

ECEN620-LOOP CHARACTERISTICS MATHEMATICS

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26. November 2023

1 Loop filter

1.1 Passive Lag-lead Filter

$$F(s) = \frac{1 + s\tau_2}{1 + s(\tau_1 + \tau_2)}$$

$$\begin{aligned}\Rightarrow H(s) &= \frac{K_{pd} \cdot K_{vco} \cdot \frac{1 + s\tau_2}{1 + s(\tau_1 + \tau_2)}}{K_{pd} \cdot K_{vco} \cdot \frac{1 + s\tau_2}{1 + s(\tau_1 + \tau_2)} + N \cdot s} \\ &= \frac{\frac{K_{pd} \cdot K_{vco} \cdot \tau_2}{N \cdot (\tau_1 + \tau_2)} \cdot \left(s + \frac{1}{\tau_2}\right)}{s^2 + \frac{1 + \frac{K_{pd} \cdot K_{vco} \cdot \tau_2}{N}}{\tau_1 + \tau_2} \cdot s + \frac{K_{pd} \cdot K_{vco}}{N \cdot (\tau_1 + \tau_2)}}\end{aligned}$$

$$\begin{aligned}\Rightarrow 2\omega_n\zeta &= \frac{1 + K_{pd} \cdot K_{vco} \cdot \tau_2}{\tau_1 + \tau_2} & \omega_n^2 &= \frac{K_{pd} \cdot K_{vco}}{N \cdot (\tau_1 + \tau_2)} \\ \Rightarrow \omega_n &= \sqrt{\frac{K_{pd} \cdot K_{vco}}{N \cdot (\tau_1 + \tau_2)}} & \zeta &= \frac{N + K_{pd} \cdot K_{vco} \cdot \tau_2}{N \cdot (\tau_1 + \tau_2) \cdot 2\omega_n} = \frac{\omega_n}{2} \cdot \left(\tau_2 + \frac{N}{K_{pd} \cdot K_{vco}}\right)\end{aligned}$$