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TUTORIAL 3 (Due Date: 16 January 2025)

1. A father, mother, 2 boys, and 3 girls are asked to line up for a photograph. Determine the number of ways they can line up if
   a) there are no restrictions $7! = 5040$
   b) the parents stand together $2!\,6! = 1440$
   c) all the females stand together $4!\,4! = 576$

2. How many words, with or without meaning can be made from letters of the word TUESDAY, assuming that no letter is repeated, if
   a) 4 letters are used at a time $^7C_4 \times 4! = 840$
   b) All letters are used at a time $7! = 5040$
   c) All letter are used but first letter is a vowel $3 \times 6! = 2160$

3. Registration numbers for a vehicle are to be made using 3 letters (using any letter of the alphabet) followed by four single-digit numbers. For example, WJC2091 is one such registration number. How many registration can be formed if
   a) The letter cannot be repeated? $26 \times 25 \times 24 \times 10 \times 10 \times 10 \times 10 = 156\,000\,000$
   b) Neither letters nor numbers can be repeated? $26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7 = 78\,624\,000$

4. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways this can be done when the committee consist of
   a) Exactly 3 girls $^4C_3 \times {}^9C_4 = 4 \times 126 = 504$
   b) At least 3 girls $(^4C_3 \times {}^9C_4) + (^4C_4 \times {}^9C_3) = 504 + 84 = 588$
   c) At most 3 girls $(^4C_0 \times {}^9C_7) + (^4C_1 \times {}^9C_6) + (^4C_2 \times {}^9C_5) + (^4C_3 \times {}^9C_4) = 36 + 336 + 756 + 504 = 1632$
   $^{13}C_7 - (^4C_4 \times {}^9C_3) = 1716 - 84 = 1632$

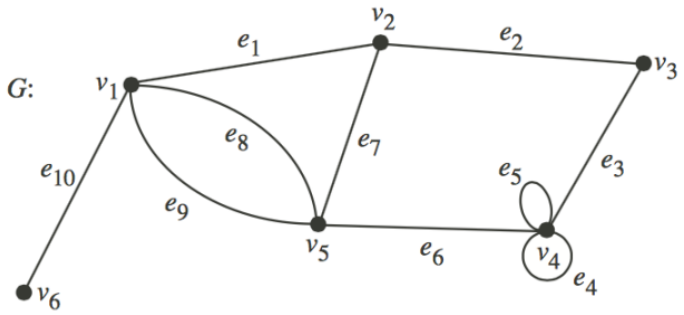
5. Assumed that there are 100 participants in a lucky draw competition. There are 3 prizes being offered which are the grand prize, second prize and third prize. The winners are randomly selected. What is the probability that Jamilah can win one of the prizes, if she participates in the competition? $\frac{3}{100} = 0.03$

6. There are 2 bags which the first bag contain 4 white balls and 3 black balls while the second bag contains 3 white balls and 5 black balls. A ball is randomly picked, what is the probability that the picked ball is a black ball from the second bag? $P(\text{Black} \mid \text{Bag 2}) = \frac{5}{8} = 0.625$

7. At a high school, 60% (0.6) of the seniors have taken an advanced calculus course. Of those who have taken advanced calculus, 80% (0.8) will apply pre-health science major when they apply for college admission . Of those who have not taken (0.4) advanced calculus, 30% (0.3) will apply for pre-health science major when they apply for college admission. A senior is selected at random. What is the probability that the senior have taken advanced calculus, given that the senior will apply for pre-health science major?
   $P(\text{Pre-Health}) = (0.6 \times 0.8) + (0.4 \times 0.3) = 0.48 + 0.12 = 0.6$
   $P(\text{Calculus} \mid \text{Pre-Health}) = \frac{P(\text{Pre-Health} \mid \text{Calculus}) \times P(\text{Calculus})}{P(\text{Pre-Health})}$
   $= \frac{0.8 \times 0.6}{0.6} = 0.8$

8. Suppose that 15% (0.15) of the population is over 60 years old, 28% (0.28) of those over 60 years old have loans, and 56% (0.56) of those less than or equal to 60 years old have loans. A person is selected at random. Find the following probabilities.
   a) The person is over 60 years old and has a loan $P(>60 \cap \text{loan}) = 0.15 \times 0.28 = 0.042$
   b) The person has no loan $P(\text{no loan}) = 1 - P(\text{loan}) = 1 - (0.15 \times 0.28 + 0.85 \times 0.56) = 1 - 0.518 = 0.482$
   c) The person has a loan, given that the person is less tha or equal to 60 years old $P(\text{loan} \mid \leq 60) = 0.56$
   d) The person is over 60, given that the person has a loan $P(>60 \mid \text{loan}) = \frac{P(>60 \cap \text{loan})}{P(\text{loan})} = \frac{0.042}{0.518} = 0.0811$

9. Given a graph (G) as shown below.



Edge	Endpoints
e_1	$\{v_1, v_2\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_3, v_4\}$
e_4	$\{v_4\}$
e_5	$\{v_4\}$
e_6	$\{v_4, v_5\}$
e_7	$\{v_2, v_5\}$
e_8	$\{v_1, v_5\}$
e_9	$\{v_1, v_5\}$
e_{10}	$\{v_1, v_6\}$

i. Write a set of vertices and the set of edges, and give a table showing the edge-endpoint function. $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$

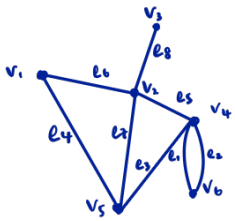
ii. Find the incidence matrix of the graph.

$I_G =$

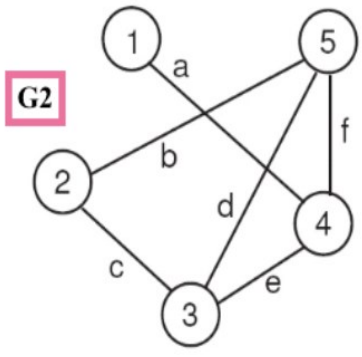
	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
v_1	1	0	0	0	0	0	0	1	1	1
v_2	1	1	0	0	0	0	1	0	0	0
v_3	0	1	1	0	0	0	0	0	0	0
v_4	0	0	0	2	2	1	0	0	0	0
v_5	0	0	0	0	0	1	1	1	1	0
v_6	0	0	0	0	0	0	0	0	0	1

10. Draw the graph G_1 represented by the incidence matrix, I(G_1) below:

$$\mathbf{I}\ (G_1) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

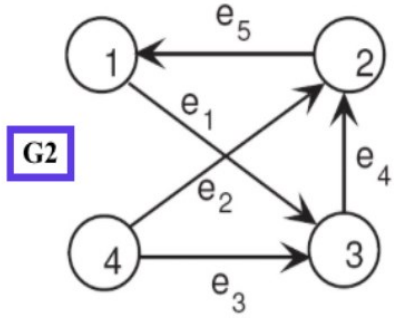


11. Find the adjacency matrix for the graph *G2* and *G3* below.



$A_{G2} =$

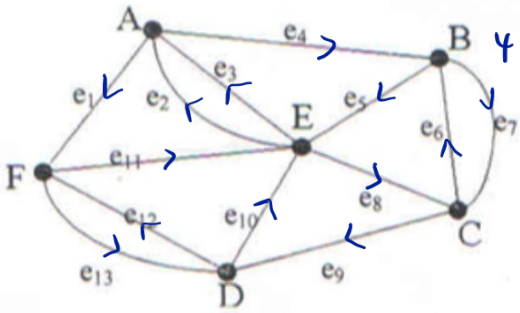
	1	2	3	4	5
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	0	1
5	0	1	1	1	0



$A_{G3} =$

	1	2	3	4
1	0	0	1	0
2	1	0	0	0
3	0	1	0	0
4	0	1	1	0

12. Given graphs below, determine whether they contain either Euler circuit or Eular trail? If yes, exhibit one and justify your answer.

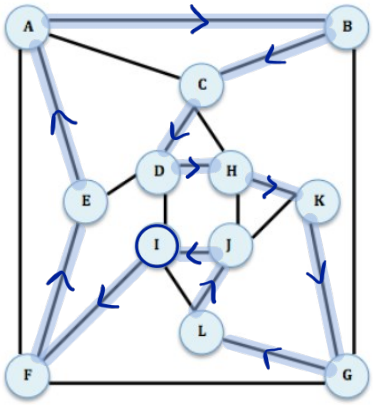


vertex	A	B	C	D	E	F
degree	4	4	4	4	6	4

This graph has an Euler circuit since all vertices have even degree.

(A, e_4, B, e_7, C, e_6, B, e_5, E, e_8, C, e_9, D, e_{10}, E, e_3, A, e_1, F, e_{13}, D, e_{12}, F, e_{11}, E, e_2, A)

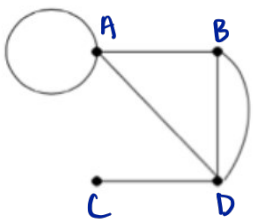
13. Check if the graph below has Hamiltonian cycle. If the graph has Hamiltonian cycle, exhibit one.



This graph has a Hamiltonian circuit.

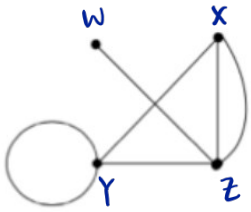
I-F-E-A-B-C-D-H-K-G-L-J-I

14. Determine whether G1 is isomorphic to G2. Justify your answer.



V = 4
e = 6

G1



V = 4
e = 6

G2

Both have 4 vertices and 6 edges.
Both are connected graph. Both have same number of loop.
Both have 2 vertices with 4 degree, 1 vertices with 3 degree and 1 vertices with 1 degree.

f(A) = Y f(B) = X f(C) = W f(D) = Z

$$A_{G1} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_{G2} = \begin{array}{c} \\ Y \\ X \\ W \\ Z \end{array} \begin{array}{c} \begin{array}{cccc} Y & X & W & Z \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

G1 and G2 are isomorphic.