

1. A recurrent neural network (RNN) receives sequences of 3-dimensional inputs and has two hidden neurons and one output neuron. The weight matrix \mathbf{U} connecting the input to the hidden layer, the weight matrix \mathbf{V} connecting the hidden output to the output layer, the hidden layer bias \mathbf{b} and the output layer bias \mathbf{c} are given by

$$\mathbf{U} = \begin{pmatrix} -1.0 & 0.5 \\ 0.5 & 0.1 \\ 0.2 & -2.0 \end{pmatrix}, \mathbf{V} = \begin{pmatrix} 2.0 \\ -1.5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0.2 \\ 0.2 \end{pmatrix}, \text{ and } \mathbf{c} = 0.5.$$

Find the output sequence for an input sequence of $(\mathbf{x}(t))_{t=1}^4$ where

$$\mathbf{x}(1) = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \mathbf{x}(2) = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}, \mathbf{x}(3) = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}, \text{ and } \mathbf{x}(4) = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \text{ if}$$

- (a) The RNN is of hidden-recurrence (Elman) type with the recurrence weight matrix \mathbf{W} connecting the previous hidden output to the current hidden layer input is given by

$$\mathbf{W} = \begin{pmatrix} 2.0 & 1.3 \\ 1.5 & 0.0 \end{pmatrix}.$$

Assume that the hidden activations are initialized to zero.

- (b) The RNN is of top-down recurrence (Jordan) type with the weight matrix \mathbf{W} connecting the previous output to the current state of hidden layer is given by

$$\mathbf{W} = \begin{pmatrix} 2.0 & 1.3 \end{pmatrix}.$$

Assume that the output activations are initialized to zero.

2. An RNN receives 3-dimensional input sequence and produces 2-dimensional output sequence. It has 5 hidden neurons and receives sequences of 100 time steps.
- Generate 8 training input sequences by drawing values uniformly between 0 and 1.0.
 - If the sequence $(\mathbf{y}(t))_{t=1}^{100}$ where $\mathbf{y}(t) = (y_k(t))_{k=1}^2 \in \mathbf{R}^2$ denotes the output sequence for an input sequence $(\mathbf{x}(t))_{t=1}^{100}$ where $\mathbf{x}(t) = (x_i(t))_{i=1}^3 \in \mathbf{R}^3$, generate the corresponding output sequences for the input training sequences as follows:

$$\begin{aligned} y_1(t) &= 5x_1(t) - 0.2x_3(t-1) + 0.1\varepsilon \\ y_2(t) &= 25x_2(t-1)x_3(t-3) + 0.1\varepsilon \end{aligned}$$

where ε is standard normally distributed random variable.

Train an RNN to learn the above sequences by using gradient descent learning. Use a learning factor $\alpha = 0.01$ and Adam optimizer.

3. Design an LSTM layer with 10 units to map the following input and output sequences:

| Input x | Output y |
|-----------|--------------|
| (1 2 5 6) | (1 3 7 11) |
| (5 7 7 8) | (5 12 14 15) |
| (3 4 5 7) | (3 7 9 12) |

Plot the learning curves at a rate $\alpha = 0.001$.

Find the output sequences for the following input sequences:

(1 2 3 4)
(4 5 6 7)

4. Design a character RNN layer with 10 GRU units to learn the sentiments about a movie, given in table 1.

Convert the input text into character ids and set the maximum text length to be 40. Train the network using gradient descent learning with the Adam optimizer and at a learning rate $\alpha = 0.001$. Use dropouts at the hidden layer of GRU at a probability $p = 0.7$.

Plot the cost and the accuracies against epochs during training.

Table 1

| <i>Input</i> | <i>Sentiment</i> |
|--|------------------|
| I did not like the movie | negative |
| The movie was not good | negative |
| I watched the movie with great interest | positive |
| I have seen better movies | negative |
| Good to see that movie | positive |
| I am not a fan of movies | negative |
| I liked the movie great | positive |
| The movie was of interest to me | positive |
| I thought they could show interesting scenes | negative |
| The movie did not have good scenes | negative |
| Family did not like the movie at all | negative |

After training, find the likelihoods of the sentiments of the following statements:

The movie was not interesting to me

I liked the movie with great interest