Intro to MPC

Autumn 2023

1 model predictive control

REFERENCE: Safety-Critical Model Predictive Control with Discrete-Time Control Barrier Function

MPC is widely used of robotic systems, such as robotic manipulation and locomotion, to achieve optimal performance while satisfying different constraints. one of the most important criteria to deploy robots for real-world tasks is safety. there is some existing work about model predictive control considering system safety. the safety criteria in the context of MPC is usually formulated as constraints in an optimization problem, such as obstacle constraints and actuation limits. one concrete scenario regarding safety criteria for robots is obstacle avoidance. the majority of literature focuses on collision avoidance using simplified models, and considers distance constraints with various euclidean norms, which we call MPC-DC in this paper.

however, these obstacle avoidance constraints under euclidean norms will not confine the robot's movement unless the robot is relatively close to the obstacles even far away from it, we usually need a larger horizon which increases the computational time in the optimization. this encourages us to formulate a new type of model predictive control, which can guarantee safety in the context of set invariance with CBF constraints confining the robot's movement during the optimization. recently, model predictive control is introduced with control lyapunov functions to ensure stability.

1.1 MPC-DC

consider the problem of regulating to the origin of a discrete-time control system described by

$$x_{t+1} = f(x_t, u_t) \tag{1}$$

where $x_t \in \mathcal{X} \subset \mathbb{R}^n$ represents the state of the system at time step $t \in \mathbb{Z}^+$, $u_t \subset \mathbb{R}^m$ is the control input, and f is locally lipschitz.

assume that a full measurement or estimate of the state x_t is available at the current time step t. then a finite-time optimal control problem is solved at time step t. when there are safety criteria, such as obstacle avoidance, the obstacles are usually formulated using distance constraints. the finite-time optimal control formulation is shown in below:

$$J_t^*(x_t) = \min_{u_{t:t+N-1|t}} p(x_{t+N|t}) + \sum_{k=0}^{N-1} q(x_{t+k|t,u_{t+k|t}}) \tag{2}$$

$$\text{s.t. } x_{t+k+1|t} = f(x+t+k|t, u_{t+k|t}), k = 0, 1, ..., N-1 \tag{3}$$

$$x_{t+k|t} \in \mathcal{X}, u_{t+k|t} \in \mathcal{U}, k = 0, 1, ..., N-1 \tag{4}$$

$$x_{t|t} = x_t \tag{5}$$

$$x_{t+N|t} \in \mathcal{X}_f \tag{6}$$

$$g(x_{t+k|t}) \geq 0, k = 0, 1, ..., N-1 \tag{7}$$

here $x_{t+k|t}$ denotes the state vector at time step $t+k$ predicted at time step t obtained by starting from the current state x_t 5, and applying the input sequence $u_{t:t+N-1|t}$ to the system dynamics 3. in 2, the terms $q(x_{t+k|t}, u_{t+k|t})$ and $p(x_{t+N|t})$ are referred to as stage cost and terminal cost respectively, and N is the time horizon. the state and input constraints are given by 4, and distance constraints for safety criteria are represented by function g, in 7, which could be defined under various euclidean norms, the terminal constraint is enforced in 6

let $u_{t:t+N-1|t} = \{u^*_{t|t}, ..., u^*_{t+N-1|t}\}$ be the optimal solution of the above finite-time optimal control at time step t. the resulting optimized trajectory using $u_{t:t+N-1}$ is referred as an open-loop trajectory. then the first element of $u_{t:t+N-1|t}$ is applied to system 1. this feedback control law is given below:

$$u(t) = u^*_{t|t}(x_t) \tag{8}$$

the finite-time optimal control problem is repeated at next time step $t+1$, based on the new estimated state $x_{t+1|t+1} = x_{t+1}$. it yields the model predictive control strategy. the resulting trajectory using 8 is referred as a closed-loop trajectory.