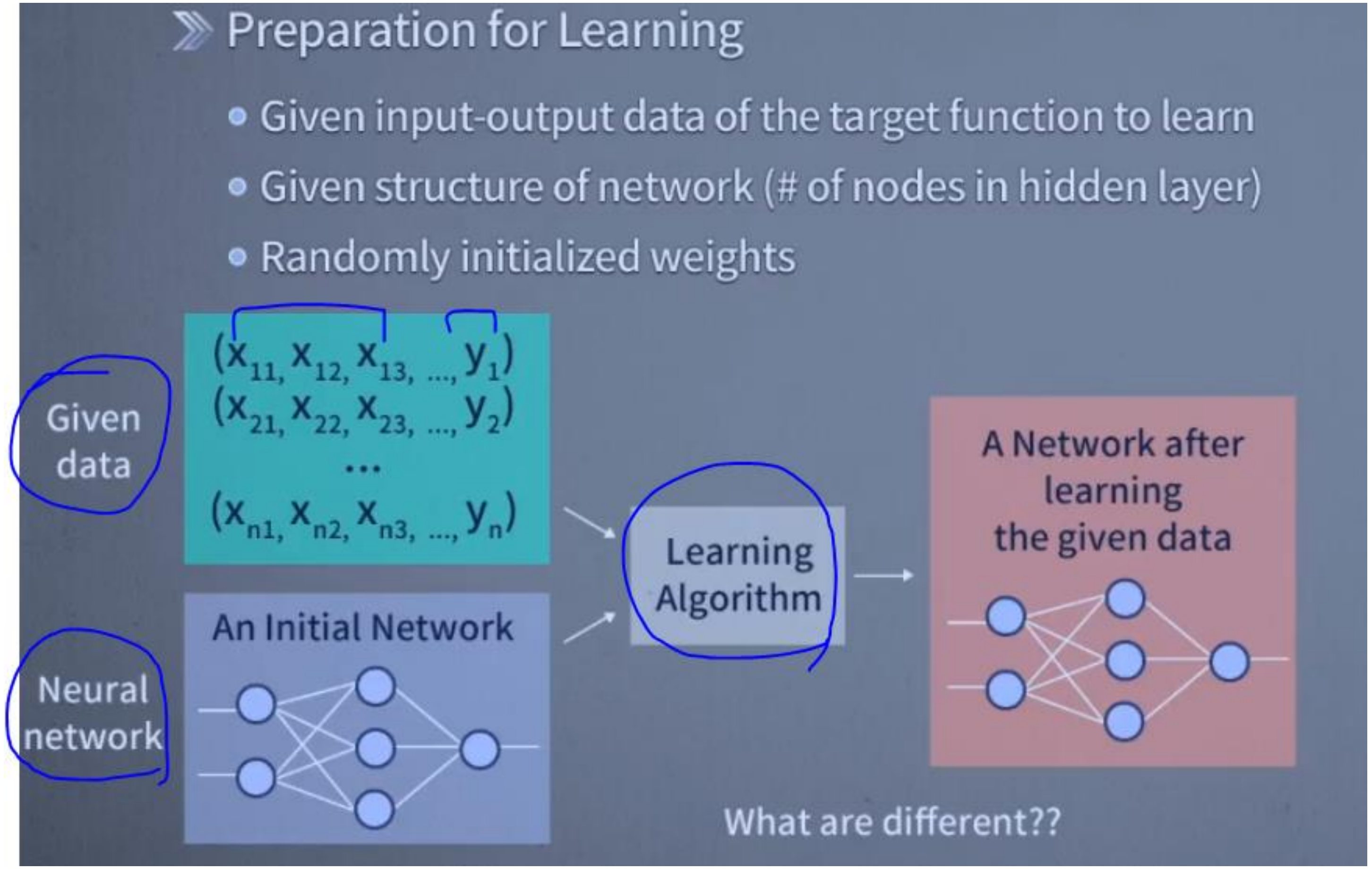
Neural Network2

Neural Network Learning

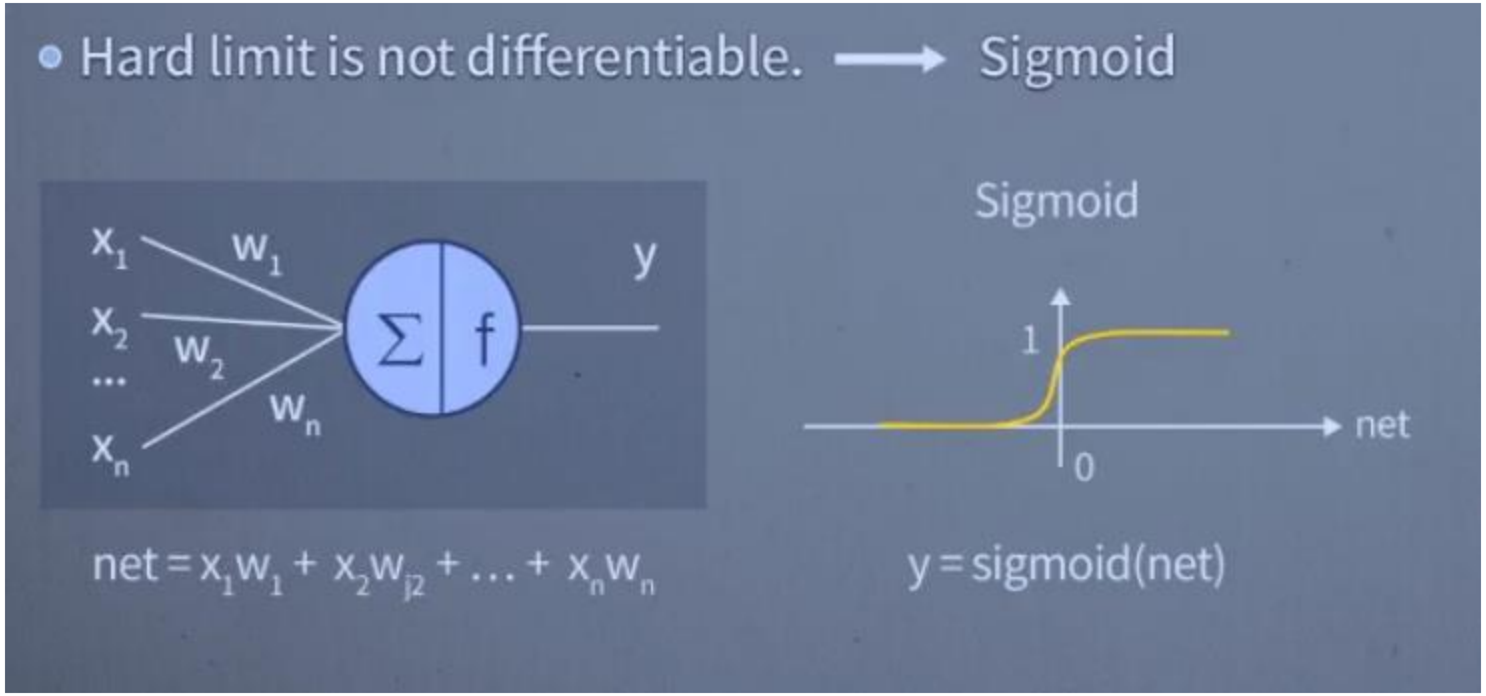
학습할 Data 준비

처음 neural network : random으로 weight 초기화



Weight 자동으로 학습(수정)

Error Back Propagation



0또는 1의 계단 함수는 불연속점(0에서)이 존재하기 때문에 미분 불가능한 구간이 생긴다. => sigmoid로 activation function 변경

Error Back Propagation

$$D_1=(x_{11}, x_{12}, \dots, x_{1d}, t_{11}, t_{12}, \dots, t_{1m}) \quad (o_{11}, o_{12}, \dots, o_{1m})$$
$$D_2=(x_{21}, x_{22}, \dots, x_{2d}, t_{21}, t_{22}, \dots, t_{2m}) \quad (o_{21}, o_{22}, \dots, o_{2m})$$
$$\dots \qquad \dots$$
$$D_N=(x_{N1}, x_{N2}, \dots, x_{Nd}, t_{N1}, t_{N2}, \dots, t_{Nm}) \quad (o_{N1}, o_{N2}, \dots, o_{Nm})$$

inputs targets Outputs of NN

입력 나와야할 값 Neural Net 결과

동일할수록 좋다

- Minimize the error

where $E_n(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{m}(t_{nk} - o_{nk})^2$

Error Back Propagation(output & hidden)

Case 1: Weights between output and hidden layer

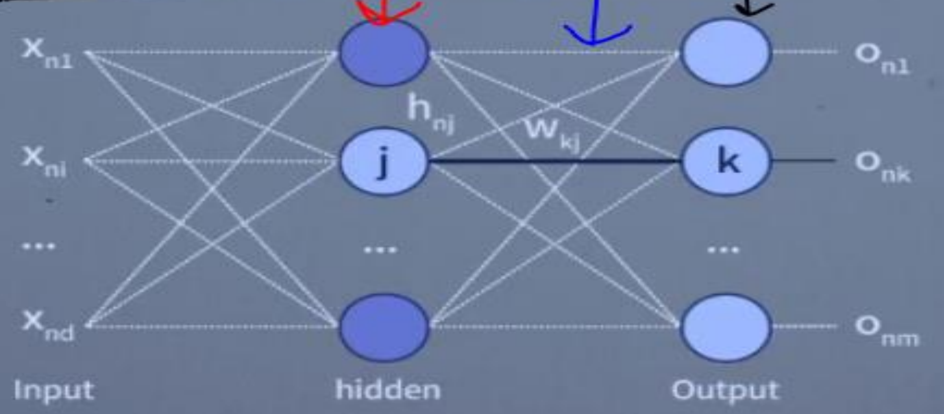
- For $D_n = (x_{n1}, x_{n2}, \ldots, x_{nd}, t_{n1}, t_{n2}, \ldots, t_{nm})$

h_{nj} = output of j−th node in hidden layer given $\mathbf{x}_n$

$$net_{nk} = h_{n1}w_{k1} + h_{n2}w_{k2} + \cdots + h_{np}w_{kp}$$

$$o_{nk} = \mathrm{sigmoid}(net_{nk})$$

$$E_n(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{m}(t_{nk} - o_{nk})^2$$



Error Back Propagation(output & hidden)

$$w_{kj} \longrightarrow net_{nk} \longrightarrow o_{nk} \longrightarrow E_n$$

$$net_{nk} = h_{n1}w_{k1} + h_{n2}w_{k2} + \cdots + h_{np}w_{kp}$$

$$o_{nk} = \text{sigmoid}(\,net_{nk})$$

$$E_n(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{m}(t_{nk} - o_{nk})^2$$

$$\frac{\partial E_n}{\partial w_{kj}} = \frac{\partial E_n}{\partial o_{nk}}\frac{\partial o_{nk}}{\partial net_{nk}}\frac{\partial net_{nk}}{\partial w_{kj}}$$

$$\frac{\partial o_{nk}}{\partial net_{nk}} = \frac{\partial\, \text{sigmoid}(\,ne}{\partial net_{nk}}$$
$$= o \quad (1 \quad o \quad)$$

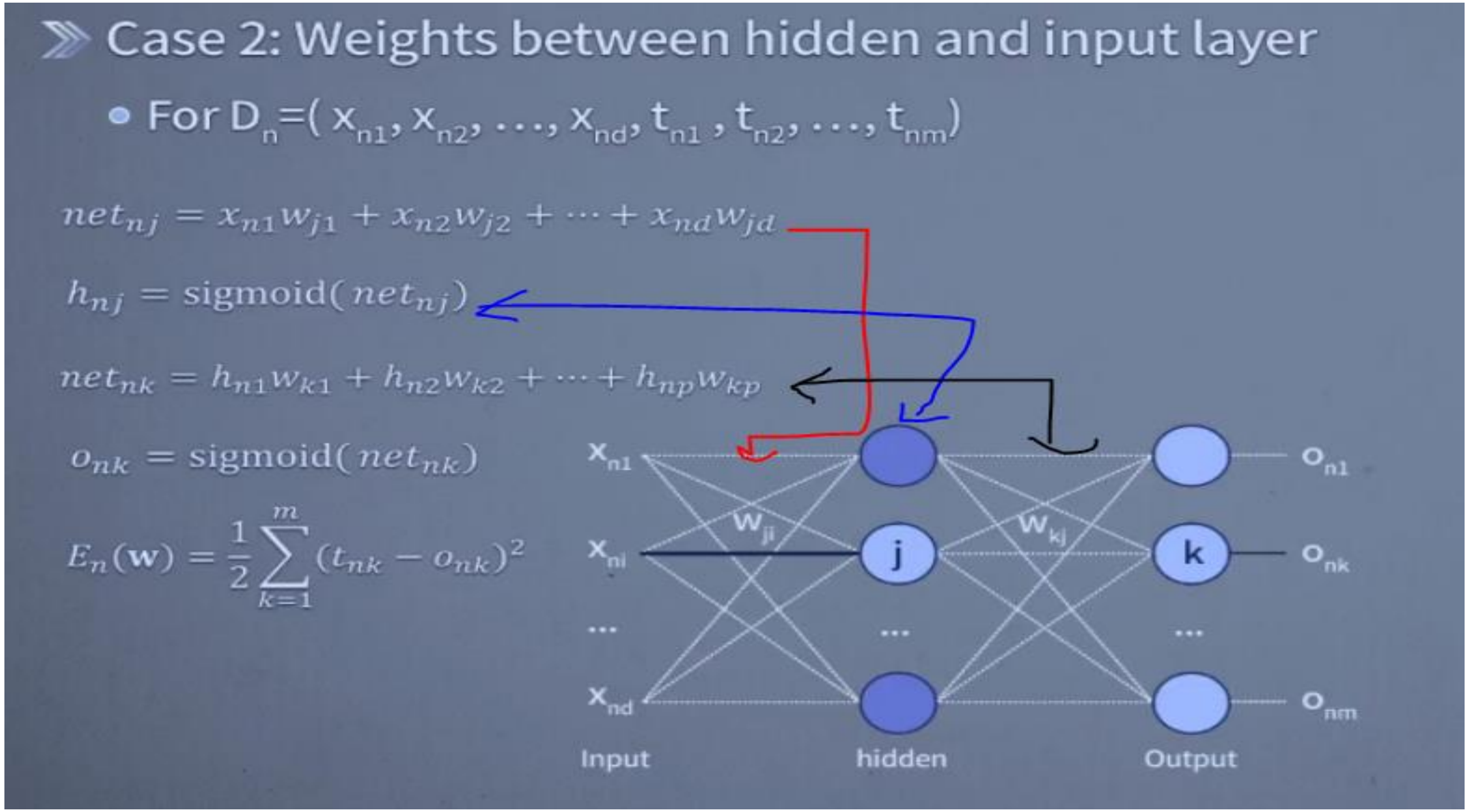
$$\frac{\partial net_{nk}}{\partial w_{kj}} = h_{nj}$$

$$\frac{\partial E_n}{\partial o_{nk}} = \frac{1}{2}\sum_{q=1}^{m}(t_{nq} - o_{nq})^2$$
$$= \frac{\partial}{\partial o_{nk}}\frac{1}{2}(t_{nk} - o_{nk})^2$$
$$= \frac{1}{2}2(t_{nk} - o_{nk})\frac{\partial(t_{nk} - o_{nk})}{\partial o_{nk}}$$
$$= -(t_{nk} - o_{nk})$$

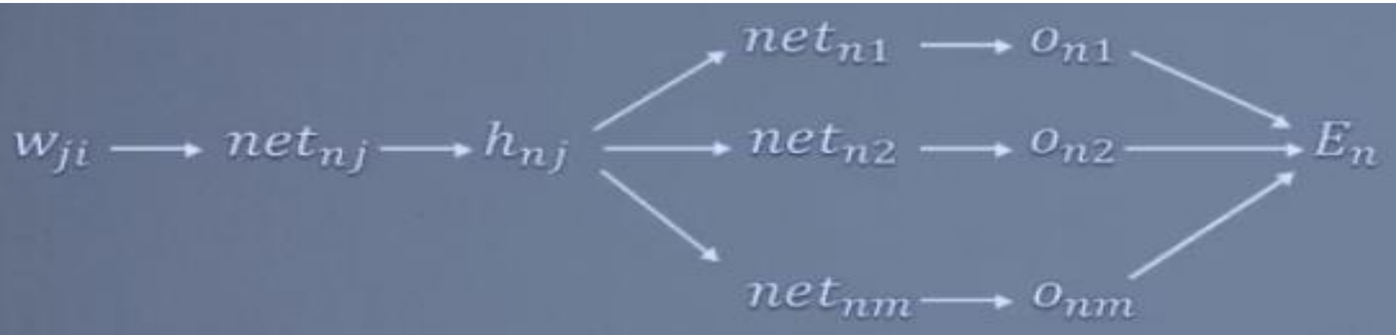
$$\frac{\partial E_n}{\partial \mathbf{w}_{kj}} = \frac{\partial E_n}{\partial o_{nk}}\frac{\partial o_{nk}}{\partial net_{nk}}\frac{\partial o_{nk}}{\partial net_{nk}} = -(t_{nk} - o_{nk})o_{nk}(1 - o_{nk})h_{nj}$$

$$\frac{\partial E}{\partial w_{kj}} = \sum_{n=1}^{N}\frac{\partial E_n}{\partial w_{kj}} = -\sum_{n=1}^{N}(t_{nk} - o_{nk})o_{nk}(1 - o_{nk})h_{nj}$$

Error Back Propagation(input & hidden)



Error Back Propagation(input & hidden)



$$net_{nj} = x_{n1}w_{j1} + x_{n2}w_{j2} + \cdots + x_{nd}w_{jd}$$

$$h_{nj} = \text{sigmoid}(net_{nj})$$

$$net_{nk} = h_{n1}w_{k1} + h_{n2}w_{k2} + \cdots + h_{np}w_{kp}$$

$$o_{nk} = \text{sigmoid}(net_{nk})$$

$$E_n(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{m}(t_{nk} - o_{nk})^2$$

Error Back Propagation(input & hidden)

$$\frac{\partial E_n}{\partial w_{ji}} = \frac{1}{2}\sum_{k=1}^{m} \frac{\partial (t_{nk} - o_{nk})^2}{\partial o_{nk}} \frac{\partial o_{nk}}{\partial net_{nk}} \frac{\partial net_{nk}}{\partial h_{nj}} \frac{\partial h_{nj}}{\partial net_{nj}} \frac{\partial net_{nj}}{\partial w_{ji}}$$

$$= \frac{1}{2}\sum_{k=1}^{m} -2(t_{nk} - o_{nk}) \cdot o_{nk}(1 - o_{nk}) \cdot w_{kj} \cdot h_{nj}(1 - h_{nj}) \cdot x_{ni}$$

$$= -h_{nj}(1 - h_{nj}) x_{ni} \sum_{k=1}^{m} w_{kj}(t_{nk} - o_{nk}) o_{nk}(1 - o_{nk})$$

$$\frac{\partial (t_{nk} - o_{nk})^2}{\partial o_{nk}} = -2(t_{nk} - o_{nk}) \qquad \frac{\partial o_{nk}}{\partial net_{nk}} = o_{nk}(1 - o_{nk}) \qquad \frac{\partial net_{nk}}{\partial h_{nj}} = w_{kj}$$

$$\frac{\partial h_{nj}}{\partial net_{nj}} = h_{nj}(1 - h_{nj}) \qquad \frac{\partial net_{nj}}{\partial w_{ji}} = x_{ni}$$

$$\frac{\partial E_n}{\partial w_{ji}} = -x_{ni} h_{nj}(1 - h_{nj}) \sum_{k=1}^{m} w_{kj}(t_{nk} - o_{nk}) o_{nk}(1 - o_{nk})$$

$$\frac{\partial E}{\partial w_{ji}} = \sum_{n=1}^{N} \frac{\partial E_n}{\partial w_{ji}} = \sum_{n=1}^{N} \left(-x_{ni} h_{nj}(1 - h_{nj}) \sum_{k=1}^{m} w_{kj}(t_{nk} - o_{nk}) o_{nk}(1 - o_{nk}) \right)$$