Assignment 2 (Due: Jun. 11, 2021)

1. (**Math**) In the augmented Euclidean plane, there is a line $x-3y+4=0$, what is the homogeneous coordinate of the infinity point of this line?

2. (**Math**) *A*, *B*, *C* and *D* are four points in 3D Euclidean space, their coordinates are $(x_i, y_i, z_i), i=1,2,3,4$, respectively. Please prove that:

$$\text{These four points are coplanar} \Leftrightarrow \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

3. (**Math**) On the normalized retinal plane, suppose that $\mathbf{p}_n$ is an ideal point of projection without considering distortion. If distortion is considered, $\mathbf{p}_n=(x, y)^T$ is mapped to $\mathbf{p}_d=(x_d, y_d)^T$ which is also on the normalized retinal plane. Their relationship is,

$$\begin{cases} x_d = x\left(1+k_1r^2+k_2r^4\right)+2\rho_1xy+\rho_2\left(r^2+2x^2\right)+xk_3r^6 \\ y_d = y\left(1+k_1r^2+k_2r^4\right)+2\rho_2xy+\rho_1\left(r^2+2y^2\right)+yk_3r^6 \end{cases}$$

where $r^2=x^2+y^2$

For performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian matrix of $\mathbf{p}_d$ w.r.t $\mathbf{p}_n$, i.e.,

$$\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T}$$

It should be noted that in this question $\mathbf{p}_d$ is the function of $\mathbf{p}_n$ and all the other parameters can be regarded as constants.

4. (**Math**) In our lecture, we mentioned that for performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian of the rotation matrix (represented in a vector) w.r.t its axis-angle representation. In this question, your task is to derive the concrete formula of this Jacobian matrix. Suppose that

$\mathbf{r}=\theta\mathbf{n}\in\mathbb{R}^{3\times1}$, where $\mathbf{n}=\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ is a 3D unit vector and θ is a real number denoting the rotation angle.

With Rodrigues formula, $\mathbf{r}$ can be converted to its rotation matrix form,

$$\mathbf{R}=\cos\theta\mathbf{I}+(1-\cos\theta)\mathbf{n}\mathbf{n}^T+\sin\theta\mathbf{n}^\wedge$$

and obviously $\mathbf{R}\triangleq\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$ is a 3×3 matrix.

Denote $\mathbf{u}$ by the vectorized form of $\mathbf{R}$, i.e.,

$$\mathbf{u}\triangleq\left(R_{11},R_{12},R_{13},R_{21},R_{22},R_{23},R_{31},R_{32},R_{33}\right)^T$$

Please give the concrete form of Jacobian matrix of $\mathbf{u}$ w.r.t $\mathbf{r}$, i.e., $\frac{d\mathbf{u}}{d\mathbf{r}^T}\in\mathbb{R}^{9\times3}$.

In order to make it easy to check your result, please follow the following notation requirements,

$$\alpha\triangleq\sin\theta, \beta\triangleq\cos\theta, \gamma\triangleq1-\cos\theta$$

In other words, the ingredients appearing in your formula are restricted to $\alpha, \beta, \gamma, \theta, n_1, n_2, n_3$.

5. (**Programming**) RANSAC is widely used in fitting models from sample points with outliers. Please implement a program to fit a straight 2D line using RANSAC from the following sample points: (-2, 0), (0, 0.9), (2, 2.0), (3, 6.5), (4, 2.9), (5, 8.8), (6, 3.95), (8, 5.03), (10, 5.97), (12, 7.1), (13, 1.2), (14, 8.2), (16, 8.5) (18, 10.1). Please show your result graphically.