**CSC3831 Predictive Analytics – Machine Learning House Price Prediction**

**First Model – Linear Regression**

A grid search is performed to find the best hyperparameters for the linear regression model on the dataset. The grid search takes in as hyper-parameters; fit\_intercept, copy\_x, and n\_jobs to search over the datasets. The grid search returns a list of results for each set of hyperparameters.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Fit Intercept** | **Copy X** | **N jobs** | **MSE** | **R^2** |
| TRUE | TRUE | 1 | 0.37206 | 0.621966 |
| TRUE | TRUE | 2 | 0.37206 | 0.621966 |
| TRUE | TRUE | 3 | 0.37206 | 0.621966 |
| TRUE | TRUE | 4 | 0.37206 | 0.621966 |
| TRUE | FALSE | -1 | 0.37206 | 0.621966 |
| TRUE | FALSE | 1 | 0.372047 | 0.621979 |
| TRUE | FALSE | 2 | 0.372047 | 0.621979 |
| TRUE | FALSE | 3 | 0.372047 | 0.621979 |
| TRUE | FALSE | 4 | 0.372047 | 0.621979 |
| FALSE | TRUE | -1 | 0.372046 | 0.621981 |
| FALSE | TRUE | 1 | 0.372046 | 0.621981 |
| FALSE | TRUE | 2 | 0.372046 | 0.621981 |
| FALSE | TRUE | 3 | 0.372046 | 0.621981 |
| FALSE | TRUE | 4 | 0.372046 | 0.621981 |
| FALSE | FALSE | -1 | 0.372046 | 0.621981 |
| FALSE | FALSE | 1 | 0.372046 | 0.621981 |
| FALSE | FALSE | 2 | 0.372046 | 0.621981 |
| FALSE | FALSE | 3 | 0.372046 | 0.621981 |
| FALSE | FALSE | 4 | 0.372046 | 0.621981 |

These results suggest that the choice of hyperparameters has a minimal impact on the model's performance. The MSE scores are relatively close together and the R^2 scores are all above 0.6, indicating that the model is performing relatively well, but there is still room for improvement.

**Second Model – Multi-Layer Perceptron**  
Another grid search is performed to find the best hyperparameters for the multi-layer perceptron model. The hyper-parameters tested are number of units, activation function, number of layers. The grid search results are as follows:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Units Per Layer | Activation Function | No. of Layers | Optimizer | Epochs | Batch Size | MSE | MAE |
| 64 | relu | 2 | adam | 10 | 8 | 0.224044 | 0.316482 |
| 64 | relu | 2 | adam | 10 | 16 | 0.212253 | 0.312662 |
| 64 | relu | 2 | adam | 10 | 32 | 0.221621 | 0.321429 |
| 64 | relu | 2 | adam | 10 | 64 | 0.228725 | 0.346533 |
| 64 | relu | 2 | adam | 10 | 128 | 0.219787 | 0.322156 |
| 64 | relu | 2 | adam | 10 | 256 | 0.22963 | 0.330435 |
| 64 | relu | 3 | adam | 10 | 8 | 0.22722 | 0.33222 |
| 64 | relu | 3 | adam | 10 | 16 | 0.21132 | 0.320985 |
| 64 | relu | 3 | adam | 10 | 32 | 0.220123 | 0.319673 |
| 64 | relu | 3 | adam | 10 | 64 | 0.212145 | 0.31642 |
| 64 | relu | 3 | adam | 10 | 128 | 0.227734 | 0.320977 |
| 64 | relu | 3 | adam | 10 | 256 | 0.225559 | 0.324945 |
| 64 | relu | 4 | adam | 10 | 8 | 0.212145 | 0.327391 |
| 64 | relu | 4 | adam | 10 | 16 | 0.209875 | 0.316931 |
| 64 | relu | 4 | adam | 10 | 32 | 0.215377 | 0.321995 |
| 64 | relu | 4 | adam | 10 | 64 | 0.226254 | 0.313717 |
| 64 | relu | 4 | adam | 10 | 128 | 0.219476 | 0.319936 |
| 64 | relu | 4 | adam | 10 | 256 | 0.226489 | 0.338178 |
| 64 | relu | 5 | adam | 10 | 8 | 0.237961 | 0.338494 |
| 64 | relu | 5 | adam | 10 | 16 | 0.215081 | 0.319046 |
| 64 | relu | 5 | adam | 10 | 32 | 0.21513 | 0.324387 |
| 64 | relu | 5 | adam | 10 | 64 | 0.209051 | 0.318896 |
| 64 | relu | 5 | adam | 10 | 128 | 0.215715 | 0.322011 |
| 64 | relu | 5 | adam | 10 | 256 | 0.248842 | 0.360523 |
| 64 | tanh | 2 | adam | 10 | 8 | 0.215241 | 0.317689 |
| 64 | tanh | 2 | adam | 10 | 16 | 0.238018 | 0.341173 |
| 64 | tanh | 2 | adam | 10 | 32 | 0.232614 | 0.325855 |
| 64 | tanh | 2 | adam | 10 | 64 | 0.248103 | 0.338166 |
| 64 | tanh | 2 | adam | 10 | 128 | 0.251979 | 0.346708 |
| 64 | tanh | 2 | adam | 10 | 256 | 0.273365 | 0.365192 |
| 64 | tanh | 3 | adam | 10 | 8 | 0.225148 | 0.318866 |
| 64 | tanh | 3 | adam | 10 | 16 | 0.226702 | 0.332043 |
| 64 | tanh | 3 | adam | 10 | 32 | 0.235575 | 0.329446 |
| 64 | tanh | 3 | adam | 10 | 64 | 0.221118 | 0.325245 |
| 64 | tanh | 3 | adam | 10 | 128 | 0.235043 | 0.342608 |
| 64 | tanh | 3 | adam | 10 | 256 | 0.247696 | 0.353213 |
| 64 | tanh | 4 | adam | 10 | 8 | 0.243508 | 0.355629 |
| 64 | tanh | 4 | adam | 10 | 16 | 0.233445 | 0.340998 |
| 64 | tanh | 4 | adam | 10 | 32 | 0.224969 | 0.327489 |
| 64 | tanh | 4 | adam | 10 | 64 | 0.225109 | 0.322122 |
| 64 | tanh | 4 | adam | 10 | 128 | 0.238279 | 0.326479 |
| 64 | tanh | 4 | adam | 10 | 256 | 0.256057 | 0.355652 |
| 64 | tanh | 5 | adam | 10 | 8 | 0.239673 | 0.329573 |
| 64 | tanh | 5 | adam | 10 | 16 | 0.235766 | 0.34191 |
| 64 | tanh | 5 | adam | 10 | 32 | 0.221314 | 0.31752 |
| 64 | tanh | 5 | adam | 10 | 64 | 0.223995 | 0.318587 |
| 64 | tanh | 5 | adam | 10 | 128 | 0.234465 | 0.330605 |
| 64 | tanh | 5 | adam | 10 | 256 | 0.243001 | 0.338845 |
| 128 | relu | 2 | adam | 10 | 8 | 0.208366 | 0.305084 |
| 128 | relu | 2 | adam | 10 | 16 | 0.209042 | 0.310396 |
| 128 | relu | 2 | adam | 10 | 32 | 0.210125 | 0.307196 |
| 128 | relu | 2 | adam | 10 | 64 | 0.217805 | 0.311788 |
| 128 | relu | 2 | adam | 10 | 128 | 0.216145 | 0.326357 |
| 128 | relu | 2 | adam | 10 | 256 | 0.21918 | 0.326501 |
| 128 | relu | 3 | adam | 10 | 8 | 0.223651 | 0.335616 |
| 128 | relu | 3 | adam | 10 | 16 | 0.207523 | 0.308899 |
| 128 | relu | 3 | adam | 10 | 32 | 0.225145 | 0.326913 |
| 128 | relu | 3 | adam | 10 | 64 | 0.203492 | 0.30959 |
| 128 | relu | 3 | adam | 10 | 128 | 0.212474 | 0.321558 |
| 128 | relu | 3 | adam | 10 | 256 | 0.223571 | 0.328741 |
| 128 | relu | 4 | adam | 10 | 8 | 0.234412 | 0.319751 |
| 128 | relu | 4 | adam | 10 | 16 | 0.206783 | 0.306865 |
| 128 | relu | 4 | adam | 10 | 32 | 0.213773 | 0.31956 |
| 128 | relu | 4 | adam | 10 | 64 | 0.214953 | 0.323949 |
| 128 | relu | 4 | adam | 10 | 128 | 0.213828 | 0.326555 |
| 128 | relu | 4 | adam | 10 | 256 | 0.231709 | 0.322284 |
| 128 | relu | 5 | adam | 10 | 8 | 0.224689 | 0.319643 |
| 128 | relu | 5 | adam | 10 | 16 | 0.221376 | 0.320123 |
| 128 | relu | 5 | adam | 10 | 32 | 0.216928 | 0.316637 |
| 128 | relu | 5 | adam | 10 | 64 | 0.219234 | 0.328439 |
| 128 | relu | 5 | adam | 10 | 128 | 0.210652 | 0.31016 |
| 128 | relu | 5 | adam | 10 | 256 | 0.217441 | 0.32655 |
| 128 | tanh | 2 | adam | 10 | 8 | 0.255132 | 0.347482 |
| 128 | tanh | 2 | adam | 10 | 16 | 0.237666 | 0.342861 |
| 128 | tanh | 2 | adam | 10 | 32 | 0.224524 | 0.32177 |
| 128 | tanh | 2 | adam | 10 | 64 | 0.223519 | 0.319779 |
| 128 | tanh | 2 | adam | 10 | 128 | 0.244833 | 0.356113 |
| 128 | tanh | 2 | adam | 10 | 256 | 0.260176 | 0.366086 |
| 128 | tanh | 3 | adam | 10 | 8 | 0.23689 | 0.334883 |
| 128 | tanh | 3 | adam | 10 | 16 | 0.236081 | 0.336716 |
| 128 | tanh | 3 | adam | 10 | 32 | 0.218373 | 0.317835 |
| 128 | tanh | 3 | adam | 10 | 64 | 0.228984 | 0.341201 |
| 128 | tanh | 3 | adam | 10 | 128 | 0.246746 | 0.35016 |
| 128 | tanh | 3 | adam | 10 | 256 | 0.238227 | 0.330724 |
| 128 | tanh | 4 | adam | 10 | 8 | 0.240521 | 0.359553 |
| 128 | tanh | 4 | adam | 10 | 16 | 0.231095 | 0.335379 |
| 128 | tanh | 4 | adam | 10 | 32 | 0.239318 | 0.343657 |
| 128 | tanh | 4 | adam | 10 | 64 | 0.227629 | 0.336927 |
| 128 | tanh | 4 | adam | 10 | 128 | 0.234796 | 0.331781 |
| 128 | tanh | 4 | adam | 10 | 256 | 0.233418 | 0.332583 |
| 128 | tanh | 5 | adam | 10 | 8 | 0.250075 | 0.347792 |
| 128 | tanh | 5 | adam | 10 | 16 | 0.256327 | 0.341721 |
| 128 | tanh | 5 | adam | 10 | 32 | 0.238161 | 0.339108 |
| 128 | tanh | 5 | adam | 10 | 64 | 0.246192 | 0.337979 |
| 128 | tanh | 5 | adam | 10 | 128 | 0.25335 | 0.346402 |
| 128 | tanh | 5 | adam | 10 | 256 | 0.241074 | 0.348754 |
| 256 | relu | 2 | adam | 10 | 8 | 0.22492 | 0.330053 |
| 256 | relu | 2 | adam | 10 | 16 | 0.21829 | 0.319995 |
| 256 | relu | 2 | adam | 10 | 32 | 0.206546 | 0.311331 |
| 256 | relu | 2 | adam | 10 | 64 | 0.222013 | 0.333738 |
| 256 | relu | 2 | adam | 10 | 128 | 0.226403 | 0.331925 |
| 256 | relu | 2 | adam | 10 | 256 | 0.21609 | 0.32517 |
| 256 | relu | 3 | adam | 10 | 8 | 0.208955 | 0.313305 |
| 256 | relu | 3 | adam | 10 | 16 | 0.213413 | 0.322922 |
| 256 | relu | 3 | adam | 10 | 32 | 0.216532 | 0.309685 |
| 256 | relu | 3 | adam | 10 | 64 | 0.233748 | 0.323179 |
| 256 | relu | 3 | adam | 10 | 128 | 0.220572 | 0.316795 |
| 256 | relu | 3 | adam | 10 | 256 | 0.207801 | 0.315225 |
| 256 | relu | 4 | adam | 10 | 8 | 0.227595 | 0.328941 |
| 256 | relu | 4 | adam | 10 | 16 | 0.222006 | 0.310008 |
| 256 | relu | 4 | adam | 10 | 32 | 0.21559 | 0.320607 |
| 256 | relu | 4 | adam | 10 | 64 | 0.228188 | 0.339168 |
| 256 | relu | 4 | adam | 10 | 128 | 0.214371 | 0.324241 |
| 256 | relu | 4 | adam | 10 | 256 | 0.218668 | 0.319749 |
| 256 | relu | 5 | adam | 10 | 8 | 0.204824 | 0.306975 |
| 256 | relu | 5 | adam | 10 | 16 | 0.2072 | 0.312006 |
| 256 | relu | 5 | adam | 10 | 32 | 0.211578 | 0.313068 |
| 256 | relu | 5 | adam | 10 | 64 | 0.207819 | 0.31816 |
| 256 | relu | 5 | adam | 10 | 128 | 0.218923 | 0.318841 |
| 256 | relu | 5 | adam | 10 | 256 | 0.209012 | 0.322161 |
| 256 | tanh | 2 | adam | 10 | 8 | 0.241161 | 0.332925 |
| 256 | tanh | 2 | adam | 10 | 16 | 0.236437 | 0.352759 |
| 256 | tanh | 2 | adam | 10 | 32 | 0.235537 | 0.345777 |
| 256 | tanh | 2 | adam | 10 | 64 | 0.222765 | 0.319905 |
| 256 | tanh | 2 | adam | 10 | 128 | 0.276093 | 0.384288 |
| 256 | tanh | 2 | adam | 10 | 256 | 0.257078 | 0.348434 |
| 256 | tanh | 3 | adam | 10 | 8 | 0.244419 | 0.347092 |
| 256 | tanh | 3 | adam | 10 | 16 | 0.240922 | 0.345308 |
| 256 | tanh | 3 | adam | 10 | 32 | 0.234029 | 0.32941 |
| 256 | tanh | 3 | adam | 10 | 64 | 0.253037 | 0.358586 |
| 256 | tanh | 3 | adam | 10 | 128 | 0.247425 | 0.341759 |
| 256 | tanh | 3 | adam | 10 | 256 | 0.238934 | 0.339356 |
| 256 | tanh | 4 | adam | 10 | 8 | 0.246199 | 0.345001 |
| 256 | tanh | 4 | adam | 10 | 16 | 0.285021 | 0.369102 |
| 256 | tanh | 4 | adam | 10 | 32 | 0.250844 | 0.354546 |
| 256 | tanh | 4 | adam | 10 | 64 | 0.248201 | 0.343142 |
| 256 | tanh | 4 | adam | 10 | 128 | 0.24275 | 0.352775 |
| 256 | tanh | 4 | adam | 10 | 256 | 0.27193 | 0.353073 |
| 256 | tanh | 5 | adam | 10 | 8 | 0.253979 | 0.347408 |
| 256 | tanh | 5 | adam | 10 | 16 | 0.289563 | 0.393264 |
| 256 | tanh | 5 | adam | 10 | 32 | 0.255788 | 0.36383 |
| 256 | tanh | 5 | adam | 10 | 64 | 0.262953 | 0.355329 |
| 256 | tanh | 5 | adam | 10 | 128 | 0.335417 | 0.427542 |
| 256 | tanh | 5 | adam | 10 | 256 | 0.250764 | 0.340192 |
| 512 | relu | 2 | adam | 10 | 8 | 0.22466 | 0.326564 |
| 512 | relu | 2 | adam | 10 | 16 | 0.20764 | 0.308138 |
| 512 | relu | 2 | adam | 10 | 32 | 0.207466 | 0.306897 |
| 512 | relu | 2 | adam | 10 | 64 | 0.204488 | 0.308437 |
| 512 | relu | 2 | adam | 10 | 128 | 0.220727 | 0.334703 |
| 512 | relu | 2 | adam | 10 | 256 | 0.225631 | 0.326718 |
| 512 | relu | 3 | adam | 10 | 8 | 0.212763 | 0.316141 |
| 512 | relu | 3 | adam | 10 | 16 | 0.216349 | 0.313481 |
| 512 | relu | 3 | adam | 10 | 32 | 0.217768 | 0.330387 |
| 512 | relu | 3 | adam | 10 | 64 | 0.209336 | 0.309705 |
| 512 | relu | 3 | adam | 10 | 128 | 0.218821 | 0.321909 |
| 512 | relu | 3 | adam | 10 | 256 | 0.209158 | 0.320715 |
| 512 | relu | 4 | adam | 10 | 8 | 0.216645 | 0.30532 |
| 512 | relu | 4 | adam | 10 | 16 | 0.218291 | 0.322156 |
| 512 | relu | 4 | adam | 10 | 32 | 0.219542 | 0.3255 |
| 512 | relu | 4 | adam | 10 | 64 | 0.210059 | 0.324888 |
| 512 | relu | 4 | adam | 10 | 128 | 0.240048 | 0.330429 |
| 512 | relu | 4 | adam | 10 | 256 | 0.213656 | 0.320748 |
| 512 | relu | 5 | adam | 10 | 8 | 0.21028 | 0.315247 |
| 512 | relu | 5 | adam | 10 | 16 | 0.215092 | 0.333958 |
| 512 | relu | 5 | adam | 10 | 32 | 0.205468 | 0.317355 |
| 512 | relu | 5 | adam | 10 | 64 | 0.215524 | 0.326526 |
| 512 | relu | 5 | adam | 10 | 128 | 0.218851 | 0.331722 |
| 512 | relu | 5 | adam | 10 | 256 | 0.211248 | 0.317664 |
| 512 | tanh | 2 | adam | 10 | 8 | 0.26916 | 0.371122 |
| 512 | tanh | 2 | adam | 10 | 16 | 0.249523 | 0.359523 |
| 512 | tanh | 2 | adam | 10 | 32 | 0.251282 | 0.347249 |
| 512 | tanh | 2 | adam | 10 | 64 | 0.249699 | 0.33647 |
| 512 | tanh | 2 | adam | 10 | 128 | 0.252707 | 0.335468 |
| 512 | tanh | 2 | adam | 10 | 256 | 0.24265 | 0.338714 |
| 512 | tanh | 3 | adam | 10 | 8 | 0.24744 | 0.345029 |
| 512 | tanh | 3 | adam | 10 | 16 | 0.260181 | 0.351524 |
| 512 | tanh | 3 | adam | 10 | 32 | 0.24503 | 0.352272 |
| 512 | tanh | 3 | adam | 10 | 64 | 0.239263 | 0.332768 |
| 512 | tanh | 3 | adam | 10 | 128 | 0.26393 | 0.350336 |
| 512 | tanh | 3 | adam | 10 | 256 | 0.240569 | 0.344233 |
| 512 | tanh | 4 | adam | 10 | 8 | 0.312941 | 0.399384 |
| 512 | tanh | 4 | adam | 10 | 16 | 0.276091 | 0.358734 |
| 512 | tanh | 4 | adam | 10 | 32 | 0.26419 | 0.363237 |
| 512 | tanh | 4 | adam | 10 | 64 | 0.272449 | 0.370399 |
| 512 | tanh | 4 | adam | 10 | 128 | 0.271018 | 0.368783 |
| 512 | tanh | 4 | adam | 10 | 256 | 0.261508 | 0.356 |
| 512 | tanh | 5 | adam | 10 | 8 | 0.438602 | 0.483205 |
| 512 | tanh | 5 | adam | 10 | 16 | 0.292322 | 0.408722 |
| 512 | tanh | 5 | adam | 10 | 32 | 0.264102 | 0.377943 |
| 512 | tanh | 5 | adam | 10 | 64 | 0.260966 | 0.357417 |
| 512 | tanh | 5 | adam | 10 | 128 | 0.271158 | 0.383269 |
| 512 | tanh | 5 | adam | 10 | 256 | 0.252574 | 0.367451 |

Figure 1 - Grid Search 1 Results – Green indicates near best result – Orange indicates best result

The majority of the best scoring models according to MSE, used 512 units per layer, with a relatively low batch size. I decided to carry on testing with a smaller space of these attributes, but with different epochs and optimizers.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Units/Layer | Act | Layers | Optimizer | Epochs | Batch\_Size | MSE | MAE |
| 512 | relu | 2 | adam | 25 | 64 | 0.200694 | 0.307518 |
| 512 | relu | 2 | adam | 50 | 64 | 0.207431 | 0.294119 |
| 512 | relu | 2 | rmsprop | 10 | 64 | 0.237384 | 0.335353 |
| 512 | relu | 2 | rmsprop | 25 | 64 | 0.271016 | 0.353882 |
| 512 | relu | 2 | rmsprop | 50 | 64 | 0.248458 | 0.327234 |
| 512 | relu | 2 | adagrad | 10 | 64 | 0.297886 | 0.389166 |
| 512 | relu | 2 | adagrad | 25 | 64 | 0.266866 | 0.364524 |
| 512 | relu | 2 | adagrad | 50 | 64 | 0.254512 | 0.353608 |
| 512 | relu | 2 | adamax | 10 | 64 | 0.212671 | 0.314056 |
| 512 | relu | 2 | adamax | 25 | 64 | 0.203614 | 0.299226 |

Figure 2 - Grid Search 2 Results – Green indicates best result

These results indicate changing the optimizer and epochs created minimal differences in the MSE score.

**Third Model – Random Forest Regression**

Once again, an exhaustive search was performed over a large space of possible hyper-parameters.

10 None 2 2 0.204426 0.792292

10 None 2 4 0.22067 0.775786

10 None 2 8 0.22303 0.773388

10 None 5 1 0.218967 0.777517

10 None 5 2 0.219281 0.777198

10 None 5 4 0.209294 0.787346

10 None 5 8 0.220742 0.775714

10 None 10 1 0.207969 0.788691

10 None 10 2 0.211008 0.785604

10 None 10 4 0.212197 0.784396

10 None 10 8 0.20981 0.786821

10 None 15 1 0.223362 0.773052

10 None 15 2 0.2147 0.781853

10 None 15 4 0.213335 0.78324

10 None 15 8 0.219237 0.777243

10 None 20 1 0.220707 0.775749

10 None 20 2 0.217372 0.779138

10 None 20 4 0.221921 0.774516

10 None 20 8 0.215733 0.780803

10 10 2 1 0.252138 0.743814

10 10 2 2 0.242202 0.753909

10 10 2 4 0.240799 0.755335

10 10 2 8 0.244633 0.751439

10 10 5 1 0.24682 0.749217

10 10 5 2 0.239446 0.756709

10 10 5 4 0.244093 0.751988

10 10 5 8 0.247924 0.748095

10 10 10 1 0.252717 0.743225

10 10 10 2 0.246019 0.750031

10 10 10 4 0.23743 0.758757

10 10 10 8 0.242588 0.753517

10 10 15 1 0.241349 0.754776

10 10 15 2 0.250417 0.745562

10 10 15 4 0.244641 0.751431

10 10 15 8 0.239028 0.757134

10 10 20 1 0.249479 0.746516

10 10 20 2 0.24443 0.751645

10 10 20 4 0.248179 0.747836

10 10 20 8 0.243689 0.752398

10 20 2 1 0.215813 0.780722

10 20 2 2 0.213614 0.782956

10 20 2 4 0.215417 0.781124

10 20 2 8 0.224081 0.772321

10 20 5 1 0.220059 0.776407

10 20 5 2 0.211665 0.784937

10 20 5 4 0.210821 0.785794

10 20 5 8 0.218665 0.777824

10 20 10 1 0.20891 0.787736

10 20 10 2 0.214518 0.782037

10 20 10 4 0.210495 0.786125

10 20 10 8 0.231209 0.765078

10 20 15 1 0.211476 0.785129

10 20 15 2 0.215766 0.78077

10 20 15 4 0.218892 0.777593

10 20 15 8 0.219618 0.776856

10 20 20 1 0.220423 0.776038

10 20 20 2 0.22353 0.77288

10 20 20 4 0.216314 0.780213

10 20 20 8 0.221995 0.77444

10 30 2 1 0.218151 0.778346

10 30 2 2 0.21766 0.778845

10 30 2 4 0.206364 0.790323

10 30 2 8 0.216594 0.779928

10 30 5 1 0.221201 0.775248

10 30 5 2 0.205765 0.790931

10 30 5 4 0.211114 0.785496

10 30 5 8 0.218473 0.778019

10 30 10 1 0.223549 0.772862

10 30 10 2 0.212073 0.784521

10 30 10 4 0.214607 0.781947

10 30 10 8 0.222756 0.773667

10 30 15 1 0.221783 0.774656

10 30 15 2 0.21587 0.780663

10 30 15 4 0.21301 0.783569

10 30 15 8 0.215103 0.781443

10 30 20 1 0.218395 0.778098

10 30 20 2 0.223805 0.772601

10 30 20 4 0.213608 0.782962

10 30 20 8 0.225014 0.771374

10 40 2 1 0.215923 0.78061

10 40 2 2 0.215825 0.780709

10 40 2 4 0.212961 0.783619

10 40 2 8 0.222113 0.77432

10 40 5 1 0.222686 0.773738

10 40 5 2 0.210998 0.785614

10 40 5 4 0.212672 0.783913

10 40 5 8 0.22062 0.775838

10 40 10 1 0.210942 0.785671

10 40 10 2 0.21443 0.782127

10 40 10 4 0.205945 0.790748

10 40 10 8 0.216464 0.780061

10 40 15 1 0.213753 0.782815

10 40 15 2 0.215378 0.781164

10 40 15 4 0.217162 0.779351

10 40 15 8 0.213668 0.782901

10 40 20 1 0.219364 0.777113

10 40 20 2 0.21708 0.779435

10 40 20 4 0.218709 0.777779

10 40 20 8 0.21846 0.778032

10 50 2 1 0.21816 0.778337

10 50 2 2 0.214489 0.782067

10 50 2 4 0.216201 0.780327

10 50 2 8 0.211893 0.784704

10 50 5 1 0.215582 0.780957

10 50 5 2 0.212949 0.783632

10 50 5 4 0.212727 0.783858

10 50 5 8 0.219819 0.776652

10 50 10 1 0.208082 0.788577

10 50 10 2 0.209613 0.787021

10 50 10 4 0.216951 0.779566

10 50 10 8 0.220786 0.775669

10 50 15 1 0.216215 0.780313

10 50 15 2 0.222574 0.773852

10 50 15 4 0.20792 0.788742

10 50 15 8 0.221777 0.774662

10 50 20 1 0.224052 0.772351

10 50 20 2 0.224961 0.771427

10 50 20 4 0.219111 0.777371

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