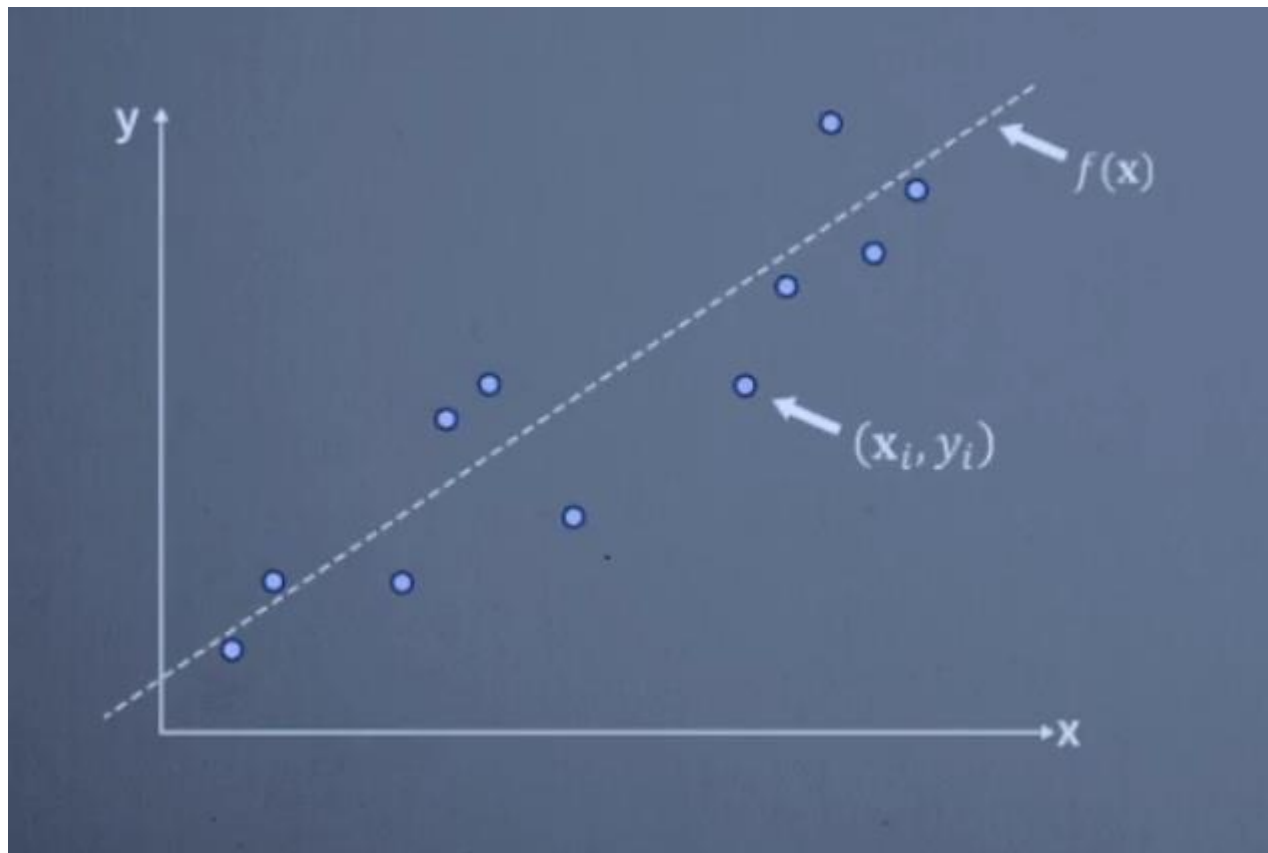


Linear Regression

Linear Regression



주어진 데이터와 오차가 최소화되는 직선 찾기
거리의 최소화 : 주어진 data와 직선까지의 거리 제곱의 합이 최소
=미분을 해서 0이 되는 지점
=1차 연립 방정식
Traing data의 X와 Y를 통해 A와 b를 구하고 w를 최종적으로 구한다.

일차 연립방정식의 해

» Let's Solve

$$\mathbf{A}\mathbf{w} = \mathbf{b}$$

$$\mathbf{A} = \begin{pmatrix} \sum_{i=1}^n x_{i0}, & \sum_{i=1}^n x_{i0}x_{i1}, & \cdots, & \sum_{i=1}^n x_{i0}x_{id} \\ \sum_{i=1}^n x_{i1}, & \sum_{i=1}^n x_{i1}x_{i1}, & \cdots, & \sum_{i=1}^n x_{i1}x_{id} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^n x_{id}, & \sum_{i=1}^n x_{id}x_{i1}, & \cdots, & \sum_{i=1}^n x_{id}x_{id} \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_d \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} \sum_{i=1}^n x_{i0}y_i \\ \sum_{i=1}^n x_{i1}y_i \\ \vdots \\ \sum_{i=1}^n x_{id}y_i \end{pmatrix}$$

» Solution is

$$\mathbf{w} = \mathbf{A}^{-1}\mathbf{b}$$

일차 연립방정식의 해

» Let's Solve

$$\mathbf{w} = (\mathbf{X}^T \mathbf{X})^{-1} (\mathbf{X}^T \mathbf{Y})$$

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$$f(\mathbf{x}_i) = w_0 x_{i0} + w_1 x_{i1} + w_2 x_{i2} + \cdots + w_d x_{id} = \sum_{j=0}^d w_j x_{ij}$$

$$\mathbf{X} = \begin{pmatrix} 1, x_{11}, x_{12}, \cdots, x_{1d} \\ 1, x_{21}, x_{22}, \cdots, x_{2d} \\ \cdots \\ 1, x_{n1}, x_{n2}, \cdots, x_{nd} \end{pmatrix} \quad \mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{pmatrix}$$

일차 연립방정식의 해

Diagram illustrating the construction of matrices X and Y from a set of data points. The data points are listed on the left:

- (2, 2)
- (3, 4)
- (6, 4)
- (7, 9)
- (8, 10)
- (10, 6)
- (14, 10)
- (15, 13)
- (16, 18)
- (17, 14)
- (18, 16)

Matrix X is constructed with the first column as 1s and the second column as the x-values:

$$X = \begin{pmatrix} 1, & 2 \\ 1, & 3 \\ 1, & 6 \\ 1, & 7 \\ 1, & 8 \\ 1, & 10 \\ 1, & 14 \\ 1, & 15 \\ 1, & 16 \\ 1, & 17 \\ 1, & 18 \end{pmatrix}$$

Vector Y is constructed with the y-values:

$$Y = \begin{pmatrix} 2 \\ 4 \\ 4 \\ 9 \\ 10 \\ 6 \\ 10 \\ 13 \\ 18 \\ 14 \\ 16 \end{pmatrix}$$

$$A = X^T X = \begin{pmatrix} 11, & 116 \\ 116, & 1552 \end{pmatrix}$$

$$b = X^T Y = \begin{pmatrix} 106 \\ 1392 \end{pmatrix}$$

$$w = (A)^{-1} b = \begin{pmatrix} 0.840708 \\ 0.834071 \end{pmatrix}$$