assets must be sold to repay an exiting tranche, a fire-sale discount, δ, is incurred. Ex-post, this loss is borne by the latest tranches; thus, sequentially rational investors will factor this risk into their ex-ante rollover decision. The end result is that each maturing tranche will require an even greater $A(t)/B(t)$ to re-invest their capital, i.e., the equilibrium level of H_m is raised to reduce exposure to rollover risk. This, in turn, increases expected losses from fire sale discounts, making all tranches worse off, ex-ante. Later, we will determine risk management covenants that will remedy this problem. A summary of variables and definitions is as follows.

Variable	Definition
$A(t)$	Total asset value
$B(t)$	Total face value of debt
$A(t)/B(t)$	(Inverse) leverage ratio of the SPV at time t
D_B	Initial face value of debt (i.e., $D_B = B(0)$)
m	Number of debt tranches, each with face value $B(0)/m$ and rollover horizon T
T	Time to maturity at issuance (i.e., rollover horizon)
δ	Percentage loss incurred on fire sale of assets when rollover is declined, i.e., recovery is $(1 - \delta)$
H_m	Leverage threshold, based on a total of m tranches, below which the maturing tranche declines to roll over debt (i.e., the maturing tranche holder will re-invest as long as $A(t)/B(t) \geq H_m$). H_m will vary with investors' acceptable level of expected losses, L_0.

In order to set intuition, we present two simple examples that (a) illustrate the effects of rollover risk; (b) explain the recursive solution procedure; and (c) demonstrate how our proposed risk management covenants are pareto-improving. Without loss of generality, we assume that the risk free rate is zero, and that asset returns are sufficient to pay interim coupons on the debt. By abstracting away from these cash flows, including dividends on equity, we highlight our main focus: the impact of rollover risk on ex-ante expected losses on the face value of debt in the SPV.

2.1 Example 1: The cost of rollover risk in a two-period framework

We begin with a simple two-period example where $T = 1$ year, i.e., each period is a half year. We assume that $A(0) = 100$ and $B(0) = 95$. Asset values may move up and down with equal probability by an up-factor of 1.025 per period, and a corresponding down factor of 0.975. We also assume an asset-liquidation discount of $\delta = 5\%$. As mentioned previously, coupon rates and the dividend rate are zero. The asset price tree for this process is depicted in **Figure 1**.

In this framework, each tranche of debt rolls over at the end of two periods from inception. Because the SPV continues indefinitely, provided it remains solvent and debt holders continue to roll over their investments, the tree repeats itself in perpetuity but may scale up or down depending on the initial portfolio value, $A(0)$. We now proceed to calculate ex-ante expected losses to debt holders under a rollover leverage threshold of $H = 1.05$.

First, we calculate the expected loss to debt holders when there is just a single tranche (i.e., $m = 1$). The process is as follows. Given a self-imposed leverage cut-off of $H_1 = 1.05$, the initial leverage ratio of $A(0)/B(0) = 100/95 = 1.0526$ satisfies this threshold and the debt holder decides to roll over his investment. At time $t = 1/2$, whether assets have risen or fallen, the debt holder has no option but to continue, given his original time to maturity of $T = 1$ year at issuance. Finally, at time $t = T = 1$, asset values fall in $A(1) = \{105.0625, 99.9375, 95.0625\}$, all of which satisfy the debt holder's leverage threshold except for the bottom-most value, which results in a leverage ratio of $95.0625/95 = 1.0007 < 1.05$. The tree with debt amounts and attendant losses for each node is shown in **Figure 2**.

In the first two scenarios, the debt holder will roll over his investment, sustaining zero losses, and the SPV will continue. However, in the final scenario, the debt holder will decline to re-invest, forcing the SPV into defeasance. Given a fire-sale discount of $\delta = 5\%$, the debt holder will be repaid $95.0625 \times 0.95 = 90.3094$, thereby sustaining a loss of 4.6906 (i.e., 4.94% of par value). Because this state occurs with probability 1/4, the ex-ante expected loss as a percentage of par is $(1/4) \times (4.6906)/95 = 1.2344\%$.

We note that H_1 does not depend on A or B. It is a level of leverage that provides a decision rule, stipulating the minimum asset-to-debt ratio in a one-tranche SPV that is acceptable to the lender. In other words, if debt holders are comfortable with an expected loss of 1.2344%, then they will select $H_1 = 1.05$ as their decision criteria with regard to whether or not they will roll over their investment when it comes due. On the other hand, if debt holders desire expected losses no greater than 1.00%, then they will choose a greater H_1 to manage their expected losses accordingly (provided it is feasible).

Next, we calculate the expected loss to debt holders when there are two tranches / tiers of debt (i.e., $m = 2$), whereby the rollover dates are staggered. We calculate these expected losses under the same decision rule: $H_2 = H_1 = 1.05$, and we will demonstrate that expected losses will now be greater than the 1.2344% expected loss we derived earlier under $m = 1$. It will then follow that investors' rollover threshold under two tranches must be greater than that under a single tranche (i.e., $H_2 > H_1$) if the latest investors wish to keep expected losses from rising. We now proceed to outline this process, which we depict graphically in **Figure 3**.

As before, each tranche has maturity $T = 1$ year, the tree has two periods, initial assets are $A(0) = 100$, and initial total debt is $B(0) = 95$, though $B(0)$ now comprises two tranches of 47.50 each. At the initial time $t = 0$, investors in the first tranche of debt (tranche 1) must decide whether to roll over their investment. Since the leverage ratio $A(0)/B(0) = 1.0526$ is greater than the investors' rollover threshold $H_2 = 1.05$, the investors continue to roll over and the total debt remains unchanged at $B(0) = D_B = 95$ and the number of tranches remains at $m = 2$.

In the following period at time $t = 1/2$, investors in the second tranche of debt (tranche 2) can now decide whether to roll over their investment. If assets have appreciated in value to $A(\frac{1}{2}) = 102.50$, then once again, the threshold $H_2 = 1.05$ is not violated and investors in tranche 2 also continue to roll over, keeping total debt at $B(\frac{1}{2}) = D_B = 95$ and the number of tranches at $m = 2$. However, if assets have depreciated in value to $A(\frac{1}{2}) = 97.50$, now $A(\frac{1}{2})/B(\frac{1}{2}) = 97.50/95 = 1.0263$, which is less than the rollover threshold $H_2 = 1.05$. In this scenario, investors will decline to roll over and a partial liquidation of assets must be undertaken to repay this exiting tranche of investors. Specifically, based on the fire-sale discount of $\delta = 5\%$, the SPV must sell $47.50/0.95 = 50$ worth of assets in order repay the exiting investors' their total face value of 47.50. Thus, the SPV now has $A(\frac{1}{2}) = 97.50 - 50 = 47.50$ and total $B(\frac{1}{2}) = 95 - 47.50 = 47.50$, with just $m = 1$ tranche remaining (see time $t = 1/2$ in **Figure 3**). We note that, now, $A(\frac{1}{2})/B(\frac{1}{2}) = 47.50/47.50 = 1$, which is less than $H_1 = 1.05$. However, investors of tranche 1 cannot yet choose to exit, because their rollover date is still 1/2 period away.

In the final period at time $t = T = 1$, investors in tranche 1 can now decide whether to withdraw or to roll over their investment. In the upper branch of the tree, asset values are sufficient in both scenarios to meet the $H_2 = 1.05$ rollover threshold. Thus, investors in tranche 1 will yet again continue to roll over their capital, and the SPV will continue operations with a total $B(1) = 95$ and $m = 2$ tranches.

In the lower branch of the tree, asset values are insufficient in both scenarios to meet the $H_1 = 1.05$ rollover threshold. Thus, investors will decide to withdraw their capital, and asset liquidations must take place to repay these investors their $B(1) = 47.50$ face value of debt. After accounting for the asset-liquidation discount of $\delta = 5\%$, we see that remaining assets are insufficient to repay these investors in full (i.e., $48.6875 \times 0.95 = 46.2531$ and $46.3125 \times 0.95 = 43.9969$). Thus, investors sustain losses of $47.50 - 46.2531 = 1.2469$ (i.e., 2.625%) and $47.50 - 43.9969 = 3.5031$ (i.e., 7.375%) in each respective scenario of the lower branch. Ultimately, the SPV ends in complete defeasance, with $A(1) = 0$, total $B(1) = 0$, and $m = 0$ tranches remaining. Thus, we see that with $m = 2$ tranches, the ex-ante expected loss to investors in tranche 1 (under a rollover decision rule of $H_2 = H_1 = 1.05$) is $(1/2)(0) + (1/4)(1.2469) + (1/4)(3.5031) = 1.1875$, which is equivalent to 2.5% of the face value of debt. This expected loss is substantially greater than the 1.2344% expected loss we found earlier under $m = 1$ tranche, suggesting that investors must increase their rollover threshold, H_m, to achieve the same level of expected losses as the number of tranches grows (i.e., H_m is increasing in m). Intuitively, the trigger level of $H_2 = 1.05$ is too low, and the investors in tranche 1 do not exit in time, effectively providing the capital for the investors in tranche 2 to subsequently exit with full payment. Here, the investors in tranche 1 should have set for themselves a greater rollover threshold $H_2 > 1.05$, opting to withdraw their capital at $t = 0$.

Overall, in the presence of rollover risk with multiple tranches, ex-ante expected losses are greater because of the risk that other tranches may choose to exit just after other investors have re-committed. In order to mitigate this, we can see that trigger levels must be set such that $H_2 > H_1$, which further increases rollover risk and also forces the SPV the operate at lower debt-to-equity ratios, thereby decreasing expected returns to investors in the equity tranche.

2.2 Example 2: A finer, four-period tree

In this section, we extend our tree to four periods over the same $T = 1$ rollover horizon as in the previous section, and we repeat the earlier examples. While the specific numbers change, the qualitative results are unaffected. Eventually, we implement the model using a continuous-time stochastic process, and the results remain unchanged. Therefore, there are no qualitative modeling differences in discrete versus continuous time.

Because each step now represents three-month quarters (rather than six-month semi-annual periods), we adjust the step sizes accordingly to increase by a factor of 1.0125 or to decrease by a factor of 0.9875 (i.e., $\pm 1.25\%$), with equal probability. This four-period tree is shown in **Figure 4**.

To begin, we calculate the ex-ante expected losses to debt holders when there is just a single tranche (i.e., $m = 1$). As before, the initial leverage ratio of $A(0)/B(0) = 1.0526$ satisfies the leverage threshold $H_1 = 1.05$, and the debt holder decides to roll over his investment. Because his lockup period is for $T = 1$ year, he has no option to withdraw until the end of the fourth quarter/period. On that date, the five possible values of assets, as shown in **Figure 4** are:

$$A(1) = \{105.0945, 102.4996, 99.9688, 97.5004, 95.0930\},$$

which correspond to leverage ratios of:

$$A(1)/B(1) = \{1.1063, 1.0789, 1.0523, 1.0263, 1.0010\}.$$

Thus, in the first three scenarios, the leverage threshold is satisfied and the debt holders continue to roll over their investment. However, $A(1)/B(1)$ does not satisfy the leverage threshold in the fourth and fifth cases, and investors will opt to withdraw their capital, sustaining losses of $95 - 97.5004 \times 0.95 = 2.3746$ and $95 - 95.0930 \times 0.95 = 4.6617$, respectively, given a fire-sale discount of $\delta = 5\%$. Since these states occur with probability 1/4 and 1/16, respectively, the expected loss as a percentage of par value is $[(1/4)(2.3746)+(1/16)(4.6617)]/95 = 0.009316$ (i.e., 0.9316%). The populated tree with debt amounts and losses sustained is shown in **Figure 5**.

Next, we calculate the expected loss to debt holders when there are two tranches / tiers of debt (i.e., $m = 2$), whereby the rollover dates are staggered and occur every $T/m = 1/2$ years. Thus, the total $B(0) = 95$ is now comprised of two tranches of 47.50 each; i.e., $B(0) = \{47.50, 47.50\}$. This new tree is depicted in **Figure 6** below. As before, we calculate these expected losses under the same decision rule: $H_2 = H_1 = 1.05$. We analyze the sequence of decisions as follows:

1. At time $t = 0$, investors in the first tranche of debt (tranche 1) must decide whether to roll over their investment. Since the leverage ratio $A(0)/B(0)$ is greater than $H_2 = 1.05$, investors in tranche 1 continue to re-invest their capital and the total face value of debt remains unchanged at $B(0) = D_B = 95$ and the number of tranches remains at $m = 2$.

2. At time $t = 1/4$, neither tranche is maturing. Thus, investors have no choice but to continue, regardless of whether asset values have increased or fallen.

3. At time $t = 1/2$, investors in the second tranche of debt (tranche 2) can now decide whether to re-invest or withdraw their capital. There are three possible scenarios at this node, with $A(\frac{1}{2}) = \{105.5156, 99.9844, 97.5158\}$, which correspond to leverage ratios of $A(\frac{1}{2})/B(\frac{1}{2}) = \{1.0791, 1.0525, 1.0265\}$, respectively.

   In the first two scenarios, investors in tranche 2 will continue to roll over their investment, since their leverage threshold of $H_2 = 1.05$ is satisfied. In these cases, total debt remains at $B(\frac{1}{2}) = D_B = 95$ and the number of tranches remains at $m = 2$.

   However, in the third scenario, these investors will opt to withdraw their capital, and a partial liquidation of assets must occur to repay this exiting tranche of investors. Specifically, based on a fire-sale discount of $\delta = 5\%$, the SPV must sell $47.50/0.95 = 50$ worth of assets to repay the exiting investors' total face value of 47.50. In this case, the SPV will now have $A(\frac{1}{2}) = 97.5158 - 50 = 47.5158$ and $B(\frac{1}{2}) = 95 - 47.50 = 47.50$, with just $m = 1$ tranche remaining. The tree continuing from this node is shown in the lower graph of **Figure 6**.

4. At time $t = 3/4$, neither tranche is maturing. Again, any remaining investors have no choice but to continue.

5. Finally, at time $t = T = 1$, investors in tranche 1 can now decide whether to re-invest or withdraw their capital. The losses realized in each of the seven possible scenarios are depicted on the terminal nodes in **Figure 6**, and aggregate to a percentage expected loss of 1.2422%. That is:

$$[(1/16)(1.2247) + (2/16)(2.3673) + (1/16)(3.4816)]/47.50 = 0.012422$$

Thus, as in Example 1, the ex-ante expected loss under $m = 2$ tranches (1.2422%) exceeds that under $m = 1$ tranche (0.9316%) due to the increased rollover risk as the number of tranches increases. Again, this increased risk suggests that new investors must impose a greater rollover threshold H_m as m increases if they wish to maintain the same level of expected losses derived under smaller m.

2.3 Mitigating rollover risk via capital structure covenants

Because rollover risk results in a non-cooperative game among tranche holders, the ex-ante expected loss to each tranche is higher as the number of tranches increases. As we have seen in the prior examples, losses compound as the inherent losses due to asset risk interact with fire-sale losses incurred when a subsequent tranche declines to roll over, prompting cautious investors to require ever higher rollover thresholds as the number of tranches increases. In order to mitigate this risk, we propose that the SPV implement a leverage trigger at which point all tranches are partially repaid in equal amounts via a partial deleveraging of the SPV. This SPV threshold, K, is lower than H_m (otherwise the SPV would be unnecessarily liquidated when investors are satisfied with the extant risks), and applies even when no rollover is due.

To illustrate, we consider a SPV threshold of $K = 1.045$ on our four-period tree under $m = 1$ tranche. That is, if $A/B \leq 1.045$ at any time t, assets are liquidated to repay debt holders, regardless of whether or not time t is a rollover date. The resultant tree of asset values and losses incurred is shown in **Figure 7**. We note that there are two nodes on the tree where the leverage trigger is breached, i.e., in the lowest nodes at time $t = 1/4$ and time $t = 3/4$. For instance, possible asset values at time $t = 1/4$ are $A(\frac{1}{4}) = \{101.25, 98.75\}$, with leverage ratios $A(\frac{1}{4})/B(\frac{1}{4}) = \{1.0658, 1.0395\}$. Thus, the SPV threshold is breached in the bottom node, and assets are liquidated to repay the debt holders, who incur a loss of $95 - 98.75 \times 0.95 = 1.1875$. Similarly, in the SPV threshold is breached in the bottom node at time t=3/4, and debt holders incur a loss of $95 - 98.7346 \times 0.95 = 1.2022$. No losses are incurred in all other nodes of the tree.

Overall, the ex-ante expected percentage loss is $[(1/2)(1.1875) + (1/8)(1.2022)]/95 = 0.7832\%$. We see that this expected loss is lower than that in the case of one tranche when the threshold covenant, K, is not applied, as previously shown in **Figure 5** (i.e., under $K = 0$, the expected loss was 0.9316%). Equivalently, the threshold covenant allows investors to impose a lower rollover threshold when deciding whether to reinvest or withdraw their capital. Thus, by imposing a capital structure covenant on the SPV, we effectively mitigate rollover risk and the increasing costs borne by investors due to rollover risk.

We now proceed to demonstrate the expected losses when employing this remedy under $m = 2$ tranches, which we show graphically in **Figure 8**. As before, investors in tranche 1 decide to reinvest their capital at time $t = 0$, since $A(0)/B(0) > 1.05$. In the following period at time $t = 1/4$, possible asset values are $A(\frac{1}{4}) = \{101.25, 98.75\}$, with leverage ratios $A(\frac{1}{4})/B(\frac{1}{4}) = \{1.0658, 1.0395\}$. Because the SPV threshold is breached in the bottom node, the SPV must undergo partial deleveraging, repaying the equivalent of one tranche of debt to all investors pari passu. That is, $47.50/0.95 = 50$ worth of assets must be liquidated to repay 47.50 worth of debt. This 47.50 will be disbursed equally across investors of both tranches, regardless of their respective rollover dates. Thus, the SPV continues with $m = 2$ tranches of debt, but with $B(\frac{1}{4}) = \{23.75, 23.75\}$, and $A(\frac{1}{4}) = 98.75 - 50 = 48.75$. Despite this partial deleveraging, we see that in continuing the tree from this node further deleveraging is required and the SPV enters total defeasance.

We populate the entire tree following this procedure (as depicted in **Figure 8**), and we see that ultimately, ex-ante percentage expected losses are 0.7832% (i.e., identical to that under the $m = 1$ tranche case). That is:

$$[(1/16)(0.6234) + (1/16)(1.7809) + (1/4)(0.6086) + (1/4)(1.7664)]/95 = 0.7832\%$$

Thus, in employing the proper SPV threshold, the two-tranche case does not entail any additional expected losses from rollover risk created by the adversarial game among the investors in tranches of varying rollover dates. These examples show that this capital-structure covenant reduces the expected losses to debt tranche holders, and dominates the setting when there are no covenants. In the following section we generalize the discrete time tree model to a continuous time stochastic process, and also show that the covenant not only reduces expected losses for debt holders but also for equity holders, who are the residual claimants, resulting in a Pareto-improving SPV structure.

3 Comprehensive Model

In this section, we generalize the model outlined in the preceding section using simulations with continuous time stochastic processes. Since most SPVs were structured as pools of mortgage debt securities, we employ a mean-reverting asset price process, rather than a process such as the geometric Brownian motion, which is used for equity portfolios. In addition, we solve the equity holders' problem, whose objective is to maximize the extent of debt funding (thereby maximizing their expected returns) while keeping expected losses at a level that is acceptable to debt holders. This analysis will deliver the equilibrium capital structure for the SPV, accounting for rollover risk in the face of multiple tranches with varying rollover dates. We will then show how this structure is improved when our proposed capital-structure covenant is included in the ex-ante design of the SPV.

3.1 Mean-reverting asset process

The assets in the SPV evolve according to the following stochastic differential equation (SDE):

$$dA(t) = k[\theta - A(t)]\, dt + \sigma\, dW(t) \tag{1}$$

Here k is the rate of mean reversion, θ is the long run mean of the asset process, with volatility σ. Given time interval h, the solution to this SDE is:

$$A(t+h) = A(t)e^{-kh} + \theta(1 - e^{-kh}) + \sigma \int_0^h e^{-k(h-s)}\, dW(s) \tag{2}$$

At time t, conditional on $A(0)$, $A(t)$ is normally distributed with mean and variance as follows:

$$\text{Mean: } \alpha(t) = E[A(t)|A(0)] = A(0)e^{-kt} + \theta(1 - e^{-kt}) \tag{3}$$

$$\text{Variance: } \beta(t) = Var[A(t)|A(0)] = \frac{\sigma^2}{2k}(1 - e^{-2kt}) \tag{4}$$

Based on these parameters, the probability density function of asset values at time t is:

$$f[A(t)|A(0)] = \frac{1}{\sqrt{2\pi\beta(t)}} \exp\left[-\frac{(A(t) - \mu(t))^2}{2\beta(t)}\right] \tag{5}$$

By setting $\theta > A(0)$, we can inject an upward drift to capture expected returns from the assets. However, in general, θ is set to par value, so that assets are pulled to par as is the case with fixed-income securities.

3.2 Characterizing the decision to roll over a debt tranche

We specify the number of tranches to be m. In our model, we simulate asset prices over time across many paths, and as each tranche matures, investors in that tranche decide to roll over their capital only if the assets to total debt ratio is greater than H_m, noting also that $H_m > 1$. H_m is determined as the minimum possible rollover decision rule such that ex-ante expected (%) losses do not exceed an acceptable level, L_0. In doing so, investors must account for the risk that other tranches may fail to roll over, additional losses from asset-liquidation discounts (δ), and the risk inherent in the asset pool itself.

We solve for H_m using a sequential, recursive approach, as was done in the simple tree examples in the preceding section. We begin by solving for H_1 under the $m = 1$ tranche case. That is, we seek to find the minimum threshold rule that provides investors with an ex-ante percentage expected loss that is no greater than L_0, where the ex-ante expected loss is defined as:

$$EL_1 = \int_{-\infty}^{D_B \cdot H_1} \max[0, D_B - (1 - \delta)A(T)] \cdot f[A(T)|A(0)]\, dA \tag{6}$$

In the equation above, losses occur at the terminal date when the single tranche does not rollover, i.e., when $A(T)/B(0) < H_1$. Thus, we integrate over the range $A(T) \leq B(0) \cdot H_1$. We note that the fire sale discount, δ, is taken into account as well. The percentage expected loss is then defined as

$$PL_1 = \frac{EL_1}{D_B} \equiv L_0 \tag{7}$$

The *objective function* of the SPV designer (i.e., the equity holders) is to maximize the amount of initial debt that can be raised, $B(0) = D_B$, subject to the expected percentage loss on the debt being capped at a given level L_0, so that the debt capital can be placed with investors. Thus, we search over a range of potential H_1 values, from 1.0 and onward, and we set the initial debt $D_B = A(0)/H_1$, using the above formula to calculate the ex-ante expected loss under this leverage threshold and initial capital structure. At very high levels of debt, i.e., low levels of H_1, expected percentage loss is likely to exceed L_0, resulting in an unsustainable structure. We then increase H_1 by small increments, thereby reducing the extent of debt funding and decreasing default risk, until we achieve $PL_1 = L_0$.

Having computed H_1, the second step is to solve for H_2, investors' rollover-decision leverage threshold under $m = 2$ tranches, again searching over a range of potential H_2 values until we find the minimum threshold that results in an ex-ante expected loss of L_0. For the $m = 2$ tranche SPV, we assume that each tranche at inception has a rollover horizon of T years, with each maturing $T/m = T/2$ years apart (i.e., rollover dates occurring at $t = 0, T/2, T, ...$). At time $t = 0$, investors in tranche 1 must decide whether to roll over their capital, which they base on whether $A(0)/B(0) > H_2$. We assume, without loss of generality, that we begin with sufficient equity at $t = 0$ to met this threshold. Then, at time $t = T/2$, investors in tranche 2 are faced with whether to roll over their capital, which they base on whether $A(\frac{T}{2})/B(\frac{T}{2}) > H_2$. This decision entails two possible outcomes:

1. If the investors in tranche 2 decide to re-invest at time $t = T/2$, then the SPV continues its operations until time $t = T = 1$, at which point investors in tranche 1 are again faced with the decision to re-invest or withdraw based on whether $A(T)/B(T) > H_2$.

2. If the investors in tranche 2 opt to withdraw their capital at time $t = T/2$, then they must be repaid $B(\frac{T}{2})/2$, whereby the SPV must liquidate $\frac{B(\frac{T}{2})/2}{1-\delta}$ worth of assets to make these payments. From this point on, the remaining assets evolve under the same process, but there is only $m = 1$ tranche remaining and the face value of liabilities is reduced to $B(\frac{T}{2}) - B(\frac{T}{2})/2 = B(\frac{T}{2})/2$.

   Then, at time $t = T = 1$, investors in tranche 1 are again faced with the decision to re-invest or withdraw based on whether $A(T)/B(T) > H_1$. Note that the investors now revert to decision rule H_1 because there is only $m = 1$ tranche remaining at this point. In this case, losses to investors in tranche 1 are higher, because they have been subjected to funding withdrawal by the investors in tranche 2, which is the cost of the adversarial game that arises in the presence of rollover risk under multiple tranches of varying rollover dates.

The ex-ante expected loss to investors (in tranche 1) under $m = 2$ tranches may be expressed as:

$$
\begin{aligned}
EL_2 \; = \; & \int_{D_B \cdot H_2}^{\infty} EL_1 \left[H_2; A(\frac{T}{2}); D_B/2; \theta; \sigma; T/2 \right] \cdot f[A(\frac{T}{2})|A(0)] \, dA \\
+ \; & \int_{-\infty}^{D_B \cdot H_2} EL_1 \left[H_1; A(\frac{T}{2}) - \frac{D_B/2}{1-\delta}; D_B/2; \gamma \cdot \theta; \gamma \cdot \sigma; T/2 \right] \times \\
& f[A(\frac{T}{2})|A(0)] \, dA
\end{aligned} \tag{8}
$$

where $\gamma = \frac{A(\frac{T}{2}) - \frac{B(0)/2}{1-\delta}}{A(\frac{T}{2})}$, which re-scales the asset-process parameters, θ and σ, in the event of partial liquidation along the way.

The two lines of the equation above correspond to the two cases outlined above. If rollover occurs at time $t = T/2$, then the function EL_1 is called with parameter H_2 (and full pool of assets, $A(\frac{T}{2})$), whereas if tranche 2 declines to rollover, then EL_1 is called with parameter H_1 (and reduced asset pool, $A(\frac{T}{2}) - \frac{B(0)/2}{1-\delta}$), as only one tranche remains. Since the computation of EL_2 requires calling the function for EL_1, the recursive nature of the approach is revealed.

Furthermore, the first line of the equation emphasizes the source of the adversarial relation between investors in tranche 1 and those in tranche 2. Specifically, in the case that investors in tranche 2 decide to roll over their investment, we see that the expected loss function to investors in tranche 1 depends on the full pool of assets at time $t = T/2$, $A(\frac{T}{2})$, though, their stake comprises only half of the debt pool. That is, at each roll over date, investors in the maturing tranche risk the possibility that their re-invested capital is facilitating a clean exit for the subsequent tranche.

In our implementation, which we present in the following section, we implement the integrals above from simulated paths, which allows easy tracking of remaining tranches and any losses sustained. We also note that in the function EL_2, values are scaled for one tranche by dividing appropriately by $m = 2$, the number of tranches. Then, the expected percentage loss is expressed as:

$$
PL_2 = \frac{EL_2}{B(0)/2} \equiv L_0 \tag{9}
$$

Having determined H_1 and H_2, we proceed to find H_3, and so on, in similar manner. As m increases through values $1, 2, 3, \ldots$, the equations involve additional nested integrals, and tractability is achieved by computing expected losses under simulated paths. We use the `R` programming language for the simulations, and even over 10,000 simulated paths, the root finder in `R` is extremely efficient in finding the values of $H_1, H_2, \ldots, H_m$, that ensure that ex-ante percentage expected losses are capped at L_0.[4] In the next subsection, we demonstrate several numerical examples to solidify intuition for the model, and we discuss additional results and insights.

3.3 Suboptimal rollover trigger levels with multiple tranches

We now implement the full simulation model to examine the costs of rollover risk and the acceptable extent of debt funding in a SPV, accounting for the adversarial game among investors across multiple tranches. We first explore, under varying asset-liquidation discounts $\delta = \{10\%, 5\%, 3\%, 2\%, 0\%\}$, how the ex-ante expected losses to debt investors are affected as we increase the number of tranches, m (while keeping the rollover-decision, $H_m = H_1$, fixed). For demonstrative purposes, we first find the H_1 that results in an expected loss of 1%. Then, we fix all subsequent H_m equal to this H_1, and we calculate the expected losses to investors as m grows (but H_m does not).

We assume initial assets of $A(0) = \theta = 100$, which follows the asset process outlined earlier in Section 3.1, with mean-reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$. We assume that the initial time-to-maturity at issuance is $T = 1$ year, and we execute our simulations in incremental time steps of $h = 1$ month. Thus, we generate 10,000 simulated paths, each comprising 12-month asset processes, and we take the average loss across all 10,000 simulations to determine expected losses under each scenario. For all $m > 2$, we assume that all debt has the same maturity ($T = 1$ year) at issuance, but that each of the m tranches mature at equally spaced intervals of T/m; i.e., for $m = 2$, the tranches mature six months apart, and for $m = 3$, the tranches mature four months apart.

The results, which we present in **Table 1**, show that ex-ante expected losses increase with the number of tranches, m, when the rollover-decision leverage threshold, H_m remains fixed. For instance, under an asset-liquidation discount of $\delta = 5\%$, a rollover-decision threshold of $H_1 = 1.0680$ yields an expected loss of 1% to the investors in a one-tranche SPV with initial debt of $D_B = A(0)/H_1 = 93.6360$. That is, if investors decide to roll over their investment in this SPV as long as the asset-to-debt ratio is greater than $H_1 = 1.0680$, their ex-ante expected loss is equal to 1%.

However, we see that if investors continue to use this same decision rule as the number of tranches (m) grows, then expected losses also grow. For instance, under $m = 2$ and

[4] The root finding function we use is `uniroot`, included in the base distribution of `R`.

$m = 3$, the expected losses to investors increase to 1.38% and 1.75%, respectively. By $m = 6$ tranches, using $H_6 = H_1 = 1.0680$, the expected loss to investors reaches $2.64\% > 1\%$, indicating that investors must increase their rollover leverage thresholds (i.e., require lower levels of debt in the SPV) to maintain their original, acceptable level of expected losses. We observe similar results across the varying fire-sale discounts, δ.

Looking across the different asset-liquidation discounts, we also see that as discounts decrease (i.e., as recovery rates rise), the H_1 required to achieve a 1% expected loss decreases, and equivalently, the maximum feasible amount of debt increases. That is, as asset-liquidation discounts decrease, the SPV can sustain greater amounts of debt since the deadweight cost of rollover risk (and the losses on ensuing asset liquidations) declines.

Overall, these results demonstrate that, interestingly, diversifying debt funding sources across tranches increases rollover risk and expected losses to debt holders, and may thereby increase the cost of debt financing. Collectively, these outcomes also suggest that the SPV must assume a lower amount of debt to remain viable if its debt funding is diversified across many tranches with varying rollover dates.

3.4 Optimal rollover decisions when there is more than one tranche

As we demonstrated earlier with discrete tree examples in Section 2.1 and Section 2.2, the investors' decision to roll over is determined by H_1 when there is $m = 1$ tranche in the SPV, and $H_2 > H_1$ when there are $m = 2$ tranches. We now generalize this result, implementing the full simulation model to examine the appropriate H_m that maintains the same expected loss to investors, $L_0 = 1\%$, even as m increases.

We assume the same asset process and initial parameters as before (in Section 3.3), and again, we explore the implications under varying asset-liquidation discounts, δ. We also report the probability of experiencing a loss, which differs from the expected (%) loss in that it represents the percentage of occurrences, across our simulations, in which any amount of loss is incurred. As before, we take the average loss across all 10,000 simulated paths to obtain the expected loss to investors under each scenario.

The results, which we present in **Table 2**, show that to keep their expected losses fixed, investors demand an ever-increasing asset-to-debt ratio, H_m, to roll over their investments as the number of tranches, m, in the SPV grows. For example, under a fire-sale discount of $\delta = 5\%$, $H_1 = 1.0680$ represents the minimum rollover threshold that yields an expected loss of 1% to investors in a one-tranche SPV with initial debt maxed out to $D_B = A(0)/H_1 = 93.6360$. However, as the number of tranches (m) grows, investors must increase their minimum rollover threshold to achieve the same 1% expected loss, and equivalently, the SPV's maximum debt capacity is reduced.

Under $m = 2$ and $m = 3$, investors demand thresholds of $H_2 = 1.0756$ and $H_3 = 1.0797$, respectively, to roll over their investments, which correspond to maximum debt capacities of $D_B = \{92.9704, 92.6228\}$, respectively, for the SPV. By $m = 6$ tranches, investors demand an even greater threshold of $H_6 = 1.0859$, requiring that the asset-to-debt ratio be greater than 1.0859 to roll over their investment and thereby reducing the SPV's maximum debt capacity to $D_B = 92.0915$.

In a related analysis, we examine how varying asset volatility, $\sigma = \{5, 10\}$, impacts rollover risk and the maximum debt capacity of the SPV. The results, which we present in **Table 3**, show that for a fixed asset-liquidation discount, investors demand a greater asset-to-debt ratio, H_m, to roll over their investments when the underlying asset pool has greater price volatility. For instance, under a liquidation discount of $\delta = 5\%$ and under $m = 1$ tranche, investors require a threshold of $H_1 = 1.0680$ to roll over their capital when asset volatility is $\sigma = 5$, but they require a threshold of $H_1 = 1.1265$ when asset volatility is $\sigma = 10$. These translate to maximum debt capacities of $D_B = \{93.6360, 88.7741\}$, respectively, for the SPV. Similarly, under $m = 6$ tranches, $H_6 = 1.0859$ when $\sigma = 5$, but $H_6 = 1.1467$ when $\sigma = 10$, which translates to maximum debt capacities of $D_B = \{92.0915, 87.2086\}$, respectively.

Overall, we observe a similar pattern of H_m increasing in m across the varying fire-sale discounts, δ. That is, rollover risk increases as the number of tranches increase, thereby promoting an adversarial game among investors, who require ever greater asset-to-debt ratios to continue to roll over their investments. Furthermore, the general level of H_m is lower and the maximum debt capacity of the SPV is greater as asset volatility and fire-sale discounts decrease. That is, investors accept a lower rollover threshold, since the losses incurred due to the adversarial relation among investors are reduced when asset liquidations are less likely and are not as costly.

3.5 Covenants for managing rollover risk

Given the escalating rollover risk and expected losses as the number of tranches increases, a natural question arises as to whether we can design an ex-ante mechanism to mitigate this adversarial game among investors. Because the additional risk arises from the probability that an investor's capital will be used to fund the exit of another tranche, we propose a de-leveraging solution that repays, in part, all investors equally irrespective of their specific tranche/rollover date. Specifically, we propose that the SPV implement an ex-ante capital-structure covenant, defining a leverage trigger that, once breached, requires all tranches to be partially repaid in equal amounts (as was outlined earlier in Section 2.3), thereby protecting all tranches equally regardless of their varying rollover dates. Following this remedy, we now implement our full simulation model to examine the effects of having this ex-ante covenant in place.

Using the same asset process and initial parameters as before, we begin with the case of $m = 1$ tranche. The results, which we present in **Table 4**, show that with the implementation of a SPV threshold covenant, the rollover threshold, H_1, demanded by investors is reduced. For instance, under $\delta = 5\%$, without a SPV threshold covenant in place (i.e., $K = 0$), investors demand a rollover threshold of $H_1 = 1.0680$, which allows them to satisfy their expected loss requirement of 1%. However, with a SPV threshold covenant of $K = 1.05$ (i.e., regardless of whether the trigger is breached on a rollover date, the SPV will liquidate and repay investors if assets-to-debt ratio drops below $K = 1.05$), investors' threshold requirement drops to $H_1 = 1.0532$, allowing the SPV to maintain a higher debt capacity. Intuitively, the SPV threshold covenant acts as a stop loss for investors during the investment lock-up period, allowing them to roll over their investments at lower asset-to-debt ratios with greater confidence.

To generalize this result, we also examine the effects of implementing a SPV threshold covenant under $m = 2$ tranches (presented in **Table 5**) and under $m = 3$ tranches (presented in **Table 6**). We find that with a SPV threshold covenant in place, we can effectively mitigate the additional rollover risk and expected losses that arise as we increase the number of tranches. For instance, for $m = 2$ tranches (and under a fire-sale discount of $\delta = 10\%$), we see that without the SPV threshold in place (i.e., $K = 0$), investors' demand a rollover threshold of $H_2 = 1.1354 > H_1$ to achieve the acceptable expected loss of $L_0 = 1\%$. However, with a SPV threshold covenant of $K = 1.025$, the expected loss to investors still falls within the acceptable $L_0 = 1\%$, even when investors relax their demanded threshold to $H_2 = 1.1273 = H_1$. We make similar observations under $m = 3$ tranches.

Overall, with the introduction of an effective, ex-ante SPV threshold covenant K, investors no longer demand an ever-increasing H_m to roll over their capital. Thus, the maximum debt capacity of the SPV can be maintained, even as it increases the number of tranches for greater funding diversity.

4 Concluding Comments

Special purpose vehicles (SPVs) are characterized by high leverage, illiquid assets that incur asset-sale discounts, and tranched liabilities that are susceptible to rollover risk. In the recent financial crisis, SPVs experienced triple-witching: falling asset values leading to unsustainable leverage, sharp increases in asset illiquidity and greater fire-sale discounts, and withdrawal of capital leading to a death spiral as SPVs defeased concurrently.

We present a model that incorporates these features, and we suggest a covenant-based remedy that results in timely deleveraging and mitigates rollover risk, thereby decreasing the ex-ante expected losses for both equity holders and debt holders. We do this in an adversarial model where debt tranche holders compete with one other in making rollover decisions, and equity holders attempt to maximize the debt in the SPV capital structure, accounting for the game between debt holders, and the optimal rollover decisions of varying tranche holders.

Overall, designing a viable SPV is a complex problem, requiring decisions not only along the suitable portfolio composition, leverage (asset-to-debt ratio), and asset-liability gap of the SPV, but also along finer details of the funding sources with regard to the diversity of sources and rollover dates. As we have seen, diversifying financing sources by having many tranches of debt exacerbates the adversarial game among senior debt holders with otherwise equal priority, makes debt more costly, and, thus, may not be optimal for either debtholders or equityholders in the SPV. However, these problems may be mitigated via our suggested covenent structure. Further research will no doubt uncover other solutions to the optimal SPV design problem, and some, such as contingent capital are particularly ripe for exploration.

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Table 1: SPV Structures and Expected Losses. This table shows the expected losses to investors as the number of tranches, m, increases (but their rollover threshold rule, H_m, does not). H_m represents the minimum asset-to-debt ratio required for investors in an m-tranche SPV to continue to roll over their investment. Expected (%) losses are calculated as the average loss incurred across all simulation paths, and the probability of loss is simply the percentage of simulated occurrences in which any loss occurs. We report these expected losses under varying recovery rates of $\{90\%, 95\%, 97\%, 98\%, 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.1273	88.7078	36.25%	1.00%
	2	1.1273	88.7078	30.94%	1.38%
	3	1.1273	88.7078	27.87%	1.75%
	4	1.1273	88.7078	24.93%	2.05%
	6	1.1273	88.7078	21.81%	2.64%
Recov = 95%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0680	93.6360	30.94%	1.38%
	3	1.0680	93.6360	27.87%	1.75%
	4	1.0680	93.6360	24.93%	2.05%
	6	1.0680	93.6360	21.81%	2.64%
Recov = 97%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0459	95.6074	36.26%	1.00%
	2	1.0459	95.6074	30.94%	1.38%
	3	1.0459	95.6074	27.87%	1.75%
	4	1.0459	95.6074	38.90%	2.05%
	6	1.0459	95.6074	49.34%	2.64%
Recov = 98%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0353	96.5939	36.26%	1.00%
	2	1.0353	96.5939	30.97%	1.38%
	3	1.0353	96.5939	44.24%	1.75%
	4	1.0353	96.5939	24.93%	2.05%
	6	1.0353	96.5939	21.83%	2.64%
Recov = 100%	m	Let $H_m = H_1$	Initial D_B	% prob loss	% E(loss)
	1	1.0146	98.5639	36.24%	1.00%
	2	1.0146	98.5639	30.93%	1.38%
	3	1.0146	98.5639	27.87%	1.75%
	4	1.0146	98.5639	24.93%	2.05%
	6	1.0146	98.5639	21.80%	2.64%

Table 2: SPV Structures and Maximum Debt Capacity, Under Varying Asset-Liquidation Discounts. This table shows the maximum debt capacity, D_B, of the SPV as the number of tranches, m, increases, and investors uniformly demand expected losses to be capped at $L_0 = 1\%$. Min H_m represents the minimum rollover threshold (asset-to-debt ratio) required for investors in an m-tranche SPV to continue to roll over their investment, given their level of acceptable expected losses. Max D_B is then the corresponding maximum debt capacity of the SPV. Expected (%) losses are calculated as the average loss incurred across all simulation paths, and the probability of loss is simply the percentage of simulated occurrences in which any loss occurs. We report these statistics under varying recovery rates of $\{90\%, 95\%, 97\%, 98\%, 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1273	88.7078	36.25%	1.00%
	2	1.1354	88.0772	24.52%	1.00%
	3	1.1396	87.7479	18.91%	1.00%
	4	1.1421	87.5547	27.49%	1.00%
	6	1.1462	87.2443	10.80%	1.00%
Recov = 95%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0756	92.9704	24.52%	1.00%
	3	1.0797	92.6228	18.91%	1.00%
	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%
Recov = 97%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0459	95.6074	36.26%	1.00%
	2	1.0534	94.9269	24.52%	1.00%
	3	1.0574	94.5705	35.05%	1.00%
	4	1.0597	94.3656	27.51%	1.00%
	6	1.0635	94.0314	34.74%	1.00%
Recov = 98%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0353	96.5939	36.26%	1.00%
	2	1.0427	95.9067	24.53%	1.00%
	3	1.0466	95.5449	18.91%	1.00%
	4	1.0489	95.3361	23.47%	1.00%
	6	1.0527	94.9974	10.78%	1.00%
Recov = 100%	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0146	98.5639	36.24%	1.00%
	2	1.0218	97.8640	24.53%	1.00%
	3	1.0257	97.4977	18.91%	1.00%
	4	1.0279	97.2833	15.42%	1.00%
	6	1.0316	96.9361	10.78%	1.00%

Table 3: SPV Structures and Maximum Debt Capacity, Under Varying Asset-Volatilities. This table shows the maximum debt capacity, D_B, of the SPV as the number of tranches, m, increases, and investors uniformly demand expected losses to be capped at $L_0 = 1\%$. Min H_m represents the minimum rollover threshold (asset-to-debt ratio) required for investors in an m-tranche SPV to continue to roll over their investment, given their level of acceptable expected losses. Max D_B is then the corresponding maximum debt capacity of the SPV. Expected (%) losses are calculated as the average loss incurred across all simulation paths, and the probability of loss is simply the percentage of simulated occurrences in which any loss occurs. We report these statistics under varying asset volatilities of $\sigma = \{5, 10\}$ and varying recovery rates of $\{95\%, 97\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = \{5, 10\}$; $T = 1$ year; and time step $h = 1$ month. Initial debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recovery = 95%					
$\sigma = 5$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0680	93.6360	36.25%	1.00%
	2	1.0756	92.9704	24.52%	1.00%
	3	1.0797	92.6228	18.91%	1.00%
	4	1.0820	92.4188	27.49%	1.00%
	6	1.0859	92.0915	19.06%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1265	88.7741	20.66%	1.00%
	2	1.1315	88.3755	14.36%	1.00%
	3	1.1371	87.9468	10.73%	1.00%
	4	1.1399	87.7269	16.89%	1.00%
	6	1.1467	87.2086	6.32%	1.00%

Recovery = 97%					
$\sigma = 5$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.0459	95.6074	36.26%	1.00%
	2	1.0534	94.9269	24.52%	1.00%
	3	1.0574	94.5705	35.05%	1.00%
	4	1.0597	94.3656	27.51%	1.00%
	6	1.0635	94.0314	34.74%	1.00%
$\sigma = 10$	m	Min H_m	Max D_B	% prob loss	% E(loss)
	1	1.1032	90.6432	20.67%	1.00%
	2	1.1082	90.2375	14.37%	1.00%
	3	1.1136	89.7968	10.72%	1.00%
	4	1.1164	89.5735	8.68%	1.00%
	6	1.1230	89.0455	29.05%	1.00%

Table 4: Enhancing Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of $m = 1$ Tranche. This table shows the maximum debt capacity, D_B, of a one-tranche SPV, now with a threshold covenant K in place, where investors uniformly demand expected losses to be capped at $L_0 = 1\%$. The SPV Threshold, K, represents the asset-to-debt ratio that, once breached, requires asset liquidation to repay investors (regardless of whether the breach occurred on a rollover date). Min H_1 represents the minimum rollover threshold (asset-to-debt ratio) required for investors in the one-tranche SPV to continue to roll over their investment, given their level of acceptable expected losses $L_0 = 1\%$. Max D_B is then the corresponding maximum debt capacity of the SPV. Expected (%) losses are calculated as the average loss incurred across all simulation paths. We report these statistics under varying recovery rates of $\{95\%, 97\%, 98\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 95%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0680	93.6360	1.00%
	1.040	1.0679	93.6468	1.00%
	1.045	1.0620	94.1591	1.00%
	1.050	1.0532	94.9515	1.00%
Recov = 97%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0459	95.6074	1.00%
	1.020	1.0447	95.7201	1.00%
	1.025	1.0375	96.3866	1.00%
	1.029	1.0303	97.0630	1.00%
Recov = 98%	SPV Threshold (K)	Min H_1	Max D_B	% E(loss)
$m = 1$	None	1.0353	96.5939	1.00%
	1.010	1.0335	96.7591	1.00%
	1.015	1.0260	97.4638	1.00%
	1.019	1.0190	98.1354	1.00%

Table 5: Enhancing Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of $m = 2$ Tranches. This table shows the expected losses and debt capacity, D_B, of a two-tranche SPV, now with an ex-ante SPV threshold covenant, K, in place. The SPV Threshold, K, represents the asset-to-debt ratio that, once breached, requires partial asset liquidation, with the proceeds split equally across all investors regardless of their respective rollover dates (see Section 3.5 for details). H_m represents the minimum asset-to-debt ratio required for investors in an m-tranche SPV to continue to roll over their investment. Expected (%) losses are calculated as the average loss incurred across all simulation paths. We report these statistics under varying recovery rates of $\{90\%, 95\%, 97\%, 98\% \, 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.1273	1.1354	88.0772	1.00%
	1.000	1.1273	1.1354	88.0772	0.88%
	1.025	1.1273	1.1354	88.0772	0.79%
	1.000	1.1273	1.1273	88.7075	1.16%
	1.025	1.1273	1.1273	88.7075	0.96%
Recov = 95%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0680	1.0756	92.9704	1.00%
	1.000	1.0680	1.0756	92.9704	0.51%
	1.025	1.0680	1.0756	92.9704	0.29%
	1.000	1.0680	1.0680	93.6356	0.62%
Recov = 97%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0459	1.0534	94.9269	1.00%
	1.000	1.0459	1.0534	94.9269	0.32%
	1.025	1.0459	1.0534	94.9269	0.10%
	1.000	1.0459	1.0459	95.6078	0.37%
Recov = 98%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0353	1.0427	95.9067	1.00%
	1.000	1.0353	1.0427	95.9067	0.22%
	1.025	1.0353	1.0427	95.9067	0.05%
	1.000	1.0353	1.0353	96.5941	0.23%
Recov = 100%	SPV Threshold (K)	H_1	H_2	Initial D_B	% E(loss)
	None	1.0146	1.0218	97.8640	1.00%
	1.000	1.0146	1.0218	97.8640	0.07%
	1.000	1.0146	1.0146	98.5639	0.06%

Table 6: Enhancing Debt Capacity via an Ex-Ante SPV Threshold Covenant: The Case of $m = 3$ Tranches. This table shows the expected losses and debt capacity, D_B, of a three-tranche SPV, now with an ex-ante SPV threshold covenant, K, in place. The SPV Threshold, K, represents the asset-to-debt ratio that, once breached, requires partial asset liquidation, with the proceeds split equally across all investors regardless of their respective rollover dates (see Section 3.5 for details). H_m represents the minimum asset-to-debt ratio required for investors in an m-tranche SPV to continue to roll over their investment. Expected (%) losses are calculated as the average loss incurred across all simulation paths. We report these statistics under varying recovery rates of $\{90\%, 95\%, 97\%, 98\% \, 100\%\}$. The recovery rate is equal to one minus the asset-liquidation discount, δ. Initial simulation parameters are: $A(0) = \theta = 100$, with mean reversion parameter $\kappa = 0.5$ and asset volatility $\sigma = 5$; $T = 1$ year; and time step $h = 1$ month. Initial debt is set to maximum capacity at $D_B = A(0)/H_m$. The number of simulations paths is 10,000.

Recov = 90%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.1273	1.1354	1.1396	87.7479	1.00%
	1.000	1.1273	1.1354	1.1396	87.7479	0.65%
	1.025	1.1273	1.1354	1.1396	87.7479	0.49%
	1.000	1.1273	1.1273	1.1273	88.7080	0.99%
Recov = 95%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0680	1.0756	1.0797	92.6228	1.00%
	1.000	1.0680	1.0756	1.0797	92.6228	0.82%
	1.025	1.0680	1.0756	1.0797	92.6228	0.57%
	1.000	1.0680	1.0680	1.0680	93.6356	0.41%
Recov = 97%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0459	1.0534	1.0574	94.5705	1.00%
	1.000	1.0459	1.0534	1.0574	94.5705	0.17%
	1.025	1.0459	1.0534	1.0574	94.5705	0.06%
	1.000	1.0459	1.0459	1.0459	95.6078	0.41%
Recov = 98%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0353	1.0427	1.0466	95.5449	1.00%
	1.000	1.0353	1.0427	1.0466	95.5449	0.11%
	1.025	1.0353	1.0427	1.0466	95.5449	0.03%
	1.000	1.0353	1.0353	1.0353	96.5941	0.13%
Recov = 100%	SPV Threshold (K)	H_1	H_2	H_3	Initial D_B	% E(loss)
	None	1.0146	1.0218	1.0257	97.4977	1.00%
	1.000	1.0146	1.0218	1.0257	97.4977	0.04%
	1.000	1.0146	1.0146	1.0146	98.5639	0.06%

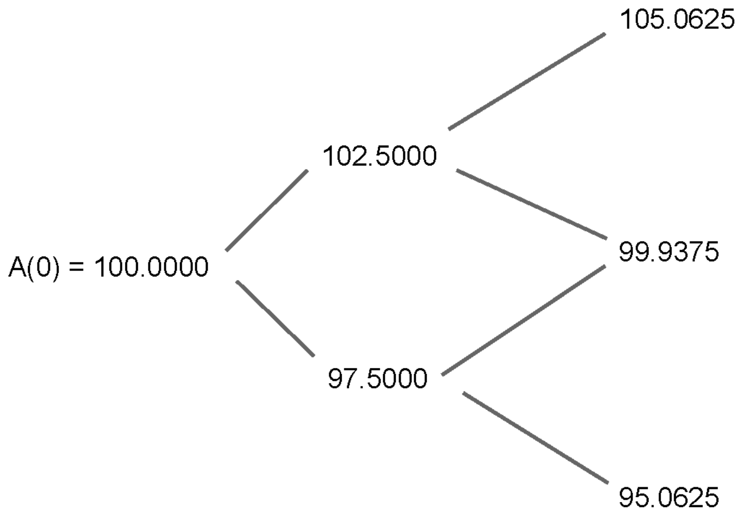

Figure 1: Two-period asset tree, $T = 1$ year. The up and down moves each period are based on a $\pm 2.5\%$ range.

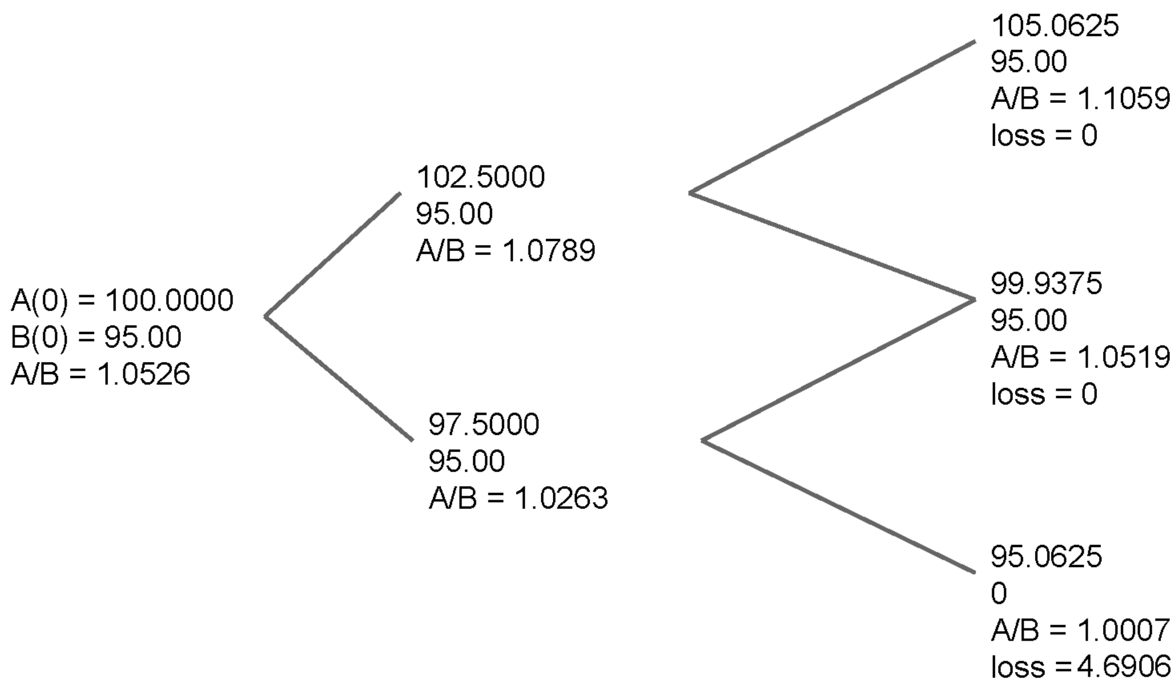

Figure 2: Two-period asset tree, $T = 1$ year, $m = 1$ tranche, $H_1 = 1.05$. Debt face values outstanding are shown below asset values for each node along the tree. Losses are shown as incurred. In this case, since just one tranche of debt is considered, the only possible loss occurs at the end of two periods, when the debt comes due for rollover.

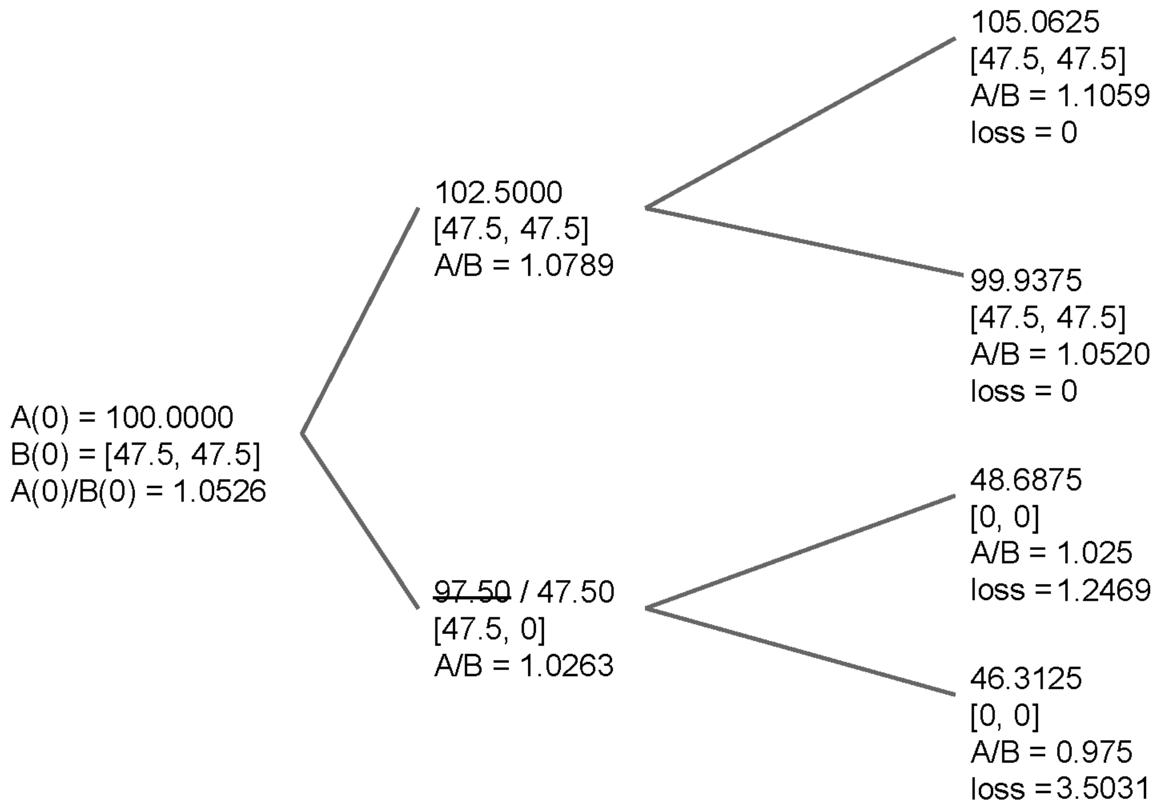

Figure 3: Two-period asset tree, $T = 1$ year, $m = 2$ tranches, $H_2 = H_2 = 1.05$. Debt face values are shown below asset values, partitioned by tranche. Losses are shown when incurred, as are reductions in debt.

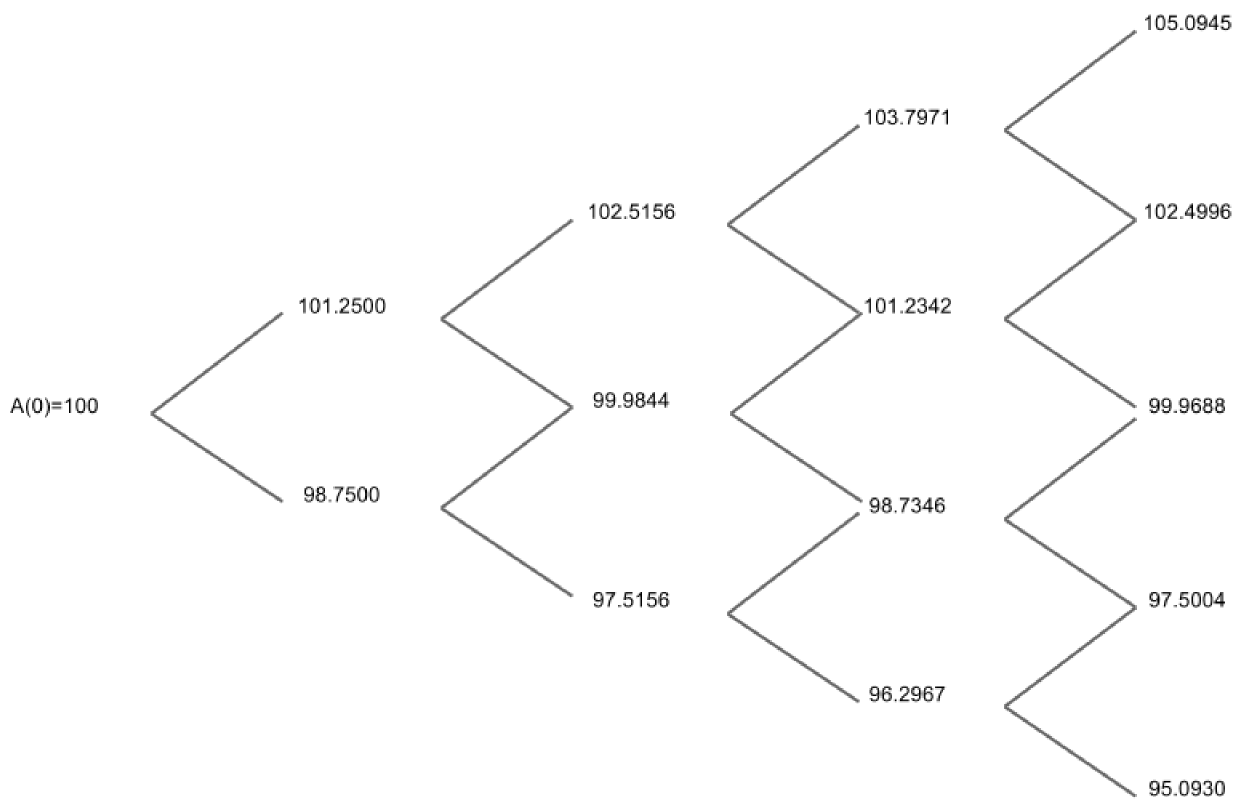

Figure 4: Four-period asset tree, $T = 1$ year. The up and down moves each period are based on a $\pm 1.25\%$ range, half that in the case of the two-period tree in the preceding examples.

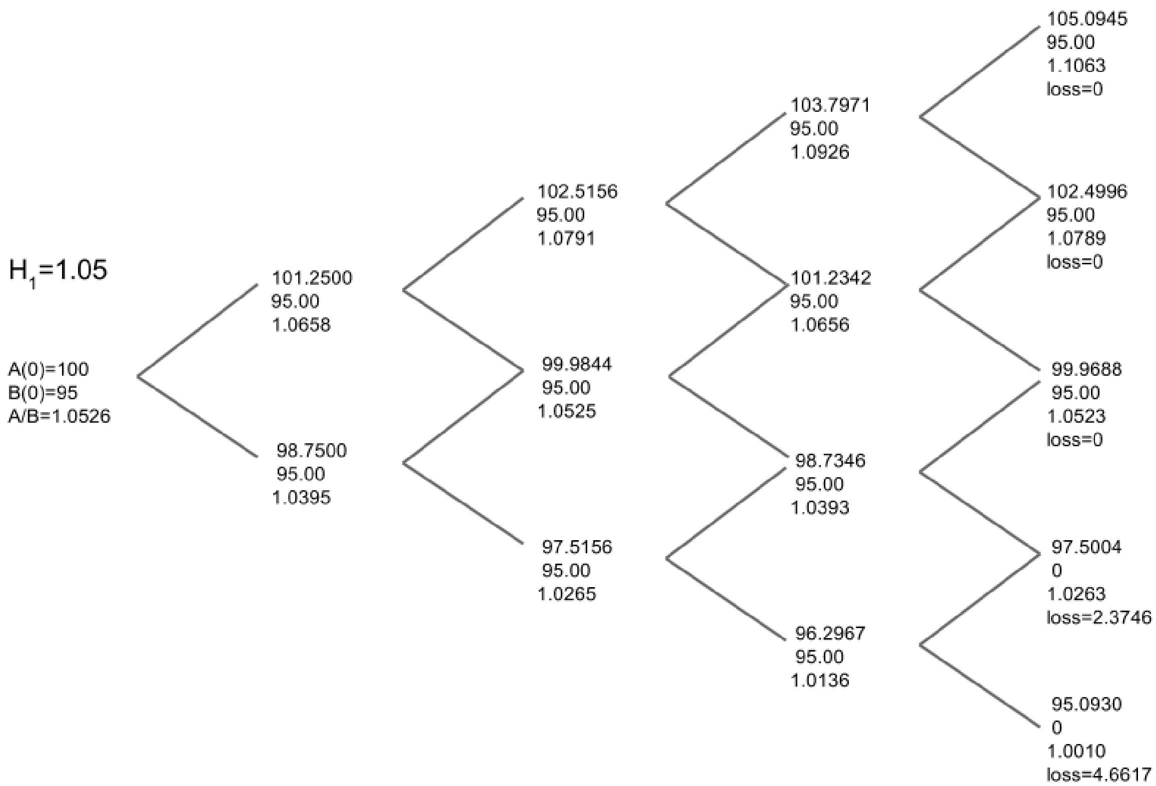

Figure 5: Four-period asset tree, $T = 1$ year, $m = 1$ tranche, $H_1 = 1.05$. Debt face values are shown below asset values. Losses are shown as incurred. Since just one tranche of debt is considered, the only possible loss occurs at the end of four periods, when the debt comes due for rollover.

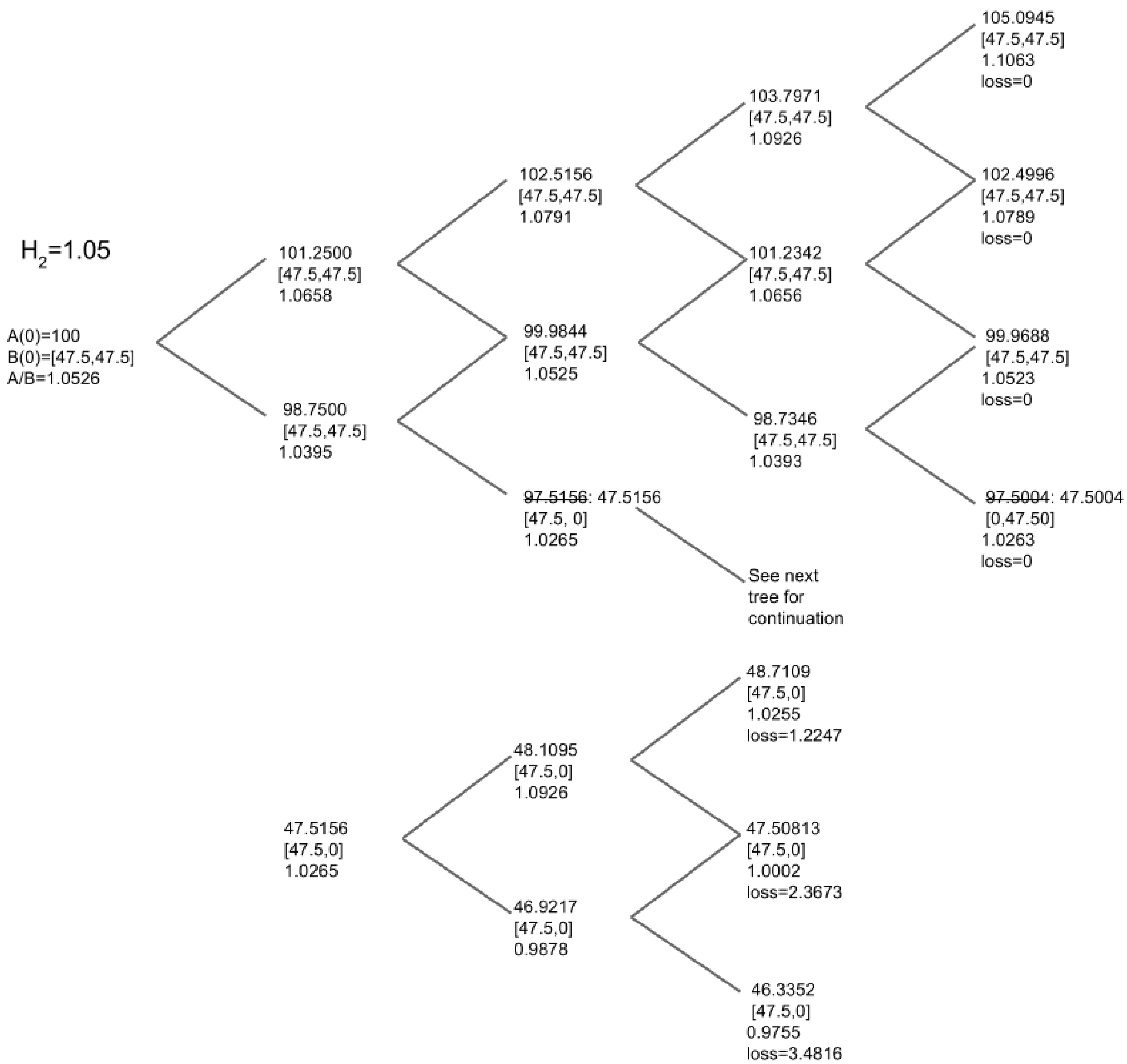

Figure 6: Four-period asset tree, $T = 1$ year, $m = 2$ tranches, $H_2 = H_1 = 1.05$. Debt face values are shown below asset values, partitioned by tranche, and losses as well as debt reductions are shown as incurred. When investors in a tranche decline to roll over, the debt value for that tranche is zeroed out, and asset values are shown before and after repayment of that particular tranche. The graph above has two sections, and the lower section continues the tree from the lowest node after two periods.

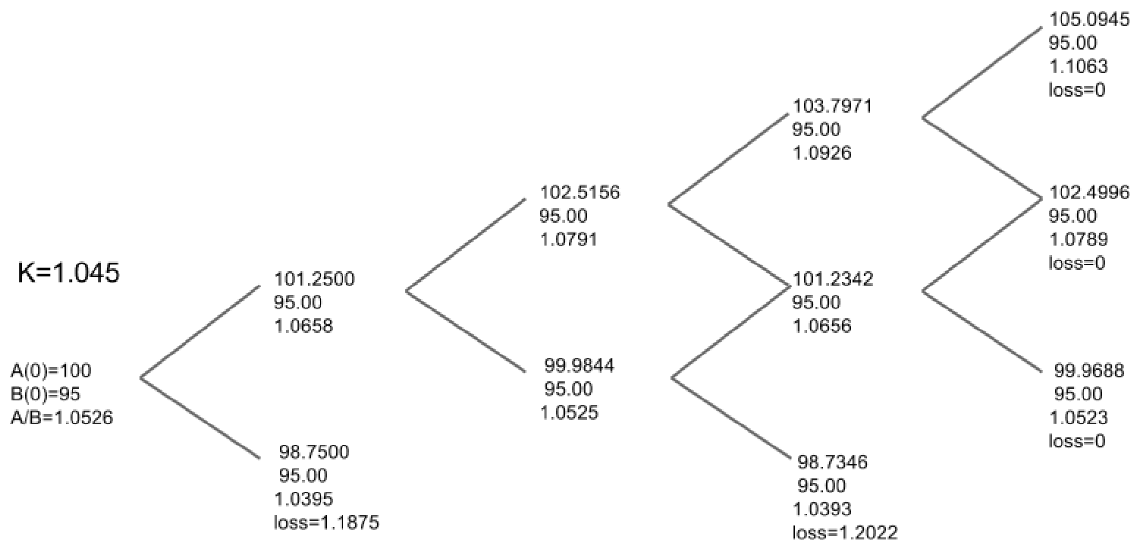

Figure 7: Four-period asset tree, $T = 1$ year, $m = 1$ tranche, $H_1 = 1.05$, SIV threshold $K = 1.045$. Debt face values and leverage ratios are shown below asset values, and losses are shown as incurred. The SPV leverage trigger, $K = 1.045$, specifies that assets are liquidated to repay debt holders if at any time $A/B \leq K$. The expected loss here is 0.78%, in contrast to the case without the covenant, where the loss is 0.93% (as was shown in **Figure 5**).

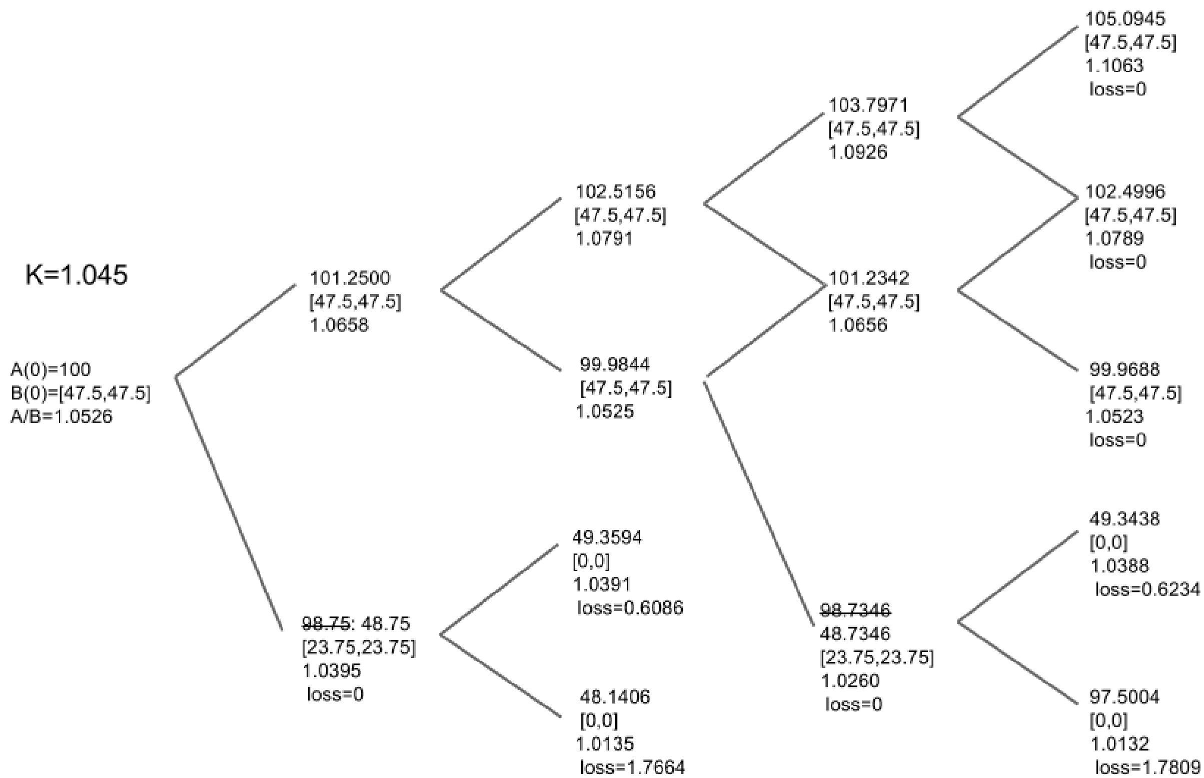

Figure 8: Four-period asset tree, $T = 1$ year, $m = 2$ tranches, $H_2 = H_1 = 1.05$, SIV threshold $K = 1.045$. Debt face values are shown below asset values, partitioned by tranche, and losses are shown as incurred. The SPV leverage trigger, $K = 1.045$, specifies a partial liquidation of assets equal to the total amount of one tranche if at any time $A/B \leq K$ with pari-passu distribution of proceeds across all debt holders without regard to their respective rollover dates. Ex-ante expected losses here are 0.78%, which is the same as that under $m = 1$ tranche.