



香港城市大學  
City University of Hong Kong

# GIFT: Conditional TWAS for fine-mapping candidate causal genes



**Speaker: Yuekai Li**

**Major: Biostatistics**

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01

# Background

## ■ Outline

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01

The relationship between traits, genes, SNPs

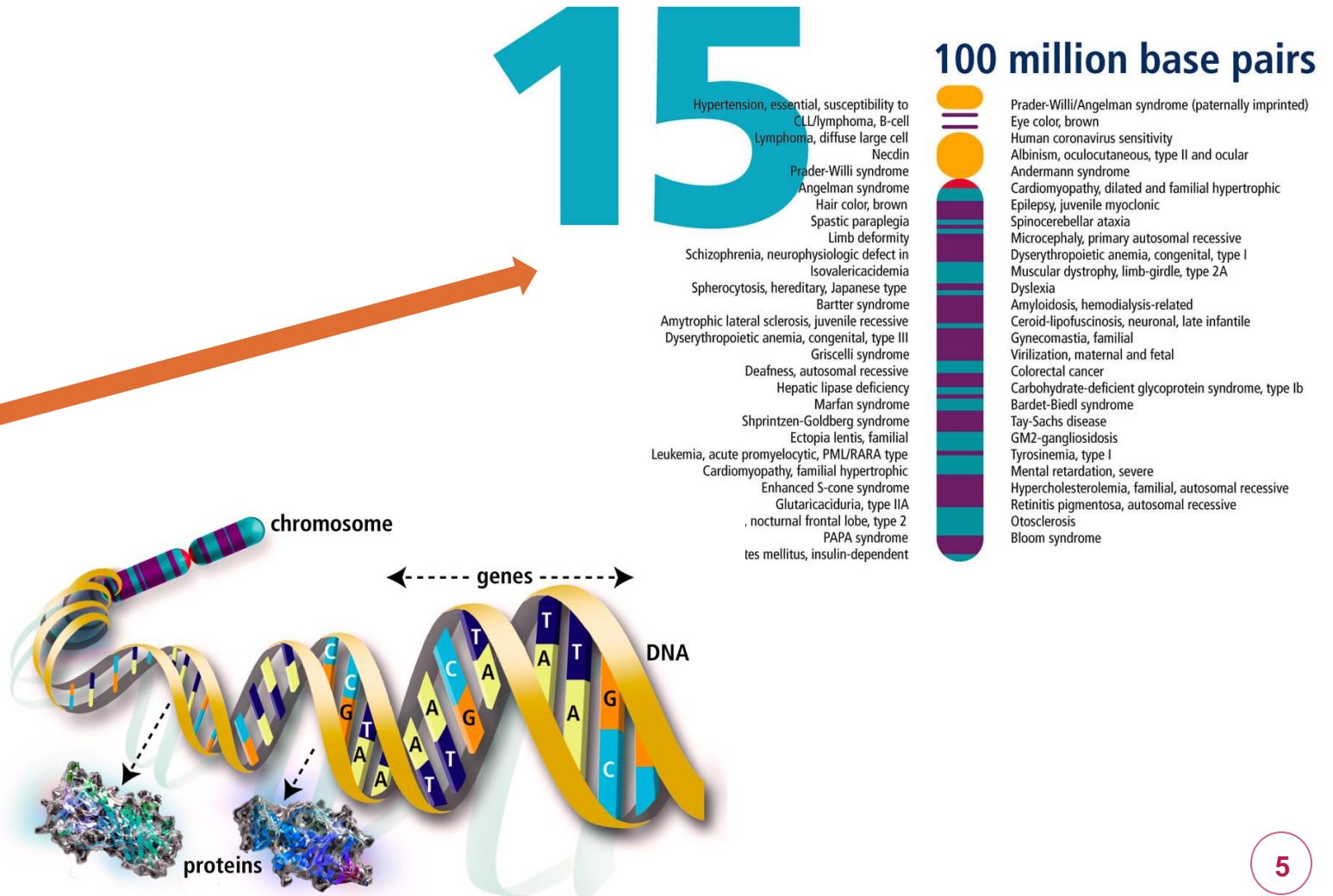
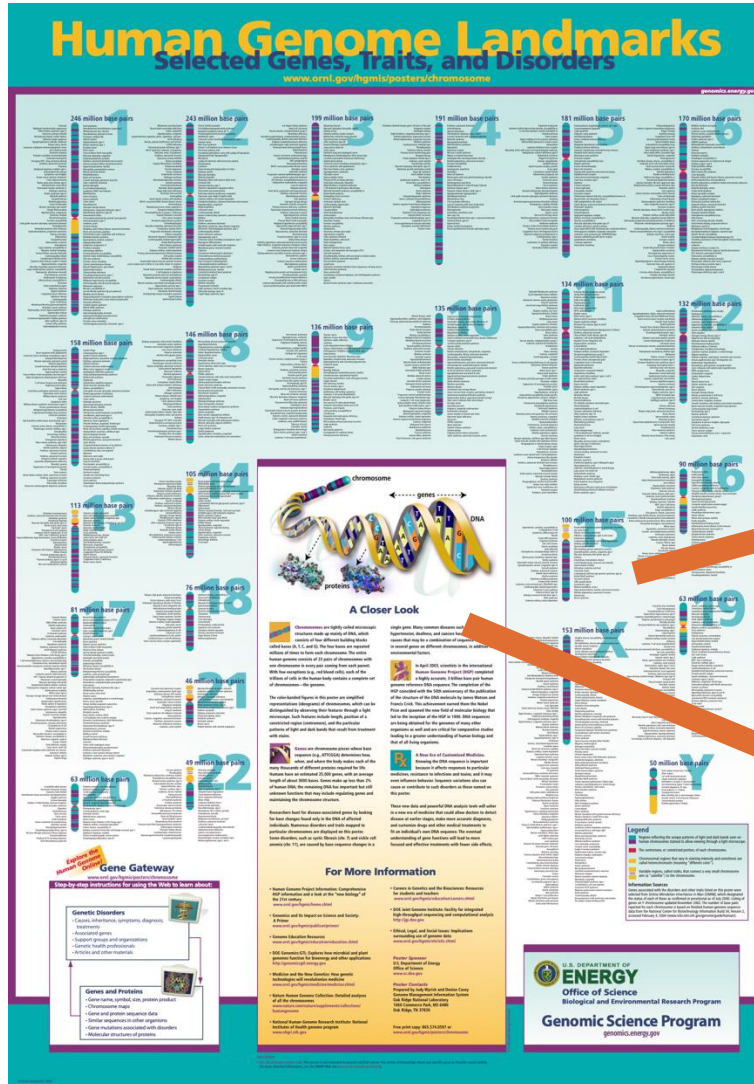
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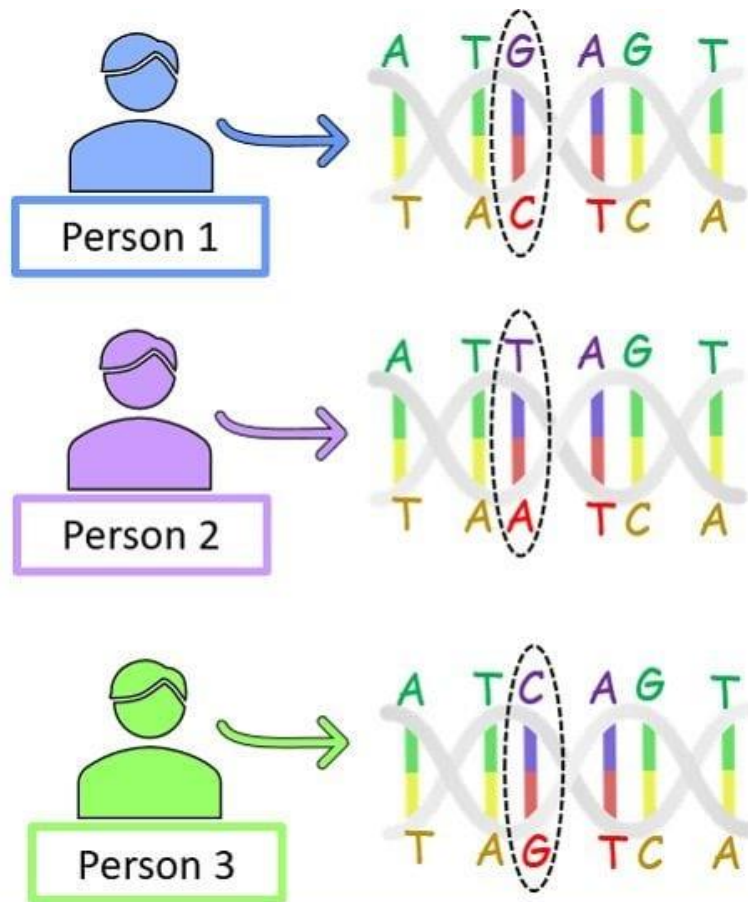
The motivation of TWAS

# ■ The relationship between traits, genes, SNPs





## ■ The relationship between traits, genes, SNPs



Single Nucleotide Polymorphism

- SNP is **the replacement of a single base pair** in the DNA sequence.
- SNP is **the most common type** of genetic variation.
- More than **600 million** SNPs have been identified across the human genome in the world's population.

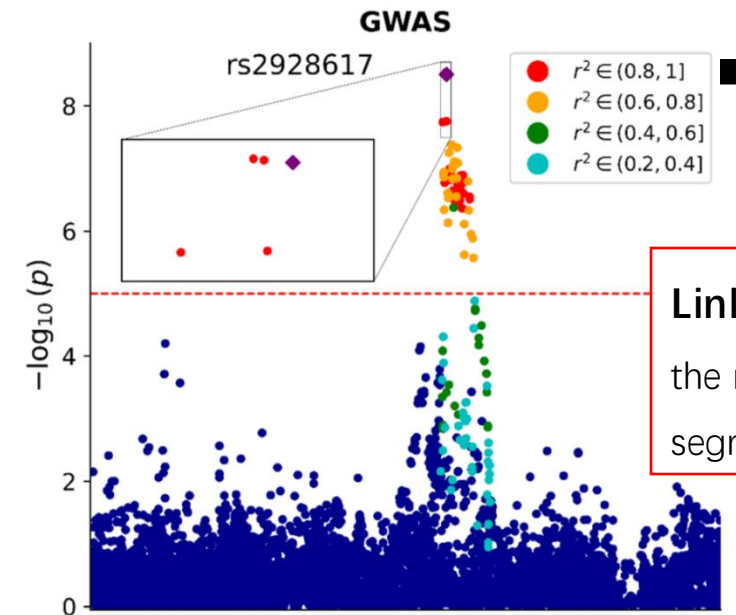
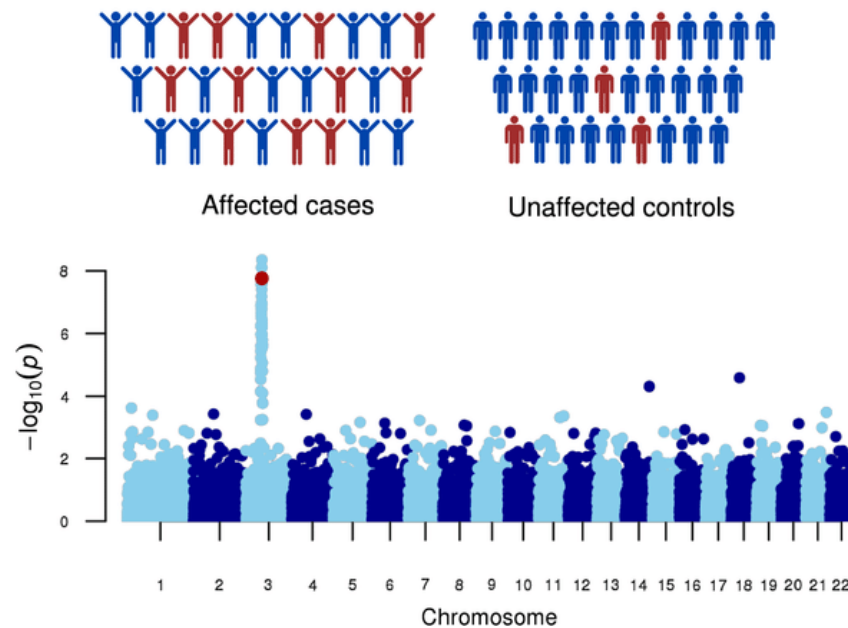
## ■ What is GWAS

Sequence Variation

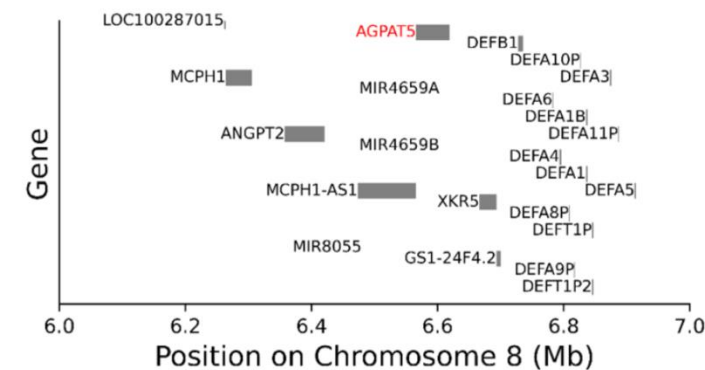
ATGCCAGTGTTC AAGATGCTTGGCCAGCTGGACGAGGGCGATGAC  
ATGCCAGTGTTC AAGATG **T**TTGGCCAGCTGGACGAGGGCGATGAC

Disease

GWAS

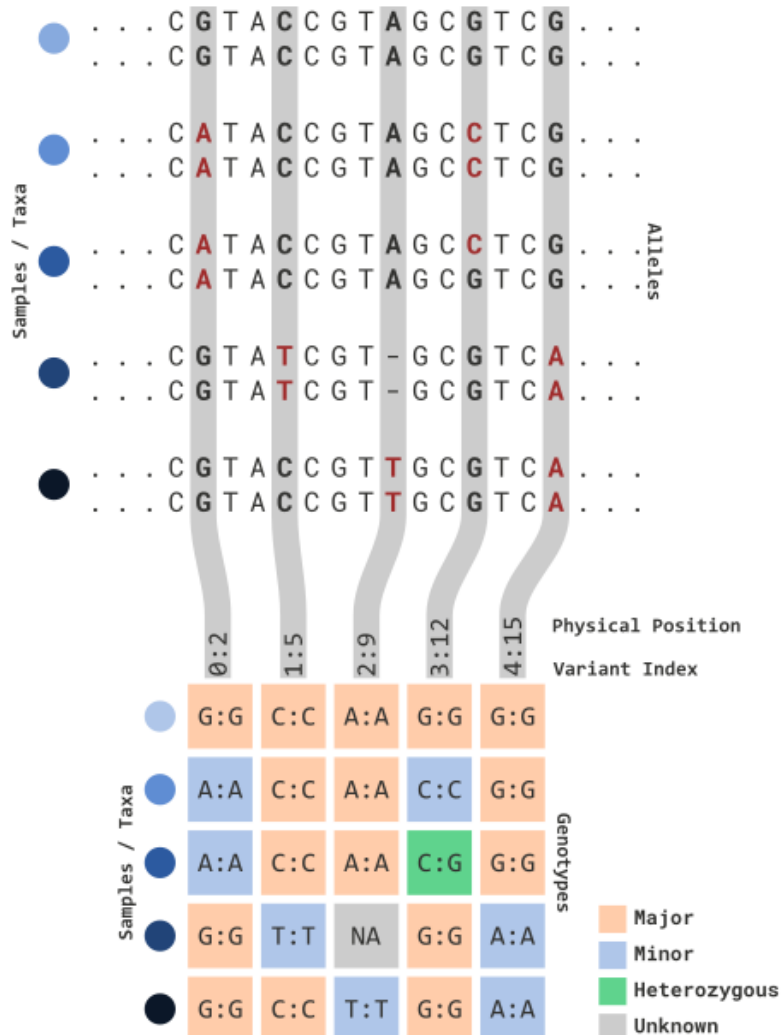


**Linkage disequilibrium (LD) :**  
the non-independent  
segregation of genetic variants.



## ■ What is GWAS

### Genotype data:



	SNP1	SNP2	SNP3	SNP4
Individual 1	AT	CG	TT	CC
Individual 2	TA	GG	GT	CA
Individual 3	TT	CC	GT	CA
Individual 4	TT	CC	GG	AA

Major=2      Heterozygous=1      Minor=0

For example, if we assume A is the major allele,  
then A:A=2, A:C/C:A=1, CC=0



## ■ What is GWAS

Linear regression models for GWAS can be written as follows:

$$Y \sim W\alpha + X_s \beta_s + g + e \quad (1)$$

Fixed effect

$$g \sim N(0, \sigma_A^2 \psi) \quad (2)$$

Its p-value measures the strength of the association between SNPs and trait.

$$e \sim N(0, \sigma_e^2 I) \quad (3)$$

$Y$  : the phenotype value

$W$  : the vector of covariates including an intercept term

$\alpha$  : the corresponding vector of effect

$X_s$  : the genotype value for the genetic variant  $s$

$\beta_s$  : the corresponding fixed effect

$g$  : the random effect that captures the polygenic effect of other SNPs

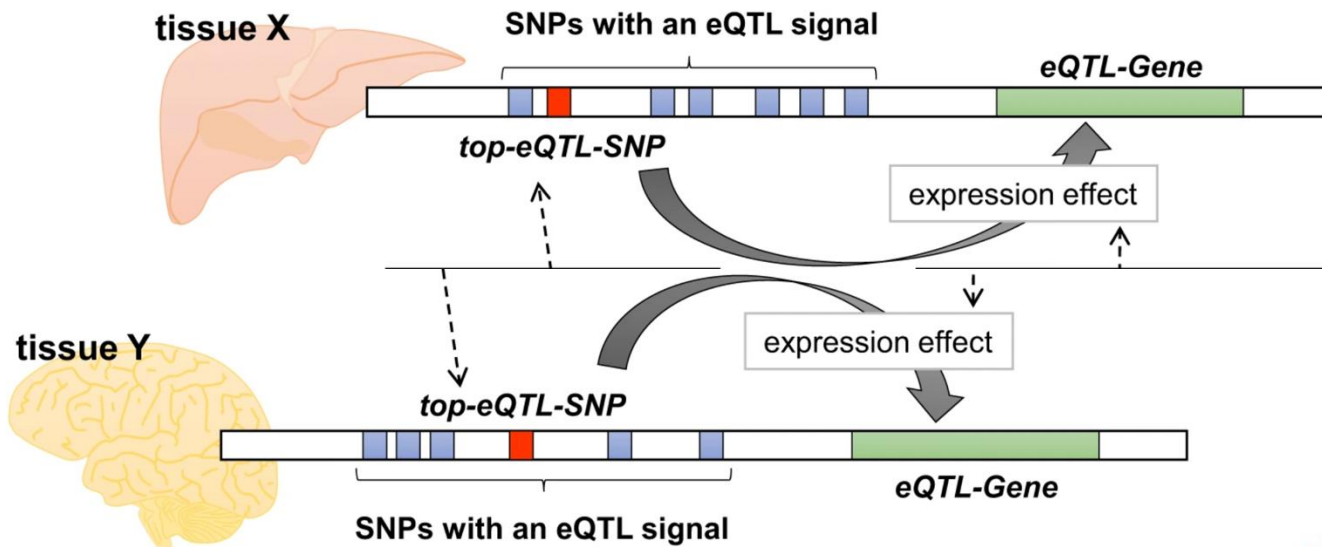
$e$  : the random effect of residual errors

$\sigma_A^2$  : the additive genetic variation of the phenotype

$\psi$  : the standard genetic relationship matrix

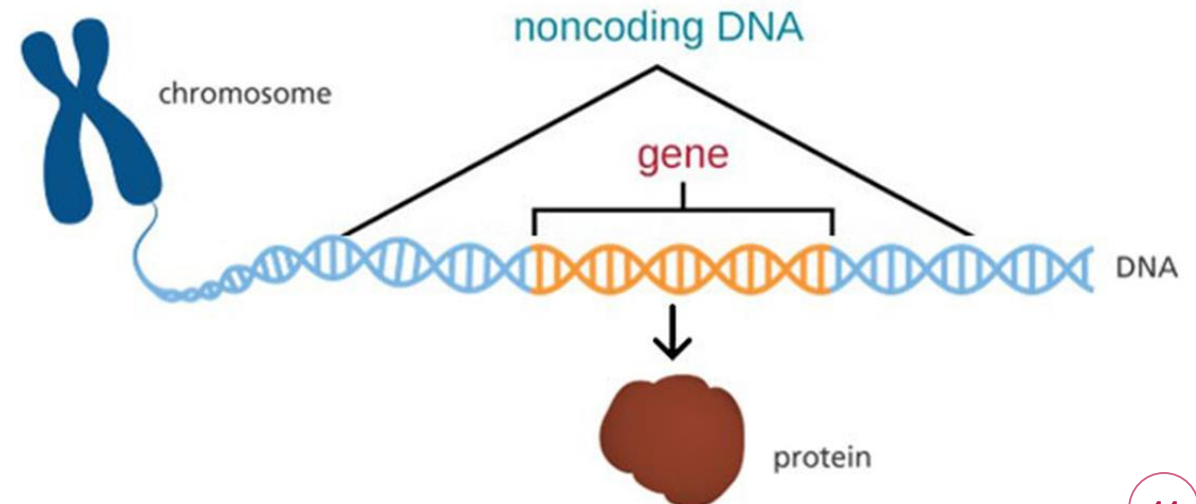
$\sigma_e^2$  : residual variance

## ■ The motivation of TWAS



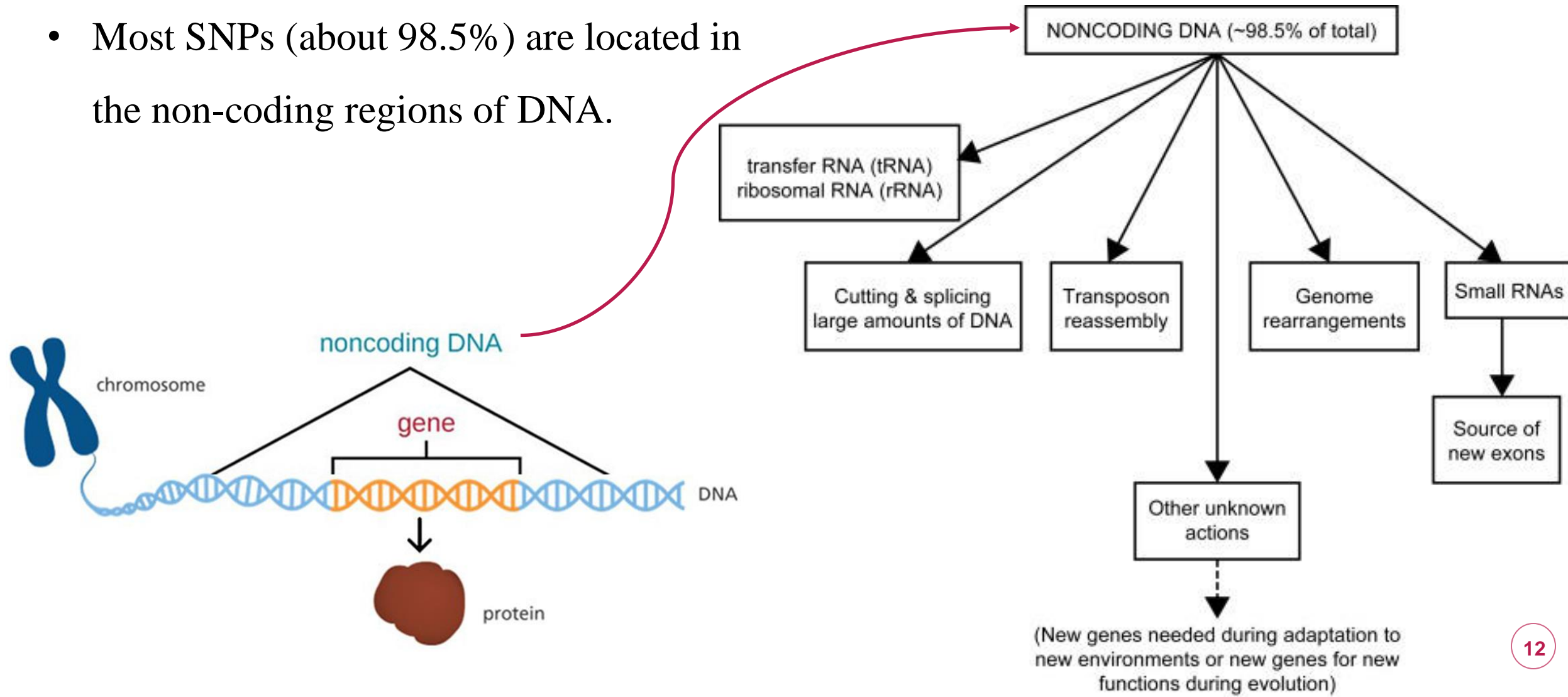
- SNPs in different tissues have different regulatory effects on gene expression.

- Most SNPs (about 98.5%) are located in the non-coding regions of DNA.



## ■ The motivation of TWAS

- Most SNPs (about 98.5%) are located in the non-coding regions of DNA.

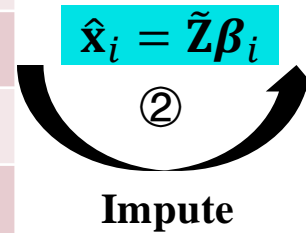


## ■ The motivation of TWAS

GWAS data  
( $\tilde{Z}$ )

Genotype data

ID	$SNP_1$	$SNP_2$	...	$SNP_p$
$id_1$	2	0	...	0
$id_2$	0	1	...	2
$id_3$	2	1	...	0
...	...	...	...	...
$id_{n_2}$	1	0	...	0

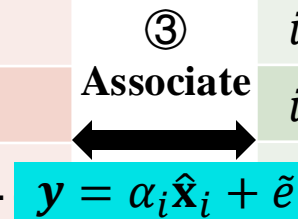


Expression data

ID	$\hat{x}_1$	$\hat{x}_2$	...	$\hat{x}_m$
$id_1$			...	
$id_2$		?	...	
$id_3$			...	
...	...	...	...	...
$id_{n_2}$			...	

Phenotype data

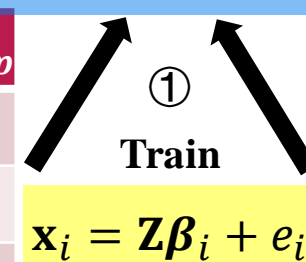
ID	Trait
$id_1$	1.23
$id_2$	4.56
$id_3$	7.89
...	...
$id_{n_2}$	2.33



$$\beta = [\beta_1, \beta_2, \dots, \beta_m]$$

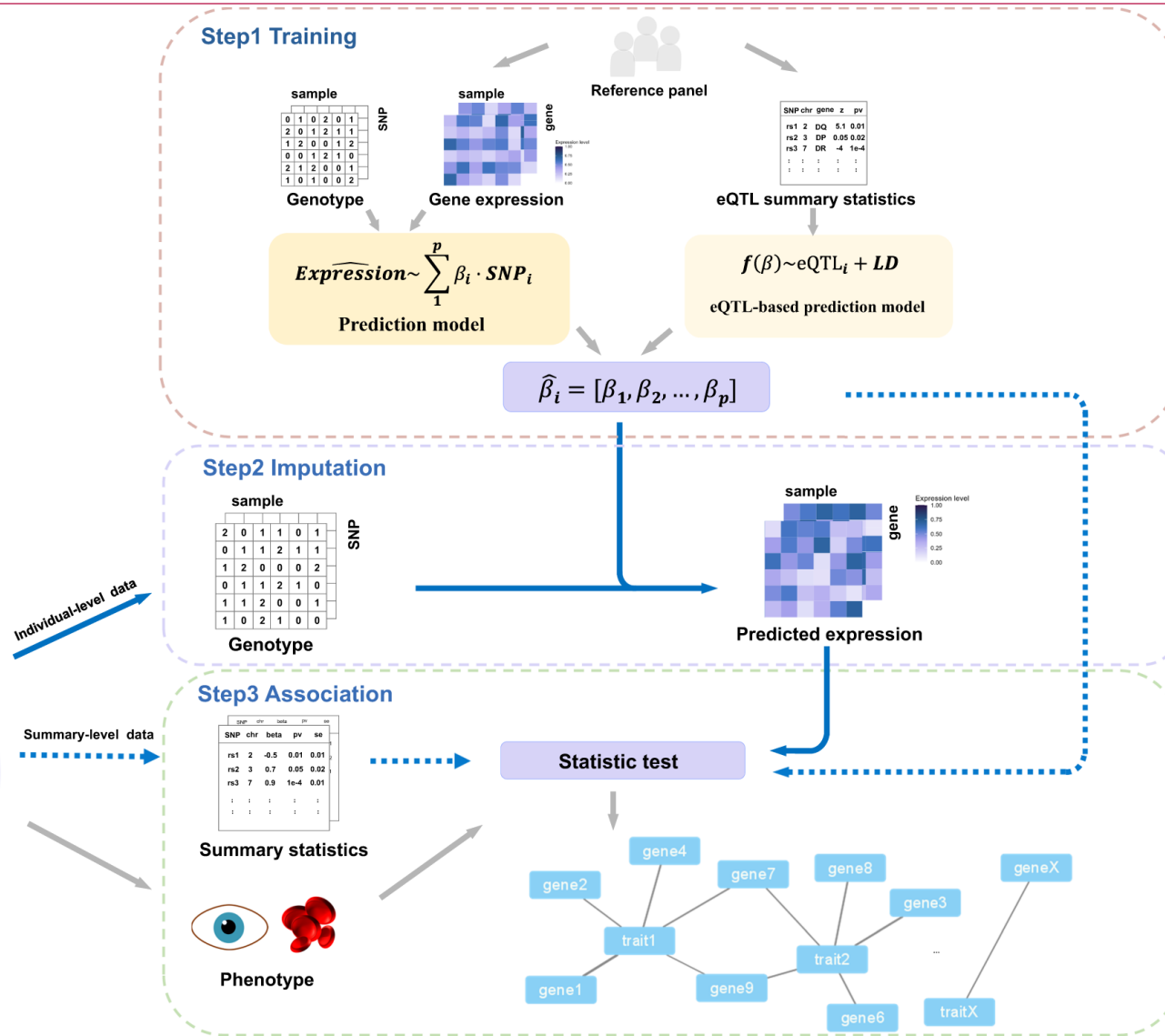
Reference panel  
( $Z$ )

ID	$SNP_1$	$SNP_2$	...	$SNP_p$
$id_1$	0	0	...	0
$id_2$	1	2	...	1
...	...	...	...	...
$id_{n_1}$	1	0	...	1



ID	$x_1$	$x_2$	...	$x_m$
$id_1$	0.1	0.5	...	1.3
$id_2$	1.2	2.2	...	0.1
...	...	...	...	...
$id_{n_1}$	0.2	0.1	...	1.0

## ■ The motivation of TWAS



**1. Training stage:** Estimate regulatory effect sizes of multiple SNPs on the gene expression level from a small reference panel with genotype and expression data.

**2. Imputation stage:** Obtain the predicted gene expression of GWAS individuals.

**3. Association stage:** Implement hypothesis tests between predicted gene expression and the target trait





## 02

# Challenges

## ■ Outline

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**LD and expression correlation lead to confounding**

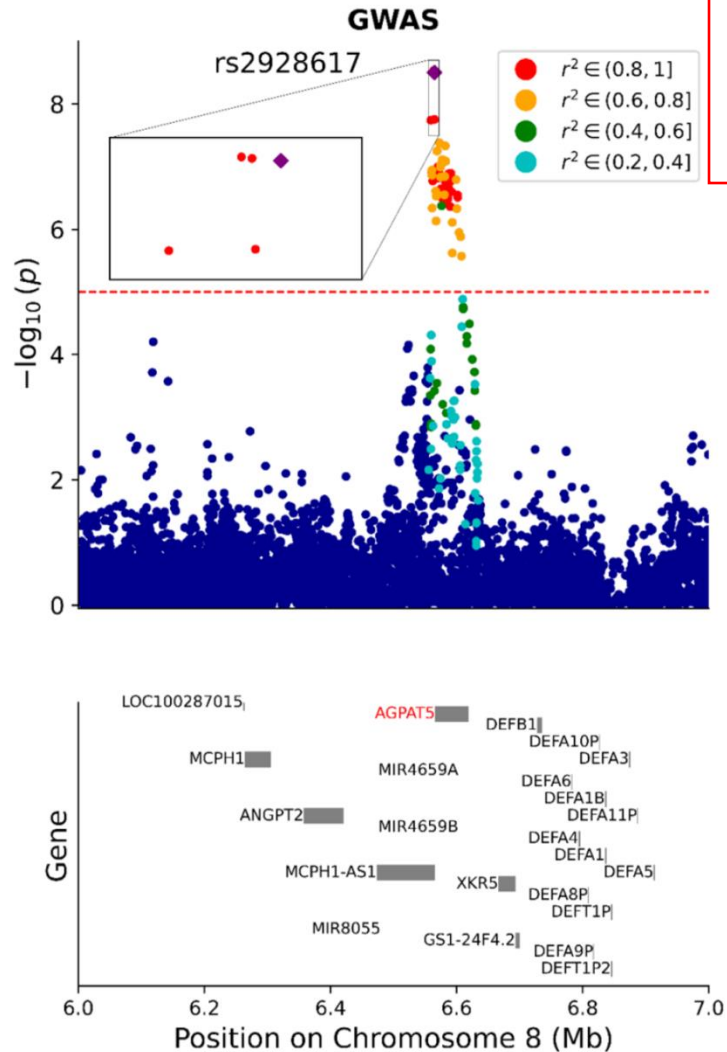


**Two-Step Inference Procedure lead to power loss**

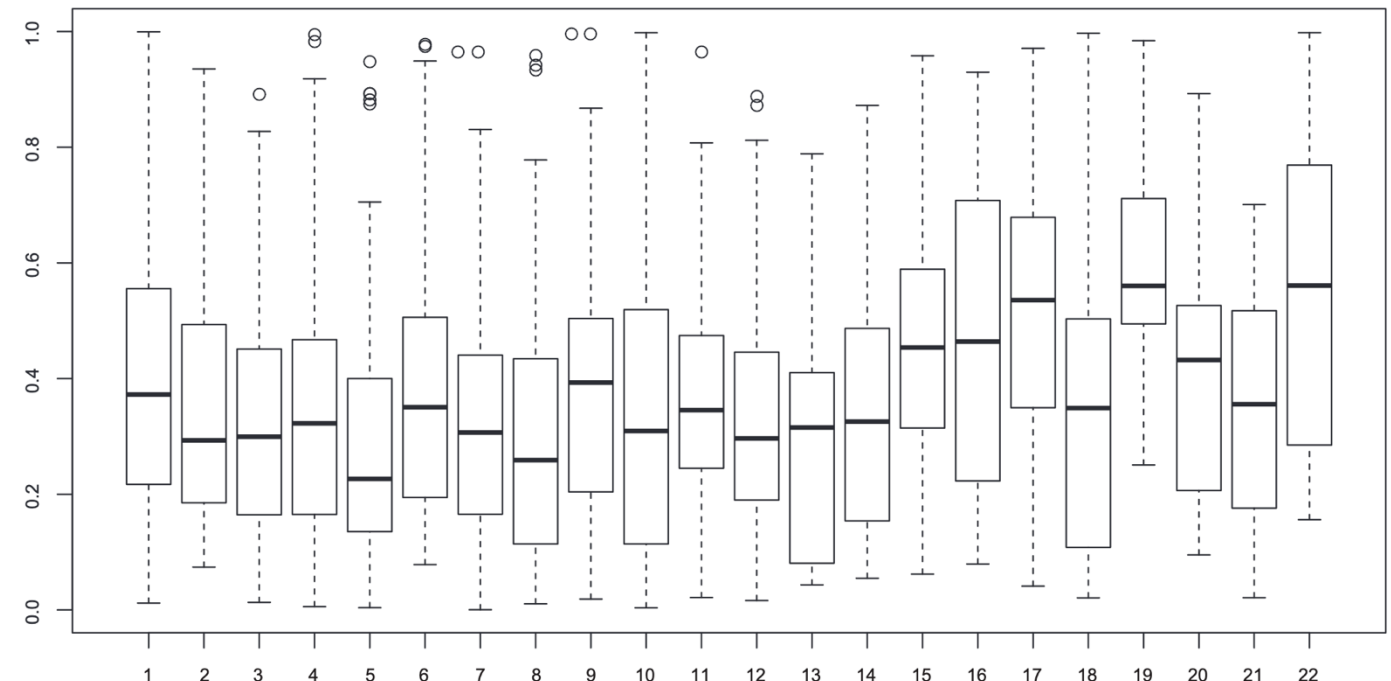
## LD and expression correlation lead to confounding

Linkage disequilibrium (LD) :

the non-independent  
segregation of genetic variants.



Boxplot displays the maximum of the absolute value of **expression correlation** estimates in each region across the 22 chromosomes

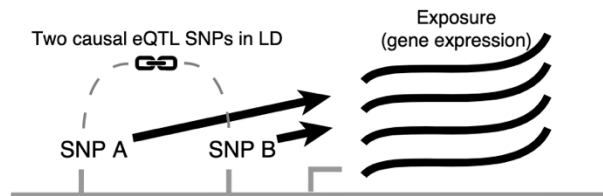


## ■ LD and expression correlation lead to confounding

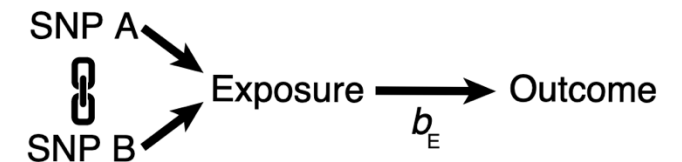
There are no unobserved exposures.  
Such situations rarely occur.



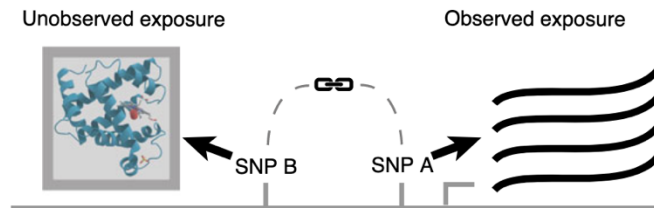
### a Causality with SNPs in LD



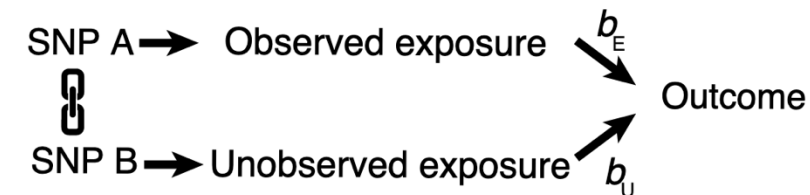
Two correlated SNPs affect the exposure and outcome, no pleiotropy



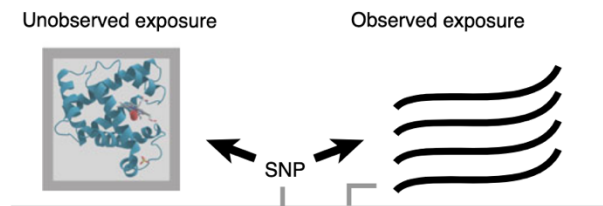
### b Causality with **pleiotropy** through LD



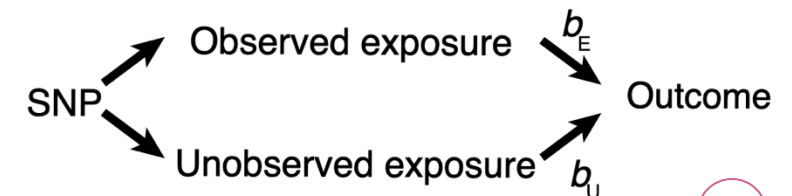
Outcome is affected through two pathways from two correlated SNPs



### c Causality with **pleiotropy** through overlap



Outcome is affected through two pathways from the same SNP



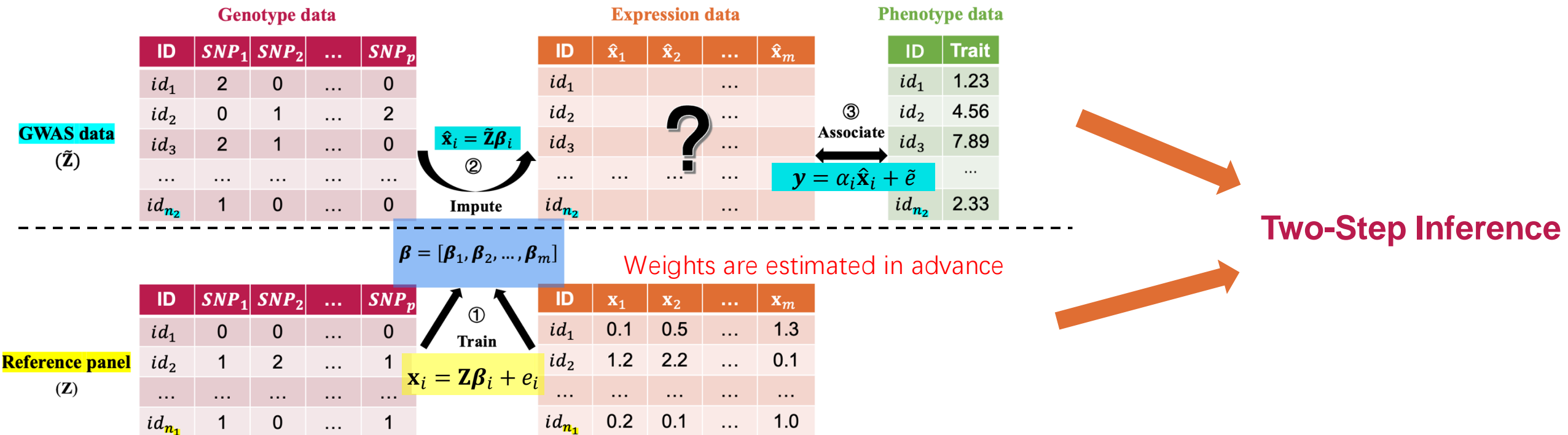
Confounding



### Pleiotropy:

A genetic variant affects the outcome through a pathway that does not involve the risk factor of interest.

## ■ Two-Step Inference Procedure lead to power loss



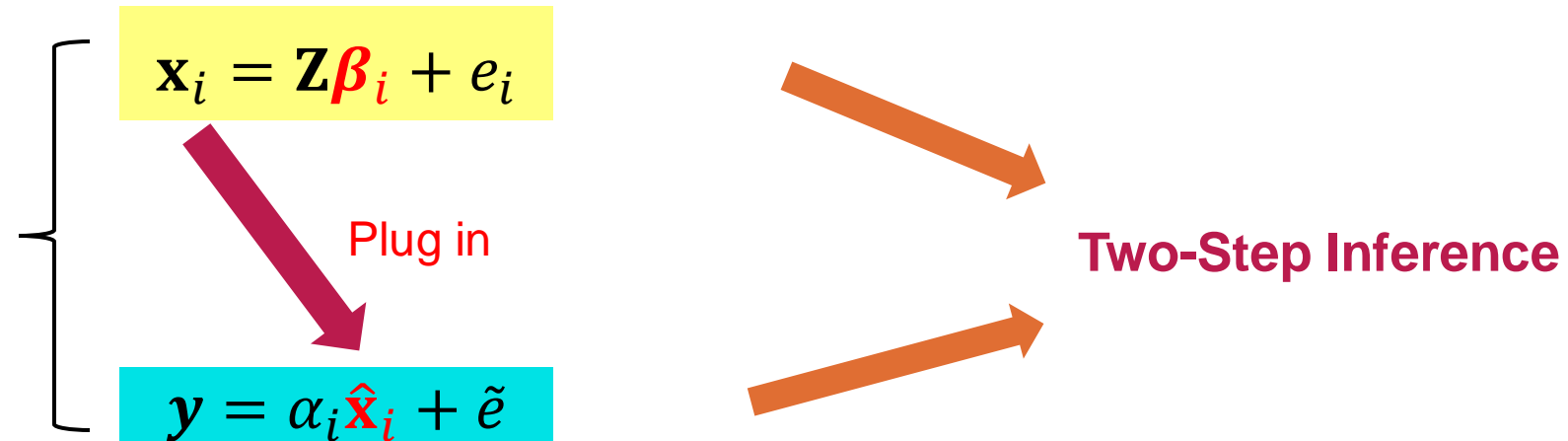
Most of the existing marginal TWAS methods consist of two separate analytical steps.

The characteristic of these methods is that they **estimate the weights  $\beta$  from the reference panel in advance**. TWAS (or TWAS fine-mapping) is performed given weights  $\beta$ .



## ■ Two-Step Inference Procedure lead to power loss

The point estimation of  $\beta_i$  has more uncertainty.



Most of the existing marginal TWAS methods consist of two separate analytical steps.

The characteristic of these methods is that they **estimate the weights  $\beta$  from the reference panel in advance**. TWAS (or TWAS fine-mapping) is performed given weights  $\beta$ .



# 03

## Methods

## ■ Outline

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01

**TWAS method timeline**

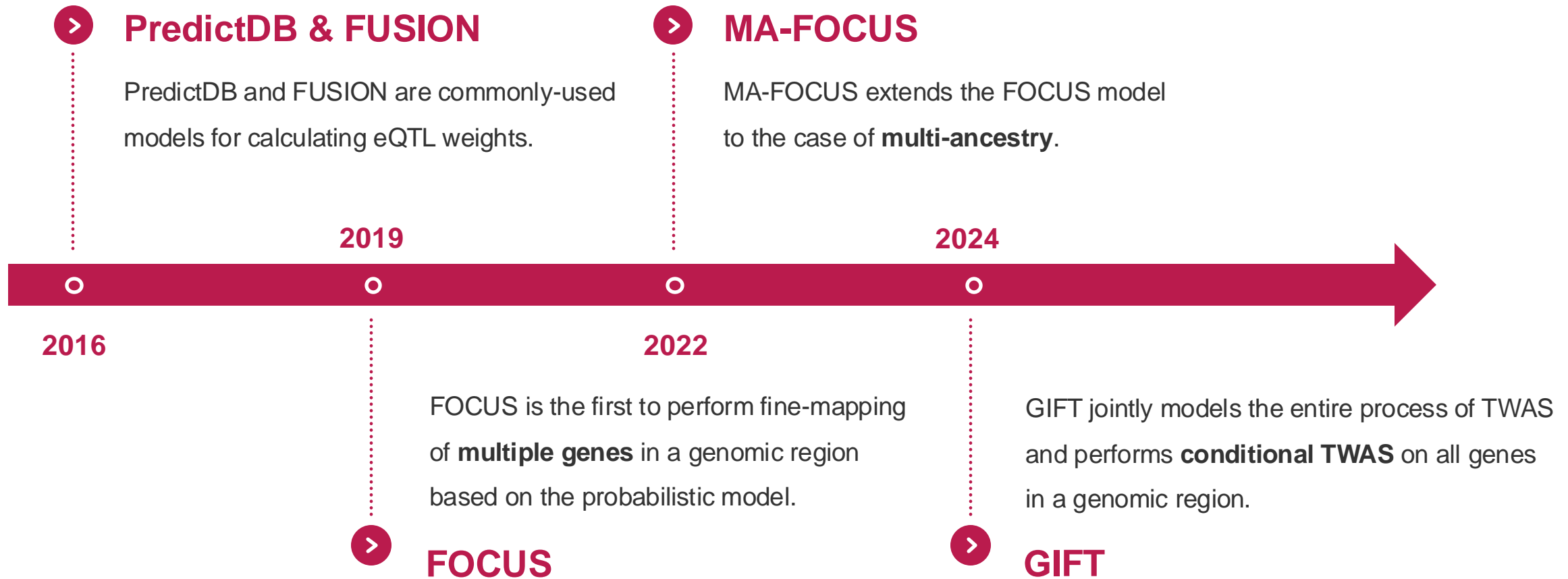
02

**The model of GIFT**

03

**The advantages of GIFT**

## ■ TWAS method timeline



## ■ The model of GIFT (individual-level)

GIFT (Gene-based Integrative Fine-mapping through conditional TWAS),

**jointly models** all  $k$  genes residing in the focal region and carries out TWAS conditional analysis:

$$\begin{cases} \mathbf{x}_i = \mathbf{Z}_i \boldsymbol{\beta}_i + \mathbf{e}_i, i = 1, \dots, k & (1) \\ \mathbf{y} = \sum_{i=1}^k \alpha_i (\tilde{\mathbf{Z}}_i \boldsymbol{\beta}_i) + \tilde{\mathbf{e}} & (2) \end{cases}$$

$\mathbf{x}_i \in \mathbb{R}^{n_1 \times 1}$ : expression vector for the  $i$ -th gene

$\mathbf{y} \in \mathbb{R}^{n_2 \times 1}$ : phenotype vector

$\mathbf{Z}_i \in \mathbb{R}^{n_1 \times p_i}$ : genotype matrix in the **reference panel** for the  $i$ -th gene

$\tilde{\mathbf{Z}}_i \in \mathbb{R}^{n_2 \times p_i}$ : genotype matrix in the **GWAS data** for the  $i$ -th gene

$\boldsymbol{\beta}_i \in \mathbb{R}^{p_i \times 1}$ : eQTL random effects on the  $i$ -th gene expression

$\alpha_i \in \mathbb{R}$ : effects of predicted expression for the  $i$ -th gene

Assumed that  $\mathbf{y}$ ,  $\mathbf{x}_i$  and each column of  $\mathbf{Z}_i$  and  $\tilde{\mathbf{Z}}_i$  have all been standardized to have a **mean of zero** and standard **deviation of 1**.



## ■ The model of GIFT (individual-level)

GIFT (Gene-based Integrative Fine-mapping through conditional TWAS),

**jointly models** all  $k$  genes residing in the focal region and carries out TWAS conditional analysis:

$$\begin{cases} \mathbf{x}_i = \mathbf{Z}_i \boldsymbol{\beta}_i + \mathbf{e}_i, i = 1, \dots, k & (1) \\ \mathbf{y} = \sum_{i=1}^k \alpha_i (\tilde{\mathbf{Z}}_i \boldsymbol{\beta}_i) + \tilde{\mathbf{e}} & (2) \end{cases}$$



Due to  $p_i > n_1$

$$\boldsymbol{\beta}_i \sim N(\mathbf{0}, \sigma_{\beta_i}^2 \cdot \mathbf{I}_{p_i})$$

$\mathbf{x}_i \in \mathbb{R}^{n_1 \times 1}$ : expression vector for the  $i$ -th gene

$\mathbf{y} \in \mathbb{R}^{n_2 \times 1}$ : phenotype vector

$\mathbf{Z}_i \in \mathbb{R}^{n_1 \times p_i}$ : genotype matrix in the **reference panel** for the  $i$ -th gene

$\tilde{\mathbf{Z}}_i \in \mathbb{R}^{n_2 \times p_i}$ : genotype matrix in the **GWAS data** for the  $i$ -th gene

$\boldsymbol{\beta}_i \in \mathbb{R}^{p_i \times 1}$ : eQTL random effects on the  $i$ -th gene expression

$\alpha_i \in \mathbb{R}$ : effects of predicted expression for the  $i$ -th gene

Using the posterior distribution of  $\boldsymbol{\beta}_i$  for eQTL effect instead of the point estimate  $\hat{\boldsymbol{\beta}}_i$

## ■ The model of GIFT (individual-level)

GIFT (Gene-based Integrative Fine-mapping through conditional TWAS),

**jointly models** all  $k$  genes residing in the focal region and carries out TWAS conditional analysis:

$$\begin{cases} \mathbf{x}_i = Z_i \boldsymbol{\beta}_i + \mathbf{e}_i, i = 1, \dots, k & (1) \\ \mathbf{y} = \sum_{i=1}^k \alpha_i (\tilde{Z}_i \boldsymbol{\beta}_i) + \tilde{\mathbf{e}} & (2) \end{cases}$$

$\mathbf{e}_i \in \mathbb{R}^{n_1 \times 1}$ : residual errors for the  $i$ -th gene,

where  $(\mathbf{e}_{l,1}, \mathbf{e}_{l,2}, \dots, \mathbf{e}_{l,k})^T \sim N_k(0, \Omega)$  for the same individual  $l$

GIFT takes the correlation of gene expressions into account.

$\tilde{\mathbf{e}} \in \mathbb{R}^{n_2 \times 1}$ : residual error with each element *i.i.d.* from the same normal distribution  $N(0, \sigma_y^2)$

## ■ The model of GIFT (summary-level)

GIFT can also be extended to perform inference using summary statistics only.

The corresponding model for summary statistics are:

$$\begin{cases} \hat{\beta}_{x_i}^* = \Sigma_{1i} \beta_i + e_{x_i}, i = 1, \dots, k \\ \hat{\beta}_y^* = \Sigma_2 (\alpha_1 \beta_1^T, \dots, \alpha_k \beta_k^T)^T + e_y \end{cases}$$

$\hat{\beta}_{x_i}^* \in \mathbb{R}^{1 \times p_i}$ : the estimates for the marginal SNP effects on the  $i$ -th gene expression

$\hat{\beta}_y^* \in \mathbb{R}^{1 \times p}$ : the estimates for the marginal SNP effects on the trait

$\Sigma_{1i} \in \mathbb{R}^{p_i \times p_i}$ : correlation matrix of all cis-SNPs for the  $i$ -th gene in the reference panel

$\Sigma_2 \in \mathbb{R}^{p \times p}$ : correlation matrix of all cis-SNPs for all the genes in the focal region in the GWAS data.

$\beta_i \in \mathbb{R}^{p_i \times 1}$ : eQTL effects on the  $i$ -th gene expression

$\alpha_i \in \mathbb{R}$ : effects of predicted expression for the  $i$ -th gene

## ■ The advantages of GIFT

01

### Conditional TWAS analysis

GIFT performs TWAS fine-mapping conditional on the effects of the other genes to avoid confounding.

02

### Joint likelihood inference framework

The joint inference framework accounts for the uncertainty in the SNP effect-size estimates on gene expression and the uncertainty in the predicted expression.

03

### PX-EM algorithm

GIFT introduces the auxiliary parameter  $\lambda$  through the parameter expansion method to significantly improve the convergence speed.



# 04

# Results



## ■ Outline

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**Data input**



**GIFT produces calibrated P values under the null simulations**

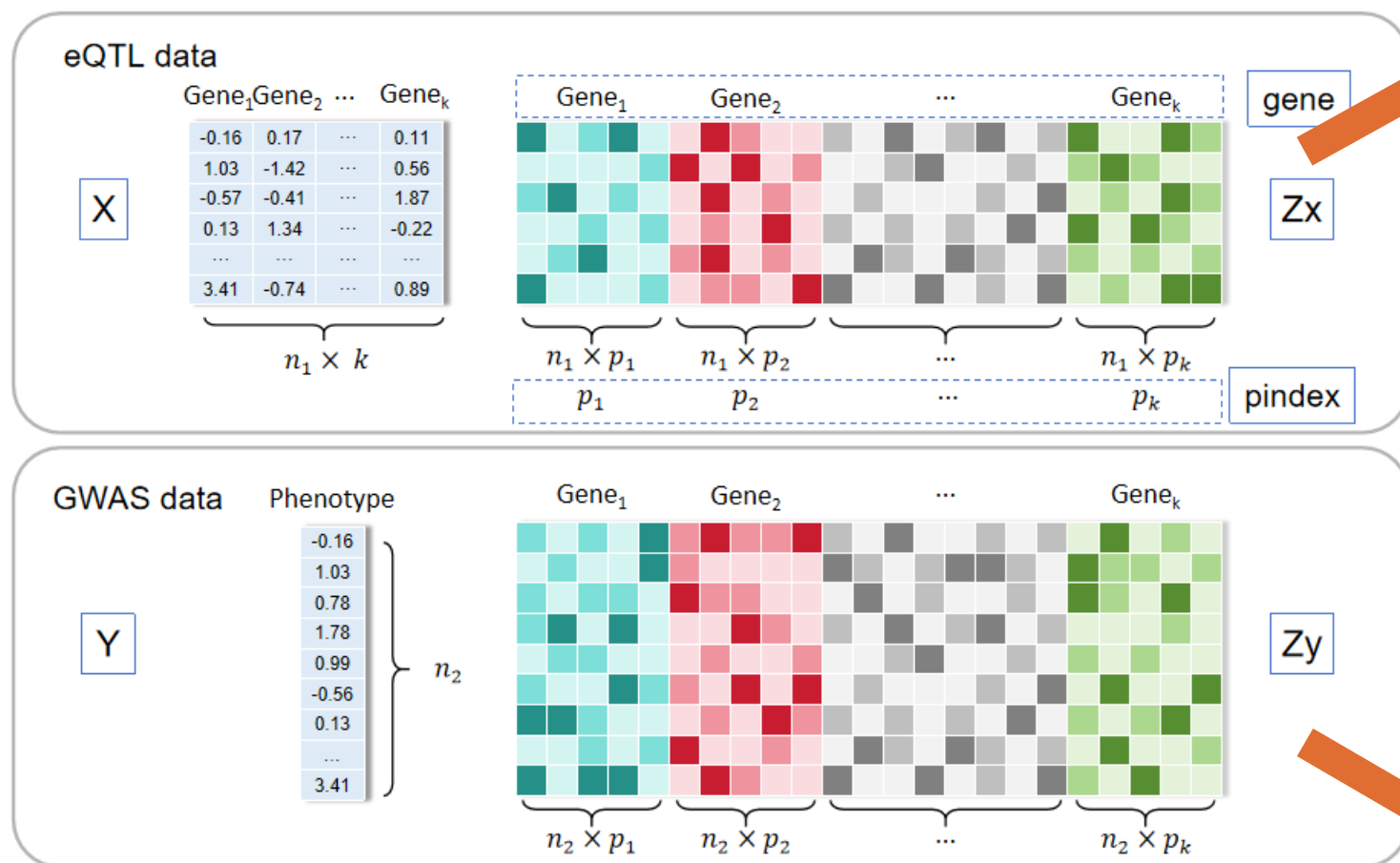


**GIFT is powerful under a range of alternative simulations**



**Real-data applications**

## ■ Data input

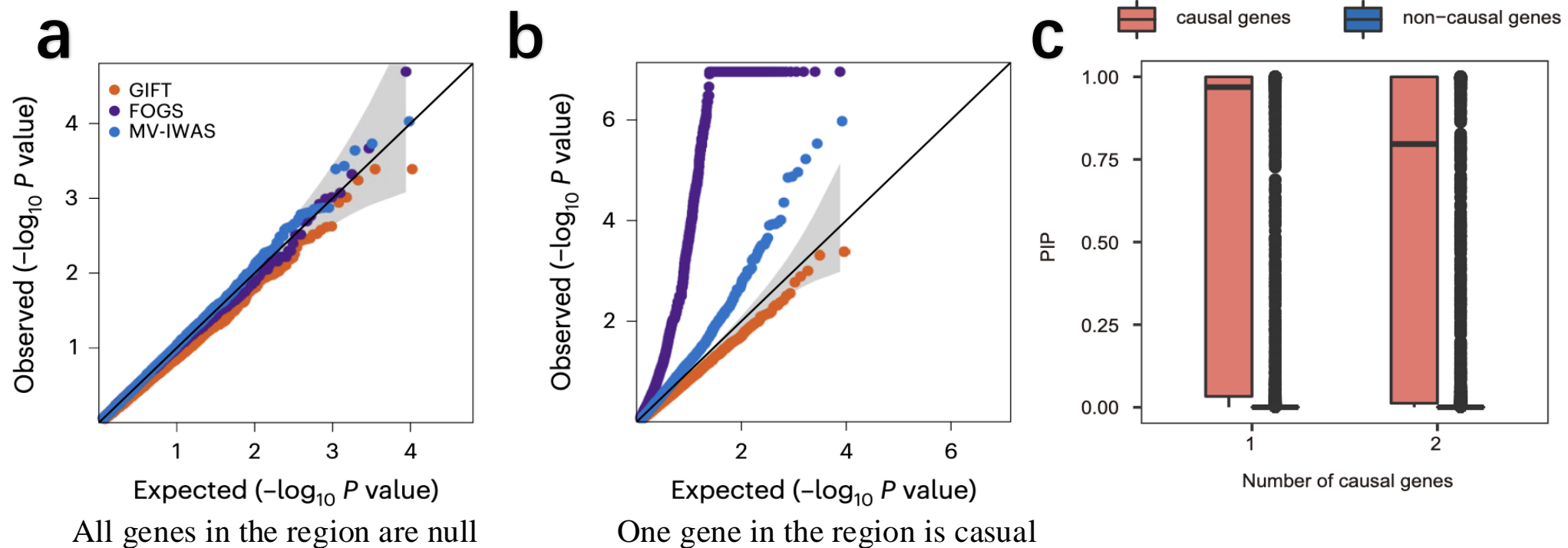


Data from GEUVADIS:  $n_1=465$

	Median	Mean	Min	Max
Genes per region ( $k$ )	8	11.75	1	100
Cis-SNPs per gene ( $p_i$ )	199	229	1	6879
Cis-SNPs per region ( $\Sigma p_i$ )	2,242	2751	2	16,655

Data from UKBiobank:  $n_2=487,298$

## ■ GIFT produces calibrated P values under the null simulations



**a, b:** Quantile-quantile plots of  $-\log_{10}(P$  values) from the [three frequentist methods](#), which are both displayed for the **non-causal genes**.

**c:** Boxplot from **FOCUS** displays the PIPs from causal genes and non-causal genes.

## ■ GIFT is powerful under a range of alternative simulations

When there is one causal gene in the region and it explains 1% of phenotypic variance, the FDR and power under the recommended thresholds from different methods as follows:

The 3 frequentist methods (GIFT, FOGS and MV-IWAS) are based on **Bonferroni's adjusted P-value threshold (0.05/m)** and the Bayesian method (FOCUS) is based on 90% credible sets.

### Bonferroni correction:

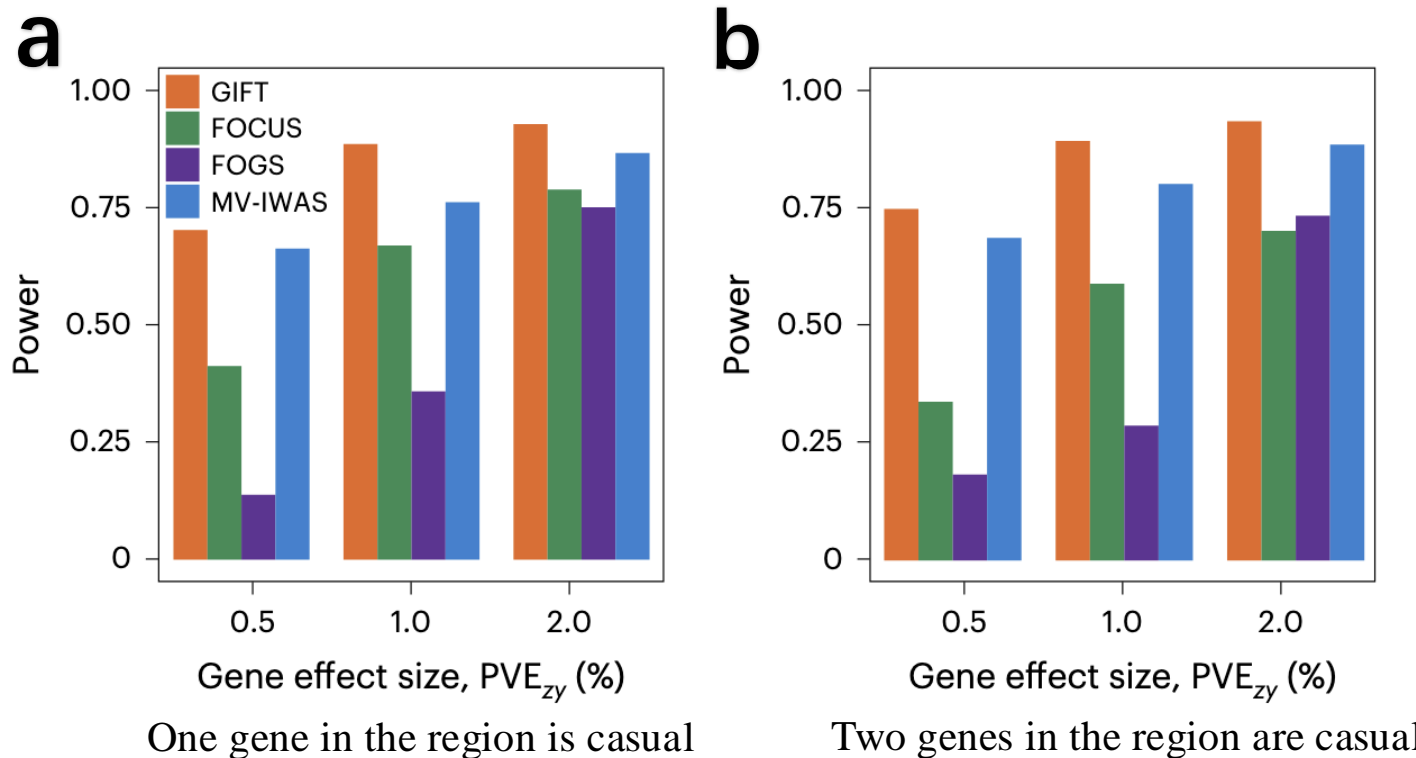
The most common way to control the familywise error rate. You will find the critical value (alpha) for an individual test by **dividing the familywise error rate (usually 0.05) by the number of tests.**

Methods	FDR	power
GIFT	↓ 0%	46.8%
FOCUS	42.1%	↑ 70.6%
FOGS	39.6%	49.6%
MV-IWAS	0.5%	56.2%

## ■ GIFT is powerful under a range of alternative simulations

As the threshold for GIFT corresponds to a much lower FDR than the other three methods, such a threshold naturally leads to a lower power for GIFT.

To allow for fair, we further computed power **based on a true FDR of 0.05**:



**a, b:** Power comparisons for different methods

based on a **true FDR** of 0.05.

## ■ GIFT is powerful under a range of alternative simulations

As the true FDR is known only in simulations but unknown for any real dataset, we also used P-value to compared power **based on the estimated FDR of 0.05**:

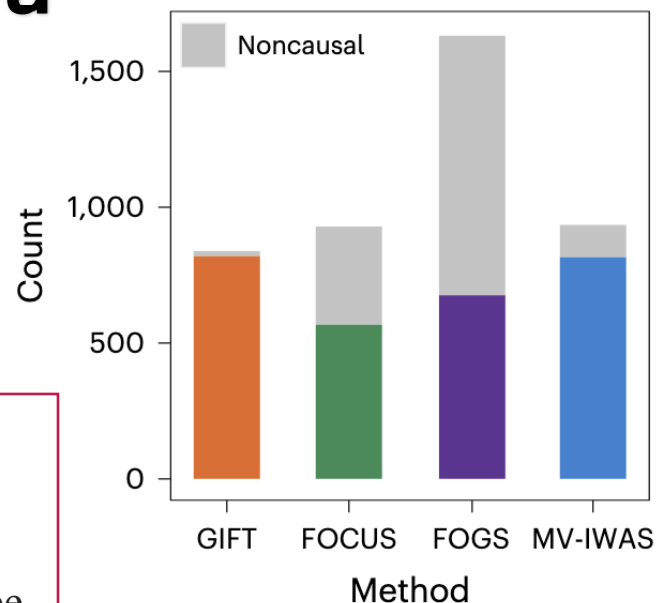
### Benjamini–Hochberg method:

Order the  $m$  hypothesis by ascending p-values, where  $P_i$  is the p-value at the  $i$ -th position with the associated hypothesis  $H_i$ . Let  $k$  be the largest  $i$  for which:

$$P_i = \frac{i}{m} q$$

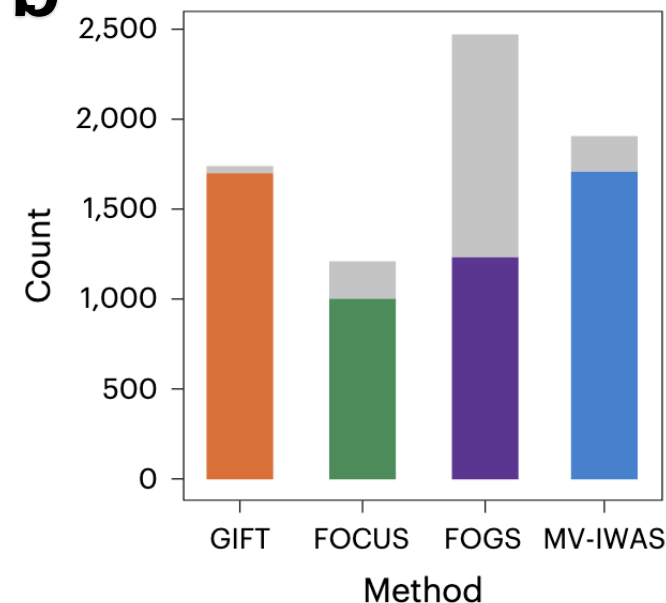
Reject hypotheses  $i = 1, 2, 3, \dots, k$ . The Benjamini–Hochberg method has been proven to control the FDR for all tests at a level of  $q$

**a**



One gene in the region is casual

**b**



Two genes in the region are casual

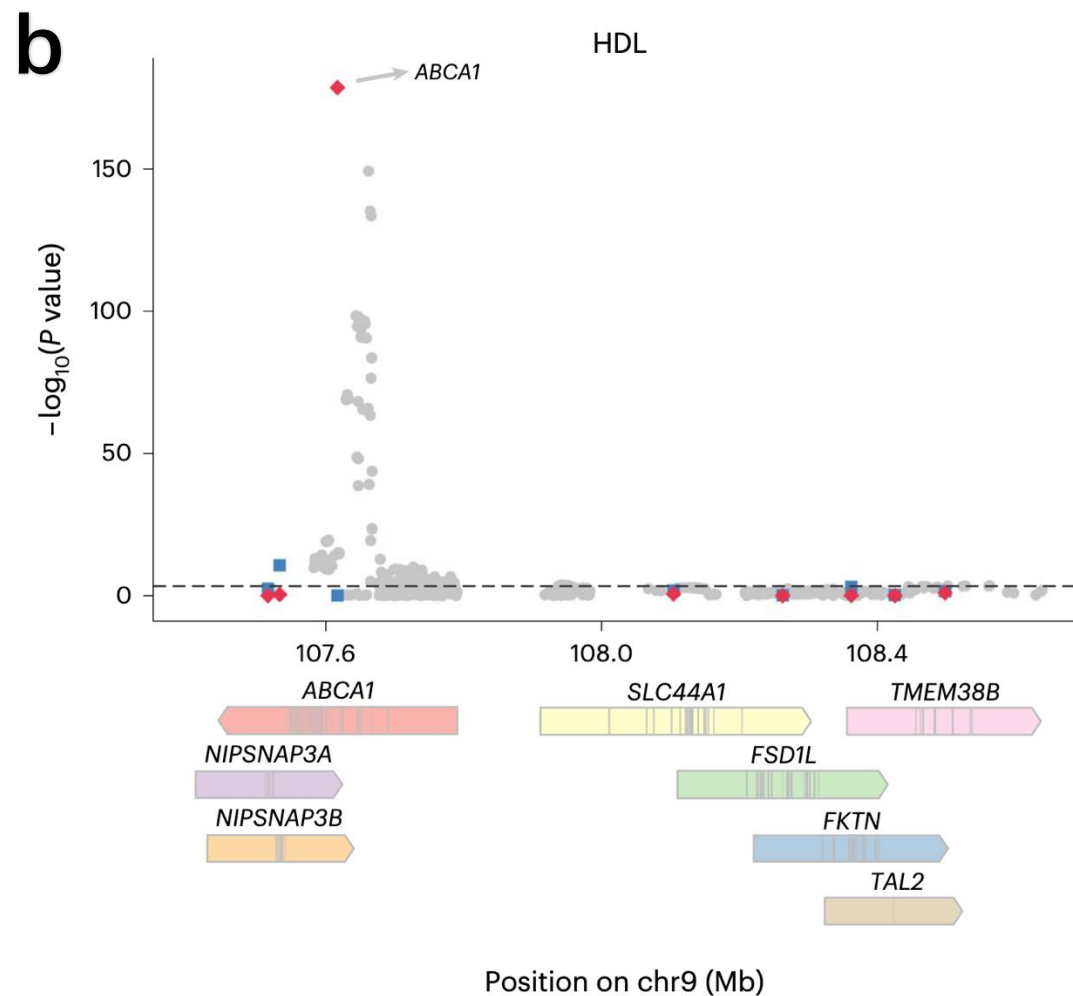
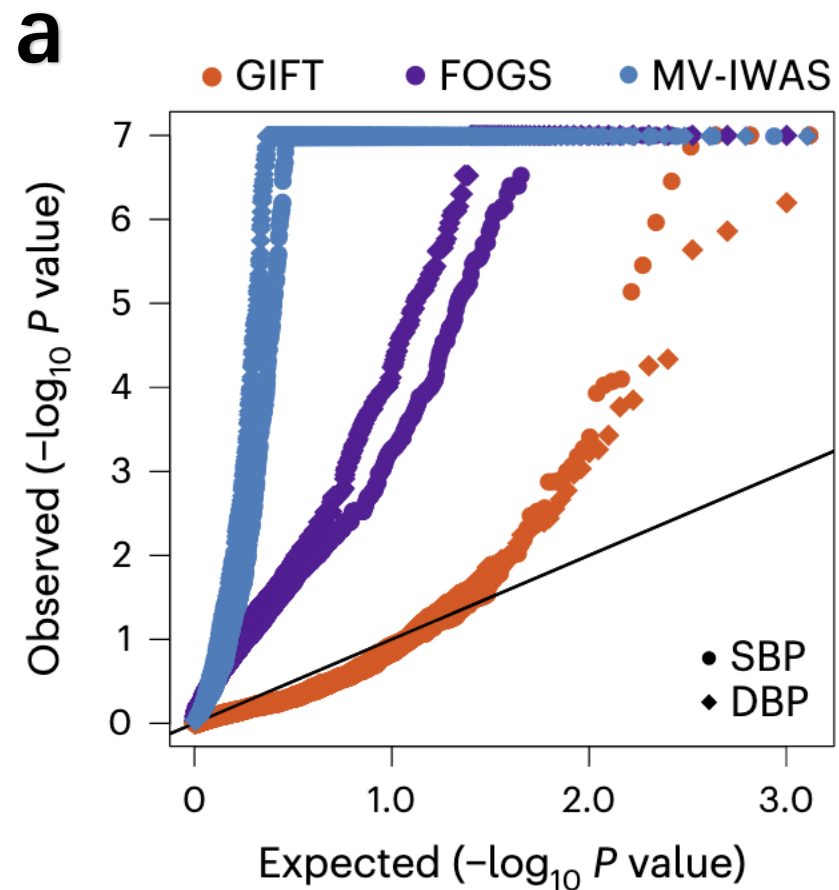
**a, b:** Number of genes identified by different methods

based on a **estimated FDR** of  $q=0.05$ .

Colors represent the number of detected causal genes.

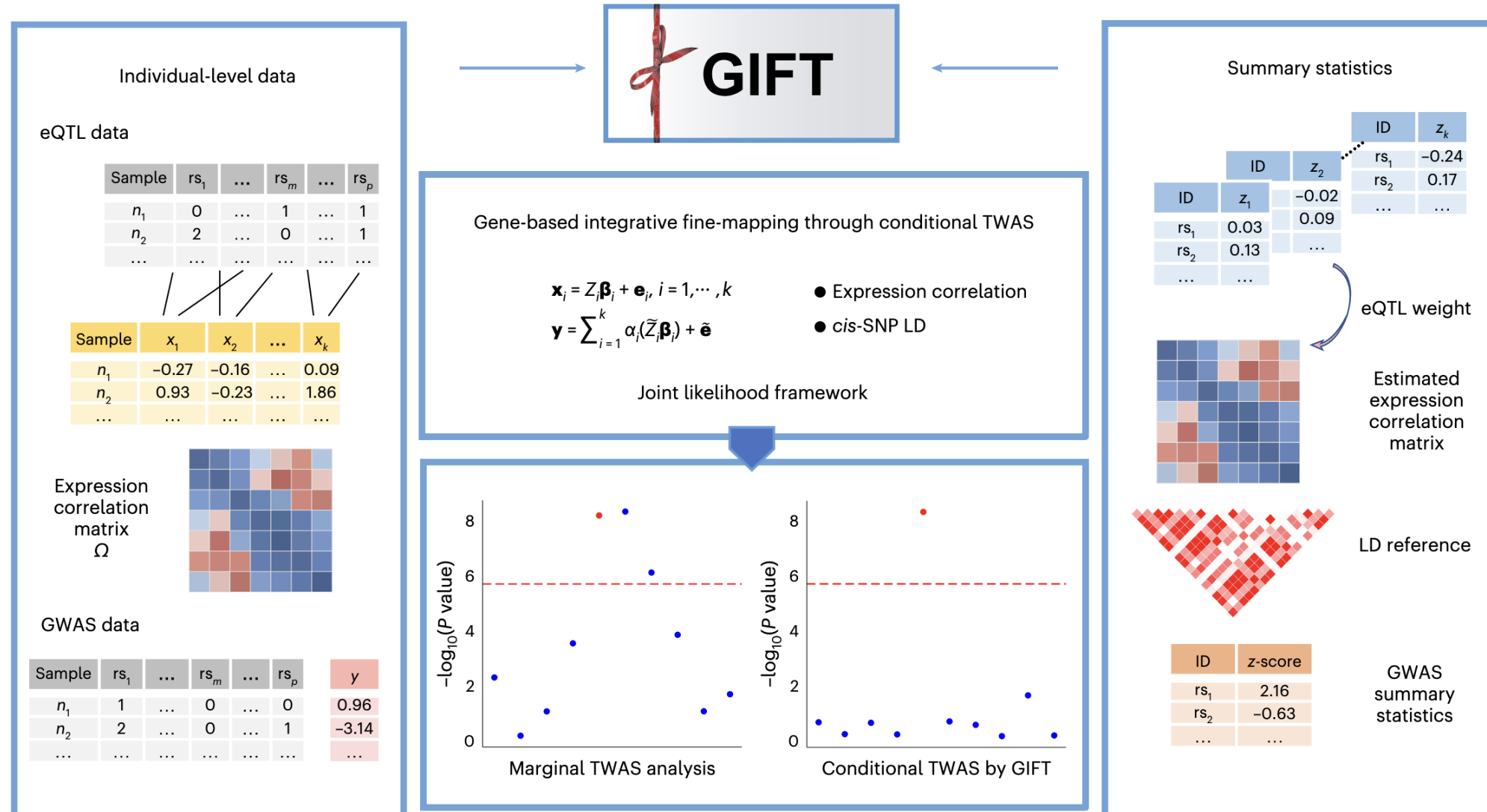
\*All of the simulation results are from 1,000 random region.

## Real-data applications





## Discussion





香港城市大學  
City University of Hong Kong

# THANK YOU



**Speaker: Yuekai Li**

**Major: Biostatistics**