Spring 2021 **Firstname M. Lastname**

University of Central Florida
College of Business

QMB 6911
Capstone Project in Business Analytics

Solutions: Problem Set #11

Chapter 1

Introduction

1.1 Introduction

In this paper I analyze the prices of fly reels. My aim is to investigate the value of fly reels produced in different locations. Does there exist a Made-in-America premium? If so, how large is it? Does the premium depend on the characteristics of the fly reels? The answers to these questions can be applied to the operational decisions of fly reel manufacturers. In particular, this analysis can be used to make recommendations on whether fly reels should be made in America or produced overseas.

In the following pages, I will investigate these questions and ultimately fit a model for the price of fly reels, allowing for endogeneity in the choice of the characteristics of fly reels and the location in which they are manufacturers produced and sold by each manufacturer. To understand the dynamics of a sample selection model, it is worthwhile to understand the research related to these questions.

1.2 Economic Theory

1.2.1 Market for Lemons

Akerlof's description of the market for lemons is a long-standing contribution to the understanding of asymmetric information.

1.2.2 Characteristic Theory

The paper by Lancaster (1966) is a contribution to economic theory in which he makes the case for a sound theory of consumer choice, in which the consumer's preferences are defined on the characteristics of different goods, rather than the quantities of uniformly-defined goods.

1.2.3 Hedonic Pricing Models

Rosen (1974) builds on the work of Lancaster (1966) by providing an empirical framework for estimating the value of products based on their characteristics.

1.3 Empirical Framework

1.3.1 Tobit Models

Heckman (1979) described sample selection models as a model specification question.

Amemiya (1984) summarized the variaous types of Tobit models and provides a review of research with applications of these models.

Lee and Trost (1978) is a well-known application to housing markets, which incorporates the fact that the residents make a decision to either own or rent a home before buying.

Chapter 2

Data Description

2.1 Data Description

By engaging an industry consultant to gather relevant and appropriate information, your firm has been able to put together data concerning 248 different fly-fishing reels, over one-half of which are produced in the United States, with the remainder being produced in Asia—either in China or Korea. These data are contained in the file `FlyReels.csv`, which is available in the `Data` folder. Each fly-fishing reel in the data set is a row, while the columns correspond to the variables whose names and definitions are the following:

Variable	Definition
`Name`	product name (a string)
`Brand`	brand name (a string)
`Weight`	weight of reel in ounces (a real number)
`Diameter`	diameter of reel in inches (a real number)
`Width`	width of reel in inches (a real number)
`Price`	price of reel in dollars (a real number)
`Sealed`	whether the reel is sealed; `"Yes"` versus `"No"` (a string)
`Country`	country of manufacture, (a string)
`Machined`	whether the reel is machined versus cast; machined=`"Yes"`, while cast=`"No"` (a string)

Chapter 3

Analysis of the Dependent Variable

3.1 Empirical Distribution Function of Fly Reel Prices

Figure 3.1 is a plot of the empirical cumulative distribution function (CDF) of fly reel prices.

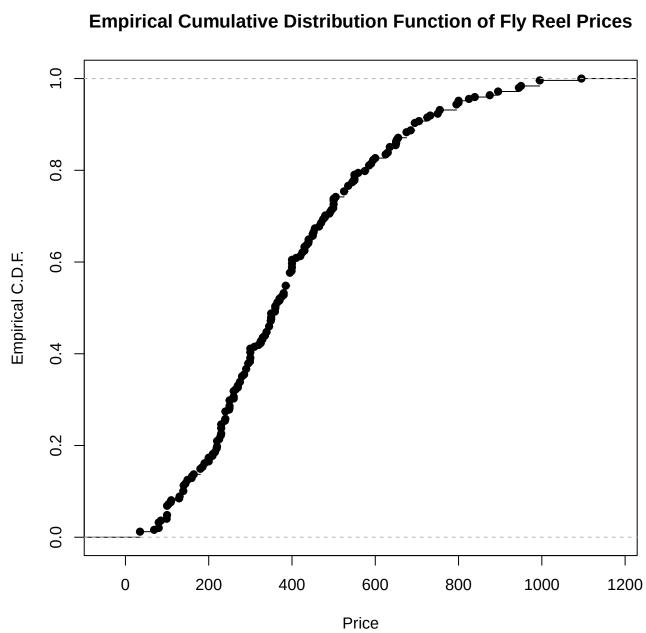


Figure 3.1: Empirical Distribution Function of Fly Reel Prices

3.2 Relative Histogram of Fly Reel Prices

Figure 3.2 is a histogram of fly reel prices.

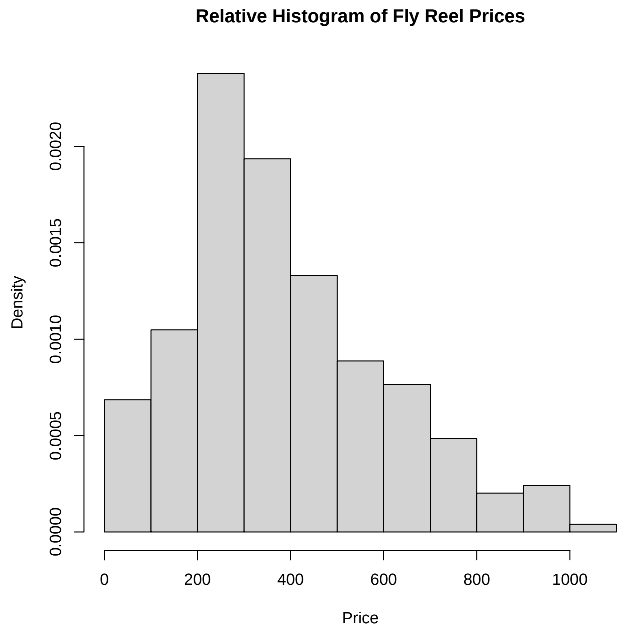


Figure 3.2: Relative Histogram of Fly Reel Prices

3.3 Probability Density Function of Fly Reel Prices

Figure 7.1 depicts the kernel-smoothed probability density function of the natural logarithm of price.

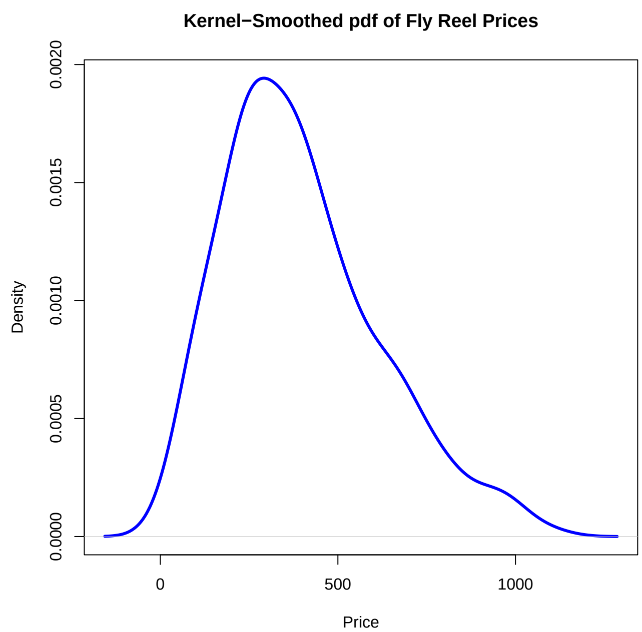


Figure 3.3: Probability Density Function of Fly Reel Prices

Chapter 4

Transforming the Dependent Variable

4.1 Probability Density Function of Fly Reel Prices

Figure 7.1 shows the kernel-smoothed probability density function of fly reel prices.

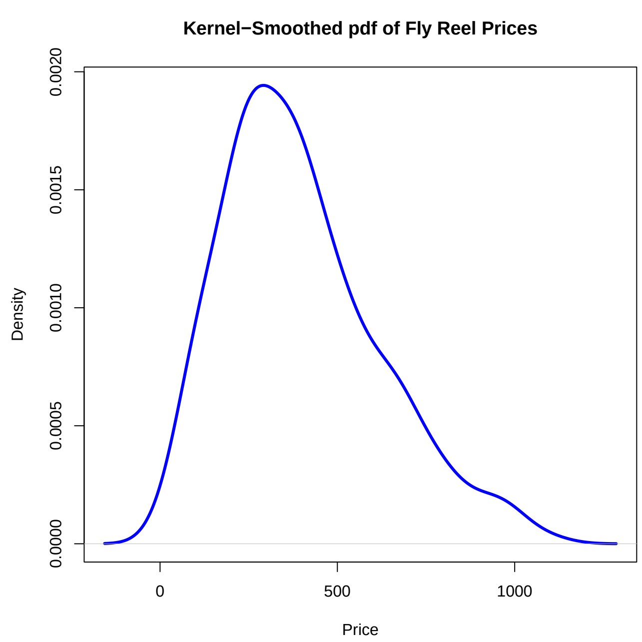


Figure 4.1: Probability Density Function of Fly Reel Prices

As a comparison, Figure 7.2 shows the kernel-smoothed probability density function of the natural logarithm of price.

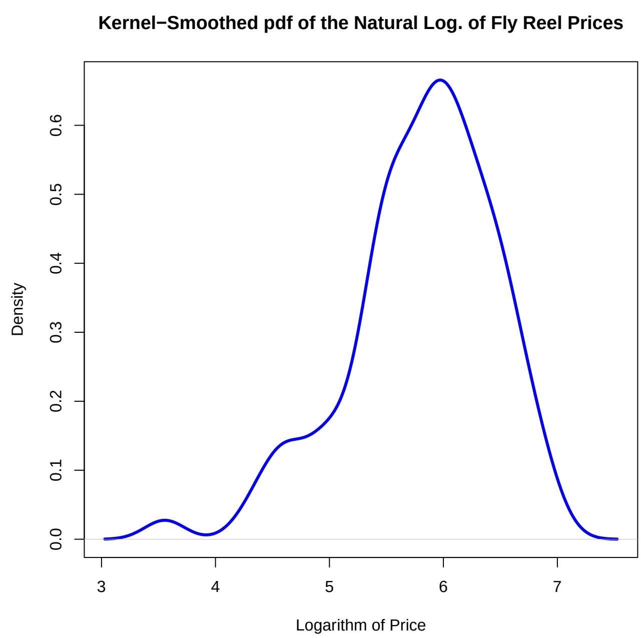
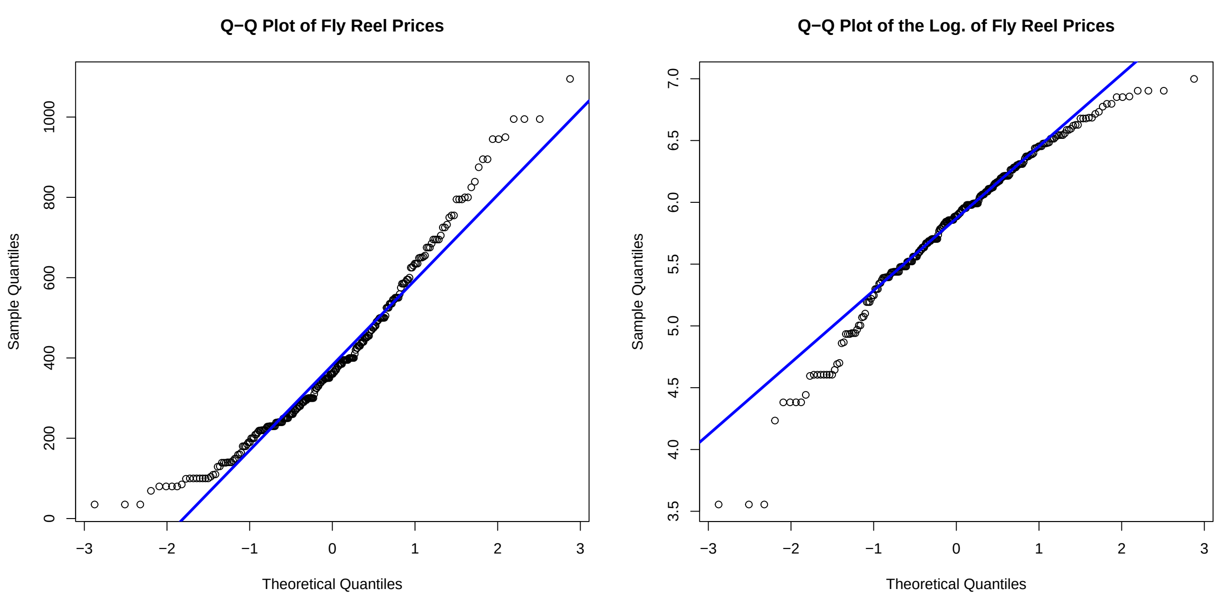


Figure 4.2: Probability Density Function of the Logarithm of Fly Reel Prices

4.2 Normality of the Original and Transformed Variables

Figure 4.8 shows a pair of Q-Q plots, comparing quantiles of the empirical distribution against the quantiles of the normal distribution. In the left panel, Figure 4.8a shows this comparison for the original level of the fly reel prices, without transformation. In the right panel, Figure 4.3b shows this comparison for the logarithmic transformation of fly reel prices, without transformation. Consistent with the pair of distributions estimated above, each plot shows a divergence from a normal distribution, suggesting that an optimal transformation might lie somewhere in the middle. The Box-Cox transformation allows for this possibility.



(a) Fly Reel Prices

(b) Transformed Fly Reel Prices

Figure 4.3: Q-QPlots of the Log. and Levels of Fly Reel Prices

4.3 Box-Cox Transformation of Fly Reel Prices

Under the Box–Cox transformation of P_n, the price of reel n is calculated as follows,

$$\Lambda(P_n) \equiv \begin{cases} \frac{P_n^\lambda - 1}{\lambda} & \text{if } \lambda > 0 \\ \log P_n & \text{if } \lambda = 0. \end{cases}$$

The following code block defines a function that performs a Box-Cox transformation.

```
# Box-Cox transformation.
Lambda_Price <- function(price, lambda) {
  if (lambda == 0) {
    return(log(price))
  } else {
    return((price^lambda - 1)/lambda)
  }
}
```

4.3.1 Log-likelihood Function

Under the Box-Cox transformation, the fly reel prices can be decomposed into a location parameter μ^0 and an error U, so

$$\Lambda(P_n) = \mu^0(\lambda) + U_n,$$

where the U_ns are independent, mean-zero, constant-variance $\sigma^2(\lambda)$, Gaussian (normal) errors. In the above equation, for clarity, the dependence of μ^0 and $\sigma^2(\lambda)$ on λ is made explicit.

The next code block defines a likelihood function for the normal distribution of the errors as a function of the parameter λ.

```
log_like_uni <- function(price, lambda) {

  # Calculate maximum likelighood estimates of the parameters.
  n <- length(price)
  lambda_price <- Lambda_Price(price, lambda)
  mu_0_lambda <- mean(lambda_price)
  sigma_2_lambda <- sum((lambda_price - mu_0_lambda)^2)/n

  # Calculate the log-likelihood from the sum of the logarithms
  # of the density of the normal distribution.
  like <- - n/2*log(2*pi*sigma_2_lambda)
  like <- like - 1/2/sigma_2_lambda*sum((lambda_price - mu_0_lambda)^2)
  like <- like + (lambda - 1)*sum(log(price))
  return(like)
}
```

As a first approximation, One can calculate the value of the log-likelihood function on a grid of values to find an optimal value of λ. The plot of this likelihood function is shown in Figure 4.4. The red points represent the values of the log-likelihood at the optimum $\lambda = 0.43$ and at $\lambda = 0$ and $\lambda = 1$.

```
# Calculate values of the log-likelihood function.
lambda_grid <- seq(-1, 2.5, by = 0.001)
like_grid <- 0*lambda_grid
for (lambda_num in 1:length(lambda_grid)) {
  like_grid[lambda_num] <- log_like_uni(price = flyreels[, 'Price'],
                                        lambda = lambda_grid[lambda_num])
}
```

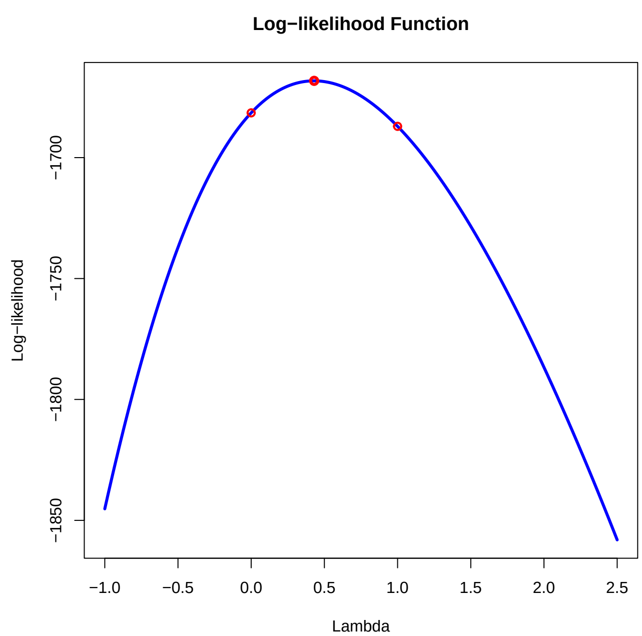


Figure 4.4: Log-likelihood Function for Box-Cox Transformation

4.3.2 Testing for an Appropriate Transformation

Now we consider the statistical properties of these estimates by calculating a likelihood ratio statistic.

```
> # Calculate likelihood ratio statistics.
> LR_stat_0 <- - 2*(like_mu_0 - like_MLE)
> print(LR_stat_0)
[1] 26.4604
> LR_stat_1 <- - 2*(like_mu_1 - like_MLE)
> print(LR_stat_1)
[1] 37.62182
>
>
> # Compare to quantile of chi-squared distribution with 1 degree of freedom.
> LR_cv_5 <- qchisq(p = 0.95, df = 1)
> print(LR_cv_5)
[1] 3.841459
>
> # Calculate p-values for these tests.
> p_value_0 <- 1 - pchisq(q = LR_stat_0, df = 1)
> print(p_value_0)
[1] 2.689959e-07
> p_value_1 <- 1 - pchisq(q = LR_stat_1, df = 1)
> print(p_value_1)
[1] 8.58782e-10
>
```

Statistically, this is evidence to reject them both. This suggests using the transformation at the MLE. However, one may want to investigate further to find out whether it is worth transforming the data. There exists a trade-of between interpretability and the accuracy of the statistical specification.

4.4 R Packages for the Box-Cox Transformation

4.4.1 Using the MASS Package

As an illustration, we calculated the likelihood ourselves. However, there exist other packages to output the estimation results for an optimal Box-Cox transformation.

One option is to use the function from the `MASS` package. This is an R package that accompanies a well-know statistics textbook and has a great reputation. In the `MASS` package, the notation is the same as for a linear model.

```
# In the MASS package, the notation is the same as for a linear model.
# i.e., summary(lm(Price ~ 1, data = flyreels))
bc_grid_MASS <- MASS::boxcox(Price ~ 1,
                             data = flyreels,
                             lambda = lambda_grid)
```

The output is plotted in Figure 4.5.

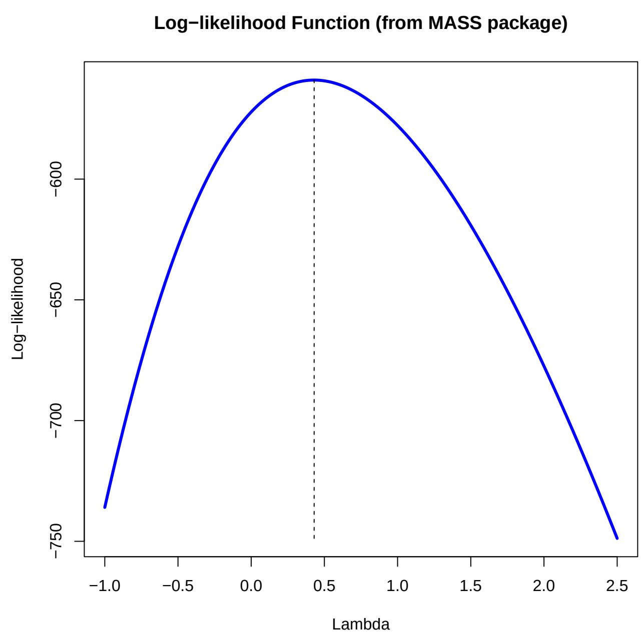


Figure 4.5: Log-likelihood Function for Box-Cox Transformation (`MASS` package)

4.5 Using the car Package

The car package is another well-known option. With this function, the optimization produces a figure automatically from the code below.

```
bc_grid_car <- car::boxCox(object = lm(data = flyreels,
                                       formula = Price ~ 1),
                           lambda = lambda_grid)
```

The output is plotted in Figure 4.6.

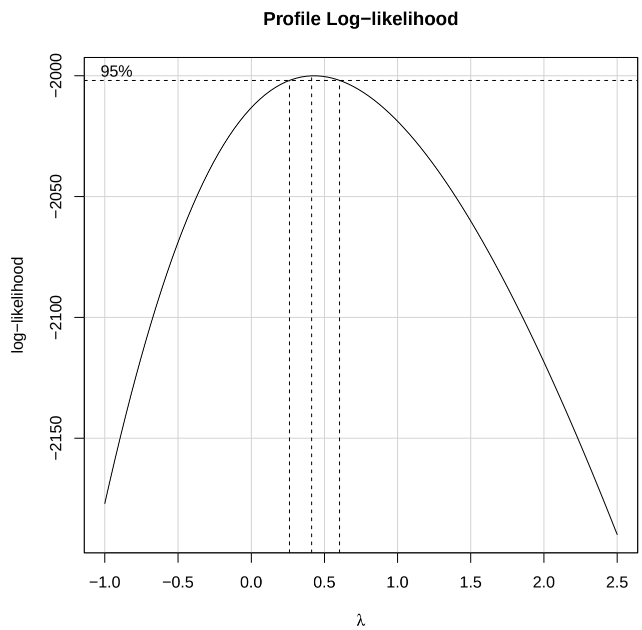


Figure 4.6: Log-likelihood Function for Box-Cox Transformation (car package)

4.6 Using the **EnvStats** Package

The `EnvStats` package is another option but it is one designed for environmental statistics. That is, it is not a generic package designed for the population of statisticians at large. For that reason, it is missing some of the features that a statistician would expect. The notation and interpretation, however, are similar, except that the straight call to `boxcox` simply does the calculation, unless you specify otherwise.

```
> # Find optimal value of lambda.
> bc_grid_ES_opt <- EnvStats::boxcox(x = flyreels[, 'Price'],
+                              lambda = range(lambda_grid),
+                              optimize = TRUE,
+                              objective.name = "Log-Likelihood")
>
> bc_grid_ES_opt$lambda
[1] 0.4295551
>
```

The output is plotted in Figure 4.7.

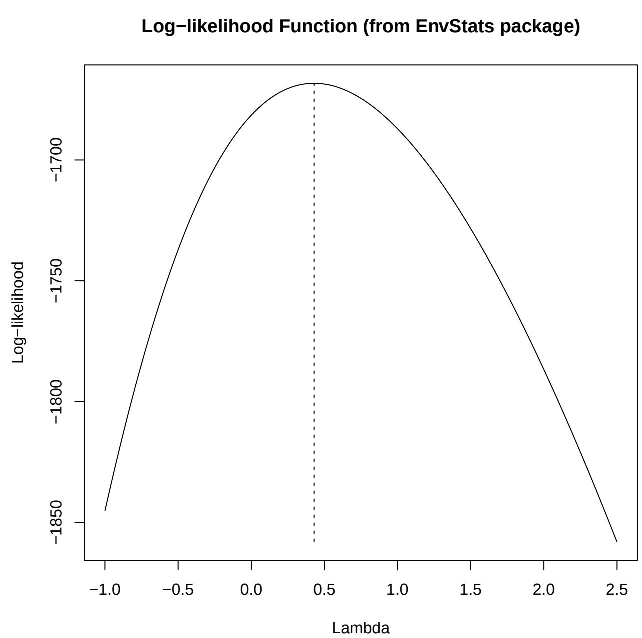


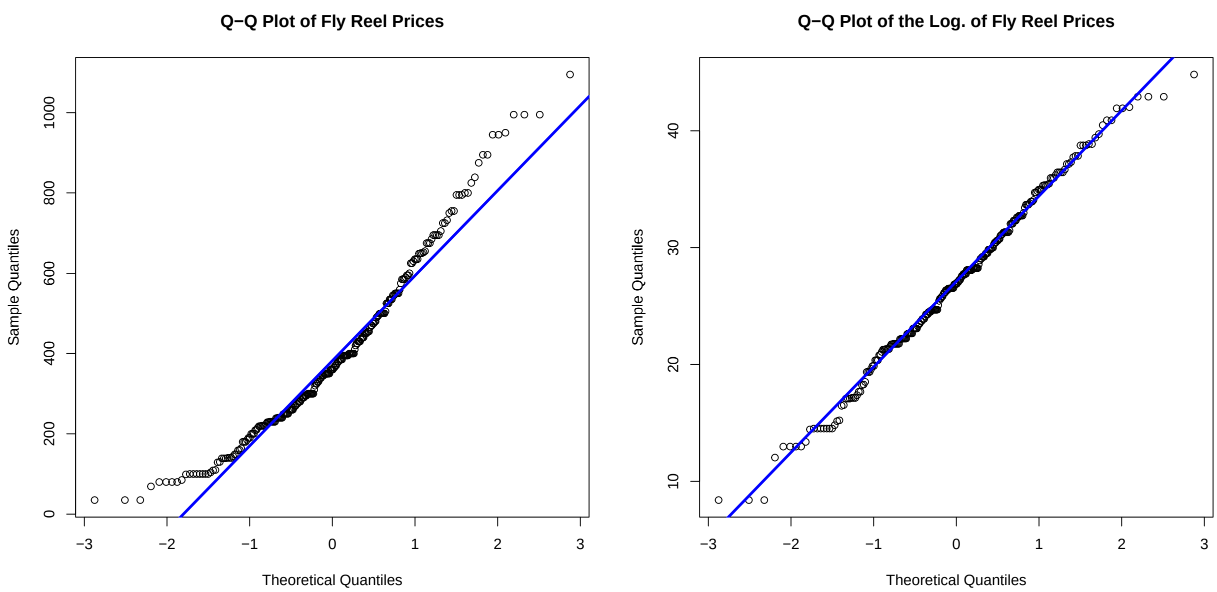
Figure 4.7: Log-likelihood Function for Box-Cox Transformation (`EnvStats` package)

4.7 Normality of the Transformed Variable

Now compare the quantiles of the distribution of the transformed variable with the original. We already plotted normal QQ plot for fly reel prices when considering the log transformation. Now we can generate a new dependent variable with the results from the estimates above.

```
# Generate new dependent variable with results from estimates above.
flyreels[, 'Trans_Price'] <- Lambda_Price(price = flyreels[, 'Price'],
                                          lambda = lambda_hat)
```

Figure 4.8 shows this comparison and the panel on the right, Figure 4.8b, shows that the quantiles of the distribution of the transformed variable nearly overlap with those of the normal distribution. From a purely statistical perspective, this provides evidence that the prices are best modeled with the transformation at the optimal $\lambda = 0.43$. From a practical point of view, however, it is still an open question whether this added complexity is warranted when other variables are added to the model.



(a) Fly Reel Prices (b) Transformed Fly Reel Prices

Figure 4.8: Q-QPlots of the Transformed Fly Reel Prices

Chapter 5

Preliminary Tabular Analysis

I analyze the data in subsets, according to country of manufacture, calculating the summary statistics for each subset and present these statistics in the LaTeX tables that follow.

5.1 Summary by Country of Manufacture

Table 5.1 lists summary statistics for numeric variables in separate columns for subsamples defined by the country of manufacture.

	China	Korea	USA
Min. Weight	3.000	7.296	12.900
Mean Weight	2.100	6.500	15.097
Max. Weight	2.540	6.459	14.800
Min. Diameter	2.500	3.935	5.250
Mean Diameter	2.750	3.925	5.500
Max. Diameter	2.700	3.878	5.430
Min. Width	0.790	1.093	1.570
Mean Width	0.7874	1.1434	1.5800
Max. Width	0.750	1.070	1.688
Min. Price	129.0	331.6	600.0
Mean Price	34.99	280.22	839.00
Max. Price	200.0	484.9	1095.0

Table 5.1: Summary by Country of Manufacture

5.2 Country of Manufacture by Brand of Fly Reel

Table 5.2 lists the frequencies of observations of each brand of fly reel by country of manufacture.

	China	Korea	USA	Total
3-TAND	15	0	0	15
Abel	0	0	15	15
Allen	0	18	7	25
Aspen	0	0	8	8
Bauer	0	0	2	2
Cheeky	11	0	0	11
ECHO	0	12	0	12
Galvan	0	0	23	23
Hatch	0	0	8	8
Loop	0	14	0	14
Nautilus	0	0	15	15
Orvis	1	0	1	2
Ross	0	0	28	28
Sage	0	6	0	6
Taylor	0	12	0	12
TFO	0	16	0	16
Tibor	0	0	4	4
Waterworks-Lamson	0	8	24	32
Totals	27	86	135	248

Table 5.2: Country of Manufacture by Brand of Fly Reel

5.3 Reel Design by Brand of Fly Reel

Table 5.3 lists the frequencies of observations of each brand of fly reel across two categorical variables: whether the reel is sealed and whether the reel is machined versus cast.

	Unsealed	Sealed	Cast	Machined	Total
3-TAND	0	15	0	15	15
Abel	9	6	0	15	15
Allen	8	17	1	24	25
Aspen	8	0	0	8	8
Bauer	0	2	0	2	2
Cheeky	6	5	6	5	11
ECHO	9	3	12	0	12
Galvan	20	3	0	23	23
Hatch	0	8	0	8	8
Loop	0	14	0	14	14
Nautilus	4	11	0	15	15
Orvis	0	2	0	2	2
Ross	21	7	0	28	28
Sage	2	4	0	6	6
Taylor	0	12	0	12	12
TFO	4	12	4	12	16
Tibor	3	1	0	4	4
Waterworks-Lamson	0	32	8	24	32
Totals	94	154	31	217	248

Table 5.3: Reel Design by Brand of Fly Reel

Chapter 6

Preliminary Graphical Analysis

6.1 Histogram and Density of Log. Fly Reel Prices

To begin visually analyzing the data, include plots of the density that were created last week.

6.1.1 All Fly Reels Together

Start with the log of prices because prices were skewed. Figure 6.1 is a histogram of the logarithm of fly reel prices, along with a rug plot and a kernel density estimate. After taking logs, we can

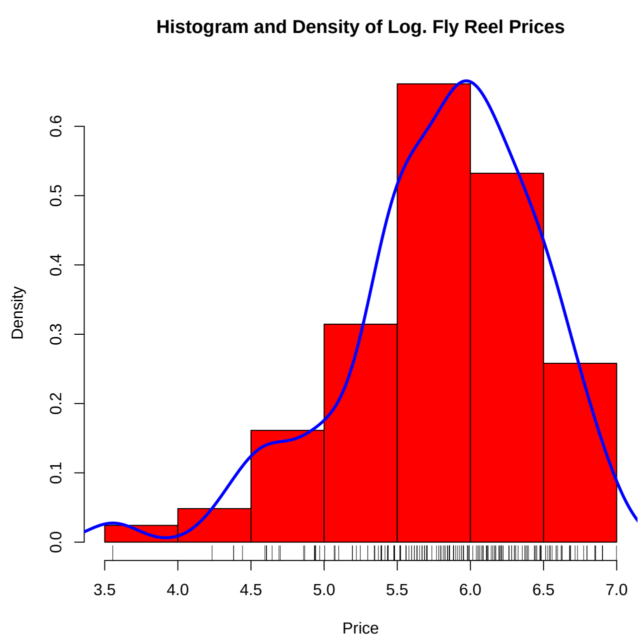


Figure 6.1: Relative Histogram of Fly Reel Prices

see that the distribution is approximately symmetric, now with a slight skew to the left. Unlike the case of the tractors, the improvement from the log transformation is not so clear, so we should investigate this further in a later problem set.

6.1.2 Comparison By Country of Manufacture

Now we investigate the prices of fly reels made in the USA compared to those made in China and Korea. Figure 6.2 shows the kernel density estimate of the prices of fly reels made in the USA in blue, those made in China in red, and those made in Korea in green. The modes of the distributions are similar, however, we observe more variability in the prices of fly reels made in Korea. The distribution of fly reels made in the USA is shifted toward the higher price range, compared to those made in other countries. This indicates mild support for a "Made in America" premium but we should also consider that it may be explained by the features of the reels made in the USA. We will investigate this further in regression analysis and other modeling approaches.

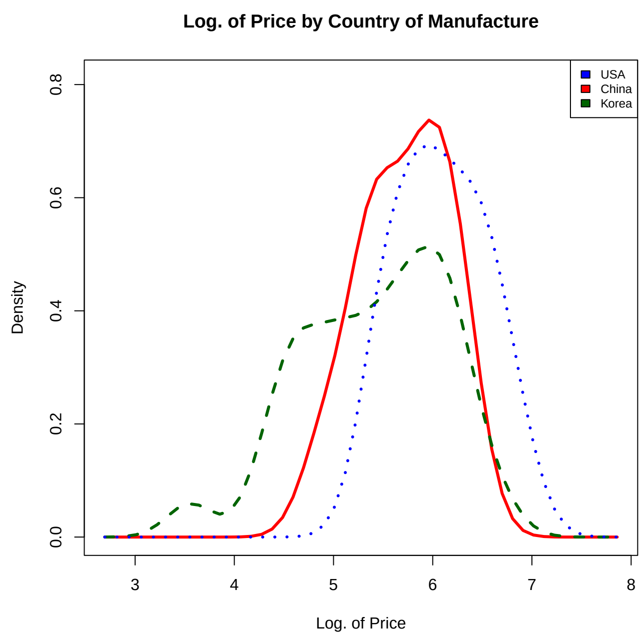


Figure 6.2: Densities of Log. Fly Reel Prices by Country of Manufacture

6.2 Scatterplot Matrices

6.2.1 Scatterplots of Numeric Variables

Figure 6.3 depicts a matrix of scatterplots of the numeric variables in the dataset.

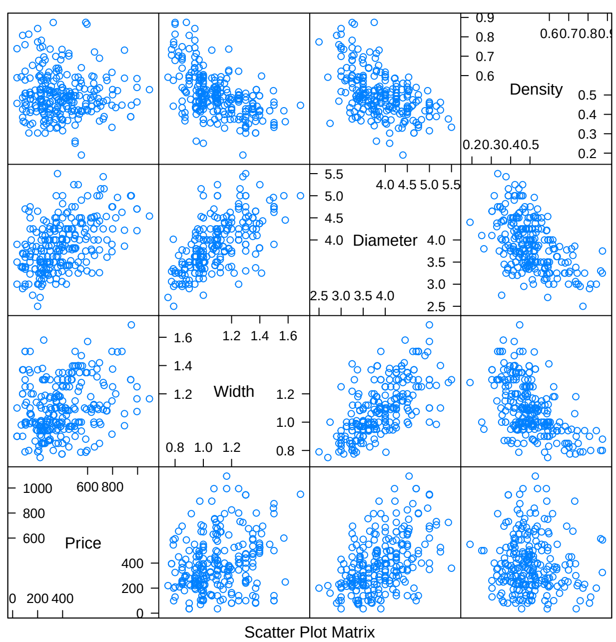


Figure 6.3: Scatterplots of Numeric Variables

6.2.2 Scatterplots with Categorical Variables

Figure 6.4 depicts a matrix of scatterplots of with categorical variables in the dataset.

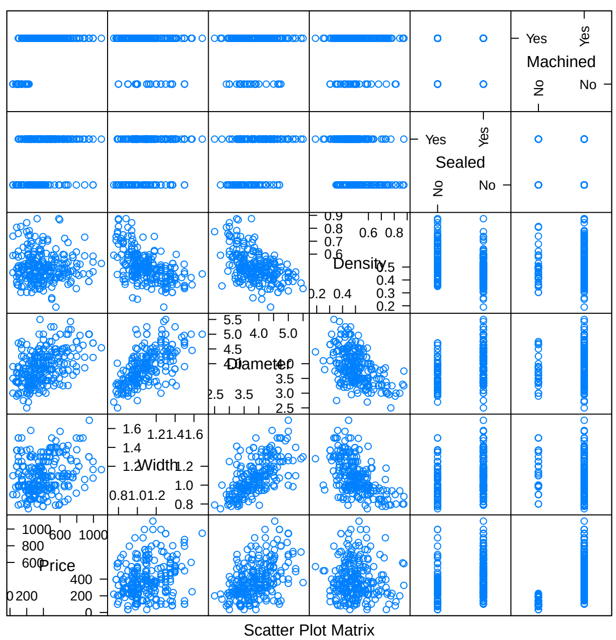


Figure 6.4: Scatterplots with Categorical Variables

Figure 6.5 depicts a matrix of scatterplots with a categorical variable for the design combinations of the fly reels: a fly reel is either sealed or unsealed and either machined or cast.

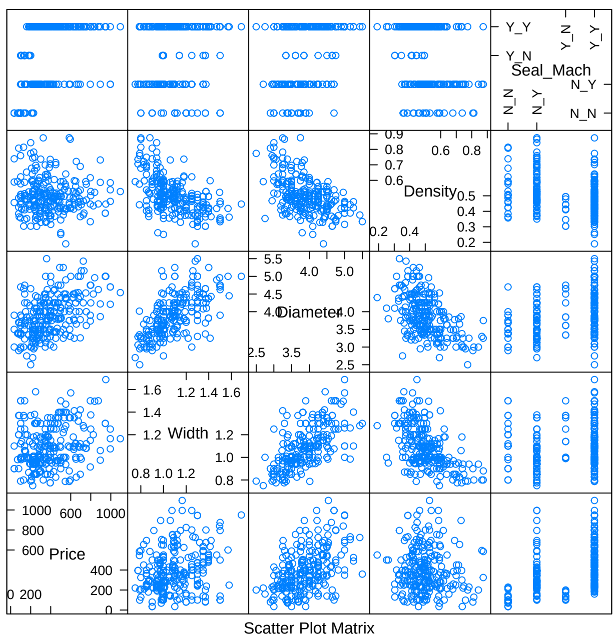


Figure 6.5: Scatterplots of Numeric Variables

6.2.3 Scatterplots by Country of Manufacture

Figure 6.6 depicts a matrix of scatterplots of the variables in the dataset with the points indicated differently by country of manufacture.

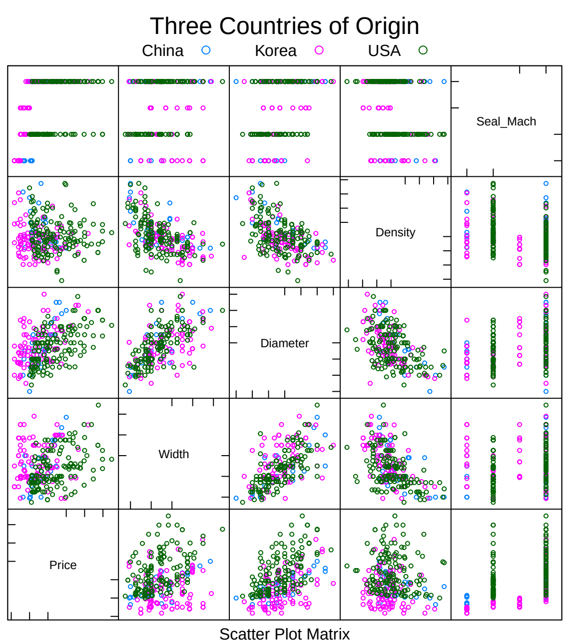


Figure 6.6: Scatterplots by Country of Manufacture

6.2.4 Dot Chart by Brand and Country of Manufacture

Now consider the average prices by brand of fly reel and country of manufacture. Figure 6.7 depicts a dot chart showing the average prices in the horizontal axis in these combinations of categories.

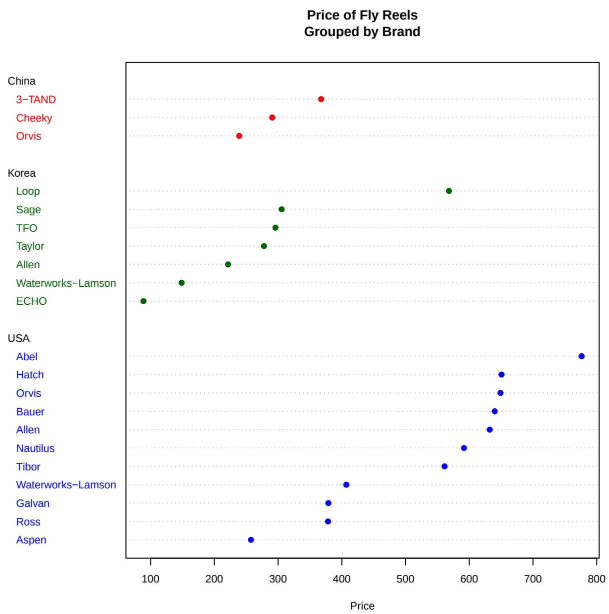


Figure 6.7: Average Prices by Brand and Country of Manufacture

We see that there exists more variety, in terms of both the number of brands and price levels within the population of American fly reel manufacturers. It's especially important that we consider the proliferation of fly reel brands at the high price points. It is worth investigating further whether those fly reels benefit from the "Made in America" premium or are simply made with more valuable characteristics.

Chapter 7

Regression Modelling with Hedonic Price Theory

7.1 Analyzing the Dependent Variable

7.1.1 Probability Density Function of Fly Reel Prices

Figure 7.1 is the kernel-smoothed probability density function of the natural logarithm of price:

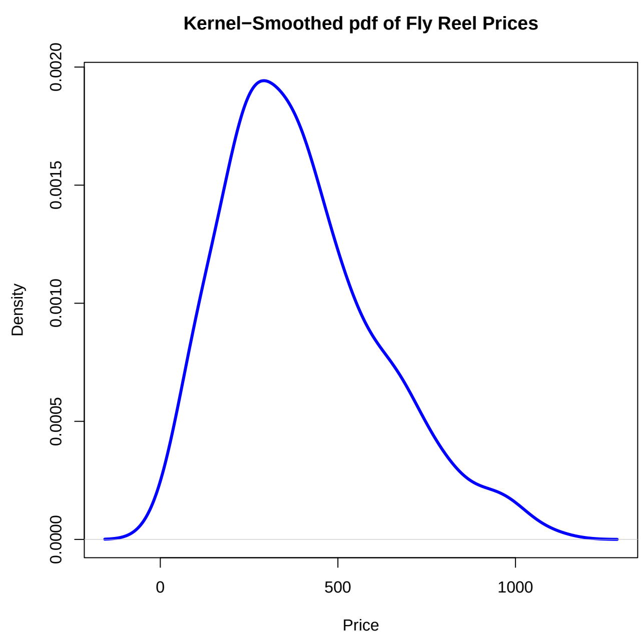


Figure 7.1: Probability Density Function of Fly Reel Prices

7.1.2 Probability Density Function of the Log. of Fly Reel Prices

Figure 7.2 is the kernel-smoothed probability density function of the natural logarithm of price:

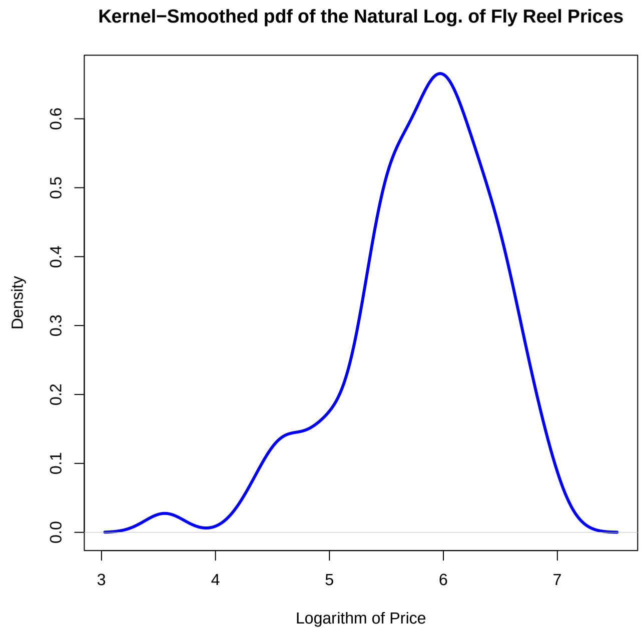


Figure 7.2: Probability Density Function of Fly Reel Prices

7.1.3 Probability Density Function of the Box-Cox Transformation of Fly Reel Prices

Figure 7.3 is the kernel-smoothed probability density function of the natural logarithm of price. Although the transformed model does look closest to normal, all three distributions are reasonably symmetric, indicating that the use of the transformation is subject to judgment. Whether it is worth using the transformation can be settled by analyzing the results of regression models with different forms of the dependent variable.

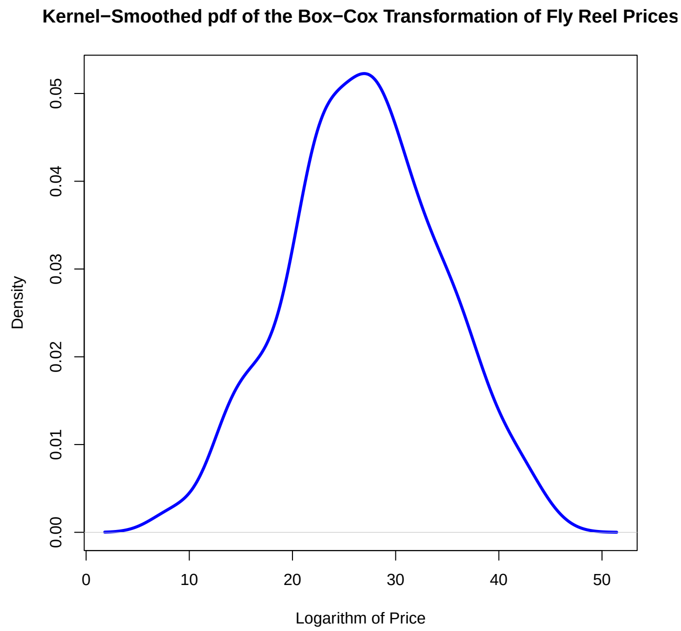


Figure 7.3: Probability Density Function of Transformed Fly Reel Prices

7.2 Regression Models for Fly Reel Prices

For the regression analysis, I create new variables. The first is the volume of each reel, calculated as the volume of a cylinder: the value π times the square of the radius of the reel, times the width of the reel. The density is then calculated as the weight divided by the volume.

```
R> flyreels[, 'Volume'] <-
   pi * (flyreels[, 'Diameter']/2)^2 * flyreels[, 'Width']
R> flyreels[, 'Density'] <-
   flyreels[, 'Weight'] / flyreels[, 'Volume']
```

7.2.1 Comparison by Transformation of Dependent Variable

Table 7.1 shows the results of a series of regression models with different definitions of the dependent variable. Model 1 uses the fly reel prices without transformation, Model 2 uses the logarithm of the fly reel prices, and Model 3 uses the Box-Cox transformation of fly reel prices, with the optimal value of the parameter in the exponent. Although the model built on the original price levels has statistically significant coefficients, the two transformed models have a better fit, with higher values of $\bar{R}^2$. Since the difference between the latter two models is marginal, it is better to model the logarithm of fly reel prices, which has the added advantage of interpretability of the coefficients, which approximately represent percentage changes in fly reel prices.

	Model 1	Model 2	Model 3
(Intercept)	-1056.01^{***}	2.09^{***}	-19.77^{***}
	(105.51)	(0.26)	(3.21)
Width	158.71^{*}	0.30	4.38^{*}
	(63.32)	(0.16)	(1.92)
Diameter	174.36^{***}	0.40^{***}	5.37^{***}
	(20.59)	(0.05)	(0.63)
Density	596.81^{***}	1.20^{***}	17.19^{***}
	(89.17)	(0.22)	(2.71)
SealedYes	144.31^{***}	0.42^{***}	5.00^{***}
	(20.43)	(0.05)	(0.62)
MachinedYes	114.40^{***}	0.76^{***}	6.53^{***}
	(30.43)	(0.08)	(0.92)
made_in_USATRUE	202.10^{***}	0.52^{***}	6.51^{***}
	(19.86)	(0.05)	(0.60)
R^2	0.64	0.74	0.71
Adj. R^2	0.63	0.73	0.70
Num. obs.	248	248	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 7.1: Regression Models with Different Dependent Variables

7.2.2 Comparison by Country of Manufacture

Table 10.2 shows the results of a series of regression models on different samples by country of manufacture. Model 1 shows the results for the sample of fly reels made in the USA and Model 2 shows the remaining fly reels made in China or Korea.

The width of the reel is insignificant in both samples and the coefficients are qualitatively similar across the samples, as well as matching in significance. This suggests that one model might be sufficient.

To test this statistically, I conduct an F-test. This compares a single model with only an indicator for the country of manufacture (the restricted model) with a separate model for each country. In this case, the full, unrestricted model has $K = 2 \times 6 = 12$ parameters, one for each variable in two models. The test that all of the coefficients are the same has $M = 6 - 1 = 5$ restrictions. The one restriction fewer accounts for the made-in-USA indicator in the full model, which allows for two separate intercepts. The F-statistic has a value of

$$\frac{(RSS_M - RSS)/M}{RSS/(N-K-1)} = \frac{(26.24962 - 25.2235)/5}{25.2235/235} = 1.912007.$$

This value is greater than 1, so we can compare it to the critical value of the F-statistic at the specified degrees of freedom for a conventional level of significance. These critical values are 3.095, 2.252, and 1.872 at the 1%, 5%, and 10% levels of significance, respectively.

This places the F-statistic between the critical values for the 5 and 10 percent levels of significance. Conclude that fly reel prices may have some difference by country of manufacture but the

	Model 1	Model 2
(Intercept)	3.35^{***}	2.03^{***}
	(0.30)	(0.47)
Width	0.28	0.39
	(0.22)	(0.23)
Diameter	0.44^{***}	0.36^{***}
	(0.07)	(0.08)
Density	1.13^{***}	1.32^{**}
	(0.25)	(0.41)
SealedYes	0.32^{***}	0.65^{***}
	(0.06)	(0.10)
MachinedYes		0.65^{***}
		(0.09)
R^2	0.54	0.75
Adj. R^2	0.52	0.73
Num. obs.	135	113

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 7.2: Regression Models by Country of Manufacture

difference is marginal. This suggests little justification for separate models by country of manufacture. We can investigate small differences between the models.

In the next section, I will consider a hybrid model with some interaction terms with separate coefficients by country of manufacture.

	Model 1	Model 2	Model 3
(Intercept)	2.01^{***}	2.13^{***}	1.89^{***}
	(0.26)	(0.30)	(0.31)
Width	0.34^{*}	0.30	0.35^{*}
	(0.16)	(0.16)	(0.16)
Diameter	0.40^{***}	0.40^{***}	0.40^{***}
	(0.05)	(0.05)	(0.05)
Density	1.21^{***}	1.14^{***}	1.40^{***}
	(0.22)	(0.32)	(0.33)
SealedYes	0.63^{***}	0.42^{***}	0.64^{***}
	(0.09)	(0.05)	(0.09)
MachinedYes	0.65^{***}	0.75^{***}	0.64^{***}
	(0.08)	(0.08)	(0.08)
made_in_USATRUE	0.75^{***}	0.47^{*}	0.91^{***}
	(0.09)	(0.19)	(0.24)
SealedYes:made_in_USATRUE	-0.30^{**}		-0.32^{**}
	(0.10)		(0.11)
Density:made_in_USATRUE		0.09	-0.29
		(0.37)	(0.39)
R^2	0.75	0.74	0.75
Adj. R^2	0.74	0.73	0.74
Num. obs.	248	248	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 7.3: Regression Models with Interaction Terms by Country of Manufacture

7.2.3 Models with Interaction Terms

Table 7.3 shows the results of a series of regression models with different specifications of interaction terms by country of manufacture. The interaction with sealed and country of manufacture is significant in both specifications Model 1 and Model 3. In contrast, the interaction with density was not significant. Furthermore, the width of fly reels is a significant predictor in this more refined model. Since all variables are significant in this model and it has the highest $\bar{R}^2$, Model 1 is the recommended model.

In terms of the production decision, there exists a substantial premium for fly reels made in the USA but this premium is half as large for sealed reels, however, this reduced premium is still smaller than the additional value for a sealed reel.

Chapter 8

Transforming the Explanatory Variables

I will revisit the recommended linear model from Problem Set #7 and the semiparametric specifications, which we covered for Problem Set #8.

Then I will further investigate any nonlinear relationships by incorporating a nonlinear but parametric specification for the value of the dimensions of the reels: the width, diameter, and density, which constitute the continuous variables in the dataset. This parametric analysis will be performed using the Box-Tidwell framework to investigate whether the value of these characteristics are best described with parametric nonlinear forms.

	Model 1
(Intercept)	2.00999^{***}
	(0.26125)
Width	0.33575^{*}
	(0.15622)
Diameter	0.39567^{***}
	(0.05076)
Density	1.21296^{***}
	(0.21948)
SealedYes	0.62731^{***}
	(0.08622)
MachinedYes	0.64934^{***}
	(0.08320)
made_in_USATRUE	0.74633^{***}
	(0.09247)
SealedYes:made_in_USATRUE	-0.29519^{**}
	(0.10092)
R^2	0.74893
Adj. R^2	0.74160
Num. obs.	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 8.1: Linear Model for Fly Reel Prices

8.1 Linear Regression Model

A natural staring point is the recommended linear model from Problem Set #7.

8.1.1 Linear Model with `Sealed*Made_in_USA` Interaction

Last week I investigated whether the functional form should include different specifications by country of manufacture. The model included the continuous variables width, diameter, and density, as well as categorical variables for country of manufacture, and whether or not the reels were sealed or machined. In addition to the indicator for the country of manufacture, the model included an indicator for an interaction between the the country of manufacture indicator and the indicator for whether the reels were sealed or unsealed. The dependent variable was chosen as the logarithm of the fly reel price, since the results were similar to those from the model with the optimal Box-Cox transformation, without the added complexity. The results of this regression specification are shown in Table 10.1.

Next, I will attempt to improve on this specification by investigating the potential for nonlinear functional forms, as we did for Problem Set #8.

8.2 Nonlinear Specifications

8.2.1 Nonparametric Specification for Width

As above, I first conducted FWL regressions to reduce the problem to two dimensions. The results are not shown here, since the comparison only verifies the conclusion of the FWL theorem.

To illustrate the fit of the linear model, Figure 9.2 shows a scatter plot of the residual log prices on residuals from the regression for width: the "excess width" of a fly reel compared to what would be expected given the other characteristics of the fly reel. The observations are shown in blue and the fitted values from the linear model are shown in red. The fit follows a straight line, as expected for a linear model.

I move directly to the nonparametric specification for the relationship between prices and width. Figure 9.2 overlays the nonparametric estimate, shown in green. The pattern has more variation in slope but closely follows the prediction from the linear model. Although the nonparametric estimate varies around the linear estimate, it appears that the linear form is also a close enough approximation.

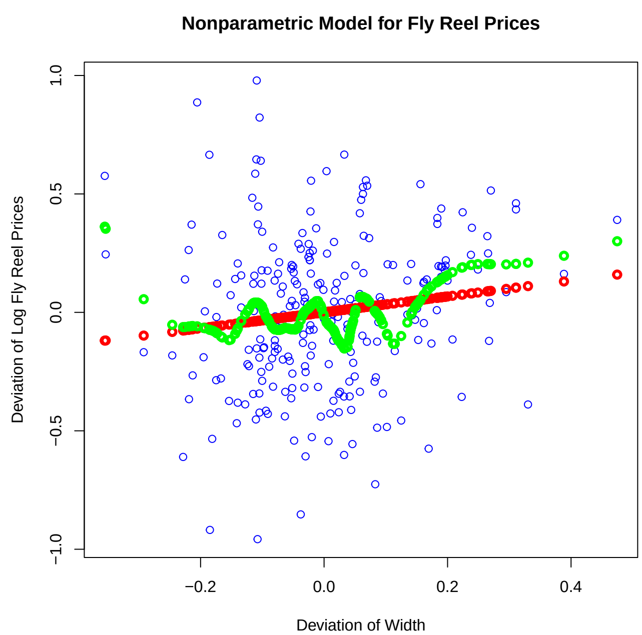


Figure 8.1: Nonparametric Model for Fly Reel Prices: Excess Width

8.2.2 Nonparametric Specification for Diameter

To illustrate the fit of the linear model, Figure 9.4 shows a scatter plot of the residual log prices on residuals from the regression for diameter: the "excess diameter" of a fly reel compared to what would be expected given the other characteristics of the fly reel. The observations are shown in blue and the fitted values from the linear model are shown in red. The fit follows a straight line, as expected for a linear model.

I move directly to the nonparametric specification for the relationship between prices and diameter. Figure 9.4 overlays the nonparametric estimate, shown in green. The pattern has more variation in slope but closely follows the prediction from the linear model. Although the nonparametric estimate varies around the linear estimate, it appears that the linear form is also a close enough approximation.

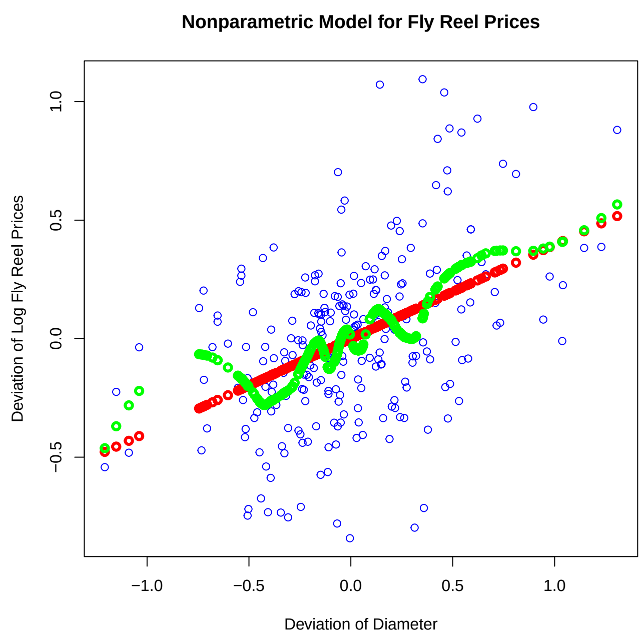


Figure 8.2: Nonparametric Model for Fly Reel Prices: Excess Diameter

8.2.3 Nonparametric Specification for Density

To illustrate the fit of the linear model, Figure 9.6 shows a scatter plot of the residual log prices on residuals from the regression for density: the "excess density" of a fly reel compared to what would be expected given the other characteristics of the fly reel. The observations are shown in blue and the fitted values are shown in red. The fit follows a straight line, as expected for a linear model.

I move directly to the nonparametric specification for the relationship between prices and density. Figure 9.6 overlays the nonparametric estimate, shown in green. The pattern has more variation in slope but closely follows the prediction from the linear model. Although the nonparametric estimate varies around the linear estimate, it appears that the linear form is also a close enough approximation.

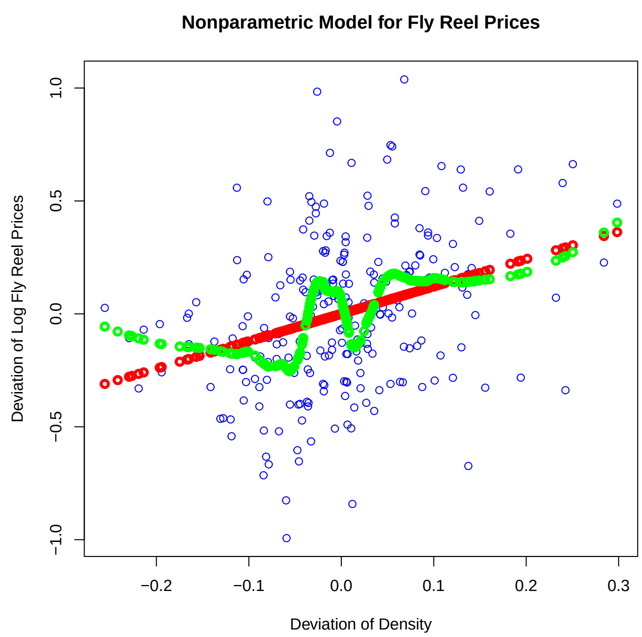


Figure 8.3: Nonparametric Model for Fly Reel Prices: Excess Density

8.3 Semiparametric Estimates

As I was building the above nonparametric models, I stored the predictions and will used them as variables in linear models. Table 9.2 shows the estimates from a set of models. Model 1 is the benchmark linear model in Table 10.1. Model 2 is a semi-parametric model with a nonparametric fit on width substituted in for the width variable. Models 3 and 4 are semi-parametric models with nonparametric fits on diameter and density, respectively. Model 5 is a maximally semiparametric model, with nonparametric fits for all continuous variables. For each of the single-variable semiparametric models, the coefficients are near one and the fits are similar to the linear model. Even with maximal flexibility, the fit of Model 5 is slightly better than the benchmark linear model. Across all models, the adjusted $\bar{R}^2$ values are all hovering around 0.75, with the full parametric model up to 0.80. All things considered, these are excellent models and the linear model is sufficient but you might recommend the full semiparametric model if you can justify the additional complexity.

One factor to keep in mind, however, is that the above semiparametric models essentially take the nonparametric functions as known, and do not account for the additional variability of the nonarametric parts of the model. The next specification estimates both linear and nonlinear parts jointly.

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	2.00999^{***}	2.34926^{***}	2.98091^{***}	3.23438^{***}	4.49531^{***}
	(0.26125)	(0.22596)	(0.22704)	(0.15261)	(0.05897)
Width	0.33575^{*}		0.92791^{***}	0.03044	
	(0.15622)		(0.12784)	(0.14082)	
Diameter	0.39567^{***}	0.43144^{***}		0.33658^{***}	
	(0.05076)	(0.04150)		(0.04671)	
Density	1.21296^{***}	1.07566^{***}	0.81613^{***}		
	(0.21948)	(0.20093)	(0.20645)		
SealedYes	0.62731^{***}	0.61960^{***}	0.70050^{***}	0.56858^{***}	0.69858^{***}
	(0.08622)	(0.08246)	(0.08226)	(0.08047)	(0.07509)
MachinedYes	0.64934^{***}	0.58954^{***}	0.71659^{***}	0.65070^{***}	0.61103^{***}
	(0.08320)	(0.07933)	(0.07938)	(0.07819)	(0.07386)
made_in_USATRUE	0.74633^{***}	0.77354^{***}	0.79615^{***}	0.70473^{***}	0.79326^{***}
	(0.09247)	(0.08855)	(0.08879)	(0.08692)	(0.08296)
SealedYes:made_in_USATRUE	-0.29519^{**}	-0.29826^{**}	-0.33376^{***}	-0.27253^{**}	-0.31356^{***}
	(0.10092)	(0.09642)	(0.09694)	(0.09500)	(0.09038)
width_np		1.11995^{***}			1.35512^{***}
		(0.21565)			(0.20456)
diameter_np			1.00650^{***}		1.00083^{***}
			(0.10926)		(0.10411)
density_np				1.03923^{***}	0.76790^{***}
				(0.12864)	(0.12582)
R^2	0.74893	0.76995	0.76755	0.77748	0.79771
Adj. R^2	0.74160	0.76324	0.76077	0.77099	0.79181
Num. obs.	248	248	248	248	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 8.2: Semiparametric Models for Fly Reel Prices

8.4 Generalized Additive Model

8.4.1 Linear Model

As an example of the output from the GAM specification, I first estimated the model with no nonlinear terms, which is essentially a linear regression.

```
Family: gaussian
Link function: identity

Formula:
log_Price ~ Width + Diameter + Density + Sealed + Machined +
    made_in_USA + made_in_USA * Sealed

Parametric coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                2.00999    0.26125   7.694 3.69e-13 ***
Width                      0.33575    0.15622   2.149  0.03262 *
Diameter                   0.39567    0.05076   7.795 1.95e-13 ***
Density                    1.21296    0.21948   5.527 8.49e-08 ***
SealedYes                  0.62731    0.08622   7.275 4.88e-12 ***
MachinedYes                0.64934    0.08320   7.805 1.84e-13 ***
made_in_USATRUE            0.74633    0.09247   8.071 3.35e-14 ***
SealedYes:made_in_USATRUE -0.29519    0.10092  -2.925  0.00378 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1


R-sq.(adj) =  0.742   Deviance explained = 74.9%
GCV = 0.10913  Scale est. = 0.10561   n = 248
```

8.4.2 Semiparametric Model

Since the results of the full semiparametric specification, in Model 5 of Table 9.2, were so promising, I estimated the model with all three continuous variables specified as nonparametric functions. The result was that all the variables—both linear and nonlinear—were statistically significant. On the other hand, the adjusted R-squared has not increased very much, from 0.742 to 0.769 under this specification, which may not justify the added complexity of the model. Perhaps more importantly, the coefficients on the linear terms are very similar across models, indicating that the models support similar conclusions relating to any business decision involving the "Made in USA" premium. With this second model, we have even more support for those conclusions and are certain that the conclusions are not coincidental results of the functional form decisions for previous models.

```
Family: gaussian
Link function: identity

Formula:
log_Price ~ s(Width) + s(Diameter) + s(Density) + Sealed + Machined +
    made_in_USA + made_in_USA * Sealed

Parametric coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                4.52027    0.06539  69.123  < 2e-16 ***
SealedYes                  0.64536    0.08257   7.816 1.95e-13 ***
MachinedYes                0.62750    0.07980   7.863 1.45e-13 ***
made_in_USATRUE            0.78661    0.08961   8.778 3.85e-16 ***
SealedYes:made_in_USATRUE -0.31047    0.09742  -3.187  0.00164 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Approximate significance of smooth terms:
              edf Ref.df      F  p-value
s(Width)    7.052  8.082  3.686 0.000411 ***
s(Diameter) 3.490  4.409 16.859  < 2e-16 ***
s(Density)  2.663  3.403  9.610 3.02e-06 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

R-sq.(adj) =  0.769   Deviance explained = 78.5%
GCV = 0.10198  Scale est. = 0.094497  n = 248
```

8.5 The Box–Tidwell Transformation

The Box–Tidwell function tests for non-linear relationships to the mean of the dependent variable. The nonlinearity is in the form of an exponential transformation in the form of the Box-Cox transformation, except that the transformation is taken on the explanatory variables.

8.5.1 Transformation of Width

Performing the transformation on the width variable produces a modified form of the linear model. This specification allows a single exponential transformation on width, rather than a linear form.

```
 MLE of lambda Score Statistic (z) Pr(>|z|)
        1.1615              0.1587   0.8739

iterations =  5
```

The R output is the statistics for a test of nonlinearity: that the exponent λ in the Box–Tidwell transformation is zero. The "MLE of lambda" statistic is the optimal exponent on horsepower. Similar to the Box-Cox transformation, with Box-Tidwell, the exponents are on the explanatory variables and are all called lambda, in contrast to the parameter τ in our class notes. The exponent is not significantly different from one. This supports a linear form for Width, confirming our result from the nonparametric analysis.

8.5.2 Transformation of Diameter

```
 MLE of lambda Score Statistic (z) Pr(>|z|)
      -0.35213             -1.2106    0.226

iterations =  4
```

This is very weak evidence for an inverse square root transformation for diameter but it is not estimated accurately. There is no evidence for a transformation with a coefficient different from 1, which suggests a purely linear relationship between log_Price and diameter. Next, I will consider the possibility of nonlinearity in the value of the density of a reel.

8.5.3 Transformation of Density

```
 MLE of lambda Score Statistic (z) Pr(>|z|)
       0.19315             -1.3448   0.1787

iterations =  10
```

Similar to Diameter, this is very weak evidence for a square root transformation for diameter but it is not estimated accurately. Conclude that the linear model is the best choice.

Chapter 9

Nonparametric Regression Models

I will revisit the recommended linear model from Problem Set #7, which included

I will investigate any nonlinear relationships by incorporating a nonparametric specification for the value of the dimensions of the reels: the width, diameter, and density, which constitute the continuous variables in the dataset. The nonparametric analysis will be performed to investigate whether the value of these characteristics are best described with nonlinear forms.

	Model 1
(Intercept)	2.00999^{***}
	(0.26125)
Width	0.33575^{*}
	(0.15622)
Diameter	0.39567^{***}
	(0.05076)
Density	1.21296^{***}
	(0.21948)
SealedYes	0.62731^{***}
	(0.08622)
MachinedYes	0.64934^{***}
	(0.08320)
made_in_USATRUE	0.74633^{***}
	(0.09247)
SealedYes:made_in_USATRUE	-0.29519^{**}
	(0.10092)
R^2	0.74893
Adj. R^2	0.74160
Num. obs.	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 9.1: Linear Model for Fly Reel Prices

9.1 Linear Regression Model

A natural staring point is the recommended linear model from Problem Set #7.

9.1.1 Linear Model with `Sealed*Made_in_USA` Interaction

Last week I investigated whether the functional form should include different specifications by country of manufacture. The model included the continuous variables width, diameter, and density, as well as categorical variables for country of manufacture, and whether or not the reels were sealed or machined. In addition to the indicator for the country of manufacture, the model included an indicator for an interaction between the the country of manufacture indicator and the indicator for whether the reels were sealed or unsealed. The dependent variable was chosen as the logarithm of the fly reel price, since the results were similar to those from the model with the optimal Box-Cox transformation, without the added complexity. The results of this regression specification are shown in Table 10.1.

Next, I will attempt to improve on this specification by investigating the potential for nonlinear functional forms.

9.2 Nonlinear Specifications

9.2.1 Nonparametric Specification for Width

As above, I first conduct FWL regressions to reduce the problem to two dimensions. The results are not shown here, since the comparison only verifies the conclusion of the FWL theorem.

To illustrate the fit of the linear model, Figure 9.1 shows a scatter plot of the residual log prices on the width of fly reels. The observations are shown in blue and the fitted values are shown in red. The variation in the fitted values results from the fact that it is not plotted against the transformed excess width variable used in the regressions. Still, the linear pattern is apparent and appears to match the data.

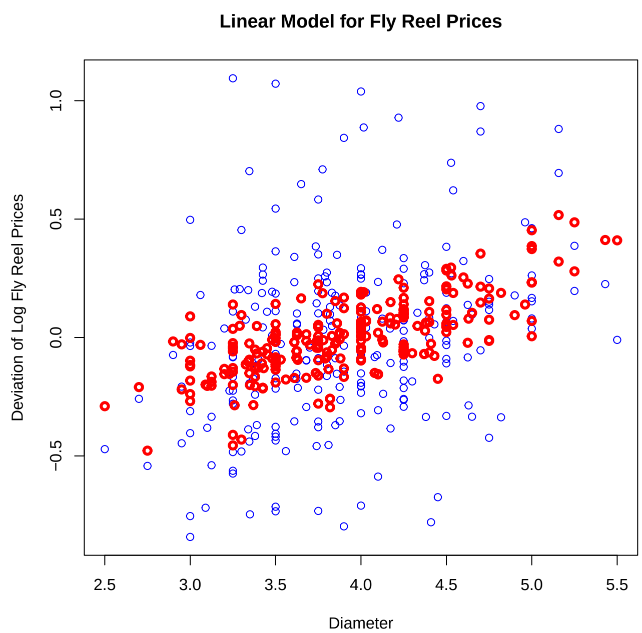


Figure 9.1: Linear Model for Fly Reel Prices vs. Width

As a comparison, Figure 9.2 augments the above by showing the plot against the residuals from the regression for width: the "excess width" of a fly reel compared to what would be expected given the other characteristics of the fly reel. The fit follows a straight line, as specified in the model. I move directly to the nonparametric specification for the relationship between prices and width. Figure 9.2 overlays the nonparametric estimate, shown in green. The pattern has more variation in slope but closely follows the prediction from the linear model. Although the nonparametric estimate varies around the linear estimate, it appears that the linear form is also a close enough approximation.

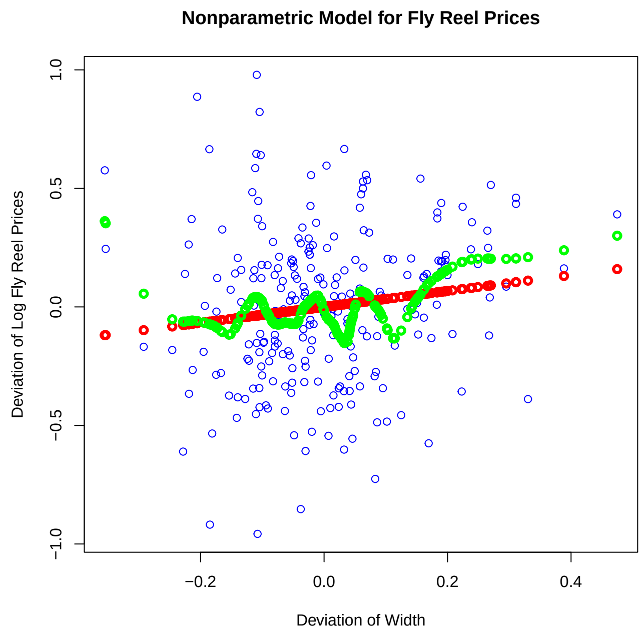


Figure 9.2: Nonparametric Model for Fly Reel Prices: Excess Width

9.2.2 Nonparametric Specification for Diameter

To illustrate the fit of the linear model, Figure 9.3 shows a scatter plot of the residual log prices on the diameter of fly reels. The observations are shown in blue and the fitted values are shown in red. The variation in the fitted values results from the fact that it is not plotted against the transformed excess diameter variable used in the regressions. Still, the linear pattern is apparent and appears to match the data.

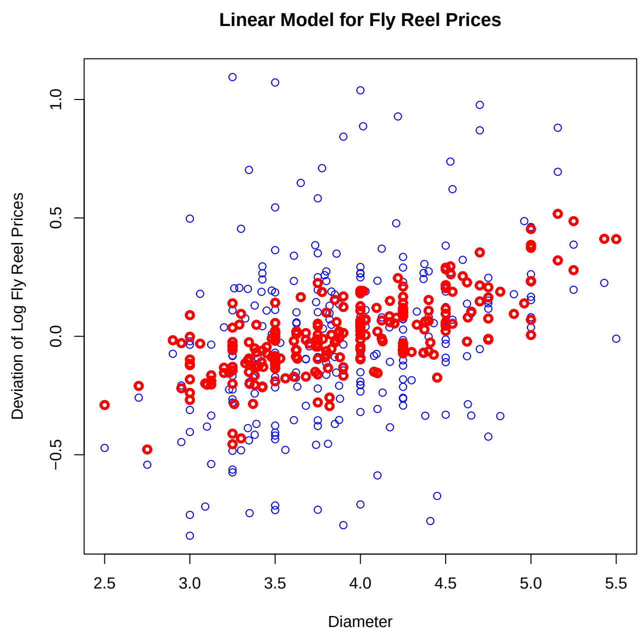


Figure 9.3: Linear Model for Fly Reel Prices vs. Diameter

As a comparison, Figure 9.4 augments the above by showing the plot against the residuals from the regression for diameter: the “excess diameter” of a fly reel compared to what would be expected given the other characteristics of the fly reel. The fit follows a straight line, as specified in the model. I move directly to the nonparametric specification for the relationship between prices and diameter. Figure 9.4 overlays the nonparametric estimate, shown in green. The pattern has more variation in slope but closely follows the prediction from the linear model. Although the nonparametric estimate varies around the linear estimate, it appears that the linear form is also a close enough approximation.

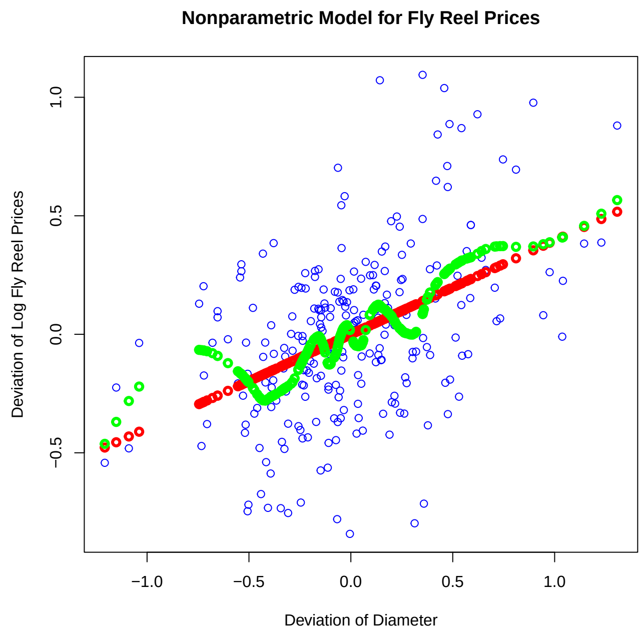


Figure 9.4: Nonparametric Model for Fly Reel Prices: Excess Diameter

9.2.3 Nonparametric Specification for Density

To illustrate the fit of the linear model, Figure 9.5 shows a scatter plot of the residual log prices on the density of fly reels. The observations are shown in blue and the fitted values are shown in red. The variation in the fitted values results from the fact that it is not plotted against the transformed excess density variable used in the regressions. Still, the linear pattern is apparent and appears to match the data.

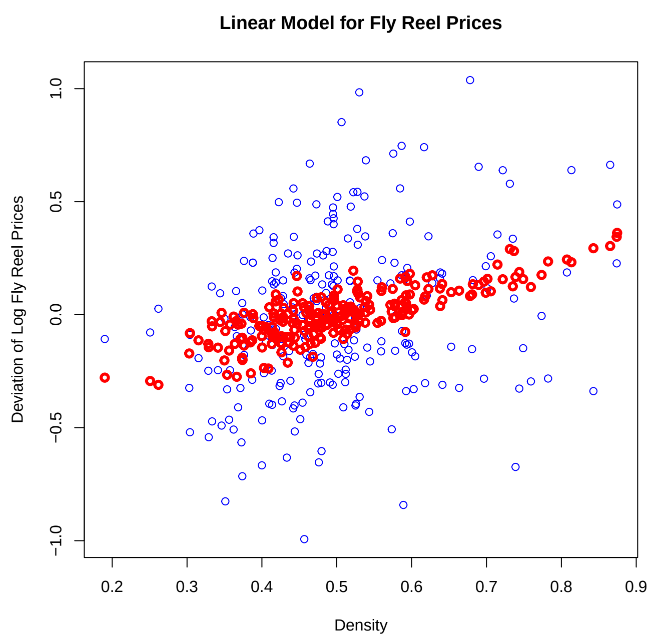


Figure 9.5: Linear Model for Fly Reel Prices vs. Density

As a comparison, Figure 9.6 augments the above by showing the plot against the residuals from the regression for density: the "excess density" of a fly reel compared to what would be expected given the other characteristics of the fly reel. The fit follows a straight line, as specified in the model. I move directly to the nonparametric specification for the relationship between prices and density. Figure 9.6 overlays the nonparametric estimate, shown in green. The pattern has more variation in slope but closely follows the prediction from the linear model. Although the nonparametric estimate varies around the linear estimate, it appears that the linear form is also a close enough approximation.

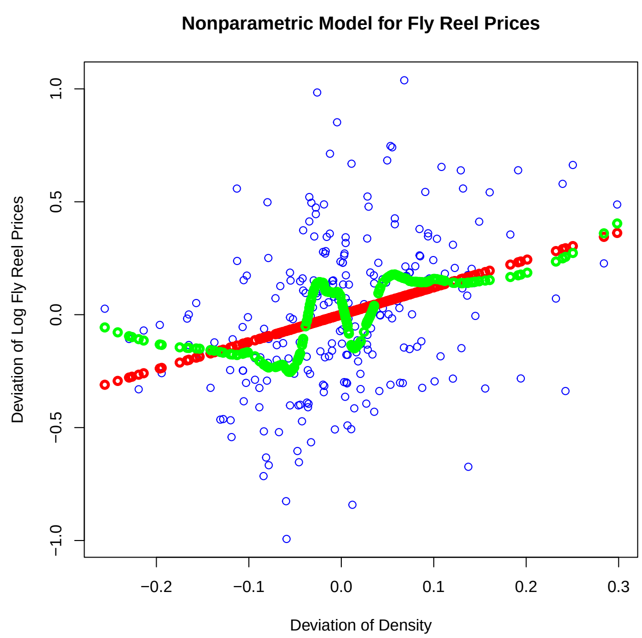


Figure 9.6: Nonparametric Model for Fly Reel Prices: Excess Density

9.3 Semiparametric Estimates

As I was building the above nonparametric models, I stored the predictions and will now use them as variables in linear models. Table 9.2 shows the estimates from a set of models. Model 1 is the benchmark linear model in Table 10.1. Model 2 is a semi-parametric model with a nonparametric fit on width substituted in for the width variable. Models 3 and 4 are semi-parametric models with nonparametric fits on diameter and density, respectively. Model 5 is a maximally semiparametric model, with nonparametric fits for all continuous variables. For each of the single-variable semiparametric models, the coefficients are near one and the fits are similar to the linear model. Even with maximal flexibility, the fit of Model 5 is slightly better than the benchmark linear model. Across all models, the adjusted $\bar{R}^2$ values are all hovering around 0.75, with the full parametric model up to 0.80. All things considered, these are excellent models and the linear model is sufficient but you might recommend the full semiparametric model if you can justify the additional complexity.

One factor to keep in mind, however, is that the above semiparametric models essentially take the nonparametric functions as known, and do not account for the additional variability of the nonarametric parts of the model. The next specification estimates both linear and nonlinear parts jointly.

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	2.00999***	2.34926***	2.98091***	3.23438***	4.49531***
	(0.26125)	(0.22596)	(0.22704)	(0.15261)	(0.05897)
Width	0.33575*		0.92791***	0.03044	
	(0.15622)		(0.12784)	(0.14082)	
Diameter	0.39567***	0.43144***		0.33658***	
	(0.05076)	(0.04150)		(0.04671)	
Density	1.21296***	1.07566***	0.81613***		
	(0.21948)	(0.20093)	(0.20645)		
SealedYes	0.62731***	0.61960***	0.70050***	0.56858***	0.69858***
	(0.08622)	(0.08246)	(0.08226)	(0.08047)	(0.07509)
MachinedYes	0.64934***	0.58954***	0.71659***	0.65070***	0.61103***
	(0.08320)	(0.07933)	(0.07938)	(0.07819)	(0.07386)
made_in_USATRUE	0.74633***	0.77354***	0.79615***	0.70473***	0.79326***
	(0.09247)	(0.08855)	(0.08879)	(0.08692)	(0.08296)
SealedYes:made_in_USATRUE	−0.29519**	−0.29826**	−0.33376***	−0.27253**	−0.31356***
	(0.10092)	(0.09642)	(0.09694)	(0.09500)	(0.09038)
width_np		1.11995***			1.35512***
		(0.21565)			(0.20456)
diameter_np			1.00650***		1.00083***
			(0.10926)		(0.10411)
density_np				1.03923***	0.76790***
				(0.12864)	(0.12582)
R^2	0.74893	0.76995	0.76755	0.77748	0.79771
Adj. R^2	0.74160	0.76324	0.76077	0.77099	0.79181
Num. obs.	248	248	248	248	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 9.2: Semiparametric Models for Fly Reel Prices

9.4 Generalized Additive Model

9.4.1 Linear Model

As an example of the output from the GAM specification, I first estimated the model with no nonlinear terms, which is essentially a linear regression.

```
Family: gaussian
Link function: identity

Formula:
log_Price ~ Width + Diameter + Density + Sealed + Machined +
    made_in_USA + made_in_USA * Sealed

Parametric coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                2.00999    0.26125   7.694 3.69e-13 ***
Width                      0.33575    0.15622   2.149  0.03262 *
Diameter                   0.39567    0.05076   7.795 1.95e-13 ***
Density                    1.21296    0.21948   5.527 8.49e-08 ***
SealedYes                  0.62731    0.08622   7.275 4.88e-12 ***
MachinedYes                0.64934    0.08320   7.805 1.84e-13 ***
made_in_USATRUE            0.74633    0.09247   8.071 3.35e-14 ***
SealedYes:made_in_USATRUE -0.29519    0.10092  -2.925  0.00378 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1


R-sq.(adj) =  0.742   Deviance explained = 74.9%
GCV = 0.10913  Scale est. = 0.10561   n = 248
```

9.4.2 Semiparametric Model

Since the results of the full semiparametric specification, in Model 5 of Table 9.2, were so promising, I estimated the model with all three continuous variables specified as nonparametric functions. The result was that all the variables—both linear and nonlinear—were statistically significant. On the other hand, the adjusted R-squared has not increased very much, from 0.742 to 0.769 under this specification, which may not justify the added complexity of the model. Perhaps more importantly, the coefficients on the linear terms are very similar across models, indicating that the models support similar conclusions relating to any business decision involving the "Made in USA" premium. With this second model, we have even more support for those conclusions and are certain that the conclusions are not coincidental results of the functional form decisions for previous models.

```
Family: gaussian
Link function: identity

Formula:
log_Price ~ s(Width) + s(Diameter) + s(Density) + Sealed + Machined +
    made_in_USA + made_in_USA * Sealed

Parametric coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                4.52027    0.06539  69.123  < 2e-16 ***
SealedYes                  0.64536    0.08257   7.816 1.95e-13 ***
MachinedYes                0.62750    0.07980   7.863 1.45e-13 ***
made_in_USATRUE            0.78661    0.08961   8.778 3.85e-16 ***
SealedYes:made_in_USATRUE -0.31047    0.09742  -3.187  0.00164 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Approximate significance of smooth terms:
              edf Ref.df      F  p-value
s(Width)    7.052  8.082  3.686 0.000411 ***
s(Diameter) 3.490  4.409 16.859  < 2e-16 ***
s(Density)  2.663  3.403  9.610 3.02e-06 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

R-sq.(adj) =  0.769   Deviance explained = 78.5%
GCV = 0.10198  Scale est. = 0.094497  n = 248
```

Chapter 10

Sample Selection Models

I will revisit the recommended linear model from Problem Set #7, which was supported in Problem Sets #8 and #9 by considering other nonlinear specifications within a Generalized Additive Model.

Then I will further investigate this nonlinear relationship by considering the issue of sample selection: fly reel manufacturers may produce fly reels in each country with specific qualities based on their perceived value to typical American customers, in ways that are not represented by the variables in the dataset.

	Model 1
(Intercept)	2.00999^{***}
	(0.26125)
Width	0.33575^{*}
	(0.15622)
Diameter	0.39567^{***}
	(0.05076)
Density	1.21296^{***}
	(0.21948)
SealedYes	0.62731^{***}
	(0.08622)
MachinedYes	0.64934^{***}
	(0.08320)
made_in_USATRUE	0.74633^{***}
	(0.09247)
SealedYes:made_in_USATRUE	-0.29519^{**}
	(0.10092)
R^2	0.74893
Adj. R^2	0.74160
Num. obs.	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 10.1: Linear Model for Fly Reel Prices

10.1 Linear Regression Model

A natural staring point is the recommended linear model from Problem Set #7.

10.1.1 Linear Model with `Sealed*Made_in_USA` Interaction

Last week I investigated whether the functional form should include different specifications by country of manufacture. The model included the continuous variables width, diameter, and density, as well as categorical variables for country of manufacture, and whether or not the reels were sealed or machined. In addition to the indicator for the country of manufacture, the model included an indicator for an interaction between the the country of manufacture indicator and the indicator for whether the reels were sealed or unsealed. The dependent variable was chosen as the logarithm of the fly reel price, since the results were similar to those from the model with the optimal Box-Cox transformation, without the added complexity. The results of this regression specification are shown in Table 10.1.

Next, I will attempt to improve on this specification, using Tobit models for sample selection.

	Model 1	Model 2
(Intercept)	3.35^{***}	2.03^{***}
	(0.30)	(0.47)
Width	0.28	0.39
	(0.22)	(0.23)
Diameter	0.44^{***}	0.36^{***}
	(0.07)	(0.08)
Density	1.13^{***}	1.32^{**}
	(0.25)	(0.41)
SealedYes	0.32^{***}	0.65^{***}
	(0.06)	(0.10)
MachinedYes		0.65^{***}
		(0.09)
R^2	0.54	0.75
Adj. R^2	0.52	0.73
Num. obs.	135	113

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 10.2: Regression Models by Country of Manufacture

10.1.2 Comparison by Country of Manufacture

Table 10.2 shows the results of a series of regression models on different samples by country of manufacture. Model 1 shows the results for the full model using the sample of fly reels made in the USA and Model 3 shows the remaining fly reels made in China or Korea. Models 2 and 4 show the estimates from a reduced model on each sample, eliminating the variable `Width`, which was not significant in either of these smaller samples.

The width of the reel is insignificant in both samples and the coefficients are qualitatively similar across the samples, as well as matching in significance. This suggests that one model might be sufficient.

	Model 1	Model 2
(Intercept)	-3.46545	3.38980^{**}
	(159.27332)	(1.06142)
Weight	0.11330	0.13348^{*}
	(0.19412)	(0.05711)
Diameter	-0.08526	
	(0.86262)	
Width	-0.71463	-2.45931^{**}
	(1.68668)	(0.81177)
Volume	-0.03294	
	(0.09631)	
Density	-2.19723	-2.02613^{*}
	(2.46062)	(0.98455)
SealedYes	-1.23411^{***}	-0.68650^{***}
	(0.23449)	(0.18639)
MachinedYes	6.58401	
	(159.21535)	
AIC	267.46344	328.68383
BIC	295.57087	346.25098
Log Likelihood	-125.73172	-159.34192
Deviance	251.46344	318.68383
Num. obs.	248	248

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 10.3: Probit Models for Country-of-Manufacture Selection of Fly Reels

10.2 Sample Selection

10.2.1 Predicting the Selection into Samples

The specification in Table 10.1 assumes a linear functional form for the relationship between characteristics and prices of fly reels, without selecting into samples by brand. To investigate this relationship further, consider the set of variables that are related to whether or not a manufacturer decides to manufacture fly reels in America or overseas with the characteristics observed in the dataset.

Table 10.3 shows the estimates for a probit model to predict the selection into samples by country of manufacture. Model 1 in Table 10.3 shows a preliminary probit model to predict the selection indicator, with all the other explanatory variables in the model. American fly reel manufacture seems only to be related to whether or not the reels are sealed. Model 2 shows the result of a variable-reduction exercise to eliminate variables that are not statistically significant. These estimates provide a concise but useful model to indicate the fly reel designs that manufacturers would prefer to manufacture on American soil.

This model is used to specify the selection equation of the sample selection estimates discussed next.

10.2.2 Estimating a Sample Selection Model

Table 10.4 shows the estimates from a model that accounts for sample selection. The models are estimates from the Tobit model of type 5, which is a model specification that allows for switching of the observations in the sample into two models: one for the value of fly reels made in the USA and the other for fly reels produced elsewhere.

Each column shows the estimates from a separate model and the series of models is the result of a downward selection procedure in which a statistically insignificant variable was removed from each of the previous models in the sequence. In each model, the estimates are grouped into three categories. The first block of coefficients describe the selection model to determine whether a fly reel design would be manufactured in the USA. These coefficients are denoted by the prefix"S:". Below these lies two blocks of coefficients for the observation equations. The notation "O: ¡name of variable¿ (i)" indicates the coefficient for the particular variable in the observation equation for sample i. In this specification, the first observation equation represents fly reels made overseas (`johndeere == 0`), while equation 2 represents the fly reels made in the USA (`johndeere == 1`).

Model 1 shows the estimates from the full model. Several of the coefficients in the model are statistically insignificant and the model has numerical issues. In particular, some of the standard errors are undefined, evidenced by the missing standard error for `sigma1`. This suggests that the likelihood function is flat in some areas of the parameter space. This model has its imperfections but is a good start. Several variables are statistically insignificant but these can be removed one by one to produce a refined model. The goal will be to obtain a final model that has well-defined standard errors for all variables and, ideally, all coefficients statistically significant.

Model 2 shows the estimates from a reduced model, eliminate `Diameter` from the other-country equation. There still exists coefficients that are statistically insignificant and the numerical issue remains for `sigma1`. One step further, Model 3 eliminates `Width` from the American observation equation. Again, this model is an improvement but a numerical issue remains.

Model 4 excludes `Weight` from the selection equation. The produces a set of estimates that are numerically stable, within a strictly concave region of the likelihood function. This model, however, still contains some variables that are statistically significant. Model 5 excludes `Density` from the selection equation. Again, this model is well-behaved numerically but one statistically insignificant variable remains.

The next step is to estimate what would be Model 6 by eliminating `Width` from the selection equation. This specification is problematic, however, as it produces an error message indicating severe multicollinearity. With these numerical problems, it is better to keep the additional variable in the selection equation, even though it may be statistically insignificant. Notice that the remaining selection variable `Sealed` appears in both observation equations and there is no variable in the selection equation that is excluded. The other extreme would offer better performance, that is, having some variables in the selection equation that are not included in the observation equations.

	Model 1	Model 2	Model 3	Model 4	Model 5
S: (Intercept)	3.38999*	3.38995*	3.39163**	1.84476*	1.06496*
	(1.52501)	(1.39731)	(1.24301)	(0.88778)	(0.49241)
S: Width	−2.45920	−2.45922*	−2.45815*	−0.87031	−0.59098
	(1.26540)	(1.18220)	(1.06044)	(0.51987)	(0.45219)
S: Weight	0.13399	0.13392	0.13475		
	(0.08555)	(0.08136)	(0.07909)		
S: Density	−2.02599	−2.02603	−2.02510	−0.88282	
	(1.36507)	(1.18246)	(1.15198)	(0.83583)	
S: SealedYes	−0.68657**	−0.68656**	−0.68891***	−0.52031**	−0.47216**
	(0.25772)	(0.21197)	(0.15923)	(0.17879)	(0.17222)
O: (Intercept) (1)	0.26764	0.23849	0.30737	1.89824**	2.14641***
	(1.08958)	(0.88866)	(0.78180)	(0.59989)	(0.50263)
O: Width (1)	1.73696***	1.75076***			
	(0.51414)	(0.50948)			
O: Diameter (1)	0.02868				
	(0.18074)				
O: Density (1)	1.70252	1.70122	1.73905*	1.09840*	0.79690*
	(0.97121)	(0.88239)	(0.79321)	(0.48704)	(0.37891)
O: SealedYes (1)	1.22730***	1.23870***	1.24406***	0.92288***	0.90755***
	(0.21538)	(0.22743)	(0.18759)	(0.11757)	(0.11456)
O: MachinedYes	0.61018**	0.60781**	0.63222***	0.75933***	0.76203***
	(0.21795)	(0.20627)	(0.18592)	(0.09168)	(0.09128)
O: (Intercept) (2)	3.32465***	3.32465***	3.30003***	3.49131***	3.44177***
	(0.30694)	(0.30651)	(0.30380)	(0.28482)	(0.27958)
O: Width (2)	0.14939	0.14939			
	(0.26002)	(0.25871)			
O: Diameter (2)	0.47103***				
	(0.07550)				
O: Density (2)	1.09216***	1.09216***	1.04813***	1.00088***	1.06657***
	(0.26518)	(0.26347)	(0.26497)	(0.26029)	(0.24389)
O: SealedYes (2)	0.26663**	0.26662**	0.21502***	0.27774***	0.28495***
	(0.08286)	(0.08173)	(0.06041)	(0.07395)	(0.07554)
sigma1	0.83812	0.89629	0.80395	0.52930***	0.53170***
				(0.07186)	(0.06979)
sigma2	0.31216***	0.31216***	0.33612***	0.31969***	0.31675***
	(0.04046)	(0.04029)	(0.02417)	(0.04534)	(0.04844)
rho1	−1.00000	−0.99511	−1.00000	−0.87806***	−0.88135***
	(3.66988)			(0.08675)	(0.08048)
rho2	0.39798	0.39798	0.75807***	0.46352	0.43112
	(0.44205)	(0.43892)	(0.06565)	(0.40092)	(0.48226)
O: Diameter		0.47102***	0.50888***	0.47502***	0.48018***
		(0.07531)	(0.05880)	(0.05735)	(0.05664)
O: Width			1.81227***	1.25522***	1.17131***
			(0.46808)	(0.30287)	(0.28044)
AIC	591.29253	617.14337	581.03621	513.34065	512.46369
BIC	661.56110	683.89852	644.27793	573.06894	568.67855
Log Likelihood	−275.64627	−289.57169	−272.51811	−239.67032	−240.23184
Num. obs.	248	248	248	248	248
Censored	113	113	113	113	113
Observed	135	135	135	135	135

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 10.4: Selection Models for Fly Reel Prices

I revert back to Model 5 to analyze the differences between country of manufacture, in the rightmost column of Table 10.4. First of all, this confirms that machined reels are more valuable, with a coefficient of 0.76, instead of 0.63, as was found in the separate linear regression model. It also justifies the fact that only machined reels are produced in the USA. This could relate to some advantage in American production technology or, rather, the outdated casting techniques used for cheaper reels produced overseas.

In the model for American-made reels, the coefficients on all three variables are statistically the same as those in the separate linear regression model. For the fly reels made overseas, however, `Width` replaces `Diameter` as a proxy for the size of the reels. As a consequence, density is a relatively less valuable feature. The value of sealed reels, however, is even greater, after accounting for the selection of different production techniques jointly with manufacturing location. The coefficient on sealed reels jumps to 0.90, compared to 0.65 in the linear model. This suggests a higher premium than indicated earlier, once we also consider the choice of country of manufacture. Similarly, the value of machined reels rises from 0.65 to 0.76 in the selection model, suggesting an even higher for this design when produced overseas.

In conclusion, an American fly reel producer should not consider producing cast reels, unless the purpose is to explore the change in value. I suspect, however, that this outcome has been observed in the past, so the older production techniques have been abandoned for a reason. Machined reels are also more valuable when produced overseas, so a company has to compare the difference in labor costs with the relative cost of producing machined reels overseas.

Similarly, sealed reels produce a much higher premium overseas than was originally estimated with the linear model. Reels produced overseas should be sealed, unless the cost of changing the manufacturing process would outweigh this premium. Likewise for the American reels, except the premium is one third the size.

However it is measured, the size of a reel matters: bigger reels are more valuable. A manufacturer can compare the cost of materials with the premiums attached to those dimensions when producing a reel.

Perhaps the largest difference is in the intercept term: 1.07 overseas vs 2.14 in the USA, double the size. In the linear model, the intercepts were 2.41 vs 3.48, which measures a similar percentage difference. No matter how it is measured, there exists a substantial premium for fly reels made in the USA.