SECI1013: DISCRETE STRUCTURE SEM 1 20242025

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Assignment 1 – Set Theory and Logic, Function
Group of 3, Due date: 17 November 2024

1. Let A={ 1, 3, 5, 7 }, B={ 3, 4, 5, 6 }, and C={ 5, 6, 7, 8 }.
   a) Prove that $(A \cap B) \cup (B \cap C) \subseteq B$.
   b) Prove or disprove: $(A \cup C) - B = (A - B) \cup (C - B)$.
   c) Find $A \oplus (B \cap C)$ and determine whether this result is equal to $(A \oplus B) \oplus C$.

a) $A \cap B = \{3, 5\}$
$B \cap C = \{5, 6\}$
$(A \cap B) \cup (B \cap C) = \{3, 5, 6\} \subseteq B = \{3, 4, 5, 6\}$
$(A \cap B) \cup (B \cap C) \subseteq B$

b) $A \cup C = \{1, 3, 5, 6, 7, 8\}$
$(A \cup C) - B = \{1, 7, 8\}$
$A - B = \{1, 7\}$
$C - B = \{7, 8\}$
$(A - B) \cup (C - B) = \{1, 7, 8\}$
$(A \cup C) - B = (A - B) \cup (C - B)$

c) $B \cap C = \{5, 6\}$
$A \oplus (B \cap C) = \{1, 3, 6, 7\}$
$A \oplus B = \{1, 4, 6, 7\}$
$(A \oplus B) \oplus C = \{1, 4, 5, 8\}$
$A \oplus (B \cap C) \neq (A \oplus B) \oplus C$

2. Define the following sets:
   - P={ x ∈ Z | x is a prime number less than 20 }
   - E={ x ∈ Z | x is an even number less than 20 }
   - D={ x ∈ Z | x is a divisor of 36 and less than 20 }

Using these sets:

a) Translate the statement: "All elements of PPP that are not in DDD are also in EEE" into a formal logic expression using quantifiers.

b) Prove or disprove the above statement in (a) by checking each element.

c) Prove that (P∩E)∪D=D using set theory laws.

$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$E = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$D = \{1, 2, 3, 4, 6, 9, 12, 18\}$

a) $\forall x\,((P(x) \wedge \neg D(x)) \rightarrow E(x))$

b) $P(x) \wedge \neg D(x) = \{5, 7, 11, 13, 17, 19\}$

$E(x) = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$\forall x\,((P(x) \wedge \neg D(x)) \rightarrow E(x)) = \emptyset$, so this statement is FALSE.

c) $(P \cap E) \cup D = D$

$P \cap E = \{2\}$

$\{2\} \subseteq D$

$(P \cap E) \subseteq D$

So, $(P \cap E) \cup D = D$

3. Let U be the universal set of all integers from 1 to 30. Define the following subsets of U:

- A={ x ∈ U | x is a multiple of 2}
- B={ x ∈ U | x is a multiple of 3}
- C={ x ∈ U | x is a multiple of 5}

a) Construct a Venn diagram that represents U, A, B, and C.

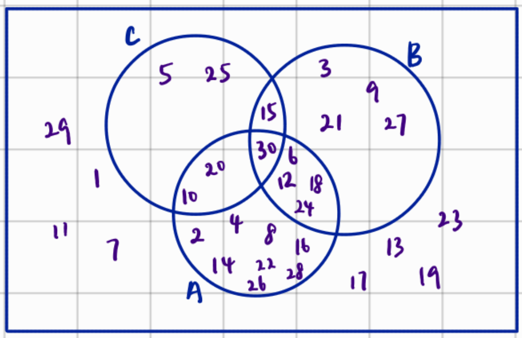
b) Identify the number of elements in each region of the Venn diagram (e.g., $A \cap B \cap C$, $A-B-C$, etc.).

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$

$B = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$

$C = \{5, 10, 15, 20, 25, 30\}$

a)



b) $|A-B-C| = 8$

$|B-A-C| = 4$

$|C-A-B| = 2$

$|(A \cap B)-C| = 4$

$|(A \cap C)-B| = 2$

$|(B \cap C)-A| = 1$

$|A \cap B \cap C| = 1$

$|U-(A \cup B \cup C)| = 8$

4. Let X={ 0, 1, 2 } and Y={ a, b }.
   a) Find the Cartesian product X×Y and list each ordered pair.
   b) Define a relation $R \subseteq X \times X$ such that $(x, y) \in R(x, y)$ if and only if $x + y$ is even. List all pairs in R.
   c) Determine if R is reflexive, symmetric, or transitive, and provide proof for each property.

a) $X \times Y = \{(0,a),(0,b),(1,a),(1,b),(2,a),(2,b)\}$

b) $0+0=0$ $\quad 1+0=0$ $\quad 2+0=2$

$0+1=1$ $\quad 1+1=2$ $\quad 2+1=3$

$0+2=2$ $\quad 1+2=3$ $\quad 2+2=4$

$R = \{(0,0),(0,2),(1,1),(2,0),(2,2)\}$

c) R is reflexive because $\{(0,0),(1,1),(2,2)\} \in R$

R is symmetric because $\{(0,2),(2,0)\} \in R$

R is transitive because $\{(0,2),(2,0)\} \in R$ $\quad (0,0) \in R$

$\{(0,0),(0,2)\} \in R$ $\quad (0,2) \in R$

$\{(2,0),(0,2)\} \in R$ $\quad (2,2) \in R$

5. Consider the following statements:

a) Prove the contrapositive of the following statement: "If $x \in Z$ is even and x^2 is also even, then x is a multiple of 4."

b) Determine whether the following statement is true or false. If true, prove it. If false, provide a counterexample:

- "For all integers *m* and *n*, if *mn* is even, then *m* is even or *n* is even."

c) Using a proof by contradiction, show that the square root of 3 is irrational.

Contrapositive:

a) If x is not a multiple of 4, then $x \in \mathbb{Z}$ is odd and x^2 is also odd.

assume x not a multiple of 4,

$x = 2k+1$ for some integer k

$x = 2k+1$ x is an odd integer

$x^2 = (2k+1)^2$

$x^2 = (2k+1)(2k+1)$

$x^2 = 4k^2+4k+1$

$x^2 = 2(2k^2+2k)+1$

$x^2 = 2m+1$ where $m = 2k^2+2k$ is an integer

x^2 is an odd integer

multiple of $4 = \{4, 8, 12, 16, \dots\}$ → even

$\neg$multiple of 4 = odd

Counter example = 2

If $x = 2$, is even

$x^2 = 4$, is also even

but $2 \neq$ multiple of 4

b) when m / n is even

$mn = (2k+1)(2k)$

$= 4k^2+2k$

$= 2(2k^2+k)$ $2k^2+k$ for some integer

$= 2x$

$\downarrow$

mn is even

TRUE

Contradiction:

c) If $\sqrt{3}$ is irrational, then $\sqrt{3}$ cannot be form $\frac{p}{q}$

Suppose that the conclusion is false. Then $\sqrt{3}$ can be form $\frac{p}{q}$

$\sqrt{3} = 1.732050808\dots$

$\sqrt{3}$ cannot be form of $\frac{p}{q}$

The stament is true.

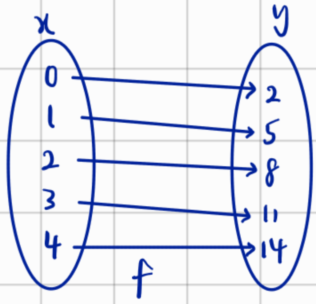
6. Let $f: Z \to Z$ be defined by $f(x) = 3x + 2$.

a) Prove or disprove that f is one-to-one.

b) Prove or disprove that f is onto.

c) Find f−1(y), if it exists, for y ∈ Z, and determine the conditions under which f has an inverse function.

a)



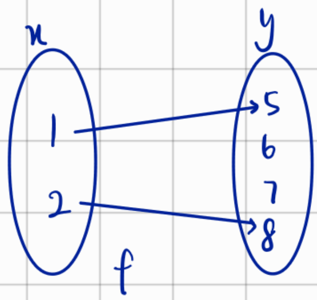
f is one to one because each element in y has at most one arrow

if $f(x_1) = f(x_2)$

$3x_1 + 2 = 3x_2 + 2$

$x_1 = x_2$

b)



f is not onto because each element in y don't have at least one arrow pointing to it

c)

$y = 3x + 2$

$3x = y - 2$

$x = \frac{y-2}{3}$

$f^{-1}(y) = \frac{y-2}{3}$

inverse function exists when $3x-1,\ x \in \mathbb{Z}$

7. Let $g: R \to R$ be defined by $g(x) = x^2$ and $h(x) = x + 1$.

a) Compute $(g \circ h)(x)$ and $(h \circ g)(x)$.

b) Determine whether $g \circ hg$ and $h \circ gh$ are one-to-one, onto, or neither.

c) If possible, find the inverse of $(h \circ g)(x)$.

a) $(g \circ h)(x) = g(h(x))$

$= g(x+1)$

$= (x+1)^2$

$= x^2 + 2x + 1$

$(h \circ g)(x) = h(g(x))$

$= h(x^2)$

$= x^2 + 1$

b) $g \circ hg = g(h(g(x)))$

$= g(h(x^2))$

$= g(x^2+1)$

$= (x^2+1)^2$

$= x^4 + 2x^2 + 1$

$x = -1$, $(-1)^4 + 2(-1)^2 + 1 = 4$

$x = 1$, $1^4 + 2(1)^2 + 1 = 4$

not one to one,

not onto because no number can point at codomain which < 1

$h \circ gh = h(g(h(x)))$

$= h(g(x+1))$

$= h(x+1)^2$

$= h(x^2 + 2x + 1)$

$= x^2 + 2x + 1 + 1$

$x = 0$, $0^2 + 2(0) + 1 + 1 = 2$

$x = -2$, $(-2)^2 + 2(-2) + 1 + 1 = 2$

not one to one,

not onto because no number can point at codomain which < 1.

c) $(h \circ g)(x) = h(g(x))$

$= h(x^2)$

$= x^2 + 1$

let $h(g(x)) = y$

$y = x^2 + 1$

$x^2 = y - 1$

$x = \pm\sqrt{y-1}$, $y \geq 1$

$f^{-1}(y) = \pm\sqrt{y-1}$, $y \geq 1$