Assignment 6

Numerical Methods

Spring 2025

Instructions

- Please work on the assignment on your own. It is okay to discuss the assignment with other students, but please write your own answer independently.
- If you use resources or code from the Internet or any other source, please be sure to acknowledge the source. Failure to do so may be considered plagiarism.
- For programming tasks, use JupyterLab (or Jupyter Notebook) with Python 3.13.x.
- Each answer to a programming question should include the code segment, the output, and your explanation. Points will be deducted if any part is missing.
- Submit both your report in PDF and the .ipynb file.

1. **[Programming]** Given the function $\boldsymbol{f(x) = e^x \cos x}$.

   (a) Plot the graph of $\boldsymbol{f(x)}$ over the interval [-5, 5]. The graph must include appropriate annotations such as axis labels, a title, and a legend.

   (b) Find a unimodal interval [a, b] where the function has a global minimum, such that $-5 \leq a < b \leq 5$, and both a, b are integers. Use scipy.optimize.minimize_scalar to test the interval [a, b], and print the values of $\boldsymbol{x, f(x)}$ at the global minimum.

   (c) Implement the Golden Section Search algorithm to find the global minimum over [a, b]. The algorithm must be computationally efficient by reusing function evaluations. For each iteration, print the following information: iteration number, $\boldsymbol{a, x_1, x_2, b, f(x_1), f(x_2)}$.

   (d) Implement the Successive Parabolic Interpolation algorithm to find the global minimum over [a, b]. For each iteration, print the following information: iteration number, $\boldsymbol{a, b, c, x, f(x), f(b)}$.

   (e) Implement Newton's method to find the global minimum over [a, b]. For each iteration, print the following information: iteration number, $\boldsymbol{x_n, f(x_n), |x_n - x_{n-1}|, |f(x_n) - f(x_{n-1})|}$.

   (f) Compare the methods from (c), (d), and (e) to determine which one converges the fastest and which one the slowest. Make sure that the methods from (b), (c), (d), and (e) all found the approximately the same global minimum within the interval [a, b], within a reasonable error tolerance.