

Ultra Wideband (UWB) localization using active CIR-based fingerprinting

Jaron Fontaine, Ben Van Herbruggen, Adnan Shahid, Sebastian Kram, Maximilian Stahlke and Eli De Poorter

Abstract—Indoor positioning systems using Ultra Wideband (UWB) achieve high positioning accuracy (<30 cm). However, traditional localization approaches require many packet exchanges (e.g. two-way ranging) or challenging clock synchronization (e.g. time difference of arrival). To remedy this, we propose active fingerprinting using the channel impulse response (CIR) from a single UWB packet received at each UWB anchor. The proposed neural network anchor-subset selection method with Savitzky-Golay filter achieves a low mean absolute error (20.9–87.0 cm), in contrast to signal strength based fingerprinting approaches that realize accuracies of 2–3 m. Finally, with CIR interpolation the data collection overhead is reduced.

Index Terms—UWB, fingerprinting, neural networks

I. INTRODUCTION

Indoor positioning systems (IPSs) are an active field of research in domains such as industry 4.0, home automation, warehouse inventory management, sports activity tracking, healthcare, etc. Wi-Fi and Bluetooth Low Energy (BLE) technologies are often utilized since they are already present in existing communication networks [1]. Typically, these systems rely on received signal strength indicators (RSSIs) to estimate the distance, which results in a 2–3 m error (the “positioning accuracy”) due to the presence of non-line-of-sight (NLOS) and multi-path fading effects [2]. More recently, phase-based receivers have been proposed to offer higher accuracy, but these solutions require more expensive hardware and antenna systems [3]. Ultra Wideband (UWB) positioning systems are gaining a lot of attention by the research community and are appearing in consumer products e.g., Apple and Samsung mobile phones. The UWB technology offers a very high temporal resolution due to the use of a high wireless bandwidth to transmit packets and is more resistant to multi-path fading effects, which are significant advantages over other competing technologies [4]. This allows UWB positioning systems to accurately (sub-1 ns) estimate the time of flight (TOF) between a positioning tag and anchor. Three main approaches are used to determine accurate 2D and 3D positions in UWB positioning systems: (i) two way ranging (TWR) (e.g. asymmetric double-sided TWR) ensures accurate TOF ranging, without clock synchronization between the tag and anchor and can mitigate processing time differences between the two ranging

devices [5] [6], (ii) time difference of arrival (TDOA) uses one-way communication, with the tag typically sending an UWB packet received by each anchor (the opposite way also exists), and calculates the difference in the time when the packet was received at each anchor [7], and (iii) angle of arrival (AOA), a technique to calculate the angle between the sending and receiving devices [8]. For 2D or 3D positioning, different approaches are used: trilateration for TWR, multilateration for TDOA and triangulation for AOA triangulation. Although these approaches have already demonstrated accurate results, they are not without drawbacks. TWR involves sending three packets between each tag-anchor pair. The amount of packets sent, increases when many anchors are used, which limits scalability, positioning update rates and energy consumption. TDOA and AOA do not require multiple packets between each tag-anchor pair. However, these approaches remain challenging, needing clock synchronization between the receiving anchors or require expensive antenna hardware to calculate TDOA and AOA, respectively. Fingerprinting approaches do not have such requirements as they typically rely on RSSI measurements at each anchor. Moreover, off-the-shelf (OTS) UWB chips, i.e. the Decawave DW1000, can provide more fine-grained information such as first- and multipath signal amplitudes, making them an excellent candidate for fingerprinting [9].

Related work for UWB fingerprinting is summarized in Table I. The authors of [10] and [11] both use feature-based data from Wi-Fi and UWB technologies, and from UWB CIRs, respectively, on simulated data to predict 2D positions, with a mean absolute error (MAE) of 65-200 cm. In [10] a large number of transmissions are required to estimate one 2D position. In [12] the ranges of UWB TWR are used to predict 2D positions in NLOS conditions, with a MAE of 6-23 cm. However, using ranges does require more UWB transmissions (3 / UWB anchor) and limits scalability. At the time of writing, only a small number of research papers use the information in channel impulse responses (CIRs) for fingerprinting [13] [14]. The authors of [13] do not predict 2D positions, but limit the precision to larger indoor zones, while also requiring TWR in their setup. Similar to the work proposed in this paper, [14] uses CIRs to predict 2D positions using deep learning and achieves a MAE of 100 cm. However, only simulated data is used for training and evaluation, and it is unclear to what extent the methodology performs well in real environments with complex signal propagation characteristics. Additionally, data from fixed anchor sets are used to fingerprint, making the solution only feasible in environments where all anchor nodes remain within the same collision domain.

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TABLE I: Related works for UWB fingerprinting shows differences in data input, prediction outputs, whether TWR was used, number of evaluated real or simulated environments, required transmissions (tx) / fingerprint (FP), ML model used and the reported accuracy.

Paper	Input	Prediction	TWR	Environments	Tx/FP	Model	Accuracy
[10]	Features	2D		Simulated	50	kNN	200 cm
[11]	Features	2D		Simulated	1	DNN	65 cm
[12]	Ranges	2D	✓	1	3/anchor	kNN	6-23 cm
[13]	CIR	Zones	✓	2	1	DNN	99%
[14]	CIR	2D		Simulated	1	NN	100 cm
This	CIR	2D		2	1	CNN	21-87 cm

In short, none of the above works focus on machine learning (ML) using raw CIR signals from OTS UWB devices to predict 2D positions. Additionally, to the best of our knowledge no previous works have addressed the challenge of missing data in NLOS conditions, as will be illustrated in Section II. To fill this gap, the contributions in this paper are as follows:

- 1) An active fingerprinting IPS method requiring a single UWB packet transmission to limit communication overhead, power consumption and anchor synchronization.
- 2) The fingerprinting IPS combines convolutional neural networks (CNNs), CIR information, an anchor-subset selection and a Savitzky-Golay smoothing filter.
- 3) Evaluation using real UWB data captured in realistic NLOS conditions with low cost OTS UWB devices and an environment modified from the one used for training.
- 4) CIR interpolation to boost the accuracy of the IPS and reduce the collection effort of labelled samples.

The remainder of this paper is structured as follows: Section II presents the system description, including the system model and data collection and Section III presents the proposed methodology. Experimental results are analysed in Section IV and is followed by conclusions and future work in Section V.

II. SYSTEM DESCRIPTION

In this section we describe the UWB IPS model which uses CIRs to train machine learning based fingerprinting models. Additionally, the performed data collection is presented.

A. System model

We define the system model as an UWB IPS which includes N anchors a_n , for $n \in [1 \dots N]$ and a tag t , as illustrated in Fig. 1. In a typical TWR scheme, $3 \times N$ UWB packets are exchanged. In the proposed active UWB fingerprinting setup, only 1 UWB packet is broadcasted from the tag. Instead of having high precision TOF information at each anchor a_n , we only collect the CIR. Additionally, whereas most fingerprinting solutions focus on the RSSI to estimate the position of a mobile tag, the CIR offers much richer information. The CIR can be defined as follows:

$$CIR_{a_n}(t) = \sum_{k=1}^K \alpha_k \delta(t - \tau_k) + n(t), \quad (1)$$

where t is the timestamp for each CIR sample (1016 samples, sampled at a temporal resolution of 1 ns); K is the number of multipath components; δ is the Dirac delta function; α_k and

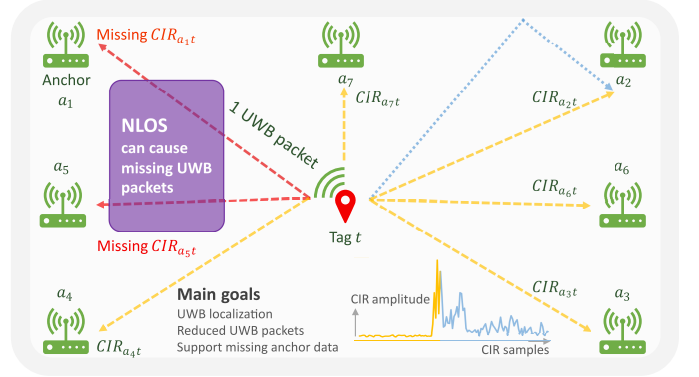


Fig. 1: The UWB fingerprinting CIR-based system model with $N = 7$ anchors. Direct paths are displayed in yellow with an example of a multi-path signal in blue. These signals construct a CIR at each anchor. A positioning solution needs to uphold missing CIRs due to NLOS (indicated in red).

τ_k are the amplitude and the time delay of the k -th multipath components, respectively; and n is the channel's additive white Gaussian noise. Not only can the RSSI of first- and multi-paths (arriving further in time through reflective surfaces) be used for fingerprinting, but also their relative time delays which are representative of a location. However, as illustrated in Fig. 1, in strong NLOS conditions not all anchors a_n will receive the UWB packet sent by the tag t if the signal attenuation was too large. Therefore, there is a need for a solution which works with M anchors, where $M < N$ in NLOS conditions. As such, in the next subsection we present the performed data collection, which includes NLOS to generate realistic conditions in order to develop and evaluate the proposed methodology in this paper.

B. Data collection

To evaluate the proposed method, we use a dataset recorded in an industrial production hall setting¹, as depicted in Fig. 3. The environment consists of an area of 300 m² and is partially enclosed by reflecting walls. Various metallic objects (metal shelves, absorber/reflector elements and an industrial vehicle) introduce reflection, absorption and scattering, so that a mixture of LOS and NLOS connections exist. After capturing one dataset in this environment, an additional dataset was produced in a modified environment. To this end, two large interfering objects (the industrial vehicle and metal storage containers) were moved to a different location the environment to create different signal propagation conditions, with changed position-related radio signatures. The recording hardware consists of a mobile tag, held by a pedestrian and 7 anchors placed around the recording area at a height of 1.5 m. This UWB hardware is based on the Decawave DW1000 and configured with a center frequency of 4 GHz and a bandwidth of 499.2 MHz. The recorded CIRs are obtained with a temporal resolution of about 1 ns. As a positioning reference system, the highly accurate QualiSys optical motion capture system with an accuracy in the 1 mm range tracks the mobile tag.

¹The dataset was part of the IPIN 2021 track 7 competition and is publicly available at <https://eval.aaloa.org/2021/call-for-competitions>

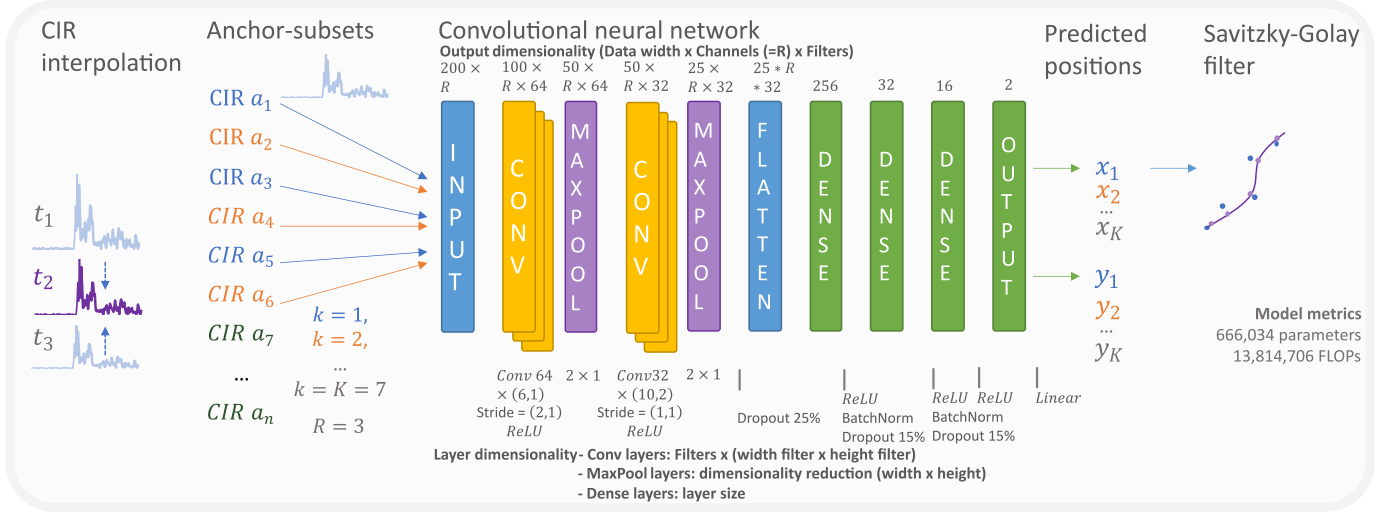


Fig. 2: Our active UWB fingerprinting includes CIR interpolation (sample reduction), anchor-subset selection (support missing CIRs), a CNN (predict tag position) and a Savitzky-Golay filter to smooth localization paths. The first CNN layer dimension is reduced by stride and maxpool, the last dimension by the convolutional filters, while the second is fixed with zero-padding.

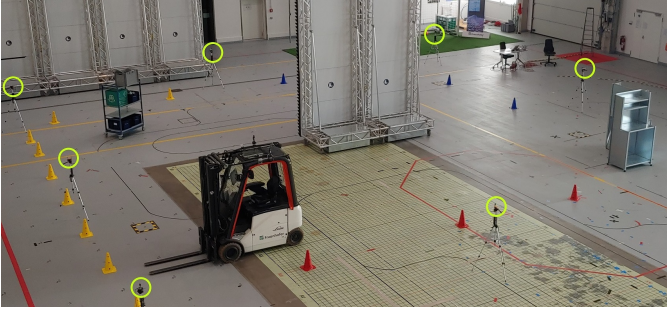


Fig. 3: Picture of the recording environment, where the anchor positions are highlighted with light green circles.

III. METHODOLOGY

In this section, we describe the proposed fingerprinting-based IPS using CIR data, as is illustrated in Fig. 2. Next to using all CIRs received at each anchor, we also propose an anchor-subset method to solve the problem when not all CIRs arrived in NLOS conditions. Additionally, we describe the CNN to make localization predictions and show how a Savitzky-Golay smoothing filter can be integrated. Finally, we propose CIR interpolation to limit the number of collected UWB packets further and reduce the data collection effort.

A. Proposed anchor-subset method

The indoor environment evaluated in this paper contains $N = 7$ UWB anchors. We can combine the CIRs from each of these anchors to estimate the position of the tag at a given time t . However, when $M < 7$ anchors receive a CIR at time t , no prediction can be made, which would be the case in large environments. Instead, we propose a method with multiple anchor-subsets and joined ML models, which contains the following steps: (i) generate K unique subsets of size R out of N anchors, (ii) for each subset k , train a model ML_k using R CIRs, (iii) once trained, predict K positions at time t with K models ML_k . As such, for each time t , there are K unique subsets of N anchors generating K positions, lowering the

probability of anchor combinations with missing CIRs at time t . During the evaluation in this paper, we found that increasing R resulted in a minor increased accuracy and chose $R = 3$ as a trade-off between accuracy and support for large environments. Additionally, we found that increasing K slightly improved the accuracy, but also the computational complexity. As such, we chose $K = 7$ as a trade-off between accuracy and complexity of the setup. The selected unique combinations using $K = 7$ and $R = 3$ from all 7 anchors yielded anchor combinations: $[1, 3, 5]$, $[2, 4, 6]$, $[3, 5, 7]$, $[4, 6, 1]$, $[5, 7, 2]$, $[6, 1, 3]$, $[7, 2, 4]$.

B. CNNs

We propose a CNN to extract features in the CIR and make predictions about the position of the tag. In typical UWB IPSs, multiple anchors N are used to determine the position of the tag. Similarly, we can combine the CIRs from R anchors and provide it as multi-dimensional input to the CNN. The architecture of the CNN used in this paper, is shown in Fig. 2. The input layer corresponds to R anchor dimensions, while the following convolutional layers ensure effective feature extraction, which have already been successfully used for NLOS detection, error correction and on simulated fingerprinting CIR data for UWB localization systems [14]–[16]. Finally, fully connected layers learn from these features and predict the x and y coordinate of the tag in the two output neurons. Starting from the model in [15], these layers were fine tuned in an experimental way to increase the performance in this use case.

C. Savitzky-Golay filter

To obtain a smooth localized trajectory, multiple filter types can be effectively used (moving averages, particle filters, Kalman filters, Savitzky-Golay filters, etc. [17]). In this paper, we selected the Savitzky-Golay filter due to its implementation simplicity (no prior information about the positioning accuracy distribution is necessary) and due to its good smoothing capabilities with low degree polynomials when considering

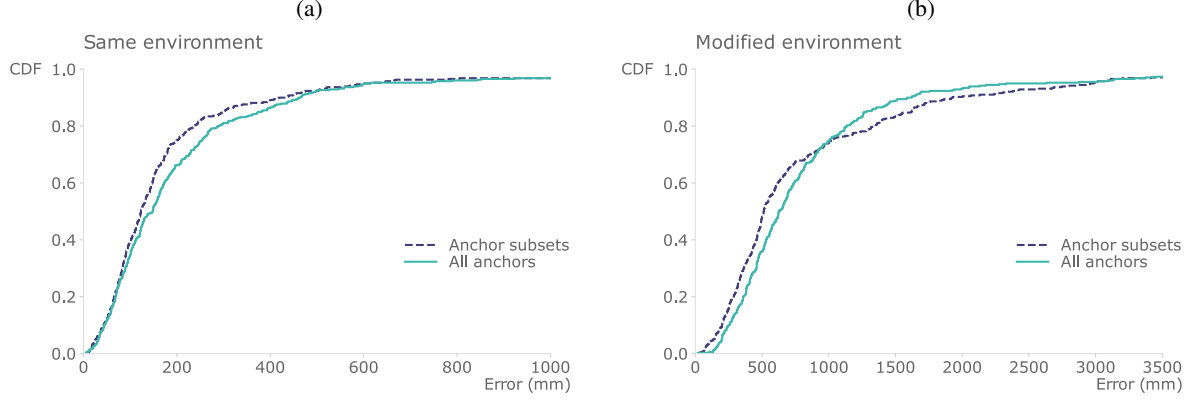


Fig. 4: Performance in the same environment and modified environment datasets with all anchors and with anchor-subsets.

a set of 2D position points along a driven trajectory [18]. For IPSs with multiple positions over a time period T , the filter becomes effective at generating a smooth path, which is expected in systems used for tracking mobile tags. More specifically, we used a filter window of length 23, which was experimentally derived, while the order of the polynomial (used to fit the samples on a polynomial curve) was set to 5.

D. Interpolation

A typical drawback of a fingerprinting-based IPS is the requirement of a dense dataset, captured at a high sampling rate, which results in a large number of positions for training [12]. We propose to facilitate data collection by interpolating CIRs between two known positions. In this paper, we investigate this by leaving out captured samples at a fixed interval, achieving a p -times data rate reduction with factors $p = [1, 2, 3]$ ($p = 1$ is no reduction). Next, we try to reconstruct the missing CIR by applying linear interpolation between two consecutive remaining CIRs. To evaluate CIR interpolation, we train models on the reduced data with factor p with and without interpolated CIRs. The results then are captured on two test sets, which contain all collected CIRs.

IV. ANALYSIS AND EXPERIMENTAL RESULTS

In this section, we first analyse the proposed methodologies where all anchors or selected anchor-subsets are used. Next, the selected anchor-subsets without and with the Savitzky-Golay filter are evaluated and finally, a sample reduction of 2x and 3x is applied without and with CIR interpolation. The ML models are trained on a dataset with 27,946 CIRs and are evaluated on (i) a test dataset in the same environment and (ii) a test dataset in a modified environment, both with 4,993 CIRs. The CNNs were trained for 500 epochs and loaded weights with the lowest MAE loss (on a 25% training data split). Furthermore, the training used a batch size of 256 and the Adam optimizer with a learning rate of 0.001.

A. All anchors vs anchor-subsets models

The performance in the same environment and modified environment (compared to the training environment) with all anchors and with anchor-subsets is illustrated in Fig.

TABLE II: The results show improvements using the Savitzky-Golay filter across the evaluation metrics (mean absolute error (MAE), second quartile (Q2), third quartile (Q3), 95th percentile (P95)) and demonstrate the benefits of CIR interpolation under sample reduction (SR) conditions. In a modified environment the accuracy decreases, but is still comparable to traditional UWB TWR.

	MAE (cm)	Q2 (cm)	Q3 (cm)	P ₉₅ (cm)
Same environment				
No filter	23.5	14.2	24.1	67.9
Filter	20.9	12.3	20.0	60.0
SR = 2x w/o interpolation	21.6	14.0	21.1	62.0
SR = 2x with interpolation	21.2	12.4	20.2	59.7
SR = 3x w/o interpolation	25.6	14.4	25.1	84.7
SR = 3x with interpolation	23.8	14.2	24.6	71.7
UWB TWR	60.1	28.6	76.0	211.1
Modified environment				
No filter	92.2	58.5	107.6	304.3
Filter	87.0	51.2	102.7	295.9
SR = 2x w/o interpolation	98.1	63.1	121.2	316.4
SR = 2x with interpolation	95.3	59.0	116.3	312.1
SR = 3x w/o interpolation	105.6	68.7	144.2	329.0
SR = 3x with interpolation	99.6	63.9	129.7	322.0
UWB TWR	66.8	25.3	88.0	237.8

4. In the same environment, an accuracy improvement can be observed for the anchor-subsets method as compared to using all anchors. In the modified environment, the result becomes more nuanced. While the accuracy still is higher for the anchor-subsets method in the lower percentiles, the situation changes around the 75th percentile. Here, using all anchor CIR information results in a higher accuracy, while the accuracy of both approaches start to again match in the higher percentiles. Still, the anchor-subset method has the ability to predict positions, even with missing CIRs (in NLOS). Hence, we choose this method for the following evaluation.

B. Evaluation of the Savitzky-Golay filter

In Table II, the results of both no filter and with filter are given in the first two rows for the same and modified environment. For comparison reasons, we also include the accuracy when using the more traditional TWR approach in the table. The usage of the Savitzky-Golay filters improves the accuracy in both environments. On average, a decreased error of 2.5-5 cm is measured, while the P₉₅ decreases up to 8.4

cm. As such, the best accuracy with the proposed fingerprinting methodology (CNN + selected anchors + Savitzky-Golay filter) in the same environment is a MAE error of 20.9 cm, which outperforms the accuracy of traditional TWR in these NLOS conditions, while requiring only 1 UWB packet. As expected, the MAE is decreased in a modified environment (in this case to 87.0 cm), due to differences in multi-path and NLOS signals, unknown to the trained CNN. Still, this error is considerably lower than the numbers reported (2-3 m) with traditional fingerprinting methods using BLE and Wi-Fi.

C. Evaluation of sample reduction and CIR interpolation

Finally, we evaluate the performance of the proposed method when the number of measured samples is significantly smaller (denoted by sample reduction (SR)) due to a lower data rate (5 and 3.3 Hz) compared to the 10 Hz present in the dataset. Table II shows the results of both $SR = 2x$ and $SR = 3x$ combined without and with CIR interpolation. The MAE increases by 0.7 cm and 11.1 cm in the same and modified environment, respectively. These numbers further increase when applying a $SR = 3$, with a MAE of 4.7 cm and 18.6 cm, in the two environments, respectively. Applying CIR interpolation to the training dataset can decrease these numbers again to 0.3 cm and 8.3 cm for $SR = 2$, while with an $SR = 3$ the numbers decrease to 2.9 cm and 12.6 cm difference, in both environments, respectively. This is an up to 6 cm and 10.3 cm increase in accuracy for $SR = 2$ and $SR = 3$, respectively, which illustrates the potential of data interpolation when limiting the number of UWB packets and reducing the required data collection effort.

V. CONCLUSIONS AND FUTURE WORK

UWB indoor positioning systems can reach a high accuracy (< 30 cm), however, they require either transmitting a large number of packets or realizing challenging (multi-hop) clock synchronization. In this paper we proposed an UWB fingerprinting positioning method using multiple CNNs trained on CIRs of unique anchor-subset combinations. After applying a Savitzky-Golay smoothing filter, we reached a MAE of 20.9-87.0 cm, which surpasses the accuracy of traditional RSSI-based fingerprinting systems in real conditions, and even outperforms UWB TWR in the considered challenging NLOS environments. Finally, we have also demonstrated sample reduction with linear CIR interpolation, which increased the accuracy up to 10.3 cm. To further improve the accuracy, tweaking the parameters R (number of input CIRs) and K (number of models) and using a higher sampling collection rate have a major impact. A minor impact is found by the choice of the smoothing filters, anchor selection (although crucial in large UWB deployments) and additional ML finetuning. With the proposed solution, tags can stay longer in sleep mode, consume less energy, deployments can be more scalable without challenging clock synchronization. Based on the outcome of this paper, future work can investigate in which scenarios selecting all or subsets of anchors can benefit the accuracy of UWB fingerprinting systems. To further reduce the number of labels, future work can explore unsupervised learning and

data augmentation, while also exploiting federated and transfer learning to quickly adapt to new environments.

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