Dynamic Risk Networks

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Boston
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[1] Joint work with Seoyoung Kim and Daniel Ostrov. Some of this research was conducted with a team at IBM Almaden Labs.

Outline

1. A review of risk metrics on networks. Using the `R` programming language.
2. A big data application to interbank loan networks for banking systemic risk in the U.S., using text mining and network analysis.
3. A new approach to systemic risk on networks.
4. An application to banking networks in India.
5. Extending the Merton (1974) model to dynamic risk networks.

Some relevant papers:

- https://srdas.github.io/midaswww2011_FINAL.pdf
- https://srdas.github.io/Papers/JAI_Das_issue.pdf

Systemic Risk

Systemic risk has some universally accepted characteristics:

1. Large impact.
2. Widespread.
3. Creates a ripple effect that endangers the viability of the economic system.

Systemic risk is an attribute of the economic system and not that of a single entity. Its easurement should have two important features:

1. Quantifiable: must be measureable on an on-going basis.
2. Decomposability (Attribution): Aggregate system-wide risk must be broken down into additive risk contributions from all entities in the system.

Financial institutions that make large risk contributions to system-wide risk are deemed "systemically important."

Dodd-Frank Act 2010

The Dodd-Frank Act and Basel III regulations characterize a systemically risky FI as one that is

1. Large;
2. Complex;
3. Interconnected;
4. Critical, i.e., provides hard to subsititute services to the economy.

The DFA does not provide quantification guidance.

What is Systemic Analysis?

1. Definition: the measurement and analysis of relationships across entities with a view to understanding the impact of these relationships on the system as a whole.
2. Challenge: requires most or all of the data in the system; therefore, high-quality information extraction and integration is critical.

Systemic Risk

1. Current approaches: use stock return correlations. Acharya, et al 2010; Adrian and Brunnermeier 2009; Billio, Getmansky, Lo 2010; Kritzman, Li, Page, Rigobon 2010.
2. Midas: uses semi-structured archival data from SEC and FDIC to construct a co-lending network; network analysis is then used to determine which banks pose the greatest risk to the system. See Burdick et al (2011).
3. Network risk metrics: combines credit and network information to construct aggregate systemic risk metrics, that are decomposable, Das (2016).

Systemic Risk (Billio, Getmansky, Lo, Pelizzon (2012)

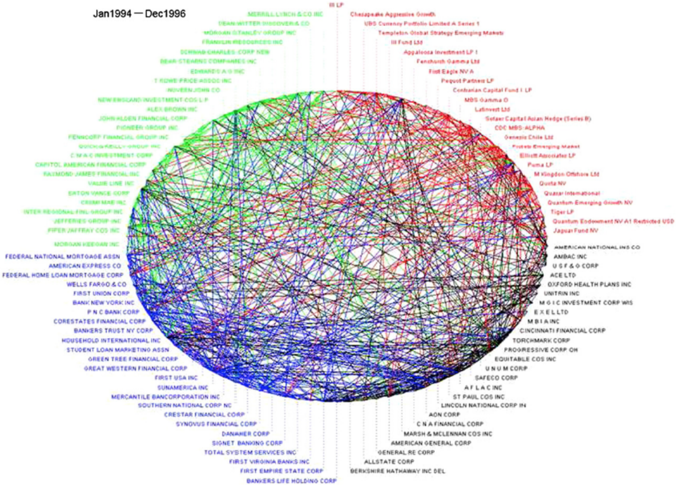


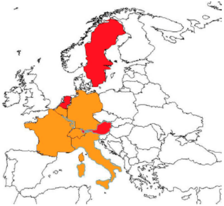
Fig. 2. Network diagram of linear Granger-causality relationships that are statistically significant at the 5% level among the monthly returns of the 25 largest (in terms of average market cap and AUM) banks, broker/dealers, insurers, and hedge funds over January 1994 to December 1996. The type of institution causing the relationship is indicated by color: green for broker/dealers, red for hedge funds, black for insurers, and blue for banks. Granger-causality relationships are estimated including autoregressive terms and filtering out heteroskedasticity with a GARCH(1,1) model.

Contagion Networks (Espinosa-Vega & Sole, IMF 2010)

Panel 1 (trigger failure)
Affected Countries: Italy.

Panel 2 (1st contagion round)
Affected Countries: Italy, France.

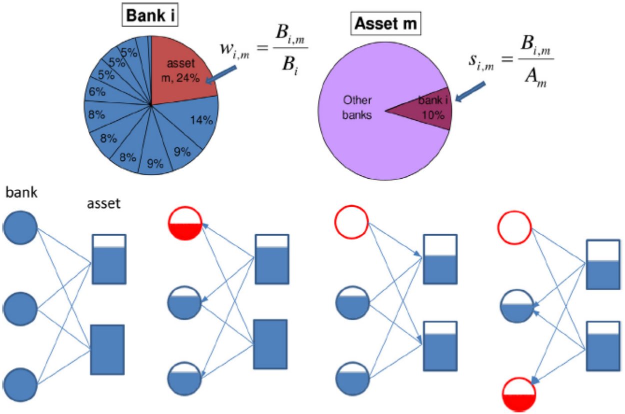
Panel 3 (2nd contagion round)
Affected Countries: Italy, France, Belgium, Germany, Switzerland.



Panel 4 (final round)
Affected Countries: Italy, France, Belgium, Germany, Switzerland, Austria, Sweden, Netherlands.

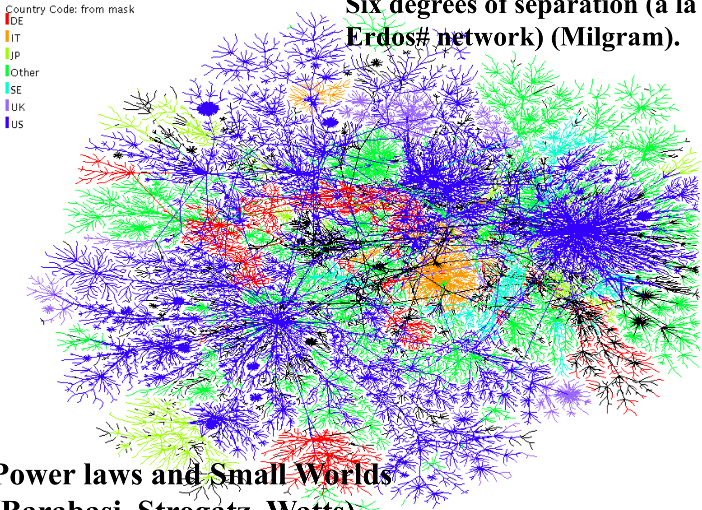
Bivalent Networks

Levy-Carciente, Kenett, Avakian, Stanley, Havlin (JBF 2015)



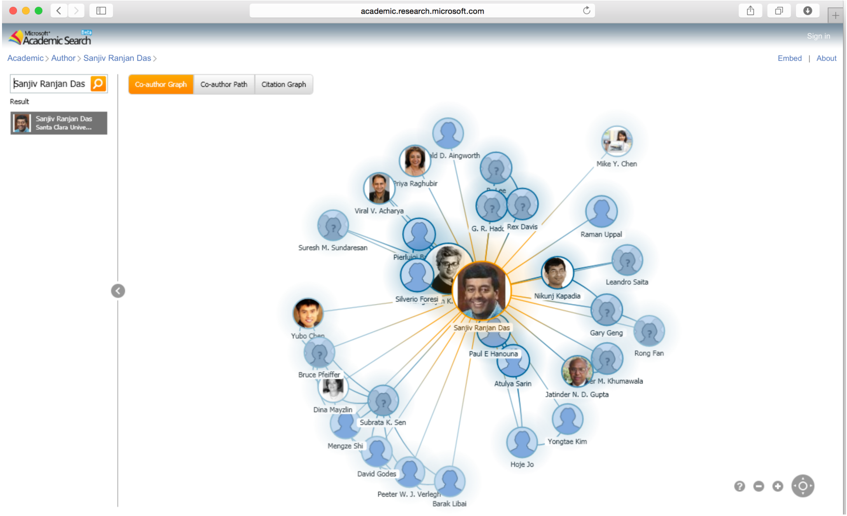
Small Worlds

Six degrees of separation (a la Erdos# network) (Milgram).

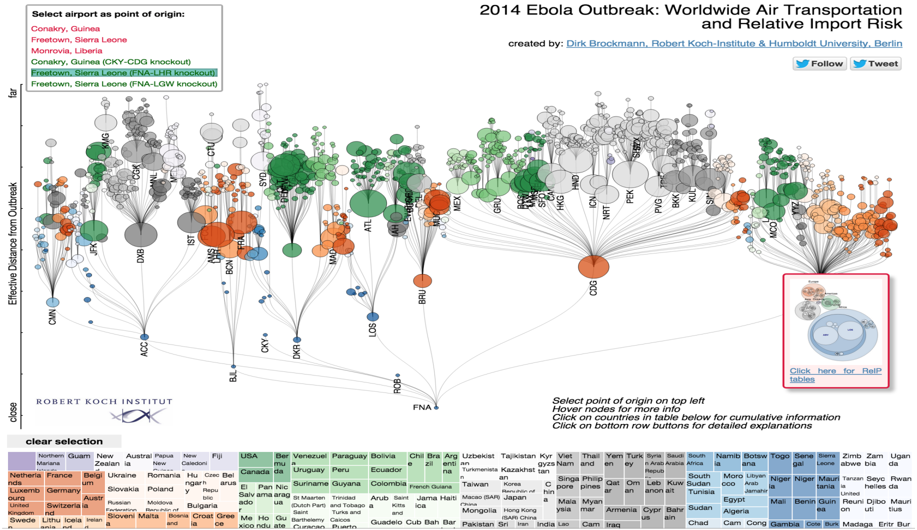


Power laws and Small Worlds (Barabasi, Strogatz, Watts).

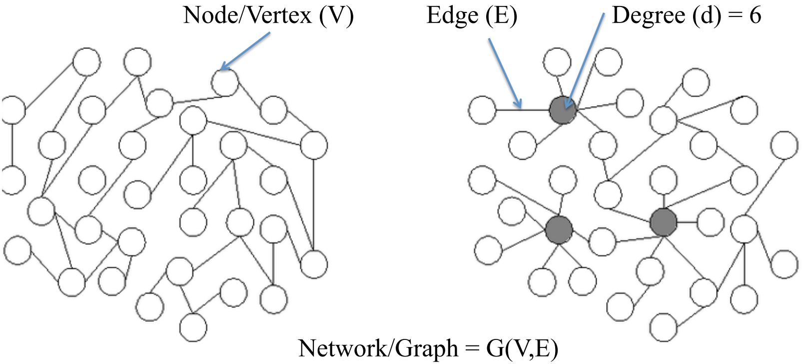
Co-Author Networks



[http://rocs.hu-berlin.de/D3/ebola/]



Graph Theory: Network Types



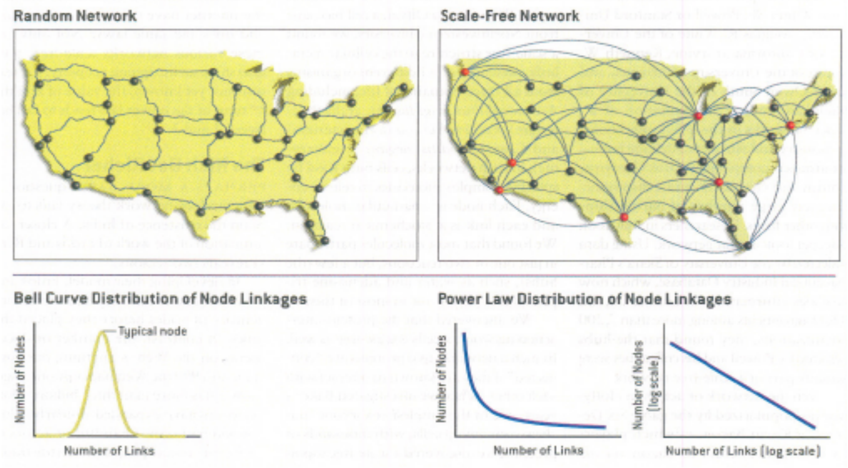
(a) Random network

$$f(d) \sim N(\mu, \sigma^2)$$

(b) Scale-free network

$$f(d) = d^{-\alpha}, \quad 2 < \alpha < 3$$

Random vs Scale-Free Graphs



Barabasi, Sciam, May 2003

Using the igraph Library in R

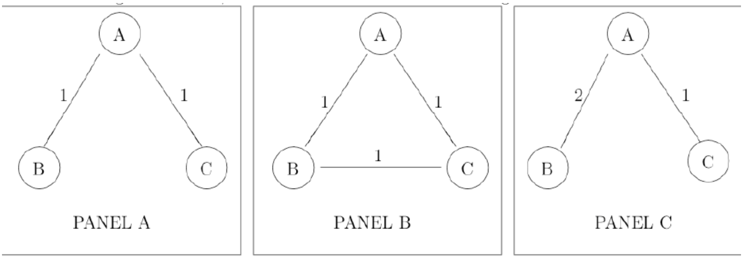
Using simple commands we generate a random graph.

```
library(igraph)

#GENERATE RANDOM GRAPH
g = erdos.renyi.game(30,0.1)
plot(g)

#COMPUTE DEGREE DISTRIBUTION
dd = degree.distribution(g)
dd = as.matrix(dd)
d = as.matrix(seq(0,max(degree(g))))
plot(d,dd,type="l")
```

And we plot (a) network, (b) degree distribution.

Centrality (Bonacich 1987)

- Also known as `PageRank` by Google.
- Adjacency matrix: $A_{ij} \in \mathcal{R}^{N \times N}$
- Influence: $x_i = \sum_{j=1}^{N} A_{ij} x_j$
- $\lambda \mathbf{x} = \mathbf{A} \cdot \mathbf{x}$
- Centrality is the eigenvector $\mathbf{x}$ corresponding to the largest eigenvalue.



$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

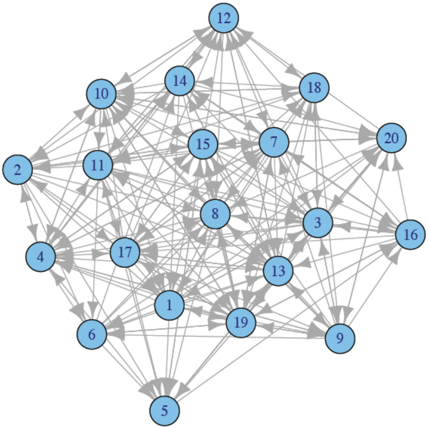
Centrality scores = {0.71, 0.50, 0.50}

Centrality scores = {0.58, 0.58, 0.58}

Centrality scores = {0.71, 0.63, 0.32}

Diameter

Longest shortest distance from a node to any other node, across all nodes.



The diameter of this graph is 2.

Fragility

- Definition: how quickly will the failure of any one node trigger failures across the network? Is network malaise likely to spread or be locally contained?
- Metric:
$$R = \frac{E(d^2)}{E(d)},$$
where d is node degree.
- Fragile if $R > 2$.
- Fragility of the sample network $= 20$

R Code for Centrality, Diameter, Fragility

```
#COMPUTE DEGREE DISTRIBUTION
dd = degree.distribution(g)
dd = as.matrix(dd)
d = as.matrix(seq(0,max(degree(g))))

#CENTRALITY
cent = evcent(g)
print(cent$vector)

#DIAMETER
print(diameter(g))

#FRAGILITY
print((t(d^2) %*% dd)/(t(d) %*% dd))
```

Weak Ties (1973)

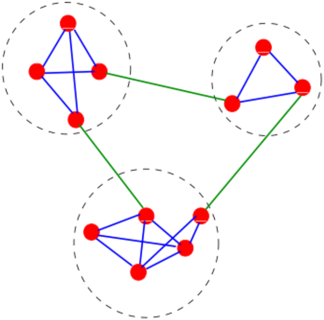
The Strength of Weak Ties[1]

Mark S. Granovetter
Johns Hopkins University

Analysis of social networks is suggested as a tool for linking micro and macro levels of sociological theory. The procedure is illustrated by elaboration of the macro implications of one aspect of small-scale interaction: the strength of dyadic ties. It is argued that the degree of overlap of two individuals' friendship networks varies directly with the strength of their tie to one another. The impact of this principle on diffusion of influence and information, mobility opportunity, and community organization is explored. Stress is laid on the cohesive power of weak ties. Most network models deal, implicitly, with strong ties, thus confining their applicability to small, well-defined groups. Emphasis on weak ties lends itself to discussion of relations *between* groups and to analysis of segments of social structure not easily defined in terms of primary groups.

Communities

- Definition: clusters of nodes that interact much more within community than across community.



- Hard computational problem.
- Fast-greedy algorithm (Girvan & Newman 2003)
- Walk-trap algorithm (Pons & Latapy 2005)

Using R for Community Detection

Consider a network of five nodes $\{A, B, C, D, E\}$, where the edge weights are as follows: $A : B = 6$, $A : C = 5$, $B : C = 2$, $C : D = 2$, and $D : E = 10$. Assume that a community detection algorithm assigns $\{A, B, C\}$ to one community and $\{D, E\}$ to another, i.e., only two communities. The adjacency matrix for this graph is

$$\{A_{ij}\} = \begin{bmatrix} 0 & 6 & 5 & 0 & 0 \\ 6 & 0 & 2 & 0 & 0 \\ 5 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 10 \\ 0 & 0 & 0 & 10 & 0 \end{bmatrix}$$

The Kronecker delta matrix that delineates the communities is

$$\{\delta_{ij}\} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Modularity

- Quasi-distance metric between community based adjacency matrix partition and one with no communities.
- Metric:

$$Q = \frac{1}{2m} \sum_K \sum_{i,j} \left[A_{ij} - \frac{d_i \times d_j}{2m} \right] \cdot \delta_{i,j}(C_k)$$

- $m = \sum_{i,j} \frac{A_{ij}}{2}$. So, $2m$ is the sum of all edges.
- Kronecker delta: $\delta_{i,j}(C_k) = 1$ if i, j are in the same community, else zero.
- Modularity matrix: $\left[A_{ij} - \frac{d_i \times d_j}{2m} \right]$
- The score takes a value ranging from -1 to $+1$ as it is normalized by dividing by $2m$. When $Q > 0$ it means that the number of connections within communities exceeds that between communities.

Computing Modularity in R

```
#MODULARITY
Amodularity = function(A,delta) {
    n = length(A[1,])
    d = matrix(0,n,1)
    for (j in 1:n) { d[j] = sum(A[j,]) }
        m = 0.5*sum(d)
        Q = 0
        for (i in 1:n) {
            for (j in 1:n) {
                Q = Q + (A[i,j] - d[i]*d[j]/(2*m))*delta[i,j]
            }
        }
        Q = Q/(2*m)
    }
```

Implementation in R

```
> A = matrix(c(0,6,5,0,0,6,0,2,0,0,5,2,0,2,0,0,0,2,0,10,0,0,0,10,0),5,5)
> delta = matrix(c(1,1,1,0,0,1,1,1,0,0,1,1,1,0,0,0,0,0,1,1,0,0,0,1,1),5,5)
> print(Amodularity(A,delta))
[1] 0.4128
```

We now repeat the same analysis using the R package.

```
> g = graph.adjacency(A,mode="undirected",weighted=TRUE,diag=FALSE)
```

We then pass this graph to the walktrap algorithm:

```
> wtc=walktrap.community(g,modularity=TRUE,weights=E(g)$weight)
> res=community.to.membership(g,wtc$merges,steps=3)
> print(res)
$membership
[1] 0 0 0 1 1

$csize
[1] 3 2

> print(modularity(g,res$membership,weights=E(g)$weight))
[1] 0.4128
```

Midas Project: Overview

Joint work with IBM Almaden[2]

- Focus on financial companies that are the domain for systemic risk (SIFIs).
- Extract information from unstructured text (filings).
- Information can be analyzed at the institutional level or aggregated system-wide.
- Applications: Systemic risk metrics; governance.
- Technology: information extraction (IE), entity resolution, mapping and fusion, scalable Hadoop architecture.

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[2] "Extracting, Linking and Integrating Data from Public Sources: A Financial Case Study," (2011), (with Douglas Burdick, Mauricio A. Hernandez, Howard Ho, Georgia Koutrika, Rajasekar Krishnamurthy, Lucian Popa, Ioana Stanoi, Shivakumar Vaithyanathan), *IEEE Data Engineering Bulletin*, 34(3), 60-67. [Proceedings WWW2010, April 26-30, 2010, Raleigh, North Carolina.]

Entity View

Midas provides an entity view around new sources of data

- Extraction and cleansing of financial entities, their resolution and linkage across multiple sources
- Uncovering non-obvious relationships between financial entities
- Computation of key financial metrics using data extracted from multiple sources of public data
- Information analyzed at the institutional level or aggregated system-wide.

Web Data

News
Blogs
Reviews

Public Data

FDIC Call Data Records
SEC Filings
OTS Thrift Financial Records

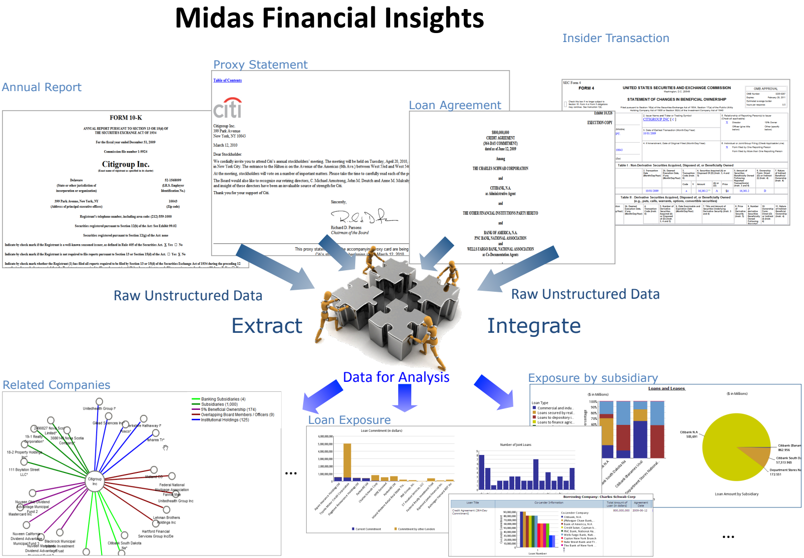
Private Data

Hoovers
D&B
FINRA
Private Wall Street Journal

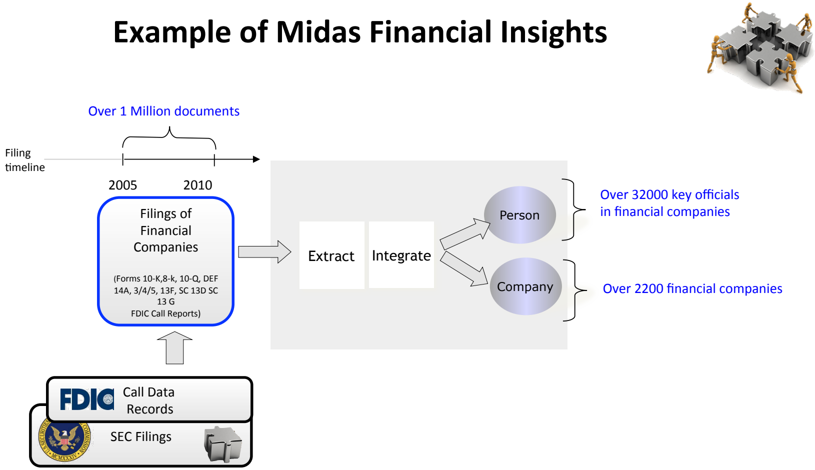
Midas Financial Insights

- Regulators
- Credit committees
- Investment analysts
- Portfolio managers
- Equity managers

Input & Output



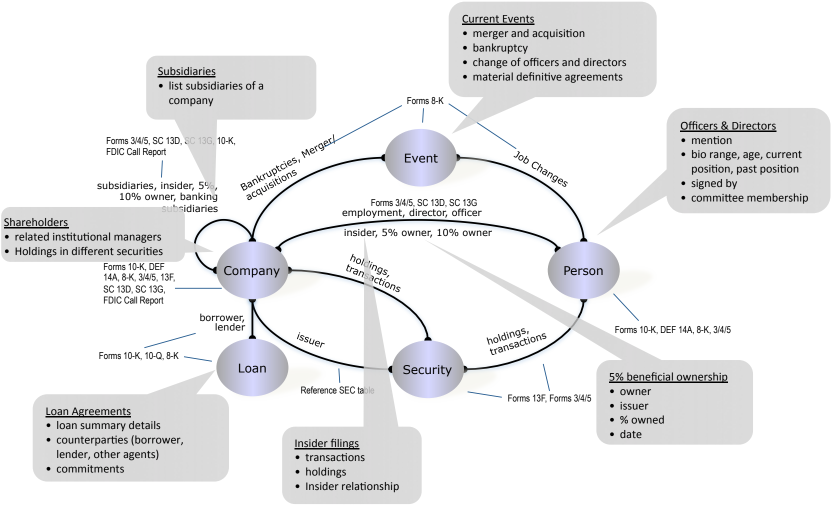
Process



Example of Midas Financial Insights
Over 1 Million documents
Filing timeline
2005
2010
Filings of Financial Companies
(Forms 10-K,8-k, 10-Q, DEF 14A, 3/4/5, 13F, SC 13D SC 13 G
FDIC Call Reports)
Extract
Integrate
Person
Company
Over 32000 key officials in financial companies
Over 2200 financial companies
FDIC
Call Data Records
SEC Filings

Data

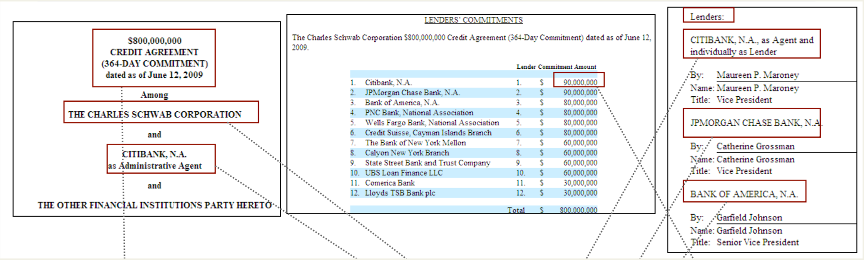
Midas provides Analytical Insights into company relationships by exposing information concepts and relationships within extracted concepts



Loan Extraction

Example Analysis : Extraction of Loan Information Data

Extract and cleanse information from headers, tables main content and signatures



$800,000,000
CREDIT AGREEMENT
(364-DAY COMMITMENT)
dated as of June 12, 2009

Among

THE CHARLES SCHWAB CORPORATION

and

CITIBANK, N.A.
as Administrative Agent

and

THE OTHER FINANCIAL INSTITUTIONS PARTY HERETO

LENDERS' COMMITMENTS

The Charles Schwab Corporation $800,000,000 Credit Agreement (364-Day Commitment) dated as of June 12, 2009.

	Lender		Commitment Amount
1.	Citibank, N.A.	1.	$ 90,000,000
2.	JPMorgan Chase Bank, N.A.	2.	$ 90,000,000
3.	Bank of America, N.A.	3.	$ 80,000,000
4.	PNC Bank, National Association	4.	$ 80,000,000
5.	Wells Fargo Bank, National Association	5.	$ 80,000,000
6.	Credit Suisse, Cayman Islands Branch	6.	$ 80,000,000
7.	The Bank of New York Mellon	7.	$ 60,000,000
8.	Calyon New York Branch	8.	$ 60,000,000
9.	State Street Bank and Trust Company	9.	$ 60,000,000
10.	UBS Loan Finance LLC	10.	$ 60,000,000
11.	Comerica Bank	11.	$ 30,000,000
12.	Lloyds TSB Bank plc	12.	$ 30,000,000
		Total	$ 800,000,000

Lenders:

CITIBANK, N.A., as Agent and individually as Lender

By: Maureen P. Maroney
Name: Maureen P. Maroney
Title: Vice President

JPMORGAN CHASE BANK, N.A.

By: Catherine Grossman
Name: Catherine Grossman
Title: Vice President

BANK OF AMERICA, N.A.

By: Garfield Johnson
Name: Garfield Johnson
Title: Senior Vice President

Id	Agreement Name	Date	Total Amount
1	Credit Agreement	June 12, 2009	$800,000,000
...			

Loan Information

Id	Company	Role	Commitment
1	Charles Schwab Corporation	Borrower	
1	Citibank, N.A.	Administrative Agent	
1	Citibank, N.A.	Lender	$90,000,000
1	JPMorgan Chase Bank, N.A.	Lender	$90,000,000
1	Bank of America, N.A.	Lender	$80,000,000
...			

Loan Company Information

Notes: Loan Document filed by Charles Schwab Corporation On Aug 6, 2009

Co-Lending Network

1. Definition: a network based on links between banks that lend together.
2. Loans used are not overnight loans. We look at longer-term lending relationships.
3. Lending adjacency matrix:

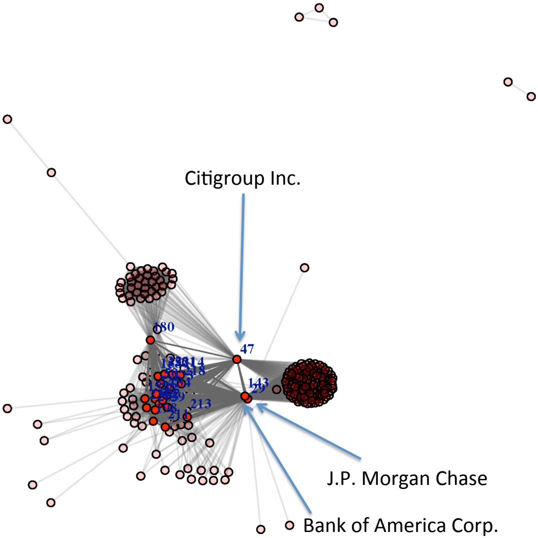
$$L \equiv \{L_{ij}\}, i,j = 1...N$$

4. Undirected graph, i.e., symmetric: $L \in R^{N \times N}$
5. Total lending impact for each bank: $x_i, i = 1...N$

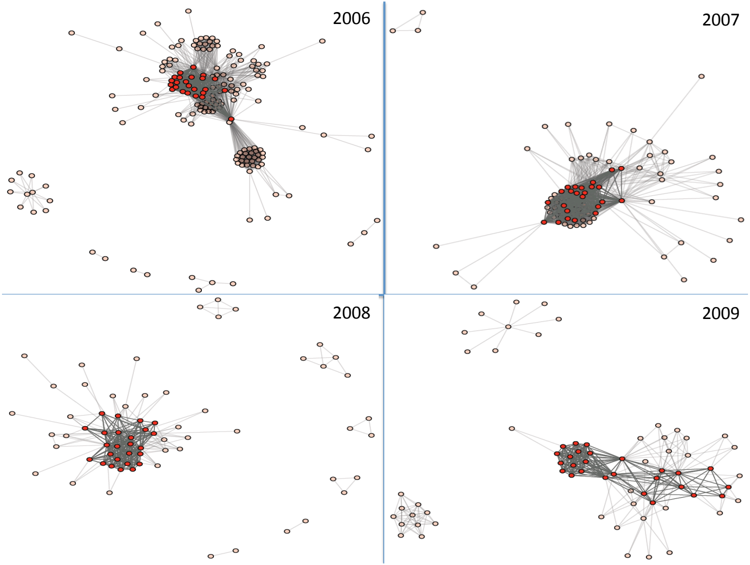
Data

- Five years: 2005 to 2009.
- Loans between FIs only.
- Filings made with the SEC.
- No overnight loans.
- Example: 364-day bridge loans, longer-term credit arrangement, Libor notes, etc.
- Remove all edge weights < 2 to remove banks that are minimally active. Remove all nodes with no edges. (This is a choice for the regulator.)

Loan Network 2005



Loan Network 2006–2009



Systemically Important Financial Institutions (SIFIs)

Year	#Colending banks	#Coloans	Colending pairs	$R = E(d^2)/E(d)$	Diam.
2005	241	75	10997	137.91	5
2006	171	95	4420	172.45	5
2007	85	49	1793	73.62	4
2008	69	84	681	68.14	4
2009	69	42	598	35.35	4

(Year = 2005)

Node #	Financial Institution	Normalized Centrality
143	J P Morgan Chase & Co.	1.000
29	Bank of America Corp.	0.926
47	Citigroup Inc.	0.639
85	Deutsche Bank Ag New York Branch	0.636
225	Wachovia Bank NA	0.617
235	The Bank of New York	0.573
134	Hsbc Bank USA	0.530
39	Barclays Bank Plc	0.530
152	Keycorp	0.524
241	The Royal Bank of Scotland Plc	0.523
6	Abn Amro Bank N.V.	0.448
173	Merrill Lynch Bank USA	0.374
198	PNC Financial Services Group Inc	0.372
180	Morgan Stanley	0.362
42	Bnp Paribas	0.337
205	Royal Bank of Canada	0.289
236	The Bank of Nova Scotia	0.289
218	U.S. Bank NA	0.284
50	Calyon New York Branch	0.273
158	Lehman Brothers Bank Fsb	0.270
213	Sumitomo Mitsui Banking	0.236
214	Suntrust Banks Inc	0.232
221	UBS Loan Finance Llc	0.221
211	State Street Corp	0.210
228	Wells Fargo Bank NA	0.198

Extensions

1. Other markets, e.g., CDS exchange. Dodd-Frank mandates conversion of all OTC contracts to be cleared through central counter parties (CCPs).
2. Inserting risk values at each node. This allows for risk assessment across the network based on severity of risk. Overcomes an essential missing component of extant network analyses.

Correlation vs Network Measures

- Correlation measures are pairwise and conditional; network measures are system-wide and unconditional.
- Correlations tend to be high in crisis periods but are not early-warning indicators of systemic risk. It is an empirical question as to whether network measures are predictive.
- Correlation measures are statistical metrics. Network measures directly model the underlying mechanics of the system because the adjacency matrix E is developed based on physical transaction activity, and the compromise vector is a function of firm quality that may be measured in multivariate ways.

Risk Networks: Definitions and Risk Score

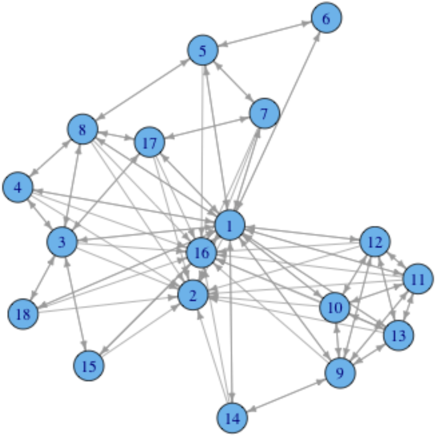
- Assume n nodes, i.e., firms, or "assets."
- Let $E \in R^{n \times n}$ be a well-defined adjacency matrix. This quantifies the influence of each node on another.
- E may be portrayed as a directed graph, i.e., $E_{ij} \neq E_{ji}$. $E_{jj} = 1$; $E_{ij} \in \{0, 1\}$.
- C is a $(n \times 1)$ risk vector that defines the risk score for each asset.
- We define the "risk score" as

$$S = \sqrt{C^\top \, E \, C}$$

- $S(C, E)$ is linear homogenous in C.

Example

Risk vector C: 0 0 1 2 2 2 2 2 1 0 2 2 2 2 1 0 1 1
Risk Score: $S = 11.62$

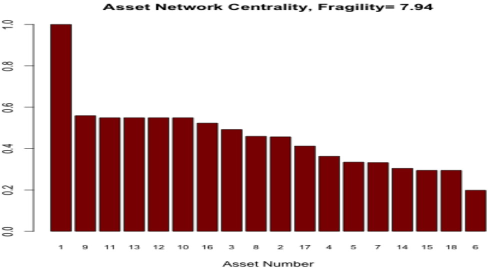


Example: Adjacency Matrix

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]	[,14]	[,15]	[,16]	[,17]	[,18]
[1,]	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
[2,]	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
[3,]	1	1	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	1
[4,]	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
[5,]	1	0	0	0	1	1	1	1	0	0	0	0	0	0	0	1	0	0
[6,]	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
[7,]	1	1	0	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0
[8,]	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	1	1	0
[9,]	1	0	0	0	0	0	0	0	1	1	1	1	1	1	0	1	0	0
[10,]	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	1	0	0
[11,]	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	1	0	0
[12,]	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	1	0	0
[13,]	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	1	0	0
[14,]	1	1	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0
[15,]	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
[16,]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
[17,]	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0
[18,]	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1

Centrality and Fragility

- Centrality is the principal eigenvector x of dimension $(n \times 1)$ such that for scalar λ: $\lambda\, x = E\, x$
- Plot:



- Fragility: for each node with degree d_j, fragility is the score given by

$$E(d^2)/E(d)$$

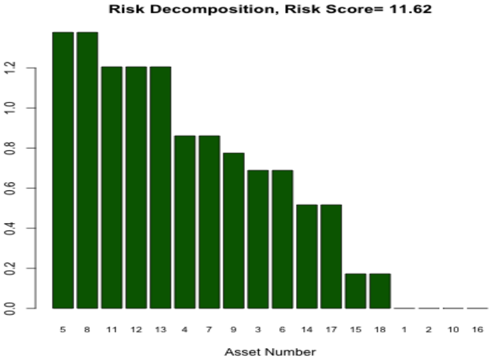
Increasing values imply a more fragile network.

Risk Decomposition

1. Exploits the homogeneity of degree one property of S.
2. Risk decomposition (using Euler's formula):

$$S = \frac{\partial S}{\partial C_1} C_1 + \frac{\partial S}{\partial C_2} C_2 + \ldots + \frac{\partial S}{\partial C_n} C_n$$

3. Plot:

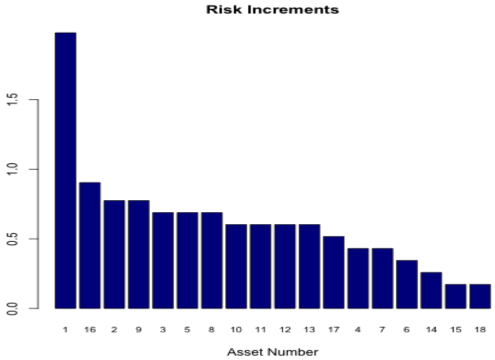


Risk Increments

- Increments are simply:

$$\mathbf{I} = \frac{\partial S}{\partial \mathbf{C}} = \frac{1}{2S}[\mathbf{EC} + \mathbf{E}^\top \mathbf{C}] \in \mathcal{R}^n$$

- Plot:



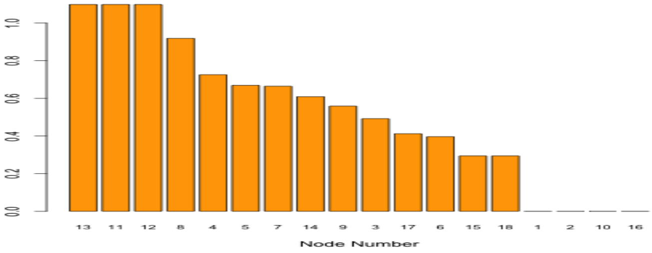
Normalized Risk Score

- Units of S are free to choose, and determined by the units of vector C, e.g., rating units, Z-score, expected loss.
- $$\bar{S} = \frac{\sqrt{C^\top E\ C}}{\|C\|} = 1.81 \tag{1}$$
  where $\|C\| = \sqrt{C^\top C}$ is the norm of vector C.
- When there are no network effects, $E = I$, the identity matrix, and $\bar{S} = 1$, i.e., the normalized baseline risk level with no network (system-wide) effects is unity.
- *Example*: Add one additional bi-directed link between nodes 6 and 12. The risk score S increases from 11.62 to 11.96, and the normalized risk score $\bar{S}$ increases from 1.81 to 1.87.
- *Example*: Keep the network unchanged, but re-allocate the compromise vector by reducing the risk of node 3 by 1, and increasing that of node 16 by 1, risk score S goes from 11.62 to 11.87, and the normalized risk score $\bar{S}$ goes from 1.81 to 1.85.

Criticality

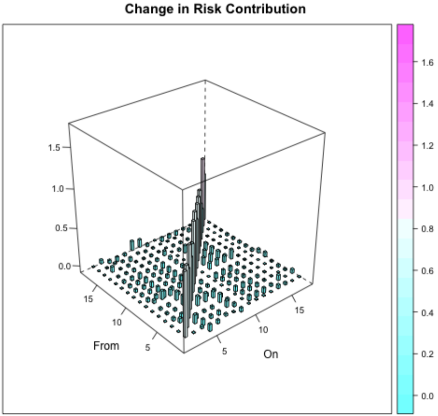
Definition: "Criticality" is compromise-weighted centrality. This new measure is defined as $y = C \times x$ where $y, C, x \in \mathcal{R}^n$. Note that this is an element-wise multiplication of vectors C and x.

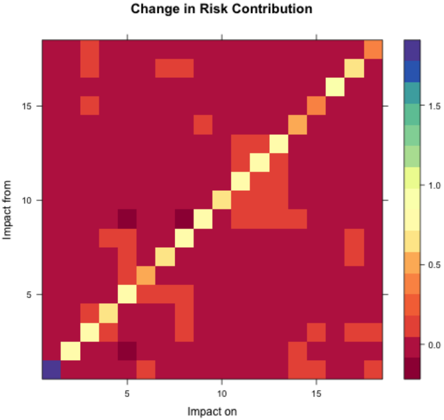
- Critical nodes need immediate attention, either because they are heavily compromised or they are of high centrality, or both.
- It offers a way for regulators to prioritize their attention to critical financial institutions, and pre-empt systemic risk from blowing up.



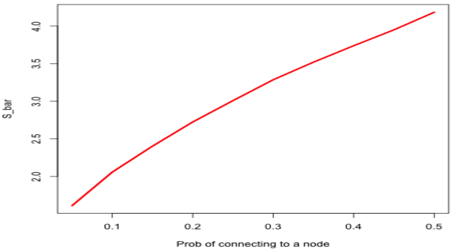
Cross Risk

Is the spill over risk from node i to node j material?



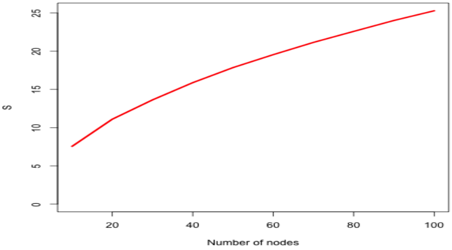


Risk Scaling



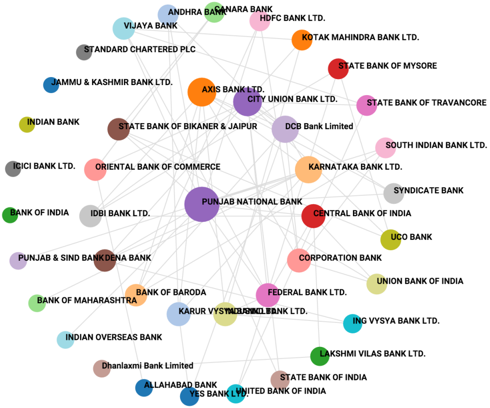
The increase in normalized risk score $\bar{S}$ as the number of connections per node increases. The plot shows how the risk score increases as the probability of two nodes being bilaterally connected increases from 5% to 50%. For each level of bilateral probability a random network is generated for 50 nodes. A compromise vector is also generated with equally likely values $\{0, 1, 2\}$. This is repeated 100 times and the mean risk score across 100 simulations is plotted on the y-axis against the bilateral probability on the x-axis.

Too Big To Fail?



Change in normalized risk score $\bar{S}$ as the number of nodes increases, while keeping the average number of connections between nodes constant. A compromise vector is also generated with equally likely values $\{0, 1, 2\}$. This is repeated 5000 times for each fixed number of nodes and the mean risk score across 5000 simulations is plotted on the y-axis against the number of nodes on the x-axis.

Systemic Risk in Indian Banks



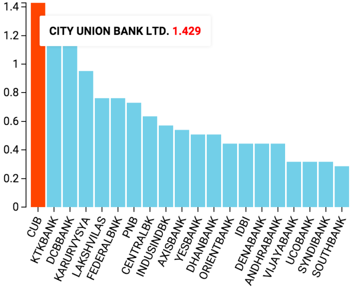
Fragility

2.91

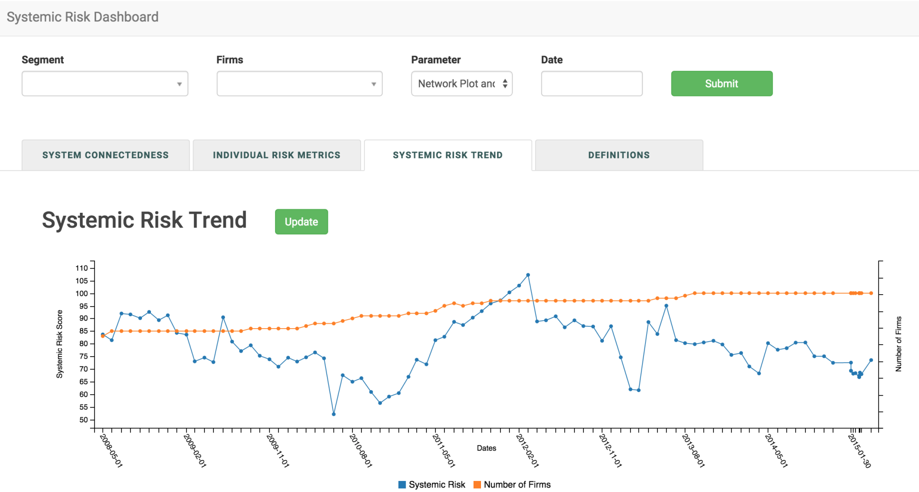
Systemic Risk Score

15.75

Risk Decomposition



Systemic Risk in India over time



Implementation

Demo

`https://shinyapps.io/srdas/SYST_Code/`

Stochastic Risk Networks in a Structural Framework

- We use the Merton (1974) model to extend the static Das (2016) model to a stochastic network setting.
- We extend each node's properties to including size, in addition to the credit score.
- To do this we normalize the S measure.
- This model can be calibrated using the same methods used for the Merton model, or variants such as the Moody's KMV model.

Definitions

Model Data (standard Merton model inputs) for each firm:

- Equity price $= \mathbf{s} = \{s_1, s_2, ..., s_n\}$
- Equity volatility $= \boldsymbol{\sigma} = \{\sigma_1, \sigma_2, ..., \sigma_n\}$
- Number of shares $= \mathbf{m} = \{m_1, m_2, ..., m_n\}$
- Risk free rate $= r$

Model Variables:

- n = number of banks in the system
- $\mathbf{a}$ = n-vector with components a_i that represent the assets in bank i (derived from s, σ, m, r).
- $\boldsymbol{\lambda}$ = n-vector with components λ_i that represent the average yearly chance of bank i defaulting (from s, σ, r).
- $\mathbf{E}$ = $n \times n$ matrix with components E_{ij} that represent the probability that if bank j defaults, it will cause bank i to default (from s, σ, r).

Model

- Define $\mathbf{c}$ to be an n-vector with components c_i that represent bank i's credit risk. More specifically, we define

$$\mathbf{c} = \mathbf{a} \odot \boldsymbol{\lambda},$$

  where $\odot$ represents component multiplication; that is, $c_i = a_i \lambda_i$.
- The aggregate systemic risk created by the n banks in our system is

$$R = \frac{\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}}}{\mathbf{1}^\top \mathbf{a}}, \qquad (2)$$

  where $\mathbf{1}$ is an n-vector of ones, so the denominator $\mathbf{1}^\top \mathbf{a} = \sum_{i=1}^{n} a_i$ represents the total assets in the n banks.
- r is linear homogenous in $\boldsymbol{\lambda}$.

Substanceless partitioning of a bank into two banks has no effect on R

Proof:

- Assume bank i's assets are artificially divided into two parts of size αa_i and $(1-\alpha)a_i$, where $\alpha \in [0,1]$, and both of these parts depend on the other's existence.
- Without loss of generality, let $i = n$, so, in our model, the new $(n+1)$-vector $\mathbf{c}$ is

$$\mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_{n-1} \\ \alpha c_n \\ (1-\alpha)c_n \end{bmatrix}$$

- The new $(n+1) \times (n+1)$ matrix $\mathbf{E}$ is

$$\mathbf{E} = \begin{bmatrix} 1 & E_{12} & \cdots & E_{1(n-1)} & E_{1n} & E_{1n} \\ E_{21} & \ddots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \ddots & \ddots & E_{(n-2)(n-1)} & \vdots & \vdots \\ E_{(n-1)1} & \cdots & E_{(n-1)(n-2)} & 1 & E_{(n-1)n} & E_{(n-1)n} \\ E_{n1} & \cdots & \cdots & E_{n(n-1)} & 1 & 1 \\ E_{n1} & \cdots & \cdots & E_{n(n-1)} & 1 & 1 \end{bmatrix},$$

- Note that $E_{(n+1)n} = E_{n(n+1)} = 1$ to reflect the fact that if one of the two parts of the divided banks defaults, the other must as well. A quick computation shows that the new $\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}}$ is equal to the old $\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}}$, and since $a_1 + \ldots + a_n = a_1 + \ldots + a_{(n-1)} + \alpha a_n + (1-\alpha) a_n$.
- The new $\mathbf{1}^\top \mathbf{a}$ is equal to the old $\mathbf{1}^\top \mathbf{a}$.
- Therefore the value of R is unchanged.

If all the assets, a_i, are multiplied by a common factor, $\alpha > 0$, it should have no effect on R.

- If a country's banks' assets all grow or all shrink in the same way, it should not affect the systemic risk of the country's banking system.
- Replace each a_i with αa_i, we replace $\sqrt{\mathbf{c}^\top \mathbf{E}\mathbf{c}}$ by $\alpha\sqrt{\mathbf{c}^\top \mathbf{E}\mathbf{c}}$, and we replace $\mathbf{1}^\top \mathbf{a}$ with $\alpha \mathbf{1}^\top \mathbf{a}$.
- Since the α then cancel in the expression for R, we have the desired property that systemic risk is unchanged.

R should increase as the banks' defaults become more connected.

- When $E_{ij} = 1$, a default in bank j guarantees a default in bank i, whereas in the opposite extreme, $E_{ij} = 0$, a default in bank j guarantees that bank i will not default. Consider the case where **a** and **c** are both held constant, so that R only depends on **E**, specifically through the expression

$$\mathbf{c}^\top \mathbf{E} \mathbf{c} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i E_{ij} c_j.$$

- The bigger the values of E_{ij} are, the larger R becomes.
- Since E_{ii} must always equal 1, R is minimized when $\mathbf{E} = \mathbf{I}$, the identity matrix, which corresponds to the (unrealistic) case where the default of any one bank guarantees that none of the other $n - 1$ banks will default.

- In the opposite extreme, R is maximized when the components of the $\mathbf{E}$ matrix are all ones, which corresponds to the case where systemic risk is maximized, since one bank failing means all the banks fail.
- When $\mathbf{E} = \mathbf{I}$, $\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}} = \sqrt{\sum_{i=1}^{n} c_i^2} = ||\mathbf{c}||_2$, the 2-norm of the vector $\mathbf{c}$, whereas when $\mathbf{E}$ is all ones, $\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}} = \sum_{i=1}^{n} c_i = ||\mathbf{c}||_1$, the 1-norm of the vector $\mathbf{c}$.

Ceteris paribus, R is minimized by dividing risk equally among n banks, maximized by putting all risk into one bank

- To analyze the effect of the spread of the credit risk on R, we first hold the total assets, $\sum_{i=1}^{n} a_i = \mathbf{1}^\top \mathbf{a}$, constant and also the total credit risk, $\sum_{i=1}^{n} c_i = \mathbf{1}^\top \mathbf{c}$, equal to a constant, c_{total}.
- To make all the banks' interactions equal, we set E_{ij} equal to the same number, e, if $i \neq j$ while, of course, keeping $E_{ii} = 1$ for all i. For the singular case where $e = 1$, all the banks act like a single bank, and so it makes no difference to R how the credit risk is spread among the banks.

- But for the general case where $e < 1$, from the definition of R we see that maximizing or minimizing R now corresponds to maximizing or minimizing $\mathbf{c}^\top \mathbf{E} \mathbf{c} = \sum_{i=1}^n c_i^2 + e \sum_{i=1}^n \sum_{j \neq i} c_i c_j$, subject to the restriction that $\mathbf{1}^\top \mathbf{c} = \sum_{i=1}^n c_i = c_{total}$.
- Since $e < 1$, it is clear that

$$\sum_{i=1}^n c_i^2 + e \sum_{i=1}^n \sum_{j \neq i} c_i c_j \leq \sum_{i=1}^n c_i^2 + \sum_{i=1}^n \sum_{j \neq i} c_i c_j = \left(\sum_{i=1}^n c_i \right)^2 = c_{total}^2.$$

- But if all the credit risk is put into one bank, we have $\sum_{i=1}^n c_i^2 + e \sum_{i=1}^n \sum_{j \neq i} c_i c_j = c_{total}^2$, the highest possible value, and so R is maximized when all the credit risk is concentrated into one bank.

- The Lagrange multiplier method tells us that we have minimized $\sum_{i=1}^{n} c_i^2 + e \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j$ subject to the restriction $\sum_{i=1}^{n} c_i = c_{total}$ when

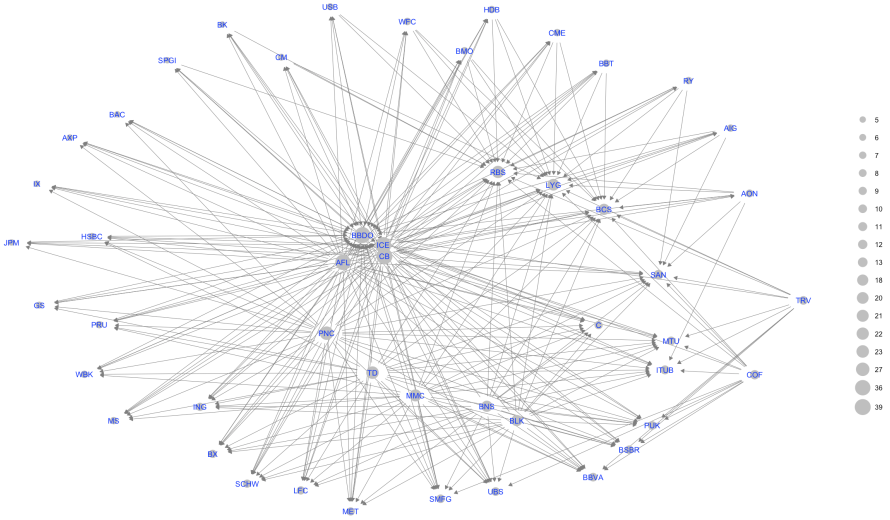
$$\frac{\partial}{\partial c_k}\left(\sum_{i=1}^{n} c_i^2 + e \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j \right) = \lambda \frac{\partial}{\partial c_k} \sum_{i=1}^{n} c_i \text{ where } k = 1, 2, ...n$$

and

$$\sum_{i=1}^{n} c_i = c_{total}.$$

- The first n equations give us that $c_1 = c_2 = ... = c_n = \frac{\lambda - 2c_{total}}{2(1-e)}$. That is, when R is minimized, all c_i have the same value. The second equation then tells us that each $c_i = \frac{c_{total}}{n}$, and so we have that R is minimized by dividing the credit risk equally among the n banks.

Network of Top 50 Financial Institutions



Computational Properties

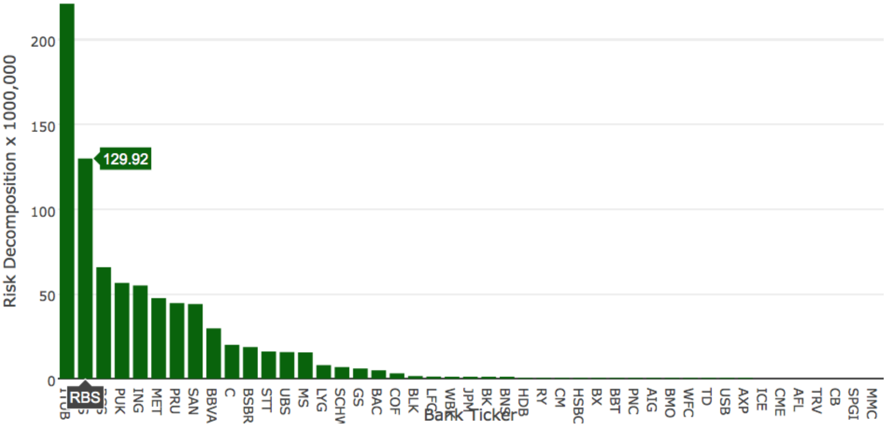
$$R = \frac{\sqrt{\mathbf{c}^\top \mathbf{E} \mathbf{c}}}{\mathbf{1}^\top \mathbf{a}}, \quad \mathbf{c} = \mathbf{a} \odot \boldsymbol{\lambda}$$

- R is linear homogeneous in $\boldsymbol{\lambda}$: Let α be any scalar constant. If we replace $\boldsymbol{\lambda}$ with $\alpha\boldsymbol{\lambda}$, it immediately follows that $\mathbf{c}$ is replaced by $\alpha\mathbf{c}$, and, by our equation for R, we see that R is replaced by αR.
- Sensitivity of R to changes in $\boldsymbol{\lambda}$: Differentiating our equation for R with respect to $\boldsymbol{\lambda}$

$$\frac{\partial R}{\partial \boldsymbol{\lambda}} = \frac{1}{2} \frac{\mathbf{a} \odot [(\mathbf{E} + \mathbf{E}^T)\mathbf{c}]}{\mathbf{1}^T \mathbf{a} \sqrt{\mathbf{c}^T \mathbf{E} \mathbf{c}}}$$

  whose components represent the sensitivity of R to changes in each bank's value of λ. This is the basis of **Risk Decomposition**, equal to $\left(\frac{\partial R}{\partial \boldsymbol{\lambda}} \cdot \boldsymbol{\lambda}\right)$, a vector containing each bank's contribution to R.

Risk Decomposition: Top 50 Financial Institutions



Real-time Implementation using Online Data

Demo

`https://shinyapps.io/srdas/SYST_Code/`

Concluding Comments

Summary

- It's a small world, and connections matter! Systemic risk measurement is about network analysis.
- Stress testing individual banks does not say anything about systemic risk unless the network is also analyzed.
- Risk networks are not easy to construct, but they are easy to analyze (`iGraph`, `networkX`).
- The Merton (1974) model may be used to model stochastic risk networks with easy to analyze properties.