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Finite Automata

Deterministic Finite Automata (DFA)

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.

- We will discuss a type of a language that can be recognized by special types of machines.

Deterministic Finite Automata (DFA)

- A deterministic finite automaton (pl. automata) is a mathematical model of a machine that accepts languages of some alphabet.

Deterministic Finite Automata (DFA)

- Deterministic Finite Automaton is a quintuple
  $M= \{ S, I, q_0, f_s, F\}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet (a finite nonempty set of symbols)
  q_0 is the initial state
  f_s is the state transition function
  F is the set of final states, subset of S.

example

- Let $M=\{ \{q_0,q_1,q_2\}, \{0,1\}, q_0, f_s, \{q_2\}\}$
  where f_s is defined as follows:

  $f_s(q_0,0)= q_1$, $f_s(q_1,1)= q_2$
  $f_s(q_0,1)= q_0$, $f_s(q_2,0)= q_0$
  $f_s(q_1,0)= q_2$, $f_s(q_2,1)= q_1$

- Note that for M:
  $S=\{q_0,q_1,q_2\}$, $I= \{0,1\}$, $F=\{q_2\}$
  q_0 is the initial state

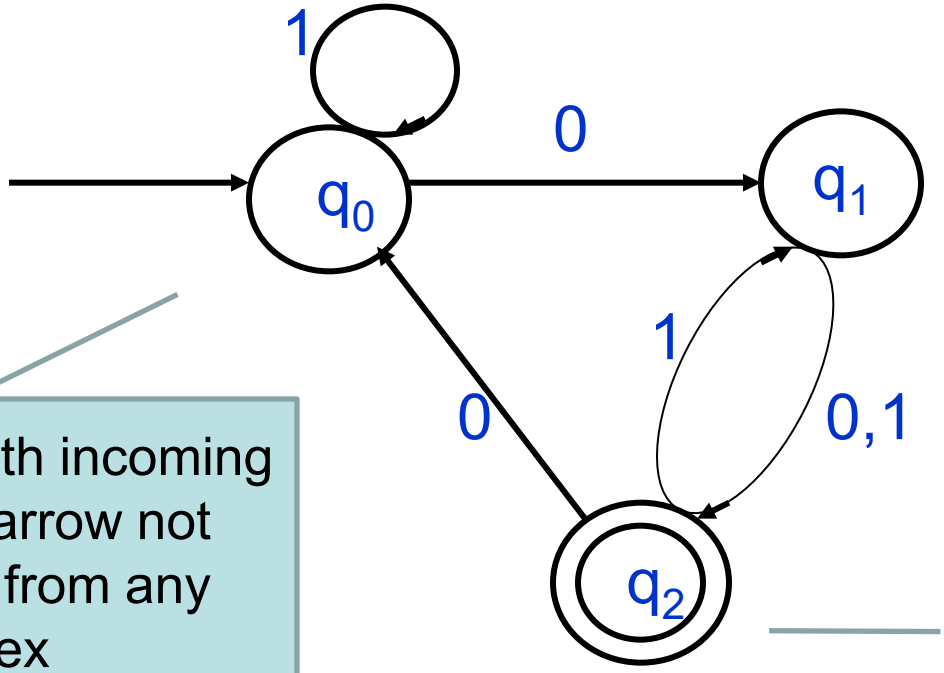
example

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

example

- The transition diagram of this DFA is,



Each state represented by a small circle labeled with the state

Initial state with incoming unlabeled arrow not originating from any vertex

Final state with a double circle

example

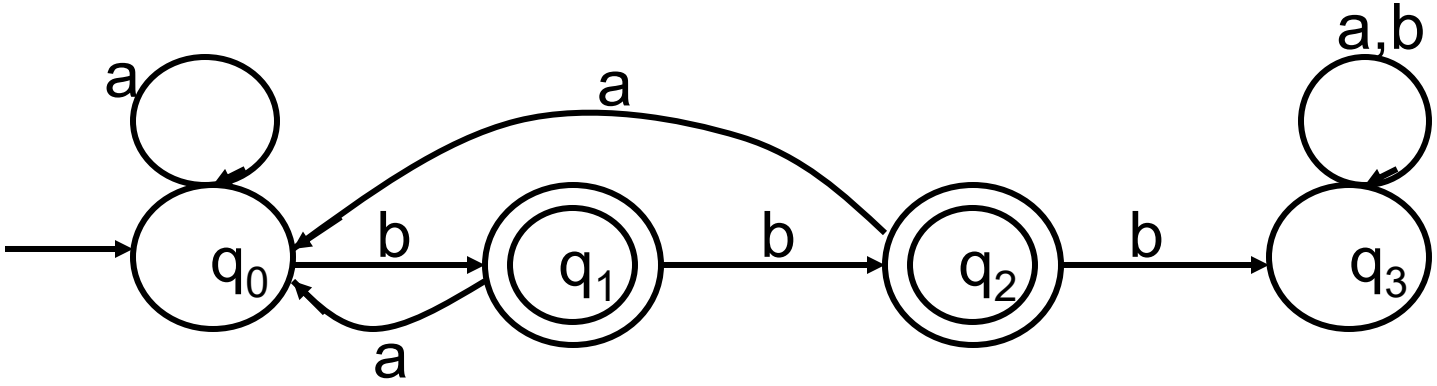
Let $M=(\{q_0,q_1,q_2,q_3\}, \{a,b\},q_0, f_s , \{q_1,q_2\})$

where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

example

- The transition diagram of this DFA is,



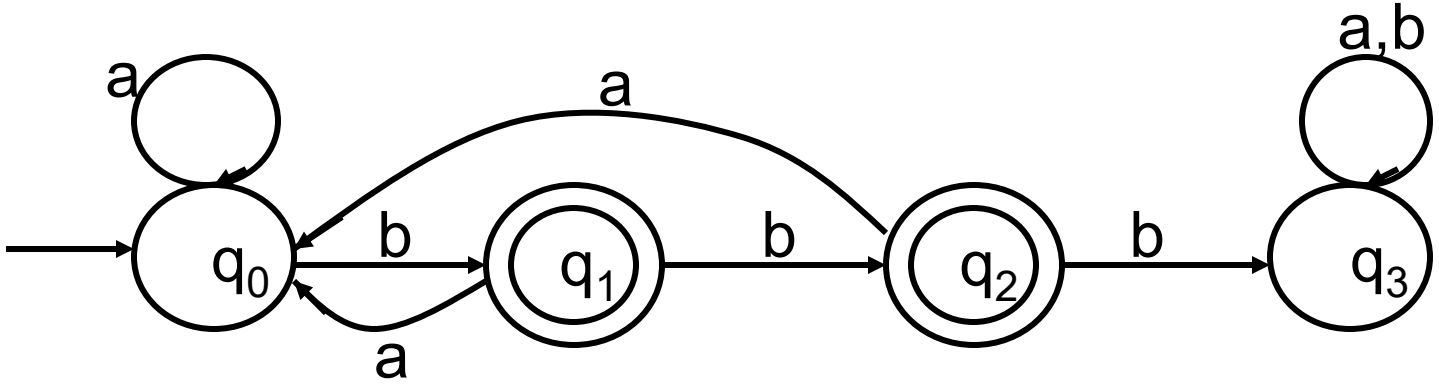
Deterministic Finite Automata (DFA)

- Let M= { S, I, q_0, f_s, F} be a DFA and *w* is an input string,
- *w* is said to be accepted by M if

$$f_s^*(q_0, w) \in F$$

- f_s^* - extended transition function for M

example

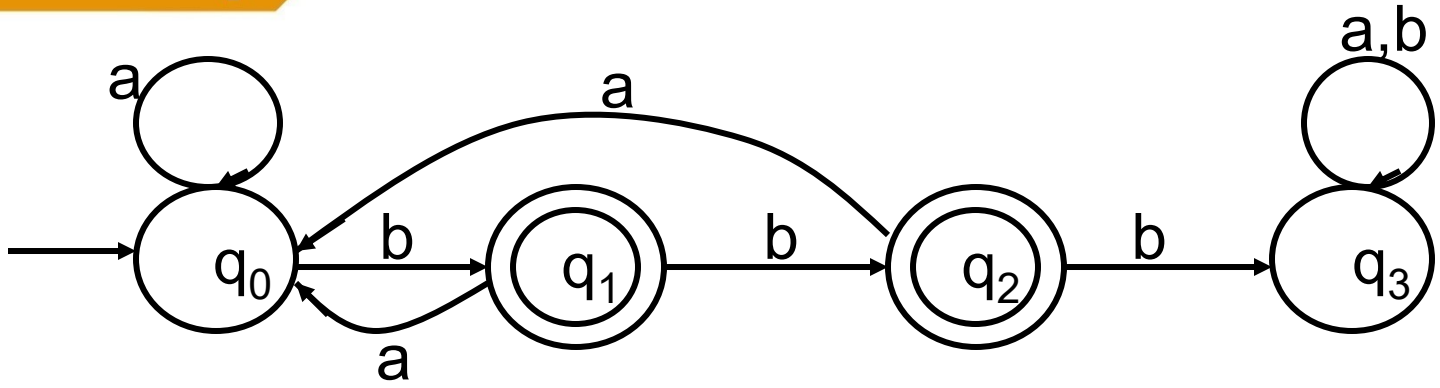


w= abb

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$$

accepted by M

example

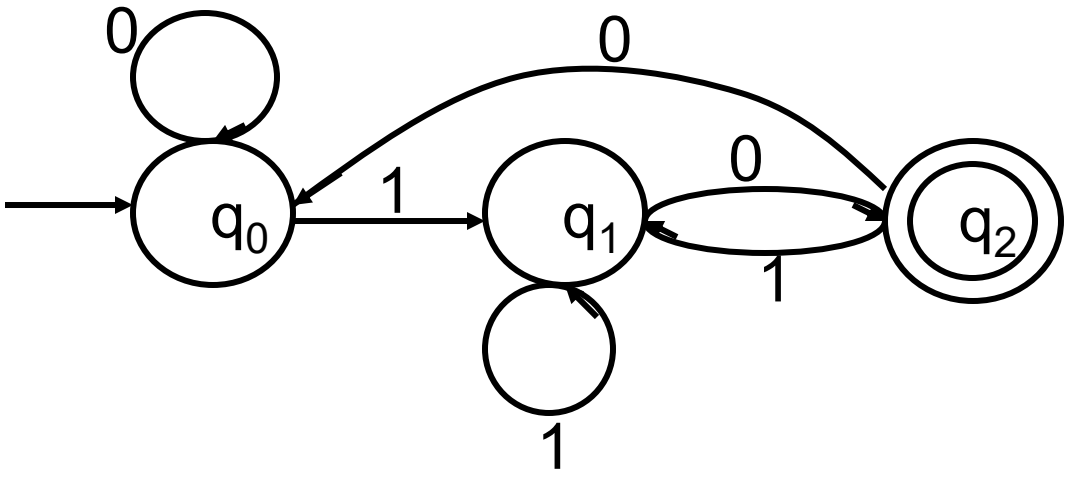


w= abba

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$$

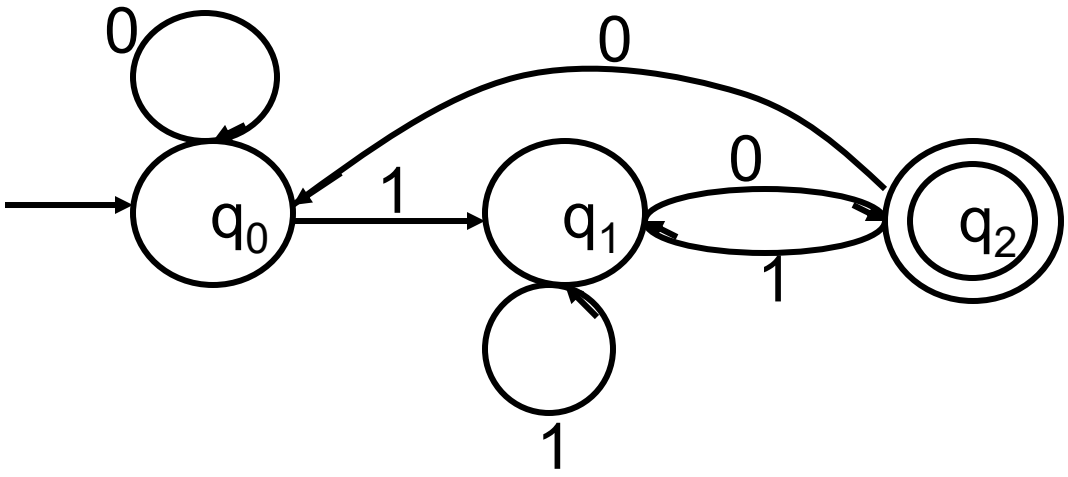
not accepted by M

example



- What are the states of M? q_0, q_1, q_2
- Write the set of input symbols. $I = \{0,1\}$
- Which is the initial state? q_0

example



- Write the set of final states. $F = \{q_2\}$
- Write the transition table for this DFA

example

The transition table, fs

f_s	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_1

example

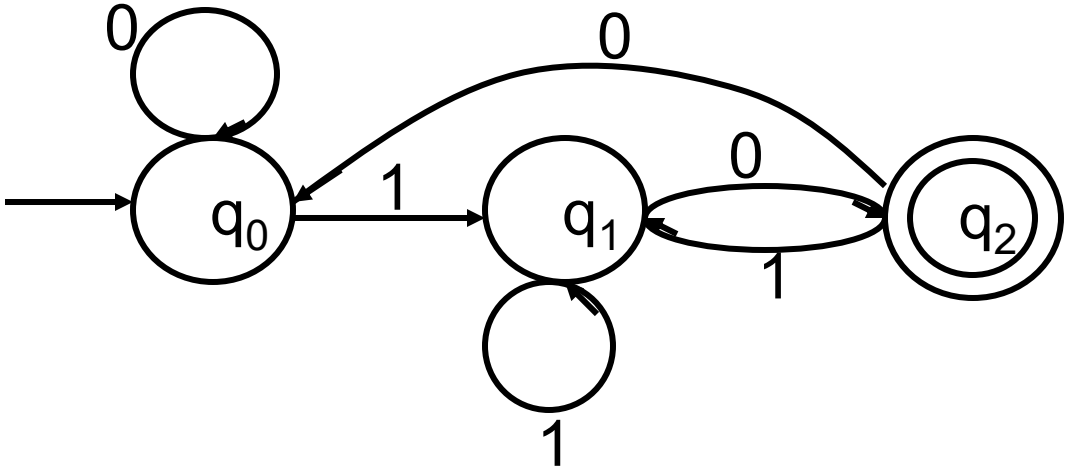
Which of the strings are accepted by M?

0111010, 00111, 111010,

0100, 1110

example

0111010

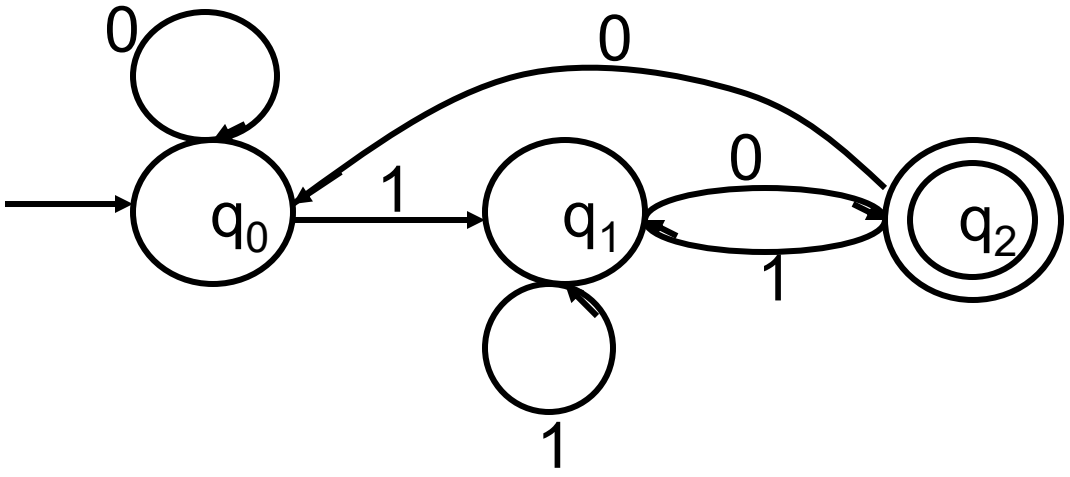


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

00111

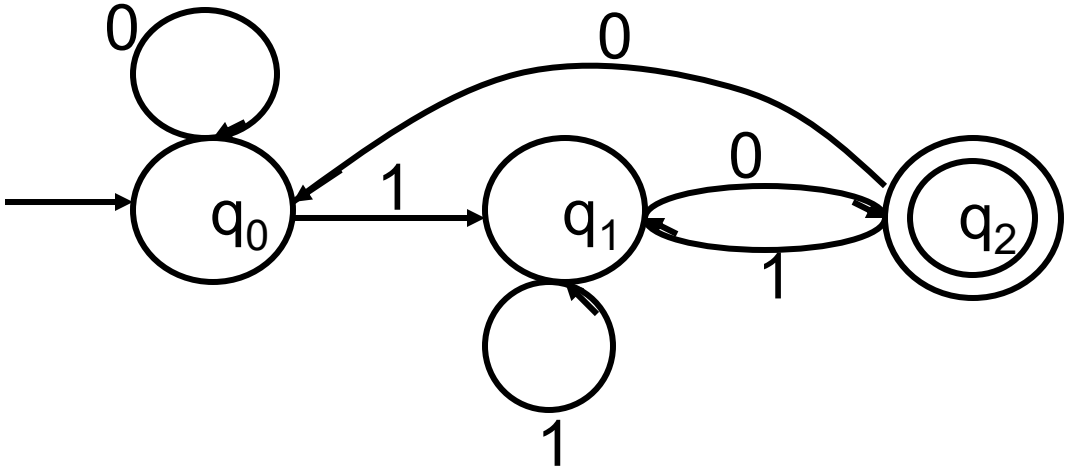


$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1$

not accepted by M

example

111010

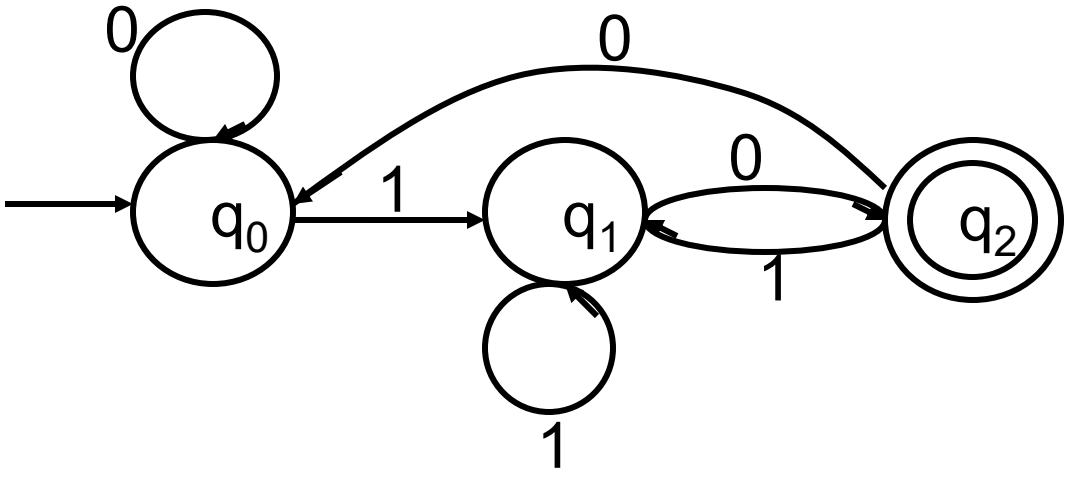


$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

0100

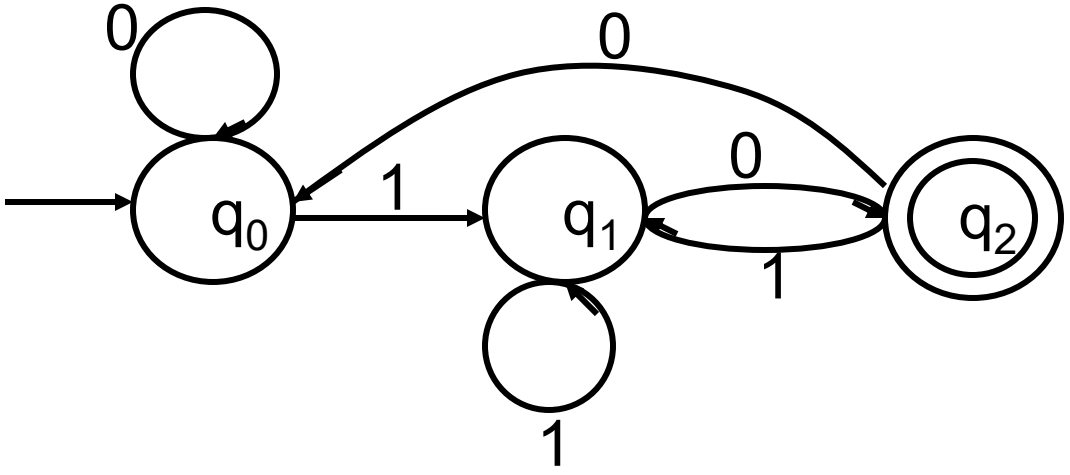


$$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_0$$

not accepted by M

example

1110



$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2$$

accepted by M

example

Construct a state transition diagram of a DFA that accepts on {a,b} that contain an even number of a's and an odd number of b's.

Example of accepted strings:

aab, baa, baaabba

example

4 states,

q_0	even num. of a's & even num. of b's.
q_1	even num. of a's & odd num. of b's.
q_2	odd num. of a's & odd num. of b's.
q_3	odd num. of a's & even num. of b's.

$S = \{q_0, q_1, q_2, q_3\}$

example

set of states, $S = \{q_0, q_1, q_2, q_3\}$

set of input symbols, $I = \{a, b\}$

initial state, q_0

final state, q_1

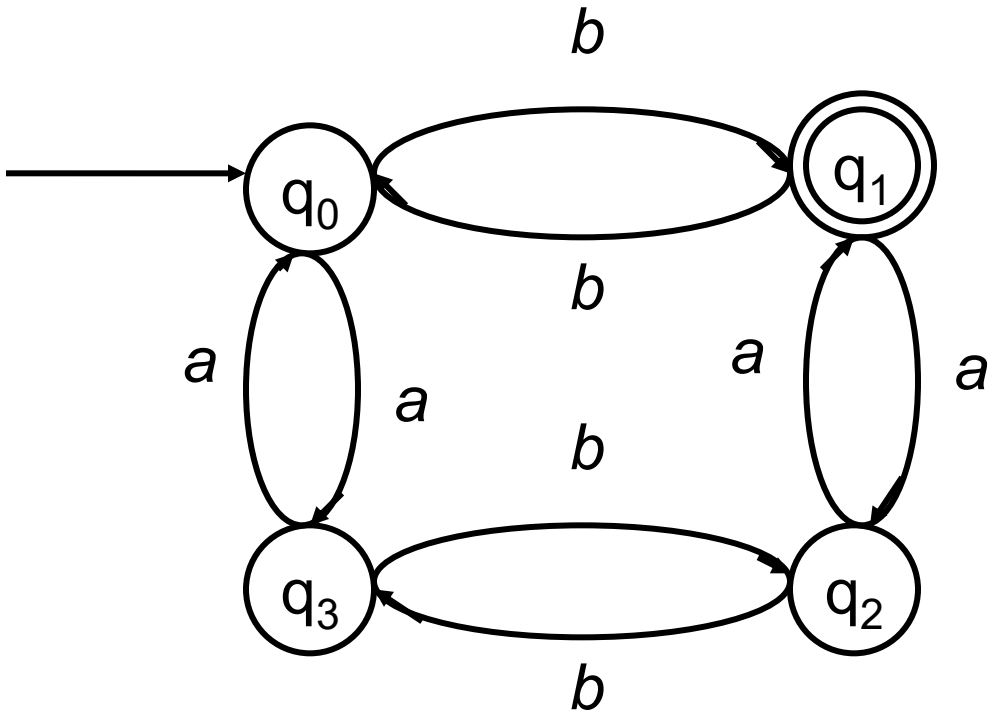
example

State transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

example

State transition diagram



exercise

Let M=(S, I, q_0, f_s, F) be the DFA such that S={q_0,q_1,q_2}, I={a,b}, F={q_2}, q_0 =initial state, and f_s is given by,

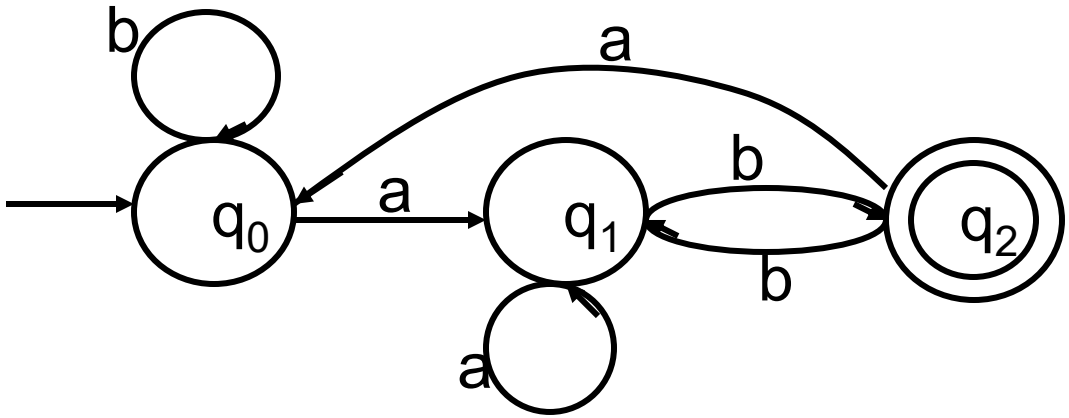
f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

Draw the state diagram of M.

Which of the strings
abaa, bbbabb, bbbaa dan
bababa
are accepted by M?

exercise

The transition diagram of M is,



Construct the transition table of M.

Which of the strings

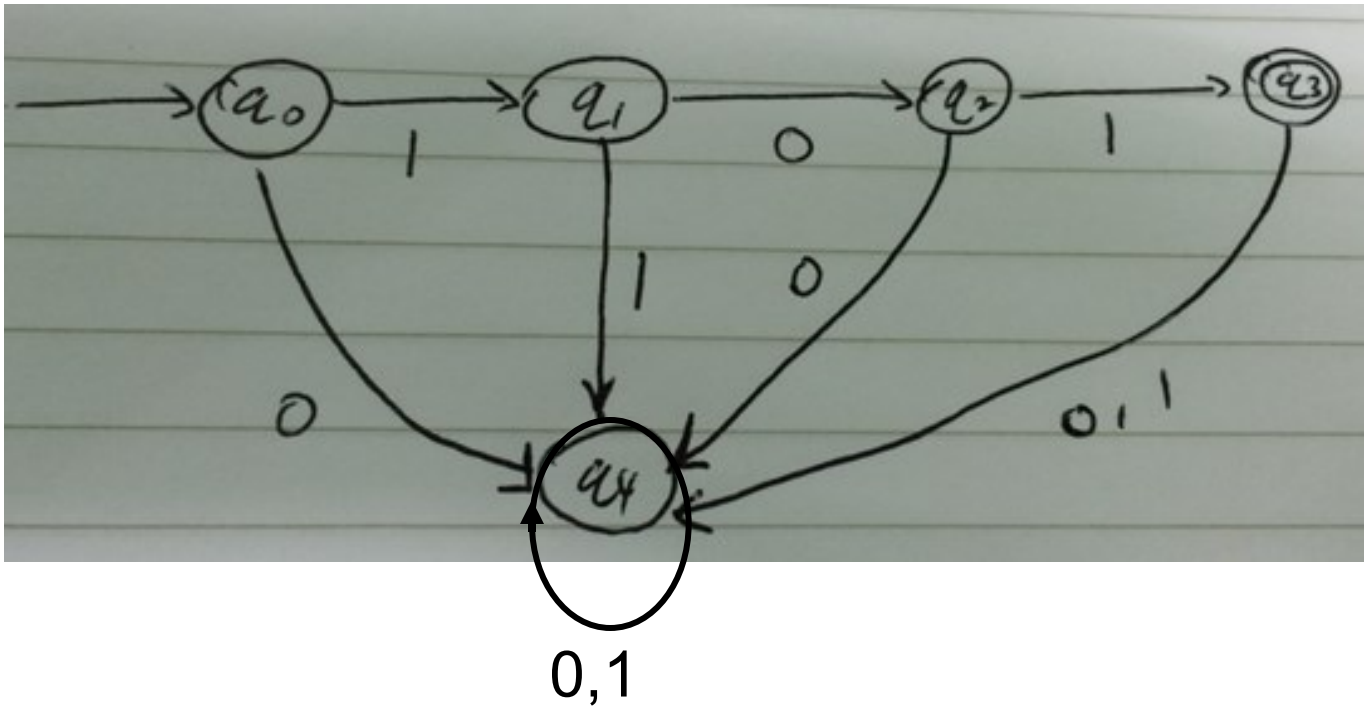
baba, baab, abab dan abaab

are accepted by M?

exercise

- Construct a state transition diagram of a DFA M with the input set {0,1} such that M accepts only the string 101.

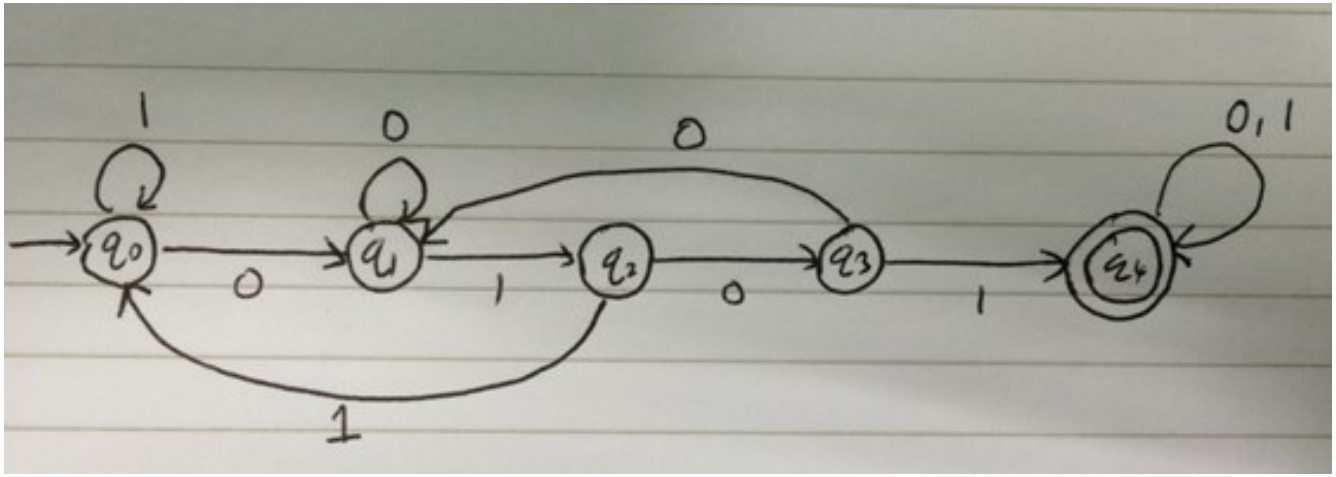
solution



Exercise

- Construct a deterministic finite automaton (DFA) that accepts the set of all bit strings that contain string ‘0101’.

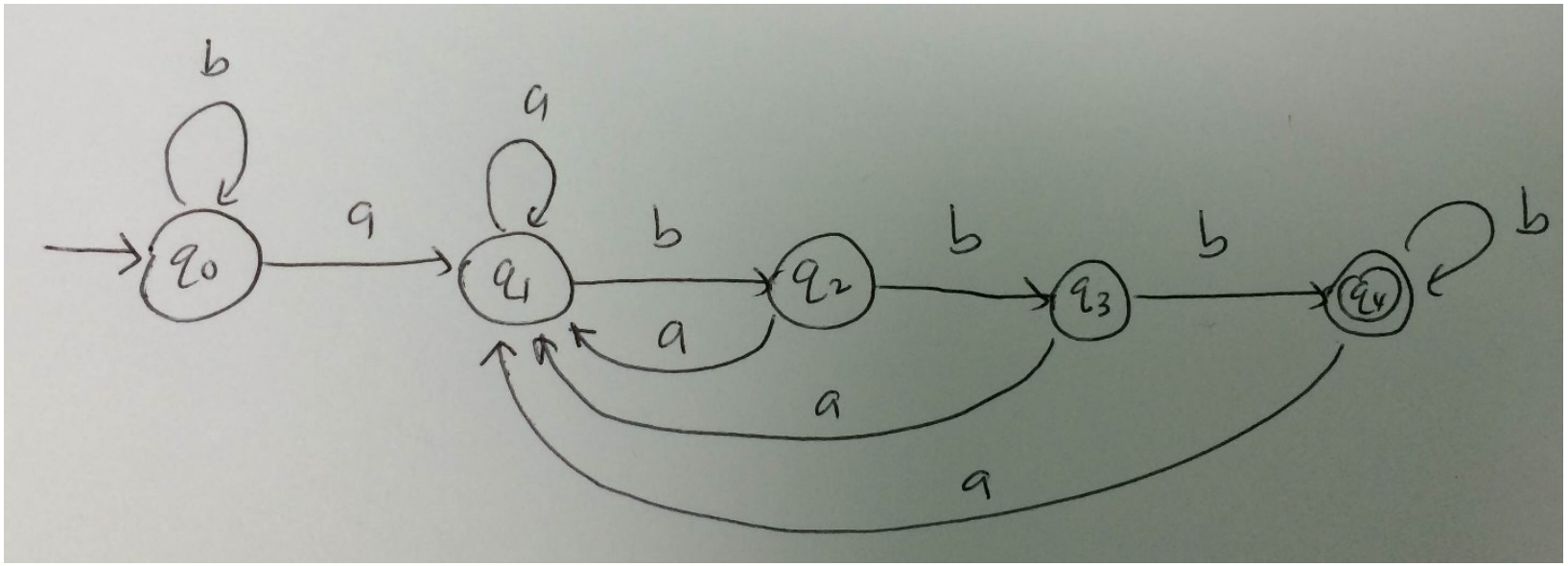
Solution



Exercise

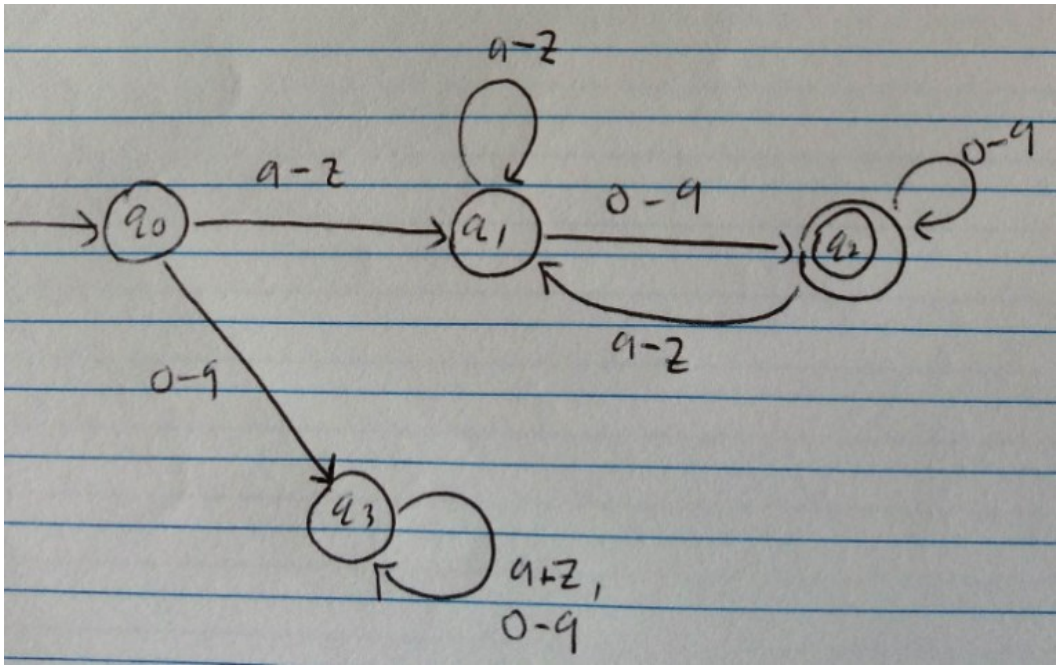
- Construct a deterministic finite automaton (DFA) that accepts all string over {a,b} that contain ab and end in bbb

Solution



Construct a state transition diagram of a DFA that accepts all string over {*a,b, … , z*, 0, 1, …, 9} that start with a letter (*a*-*z*) and end with a digit (0-9).

Example of accepted strings are *a*9, *xy*12, *a*1*b*2*c*3 ect.


a-z
a-z
0-9
0-9
a-z
0-9
q0
q1
q2
q3
a-z,
0-9

Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.

Finite State Machines (FSM)

- A finite state machine is a sextuple,
  $M= \{ S, I, O, q_0, f_s, f_o\}$
  where,
  S is a finite nonempty set of states
  I is the input alphabet
  O is the output alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function.

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM

- where,

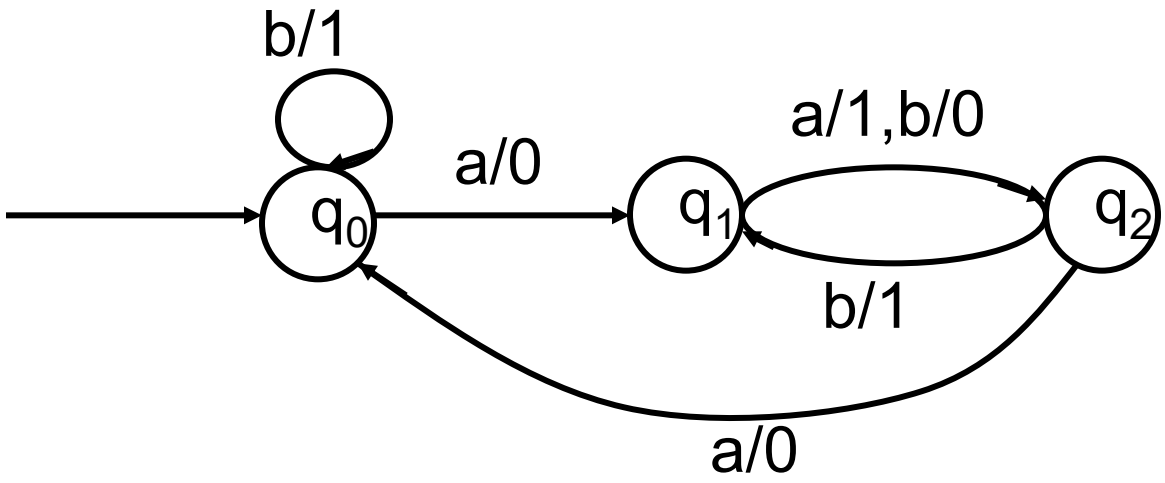
  S = {q_0,q_1,q_2},
  I = {a,b},
  O={0,1},
  q_0= initial state,

example

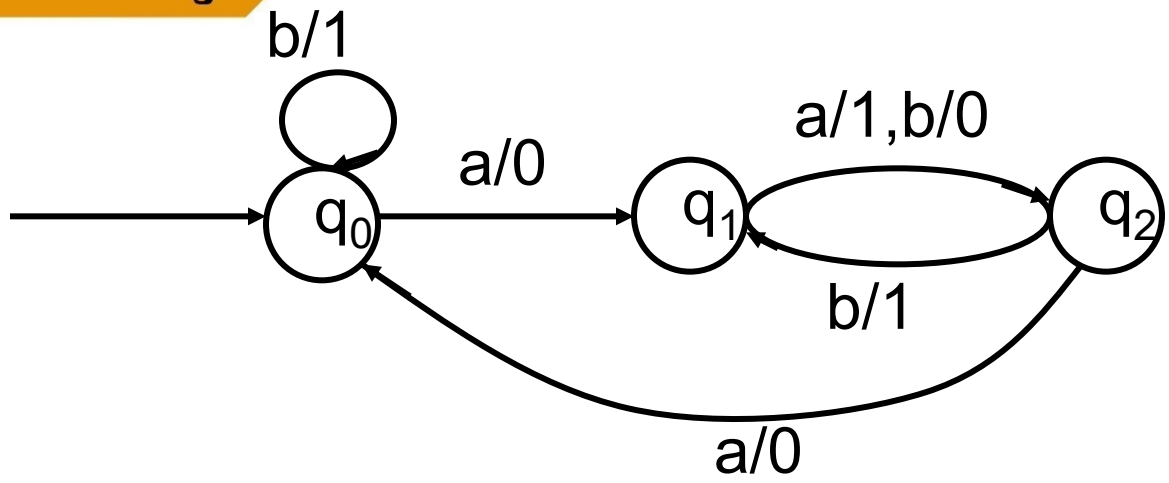
f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

example



example

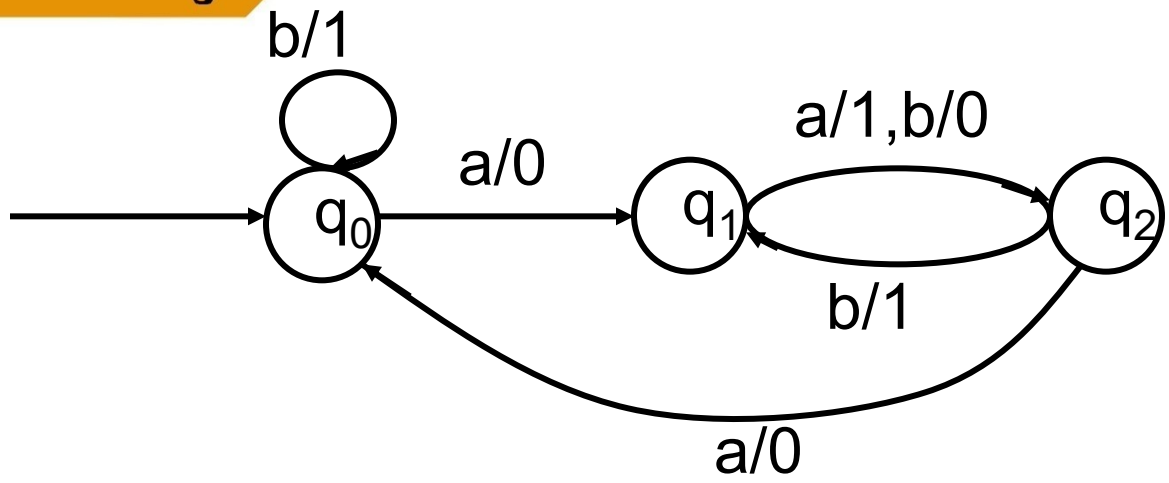


Input string: bbab

$$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2$$

Output string: 1100 Output: 0

example



Input string: bababaa

$$q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[0]{b} q_2 \xrightarrow[0]{a} q_0 \xrightarrow[1]{b} q_0 \xrightarrow[0]{a} q_1 \xrightarrow[1]{a} q_2$$

Output string: 1000101 Output: 1

example

- Let M= { S, I, O, q_0, f_s, f_o} be the FSM

- where,

  S ={q_0,q_1,q_2 ,q_3},

  I ={a,b},

  O ={0,1},

  q_0 = initial state,

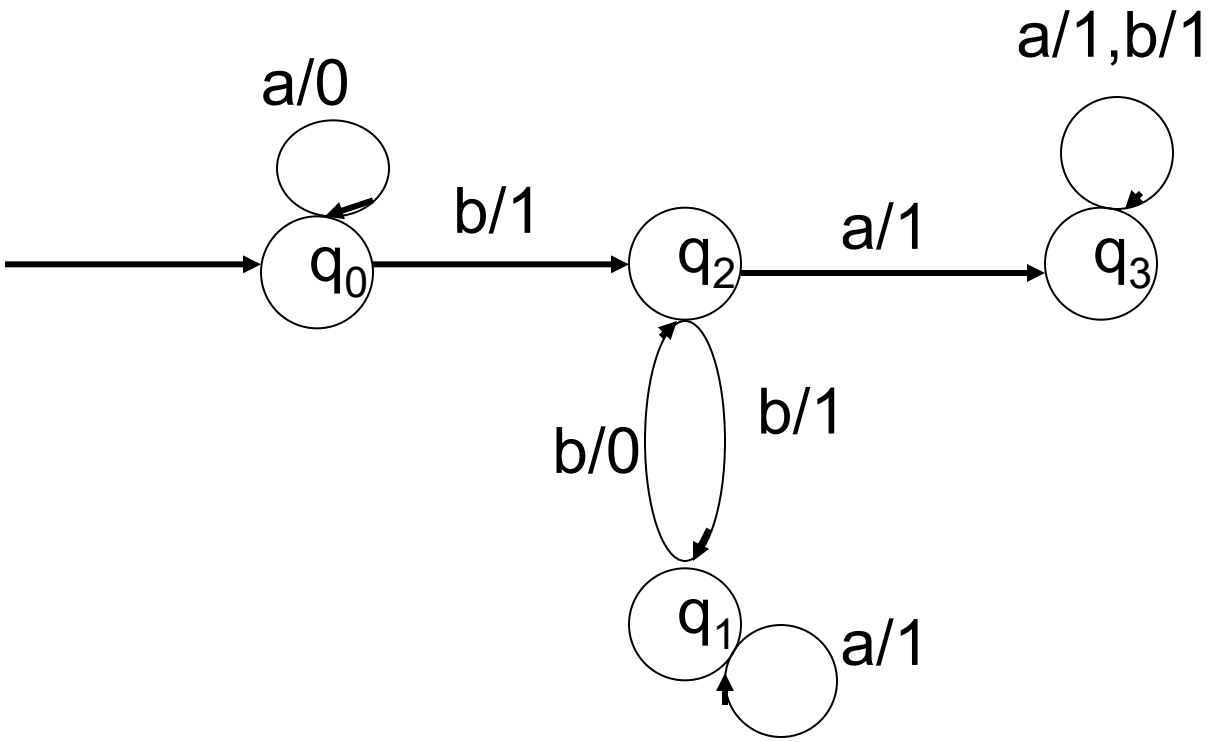
example

- f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

example

- Draw the transition diagram of M.



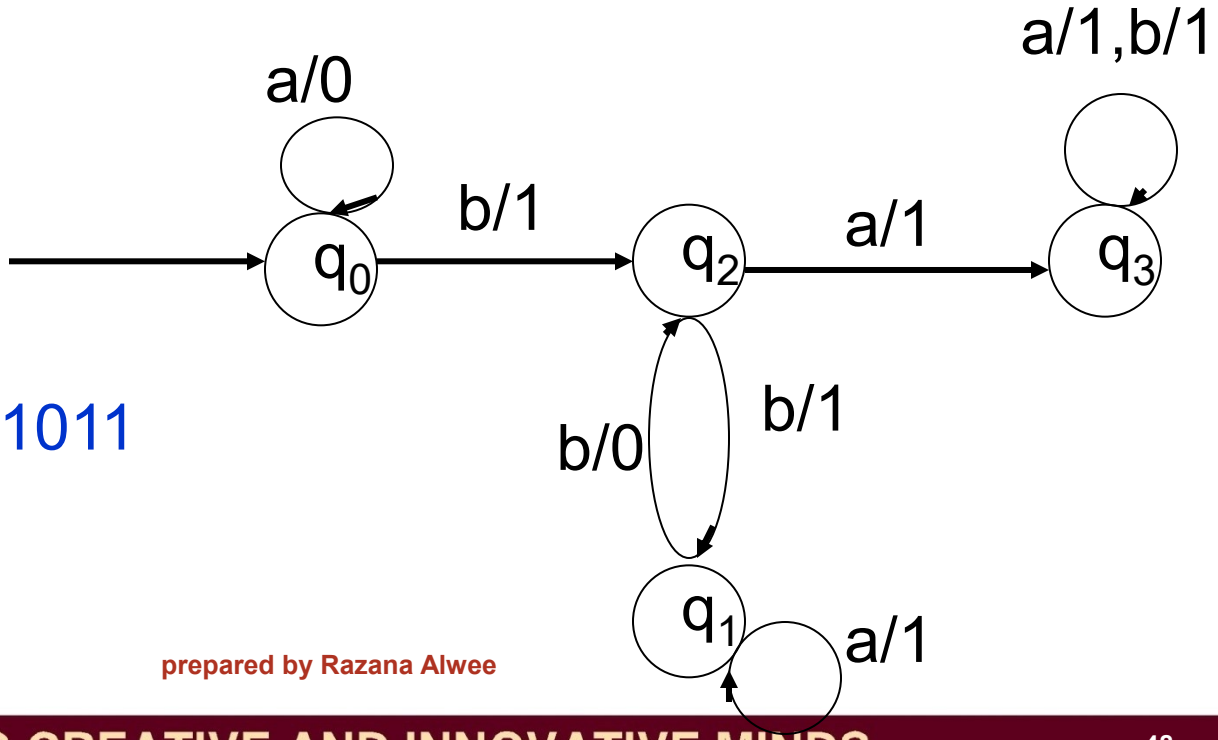
example

- What is the output string if the input string is *abbabab*?

abbabab

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_2 \xrightarrow{b} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_3 \xrightarrow{b} q_3$

0 1 1 1 0 1 1



Output string: 0111011

example

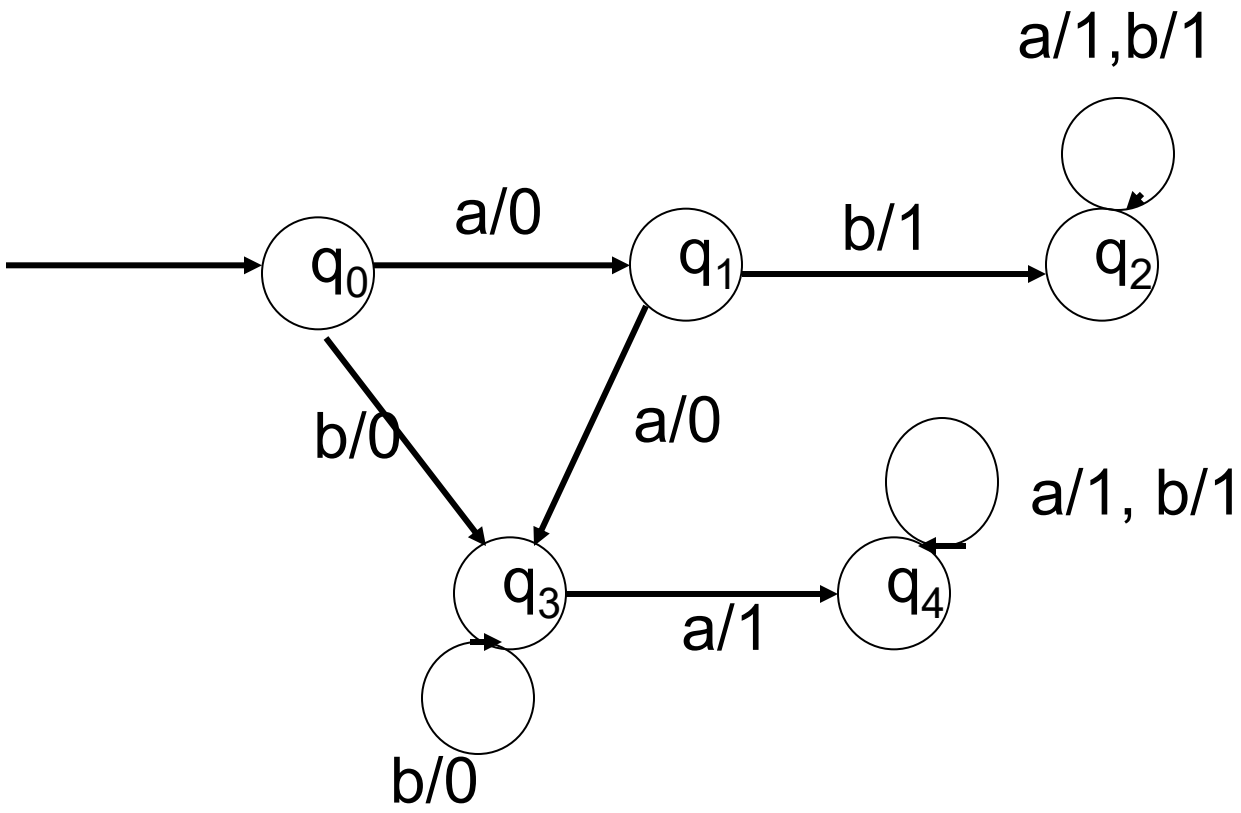
- What is the output of *abbabab*?

Output: 1

Finite State Machines (FSM)

- Let M be a FSM.
- Let x be a nonempty string in M.
- We say that x is accepted by M if and only if the output of x is 1.

example



example

- Write the transition table of M.
- What is the output string if the input string is *aaabbbb*?
- What is the output if the input string is *bbbaaaa*?

example

- Is the string *aaa* accepted by M?

- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

example

- The transition table of M.

	f_s a	b	f_o a	b
q_0	q_1	q_3	0	0
q_1	q_3	q_2	0	1
q_2	q_2	q_2	1	1
q_3	q_4	q_3	1	0
q_4	q_4	q_4	1	1

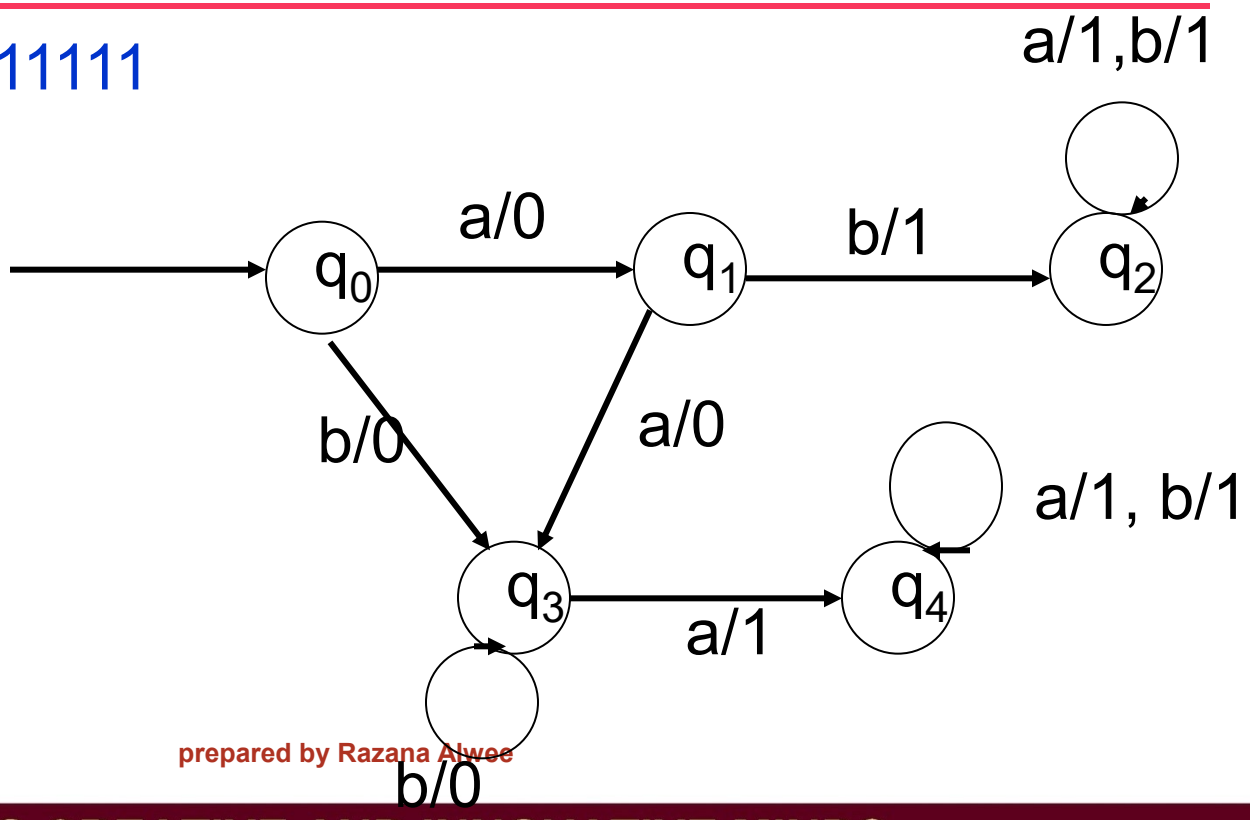
example

- What is the output string if the input string is *aaabbbb*?

aaabbbb

$q_0 \xrightarrow{a/0} q_1 \xrightarrow{a/0} q_3 \xrightarrow{a/1} q_4 \xrightarrow{b/1} q_4 \xrightarrow{b/1} q_4 \xrightarrow{b/1} q_4 \xrightarrow{b/1} q_4$

Output string: 0011111



example

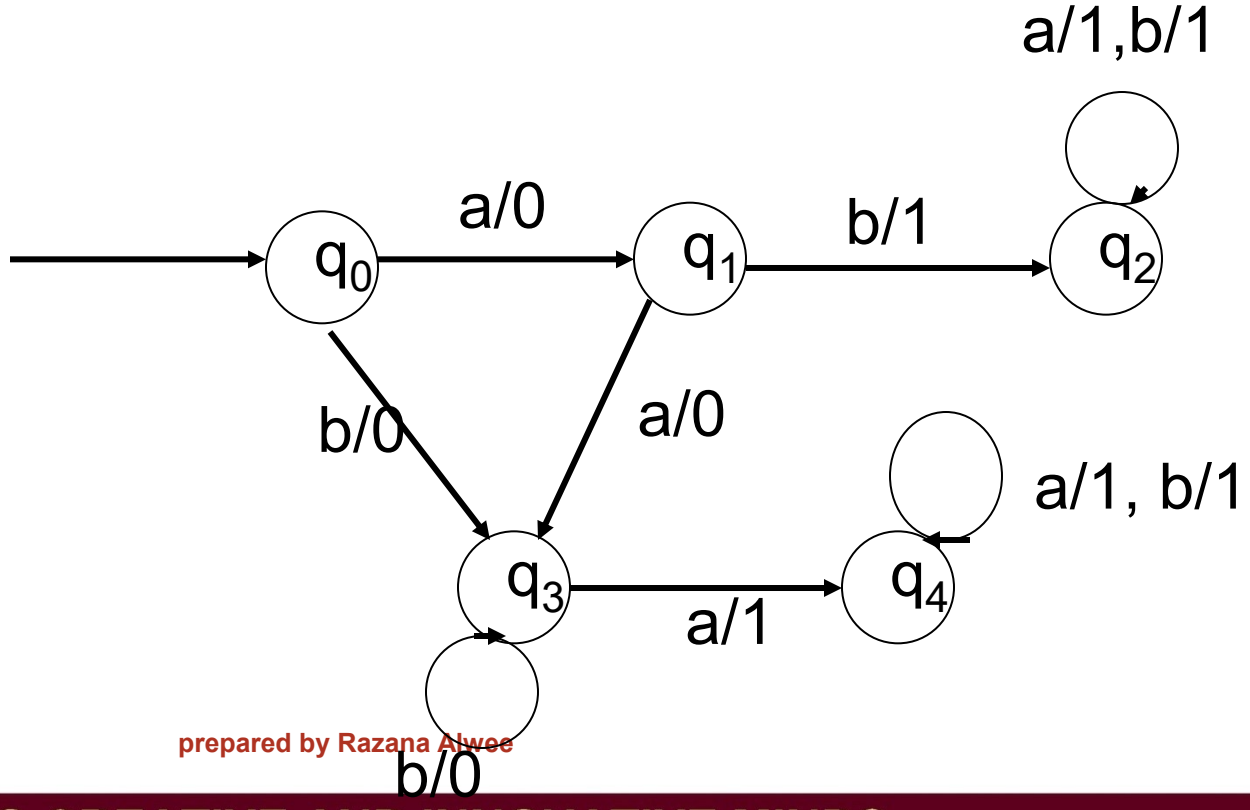
- What is the output if the input string is *bbbaaaa*?

bbbaaaa

$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4 \xrightarrow{a} q_4$

0 0 0 1 1 1 1

Output: 1



example

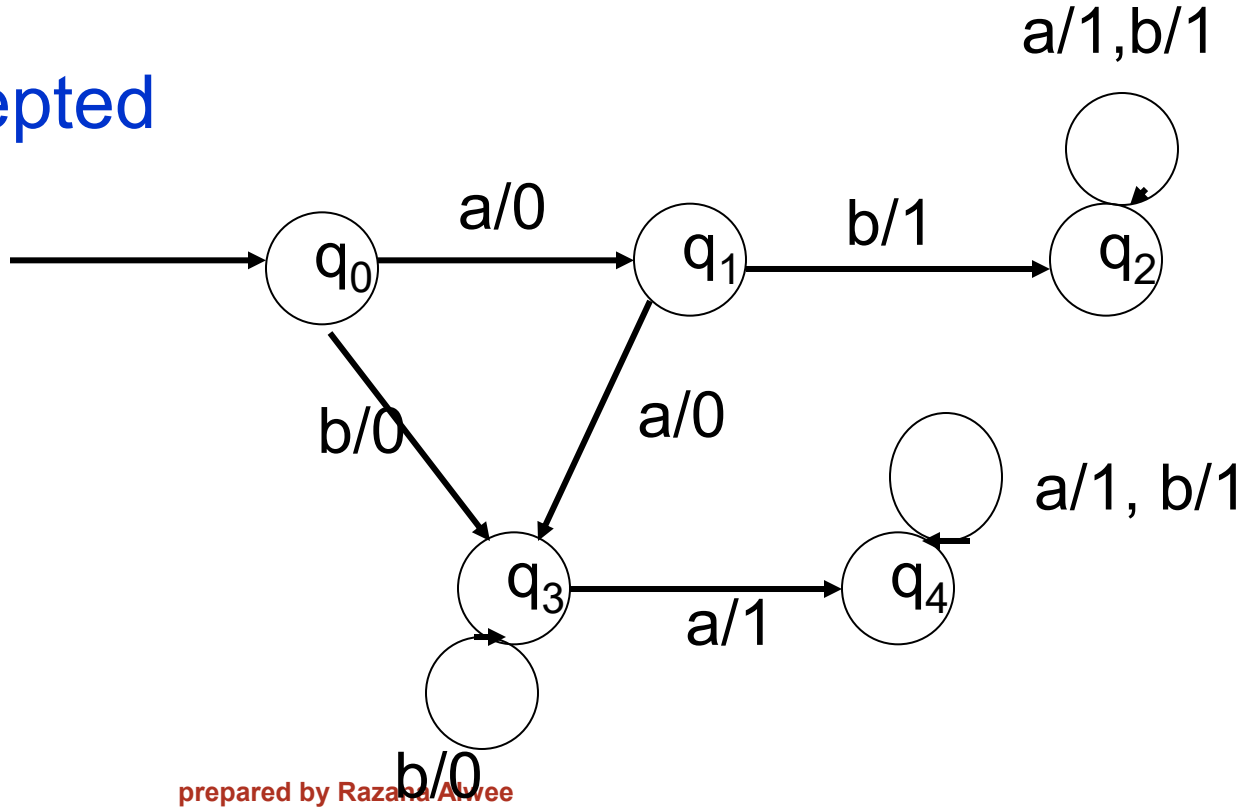
- Is the string *aaa* accepted by M?

aaa

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4$

0 0 1

Output: 1, accepted



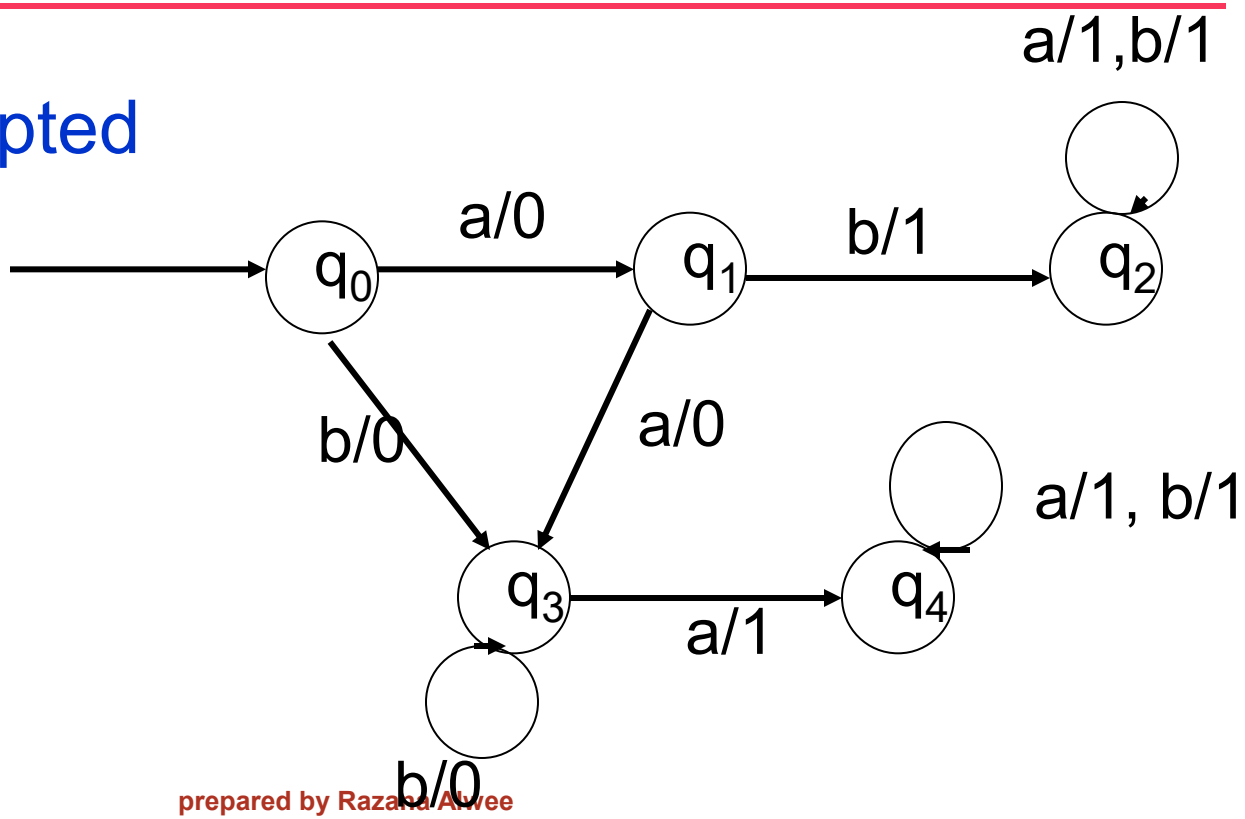
example

- Which of the strings
  ba, aabbba, bbbb, aaabbbb
  are accepted by M?

ba

$$q_0 \xrightarrow[0]{b} q_3 \xrightarrow[1]{a} q_4$$

Output: 1, accepted

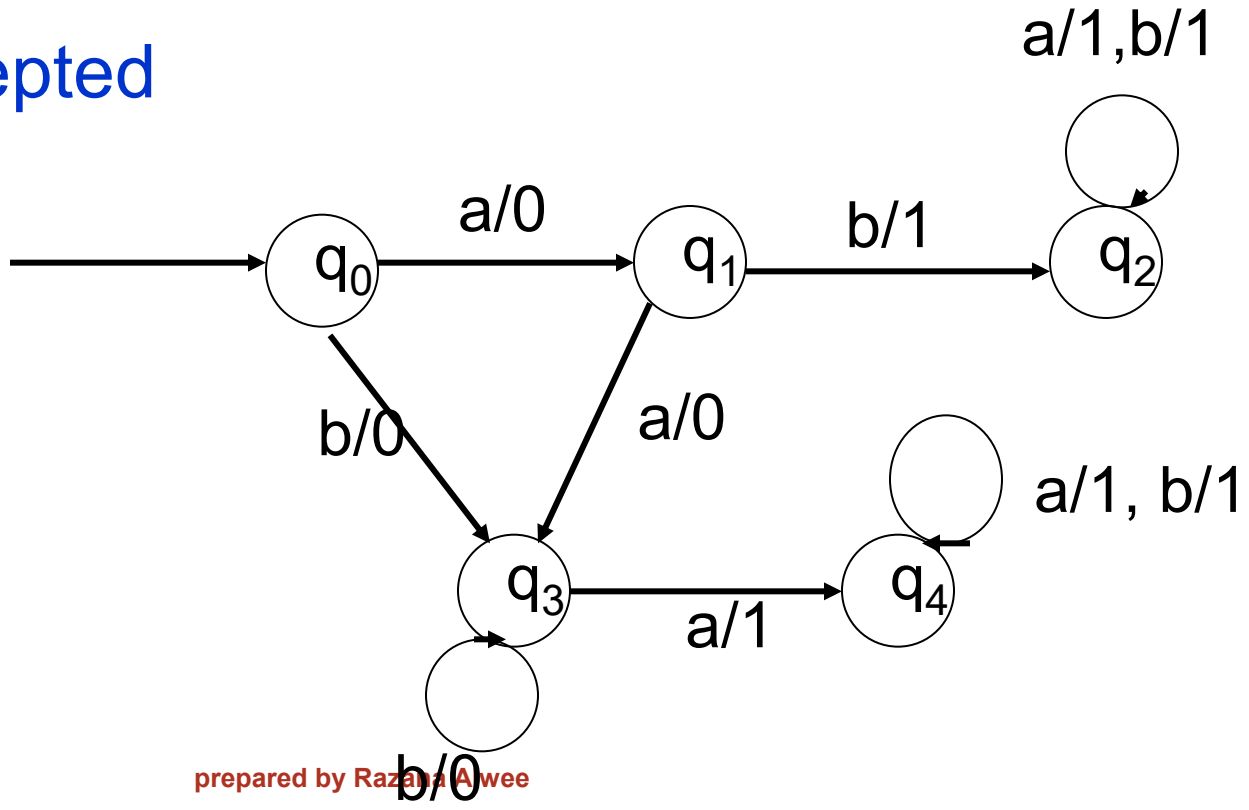


aabbba

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{a} q_4$

0 0 0 0 0 1

Output: 1, accepted

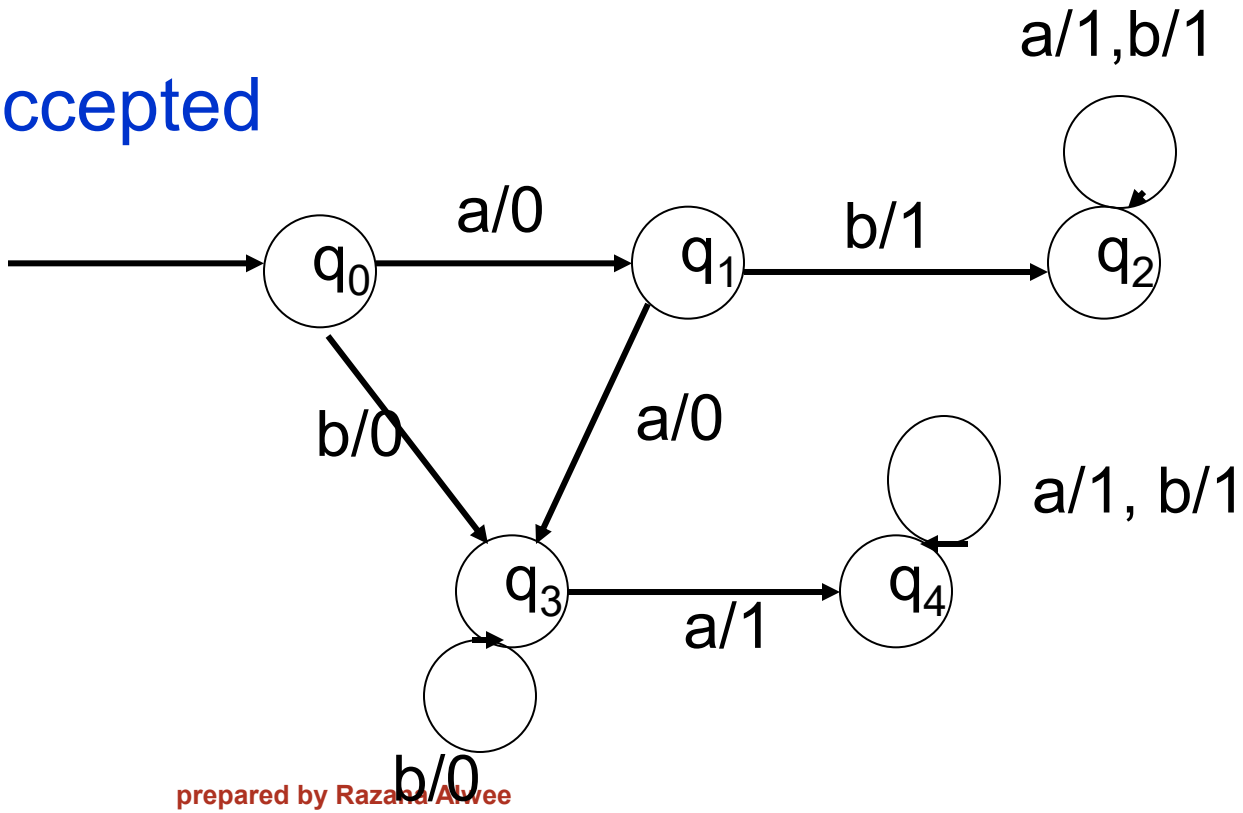


bbbb

$$q_0 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3 \xrightarrow{b} q_3$$

0 0 0 0

Output: 0, not accepted

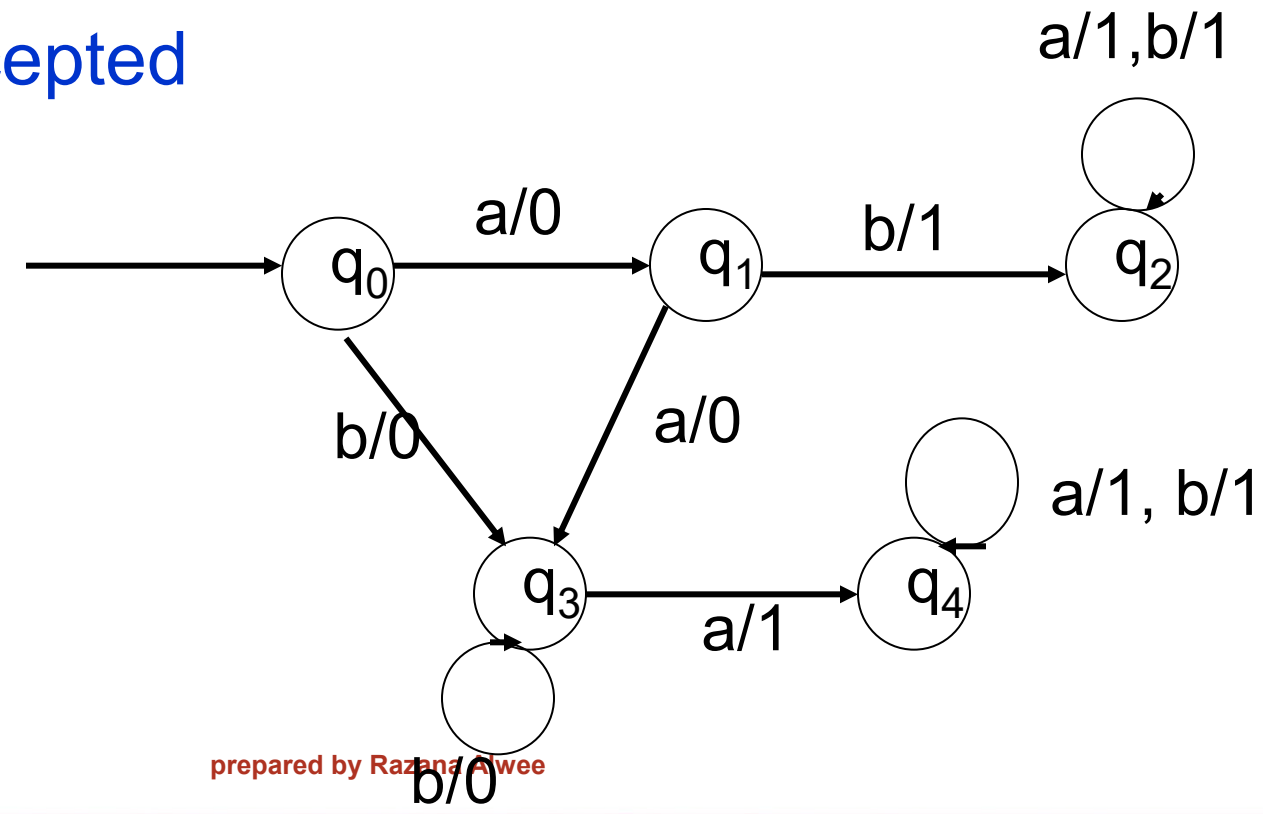


aaabbbb

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4 \xrightarrow{b} q_4$

0 0 1 1 1 1 1

Output: 1, accepted



example

- Consider a vending machine that sells candy and the cost of a candy is 50 cents.
- The machine accepts any sequence of 10-, 20-, or 50 cent coins.
- After inserting at least 50 cents, the customer can press the button to release the candy.

example

- If the customer inputs more than 50 cents, the machine does not return the change.
- After selling the candy, the machine returns to initial state.
- Construct a finite state machine that models this vending machine.

example

States,

- q_0, initial state (0)
- q_1, 10 cents
- q_2, 20 cents
- q_3, 30 cents
- q_4, 40 cents
- q_5, $\geq$ 50 cents

example

$S= \{q_0,q_1,q_2,q_3,q_4,q_5\}$,

$I = \{10,20,50,B\}$,

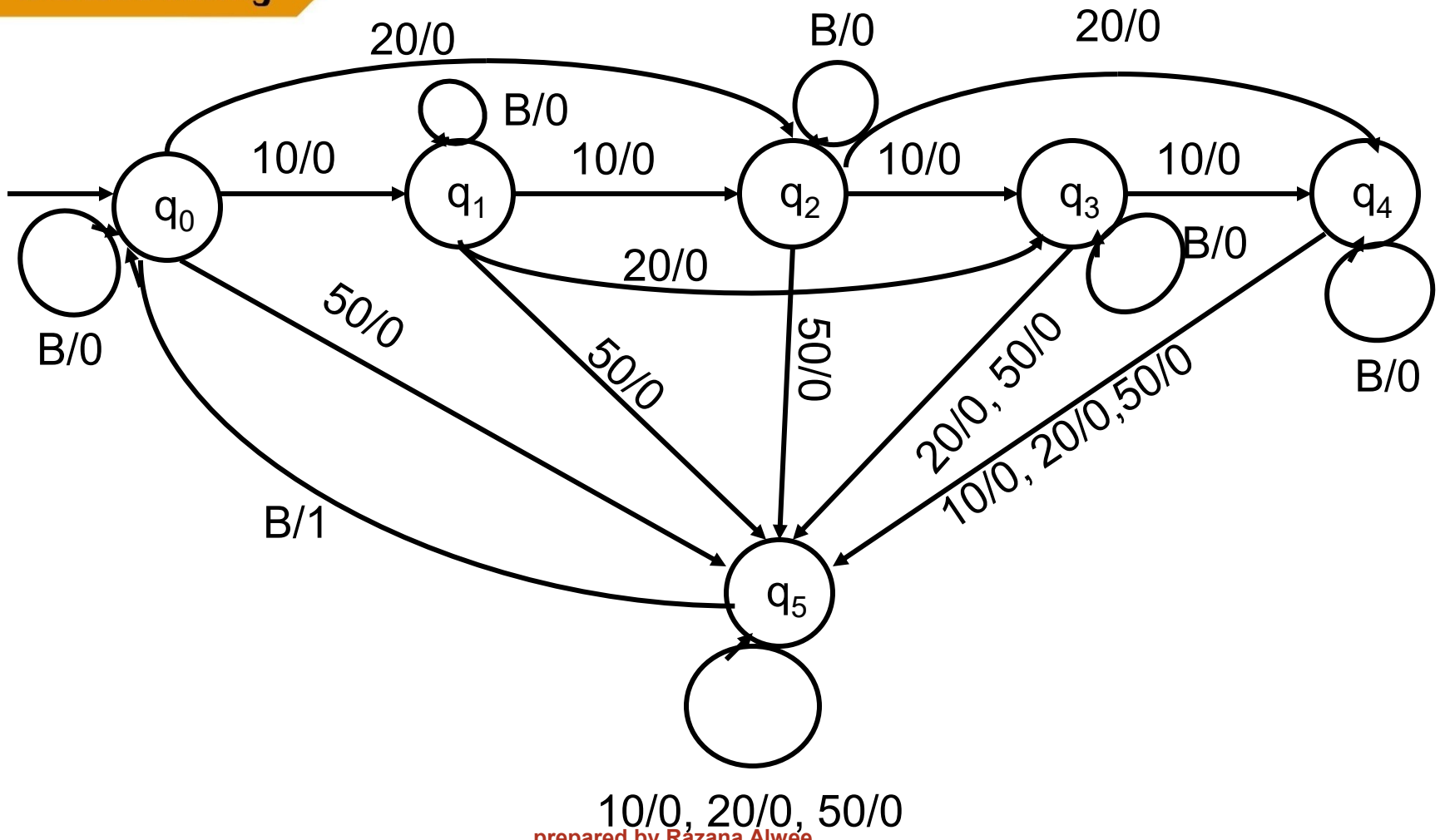
$O= \{0,1\}$,

q_0 = initial state,

example

	f_s				f_o			
	10	20	50	B	10	20	50	B
q_0	q_1	q_2	q_5	q_0	0	0	0	0
q_1	q_2	q_3	q_5	q_1	0	0	0	0
q_2	q_3	q_4	q_5	q_2	0	0	0	0
q_3	q_4	q_5	q_5	q_3	0	0	0	0
q_4	q_5	q_5	q_5	q_4	0	0	0	0
q_5	q_5	q_5	q_5	q_0	0	0	0	1

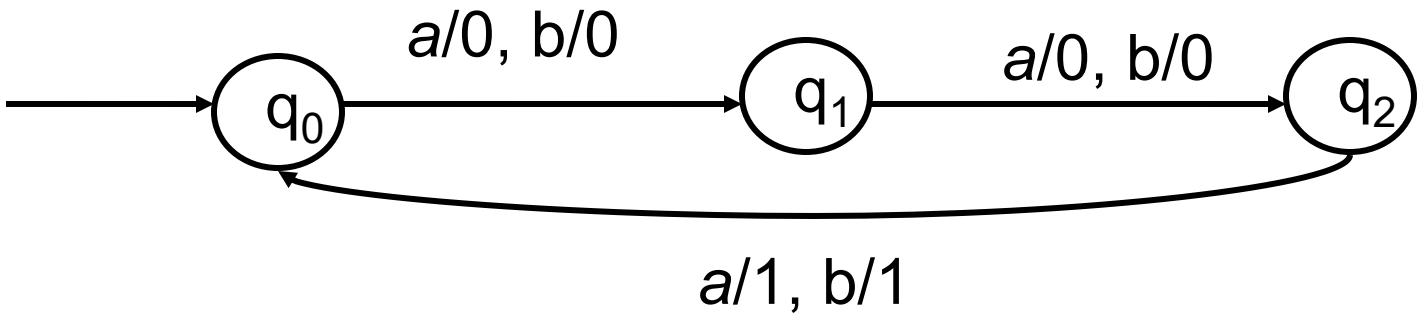
example


20/0
B/0
20/0
B/0
10/0
10/0
10/0
10/0
q_0
q_1
q_2
q_3
q_4
B/0
20/0
B/0
50/0
50/0
50/0
20/0, 50/0
10/0, 20/0,50/0
B/0
B/1
q_5
10/0, 20/0, 50/0

example

- Design a FSM, with input alphabet I={a, b}, that outputs a 1 if the number of input symbols read so far is divisible by 3.

example



exercise

Let M= { S, I, O, q_0, f_s, f_o} be a FSM

where,

S ={q_0,q_1,q_2},

I ={a,b},

O ={0,1},

q_0 = initial state,

exercise

f_s and f_0

	f_s		f_o	
	a	b	a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

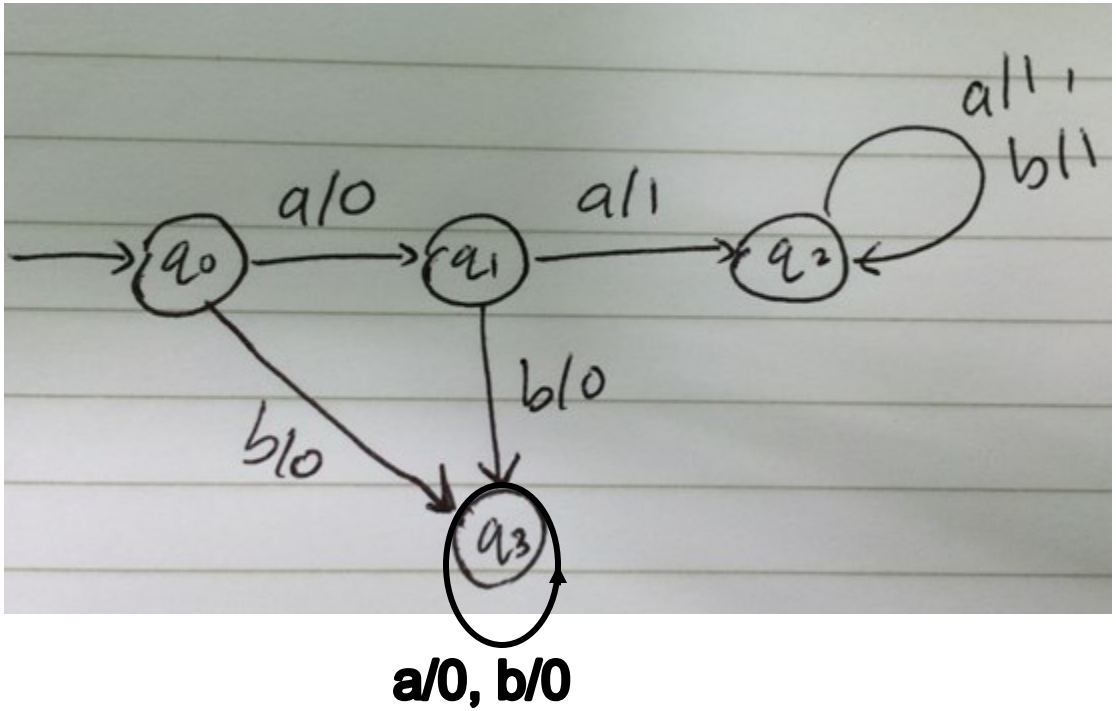
exercise

- Draw the transition diagram of M.
- What is the output string if the input string is *aabbb*?
- What is the output string if the input string is *ababab*?
- What is the output if the input string is *abbbaba*?
- What is the output if the input string is *bbbababa*?

exercise

- Design a FSM that accepts all string over {a,b} that begin with aa.
- For example: aaab, aabba, aababab

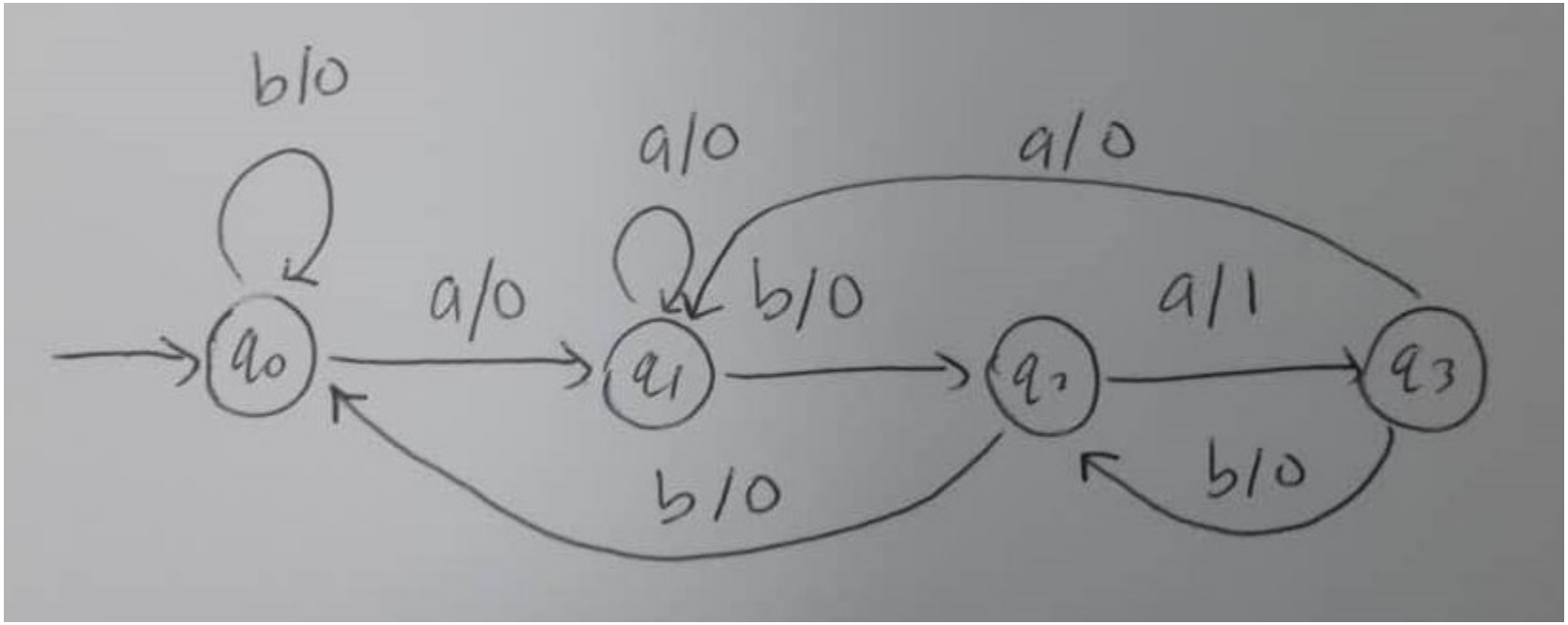
solution



Exercise

- Design a FSM that accepts all string over {a,b} that end with aba.
- For example: aaba, aababa, bbbaba

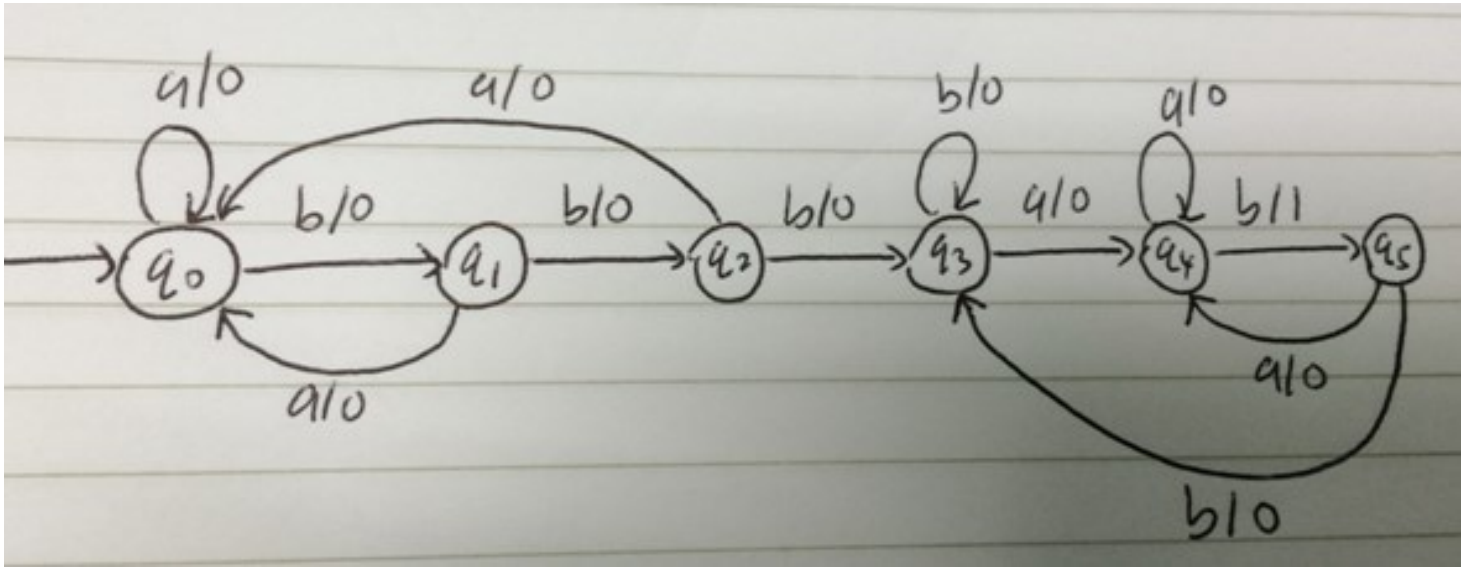
Solution



Exercise

- Design a FSM that accepts all string over {a,b} that contain bbb and end in ab

Solution



In a standard washing machine operation, there are four phases which start with Idle/Stop, Wash, Rinse and Spin. When the start/stop button is pressed, the door will be automatically locked, timer will start and the washing machine will begin to wash. After the timer end, the washing machine starts the rinse phase and timer for rinsing phase will begin. After the timer is end, the spin phase will begin. At this point, the timer will start again and after it end, operation of the washing machine is finish and it returns to Idle/Stop condition. At any time during the operation, if the start/stop button is pressed again, the washing machine will stop the operation and return to Idle/Stop condition. The door will always remain locked during the operation unless it is in Idle/Stop condition.

Based on the above washing machine operation,

a) define all the states, inputs and outputs.

b) construct a transition diagram.

States : *Idle/Stop, Wash, Rinse, Spin*
Inputs : *Start/Stop button pressed, Timer start, Timer stop*
Outputs : *Door Locked, Door Unlocked*

Inputs	B	Start/Stop button pressed
	TS	Timer start
	TE	Timer end
Outputs	DL	Door locked
	DU	Door Unlocked