1 control Lyapunov functions

let V be a positive definite function satisfying

$$\dot{V}(x, k(x)) = L_f V(x) + L_g V(x) k(x)$$

then if

$$\exists u = k(x) \quad s.t. \quad \dot{V}(x, k(x)) \leq -\gamma V(x)$$

we can prove that the system is stabilizable to $V^*(x) = 0$. γ is a monotonically increasing function and $\gamma(0) = 0$. The process of stabilizing a nonlinear system can be understood as finding an input that creates a one-dimensional stable system given by the Lyapunov function $\dot{V} \leq -\gamma(V)$

Definition 1.1 (control Lyapunov function)**.** *V is a control Lyapunov function if it is positive definite and satisfies:*

$$\inf_{u \in U} [L_f V(x) + L_g V(x) u] \leq -\gamma(V(x))$$

this definition allows for us to consider the set of all stabilizing controllers u for every point $x \in D$

$$K_{clf}(x) = \{u \in U : L_f V(x) + L_g V(x) u \leq -\gamma(V(x))\}$$

we have the following stabilization theorem for CLFs:

Theorem 1.2. *for the nonlinear control system*

$$\dot{x} = f(x) + g(x) u$$

if there exists a control Lyapunov function $V : D \to \mathbf{R}_{\geq 0}$, i.e. a positive definite function satisfying

$$\inf_{u \in U} [L_f V(x) + L_g V(x) u] \leq -\gamma(V(x))$$

then any Lipschitz continuous feedback controller $u(x) \in K_{clf}(x)$ asymptotically stabilizes the system to $x^* = 0$