

Towards LOGIGLUE: A Brief Survey and A Benchmark for Analyzing Logical Reasoning Capabilities of Language Models

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Abstract

Logical reasoning is fundamental for humans yet presents a substantial challenge in the domain of Artificial Intelligence. Initially, researchers used Knowledge Representation and Reasoning (KR) systems that did not scale and required non-trivial manual effort. Recently, the emergence of large language models (LLMs) has demonstrated the ability to overcome various limitations of formal Knowledge Representation (KR) systems. Consequently, there’s a growing interest in using LLMs for logical reasoning via natural language. This work strives to understand the proficiency of LLMs in logical reasoning by offering a brief review of the latest progress in this area; with a focus on the logical reasoning datasets, tasks, and the methods adopted to utilize LLMs for reasoning. To offer a thorough analysis, we’ve compiled a benchmark titled LOGIGLUE. This includes 24 varied datasets encompassing deductive, abductive, and inductive reasoning. Utilizing LOGIGLUE as a foundation, we have trained an instruction fine-tuned language model, resulting in LogiT5. We study single-task training, multi-task training, and a “chain-of-thought” knowledge distillation fine-tuning technique to assess the model’s performance across the different logical reasoning categories. We also assess various large language models (LLMs) using LogiGLUE, and the findings indicate that LLMs excel most in abductive reasoning, followed by deductive reasoning, while they are least effective at inductive reasoning. We aim to shed light on the capabilities and potential pathways for enhancing logical reasoning proficiency in LLMs, paving the way for more advanced and nuanced developments in this critical field.

1 Introduction

Logical reasoning enables humans to *deduce* new conclusions, *learn* (or *induce*) new abstractions and provide plausible explanations (or *abduce*) to various phenomena in the universe. The discovery of black holes serve as a compelling example of the power of logical reasoning. Scientists like Stephen Hawking used the principles of relativity and quantum mechanics to predict their existence and properties. Through formalization and logical deductions alone, they predicted the presence of these mysterious cosmic entities; fifty years before it was confirmed by MIT researchers through physical experiments. From early days, Artificial Intelligence (AI) researchers therefore have devoted significant attention towards developing logical reasoning abilities (McCarthy, 1989; Colmerauer & Roussel, 1996). Early efforts revolved around the design of formal logical languages to encapsulate

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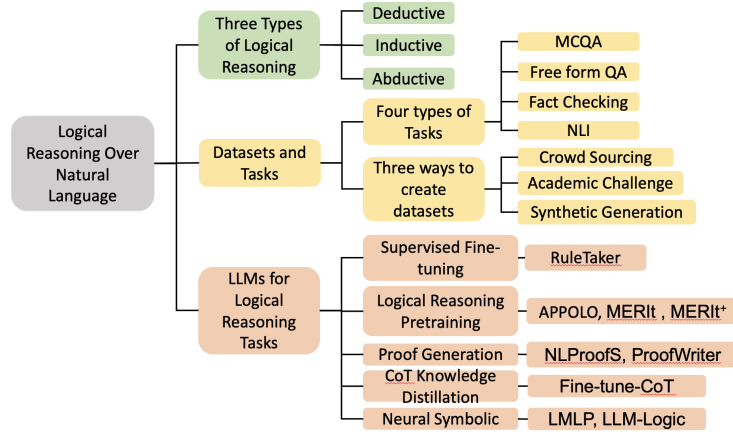


Figure 1: Logical Reasoning Survey: Datasets and Language Model Application.

rules and facts (domain knowledge), alongwith automated inference engines (Lifschitz, 2019). This paradigm necessitated expert understanding of the target domain, and, understanding of syntax and semantics of a logical language; which made knowledge *representation* and *acquisition* hard, and an expert-driven endeavor. These challenges motivated the contemporary researchers to progressively turn towards addressing logical reasoning tasks (Clark et al., 2020; Tian et al., 2021b; Han et al., 2022) by employing transformer-based (Vaswani et al., 2017) pre-trained language models (Devlin et al., 2019a; Brown et al., 2020).

The Language models (LMs) that are pretrained using objectives such as the mask language modeling (Devlin et al., 2019a) and next word prediction (Brown et al., 2020) enables them to acquire adequate syntax and semantics of language, alongside commonsense knowledge. These language models excel in numerous natural language understanding tasks, owing to the unsupervised pretraining on a vast array of unstructured text data. However, it is unclear if the current pretraining objectives are sufficient for the models to infer logical reasoning because this involves understanding structure; coupled with inductive, deductive, and abductive reasoning skills. This question has drawn intense attention and inspired different research directions to examine if LMs can learn logical reasoning ability (Wu et al., 2023; Lanham et al., 2023; Clark et al., 2020; Joshi et al., 2020). For instance, Clark et al. (2020) shows that pre-trained language models can serve as a “soft-reasoner” based on their near-perfect performance on synthetic datasets. Creswell et al. (2022) shows that large LMs are few-shot logical reasoners. On the other hand, Liu et al. (2020); Joshi et al. (2020); Han et al. (2022) show that logical reasoning remains challenging for language models. Furthermore, Wu et al. (2023); Lanham et al. (2023) shows that LLMs maybe retrieving or reciting previously seen facts and steps, instead of actually reasoning. Liu et al. (2023b) shows that while ChatGPT and GPT-4 generally perform well on some benchmarks, their performance noticeably diminishes on new or out-of-distribution datasets.

To understand, and benchmark the progress of logical reasoning abilities in the era of language models, we offer four contributions. 1) First, we provide a concise **survey** of the role that reasoning plays within current language models. 2) Based on the insights from our survey, we assemble a logical reasoning **benchmark** termed as LOGIGLUE with tasks where conclusions can be drawn through deductive, inductive, or abductive reasoning based solely on the information provided. LOGIGLUE demonstrates three distinguishing characteristics. First, it encompasses diverse logical reasoning tasks evaluation, ensuring a comprehensive assessment of a model’s performance across varied logical paradigms. Second, the unique format of each dataset within LOGIGLUE simplifies both training and evaluation processes, facilitating swift integration into research workflows. Thirdly, researchers can easily compare with established baselines, and the LOGIGLUE offers the flexibility to seamlessly integrate new datasets in the future, ensuring its lasting relevance in logical reasoning evaluation. 3) Drawing on multi-task learning and instruction-fine-tuned models, we trained Flan-T5 (Chowdhery et al., 2022) seq2seq models using LOGIGLUE’s data and evaluated the efficacy of various fine-tuning strategies. Our findings reveal that approaches involving multi-task fine-tuning and chain-of-thought knowledge distillation lead to superior performance. 4) We comprehensively assess multiple LLMs

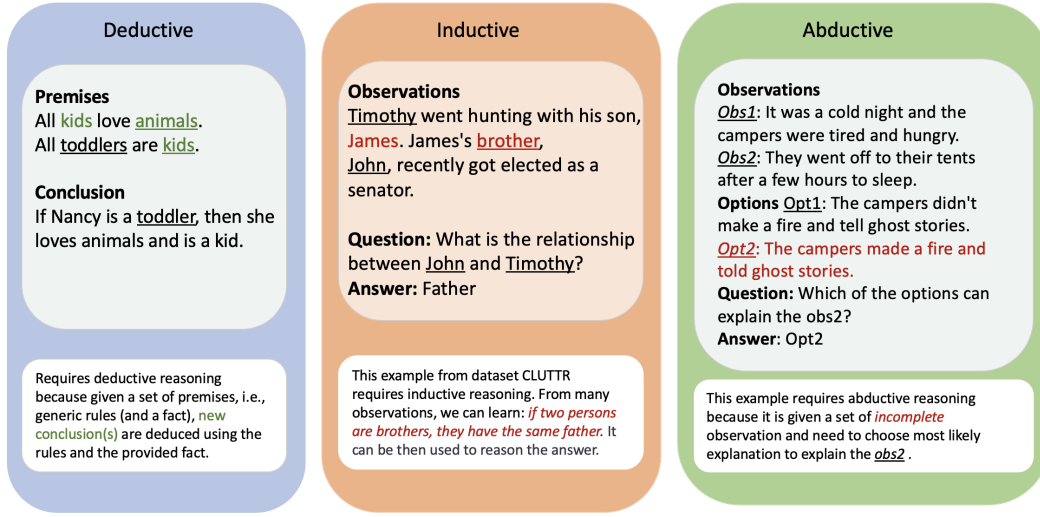


Figure 2: Examples (top) of three types of logical reasoning and explanations (bottom) correlating each example with its respective reasoning type.

performance on LogiGLUE, and find that LLMs perform the best at abductive reasoning, followed by deductive reasoning, while they are least effective at inductive reasoning.

2 A Concise Survey of Logical Reasoning in NLP

2.1 Three Types of Logical Reasoning

Deductive Reasoning. In this predominant form of reasoning, we start with a set of premises which can be facts or rules, and derive a specific conclusion based on the available premises with a valid logical derivation path. In short, deductive reasoning derives specific conclusion(s) from generic observation(s) (Byrne et al., 2019). There are two characteristics of a deductive reasoning system, *validity* and *soundness*. A conclusion is *valid* if and only if it is fully supported by the premises irrespective of the factuality of the premises. A conclusion is *sound* if and only if it is valid and the premises are true. For example in Figure 2, the conclusion “All kids love animals” is valid but is not sound. Most of the synthetic deductive reasoning datasets such as RuleTaker (Clark et al., 2020) has valid conclusions, but may not be sound as the synthetically generated rules in the premises may be untrue in the real world. Datasets such as PrOntoQA (Saparov & He, 2023b) offer a broader view, by sourcing the premise rules from a true, false and a fictional ontology.

Inductive Reasoning. For inductive reasoning, one starts with a set of observations, and derives a general conclusion that is merely true, but not certain (Heit, 2007; Sauce & Matzel, 2017). In contrast to deductive reasoning, inductive reasoning is a bottom-up reasoning process which starts from specific observations and derives a generic conclusion. Many Knowledge graph completion task requires inductive reasoning such as WN18RR¹. To apply inductive reasoning, one usually relies on a large number of observations (both positive and negative in support or against an induced rule). Since large language models are pretrained on large amount of free-text, it learns several generic patterns or conclusions, therefore reasoning inductively (even if the rules may not be represented symbolically or a human-readable fashion) (Han et al., 2023). Commonsense reasoning tasks in NLP may require both inductive and deductive reasoning.

Abductive Reasoning. Abductive reasoning typically begins with an incomplete set of observations and proceeds to derive most likely explanations for the observations to be true (Paul, 1993; Hobbs et al., 1993). Similar to inductive reasoning, this also involves uncertainty, as there can be different

¹Here, we exclude this task as we are more interested in natural language input. In this paper, we do not discuss about knowledge graph completion tasks since most of them are not in natural language forms.

explanations. While deductive reasoning is a process from known facts or rules to derive a new conclusion, while abductive reasoning is from an observation to a “guess” of what can be the reason to cause the observation. It is used more often in our daily decision-making, such as medical diagnoses based on an incomplete set of symptoms.

Remark. Both inductive and abductive reasoning inherently encompass uncertainty. In fact, deductive reasoning can also operate within the realm of uncertainty (De Raedt et al., 2007; Richardson & Domingos, 2006; Lee & Wang, 2016; Bach et al., 2017; Lee & Luo, 2018). Such reasoning paradigm uses “soft rules” to indicate the likelihood of a rule being true rather than its absolute truth. Consequently, conclusions derived may carry probabilistic true/false values. *Reasoning under uncertainty* is particularly useful because the real world is inherently unpredictable and full of unknown variables. While uncertainty is important, we found that most of the evaluation datasets are under the assumption that rules are unequivocally true, one exception is Rulebert (Saeed et al., 2021) which deviates by attributing weight values to each rule.

2.2 Logical Reasoning Tasks and Datasets

There exists various tasks connected to logical reasoning, including counterfactual reasoning (Tandon et al., 2019; Froberg & Binder, 2022), spatial reasoning (Mirzaee et al., 2021), and mathematical reasoning (Mishra et al., 2022b). These tasks often demand additional knowledge, such as understanding spatial relationships and requiring equation calculations, and therefore are not included in the following discussion.

2.2.1 Four Types of Tasks

Our survey reveals that logical reasoning datasets predominantly fall into four task formats: Multiple-choice QA (MCQA), Free-Form QA (FFQA), fact-checking (FV), and Natural Language Inference (NLI). MCQA and FFQA involve question-answering with varied answer formats (Yu et al., 2019; Liu et al., 2020; Weston et al., 2015b; Banerjee et al., 2020). Fact-Checking requires validating the truth value of a fact against given information (Clark et al., 2020; Saeed et al., 2021; He et al., 2021), while NLI entails analyzing a premise and hypothesis to determine their logical relationship (Tian et al., 2021b).

2.2.2 Dataset Creation Approach

Human Annotation via Crowdsourcing. This manner offers benefits like diverse linguistic structures and complex task designs. We find that most of the logical NLI datasets are created in this manner. However, it’s costly and often introduces biases, impacting dataset quality and inflating neural model accuracy scores (Yu et al., 2019). Additionally, creating logical reasoning datasets requires expertise, adding to the challenge.

Extraction from Academic Challenges. While crowdsourcing workers struggle to create complex logical reasoning questions due to the need for extensive training, standardized test questions can serve as a source for these datasets after preprocessing (Yu et al., 2019; Liu et al., 2020). However, the scope of these exams is narrow and results in smaller dataset sizes.

Synthetic Generation. Such a method offers a more efficient way to create large datasets compared to manual methods. It includes simulation-based approaches (Weston et al., 2015b) and rule-based methods (Clark et al., 2020; Saeed et al., 2021; Banerjee et al., 2020), where logic programs are transformed into natural language. However, this method may produce rules or facts with limited real-world applicability and simplistic language.

2.3 Language Models for Logical Reasoning over Natural Language

Recent studies on language models (LMs) for logical reasoning tasks have shown promising developments. Large language models (LLMs) demonstrate human-like abstract reasoning skills (Dasgupta et al., 2022). Creswell et al. (2022) introduced a selection-inference pipeline for contextual fact selection and question decomposition. Wei et al. (2022) highlighted the capability of language models in chain-of-thoughts (CoT) reasoning, aiding in tasks like mathematical reasoning. The next section summarizes five key trends in using LMs for logical reasoning.

Supervised Finetuning. Fine-tuning language models on downstream tasks is a standard approach for logical reasoning tasks (Clark et al., 2020; Liu et al., 2021; Tian et al., 2021a; Saeed et al., 2021; Han et al., 2022; Chan et al., 2023). This method typically employs moderate-sized models like BERT, GPT2, RoBERTa, and XLNet (Devlin et al., 2019b; Radford et al.; Liu et al., 2019; Yang et al., 2019). Transformer-based models outperform others like LSTM, partly due to their inherent commonsense and logical reasoning capabilities (Huang et al., 2019; Clark et al., 2020). Larger models tend to perform better, indicating a correlation between model depth and reasoning complexity (He et al., 2021). Despite high in-domain performance, these models often lack generalization across different reasoning depths, language levels, and domains (Clark et al., 2020; Richardson & Sabharwal, 2021; Tafjord et al., 2021; Banerjee et al., 2020), suggesting they may learn patterns rather than underlying reasoning skills (Zhang et al., 2022).

Logical Reasoning Pretraining. Language models learn syntax, semantics, and world knowledge through tasks like next-word prediction or masked language modeling, but this doesn’t ensure logical reasoning skills. To address this, researchers are developing logical reasoning-oriented pretraining tasks. APOLLO (Sanyal et al., 2022) uses two tasks: selective masked language modeling (s-MLM), masking logical words, and entailment classification for relationships in masked sentences. MERIt (Fangkai Jiao, 2022) introduces a meta-path-guided task, converting documents into graphs for self-supervised learning, distinguishing positive from negative entity-linked sentences. MERIt⁺ (Jiao et al., 2023) evolves this by optimizing the probability of positive sentences, moving away from contrastive learning.

Proof Generation. Proof generation is more challenging than answer generation but yields better out-of-domain or unseen depth reasoning performance (Saha et al., 2020; Tafjord et al., 2021). NL-ProofS, introduced by Kaiyu Yang & Chen (2022), generates step-by-step proofs using a prover, a verifier for step validity, and an algorithm to retrieve the highest validity score proof. ProofWriter (Tafjord et al., 2021) employs T5 models for proof generation, either predicting the entire proof sequence at once or iteratively generating conclusions. Proof strategies typically involve forward chaining (starting with facts and deriving inferences) or backward chaining (starting with a target and breaking it into sub-goals). Kazemi et al. (2022) found backward chaining more effective for LLMs in deductive logical reasoning tasks.

CoT Knowledge Distillation. While previous methods depend on proof annotations in datasets, it’s shown that large language models (LLMs) can generate step-by-step reasoning, akin to proofs (Saparov & He, 2023b; Liu et al., 2023c). Namgyu Ho (2022) introduced the Fine-tune-CoT (chain-of-thought) approach, which involves a three-step process. First, a large teacher model generates multi-step explanations for complex queries, which are then filtered for accuracy. Second, these explanations form reasoning samples, combining the question, rationale, and answer for comprehensive prompts. Finally, a student model is fine-tuned with these samples to enhance its reasoning capabilities. However, LLMs sometimes struggle with planning proofs, leading to incomplete proofs and potential inaccuracies.

Neural Symbolic. Pretrained Language Models (PLMs) show promise in natural language reasoning but have inconsistent failures. In contrast, symbolic solvers excel in symbolic reasoning tasks but struggle with diverse natural language inputs. A viable approach is using LLMs to convert natural language inputs into symbolic programs for symbolic solvers, reducing LLM unfaithfulness (Pan et al., 2023). Hanlin Zhang¹ (2022) use LLMs as Logic Programmers (LMLP) to reason over knowledge bases with Prolog’s backward chaining, outperforming CoT in deductive reasoning. Pan et al. (2023) developed Logic-LM for deductive reasoning and constraint programming tasks, translating natural language to formal language for symbolic engines. This paradigm is explored further, with LLMs as planners and external tools for execution (Lu et al., 2023; Sumers et al., 2023; Paranjape et al., 2023; Guan et al., 2023; Schick et al., 2023) and LLMs augmented with inputs from Logic provers Olausson* et al. (2023). Feng et al. (2023) addressed LLMs’ syntax parsing issues impacting solver accuracy by fine-tuning an LLM with an instruction-tuning dataset, thus improving parsing accuracy.

Dataset	Train size	Dev size	Test size	Synthetic	Task Type	Reasoning Type
Fine-tuning Set						
α ARCT 2019	2420	632	888	✗	MCQA	Abductive
α NLI 2019	169,654	-	1532	✗	NLI	Abductive
AbductionRule-Animal 2019	23,100	3,300	6,600	✓	FFQA	Abductive
LogicNLI 2021a	16,000	2,000	2000	✓	NLI	Deductive
ProofWriter 2021	69,814	10,158	20,058	✓	FV	Deductive
Rulebert-Union 2021	56,000	4,666	9,334	✓	FV	Deductive
FOLIO 2022	1004	204	227	✗	FV	Deductive
ANLI 2020	162,865	3,200	3,200	✗	NLI	Deductive
CLUTTR-Robust 2019	10,100	-	144	✓	FFQA	Inductive
LogiQA 2021	7,376	651	651	✗	MCQA	Mixed
Leave-out Testing Set						
AbductionRule-person 2019	-	-	4,864	✓	FFQA	Abductive
bAbi 2015a	-	-	5000	✓	FFQA	Deductive
Bird-Electricity 2021	-	-	5270	✗	FV	Deductive
NatLang 2021	-	-	8,008	✗	FV	Deductive
Winologic 2021	-	-	562	✗	FV	Deductive
WaNLI 2022	-	-	5000	✓	NLI	Deductive
Rulebert-Union 2021	-	-	5000	✓	FV	Deductive
PrOntoQA 2023a	-	-	200	✗	MCQA	Deductive
BigBench 2022	-	-	1300	✗	FFQA	Deductive
BigBench 2022	-	-	32	✗	FFQA	Inductive
bAbi 2015a	-	-	5000	✓	FFQA	Inductive
CLUTTR-Systematic 2019	-	-	10100	✓	FFQA	Inductive
ReClor 2020	-	-	500	✗	MCQA	Mixed
LogiQA 2.0 2023a	-	-	3238	✗	NLI	Mixed

Table 1: Statistics of In-domain (IID) and out-of-domain (OOD) datasets of LogiGLUE benchmark.

2.4 Survey Summary

Our survey outlines the current datasets covering three logical reasoning types across four task formats and how dataset curation impacts difficulty levels. We’ve highlighted five methods for employing LLMs in these reasoning tasks. These insights lay the groundwork for future research, guiding improvements in model performance and dataset creation. Next, we introduce a logical reasoning benchmark, aligned with established benchmarks like SuperGlue (Wang et al., 2019), BigBench (Srivastava et al., 2023), and Unicorn (Lourie et al., 2021), to thoroughly assess system capabilities. Unlike existing reasoning benchmarks such as CROW (Ismayilzada et al., 2023), our focus is evaluating diverse logical reasoning ability with limited need for implicit/external knowledge.

3 LOGIGLUE: General Logical Reasoning Benchmark

Various studies have reported differing evaluations of language models’ logical reasoning skills (Clark et al., 2020; Joshi et al., 2020; Han et al., 2022), attributing inconsistencies to the varied evaluation benchmarks and task formats employed. In response, we have compiled LOGIGLUE, a comprehensive benchmark to uniformly assess the logical reasoning capabilities of language models. This benchmark consists of 24 datasets, segmented into a fine-tuning set (10 datasets) and a testing set left out of training (14 datasets), detailed license information for which can be found in Appendix A. The fine-tuning set is designed for training on logical reasoning tasks, while the leave-out testing set aims to test the model’s ability to generalize to new, unrepresented domains beyond the fine-tuning dataset.

Diversity. Table 1 demonstrates that LOGIGLUE spans the entire spectrum of logical reasoning by encompassing all three primary reasoning types. Furthermore, LOGIGLUE incorporates datasets across four task categories and three methodologies of creation. Although we aim for varied reasoning types in both fine-tuning and leave-out testing sets, we observe a trend in the literature towards more datasets designed for deductive reasoning. This may be attributed to the feasibility of template-based generation for deductive reasoning, in contrast to the complexities involved in creating large-scale datasets for inductive and abductive reasoning.

Benchmark	# Tasks	Train	In-Domain Test	Leave-out Test	Annotation of Reasoning Type
GLoRe 2023	12	✗	✓	✓	✗
LogiTorch 2022	14	✓	✓	✗	✗
LogiBench 2023	3	✓	✓	✗	✓
CROW 2023	6	✗	✓	✓	✗
LOGIGLUE (ours)	24	✓	✓	✓	✓

Table 2: Compare LOGIGLUE with other logical reasoning benchmarks. Train: include training or not; Test: include training or not; OOD: whether include out-of-domain evaluation, Annotation: Annotation of Type of Reasoning,

Unique Format. Existing practices standardize diverse datasets into a consistent format, such as question answering or NLI styles (Mishra et al., 2022a; Lourie et al., 2021). A consistent format cross diverse datasets not only simplifies the training and testing process but also beneficial to build a single model that are capable of performing different tasks. Based on this practical reason, we convert all 24 datasets into a Sequence2Sequence format. Specifically, the input of the MCQA/FV/NLI tasks include a context, question, and answer choices, while FF tasks do not have answer options. For FV, statements serve as questions with true/false answer choices, and NLI tasks offer natural, contradictory, and entailment options. For all four types of tasks, the outputs are the answer string.

Comparison with Other Logic Reasoning Benchmark. While various initiatives aim to create a benchmark for assessing logical reasoning (Teng et al., 2023; Helwe et al., 2022; Anonymous, 2023; Ismayilzada et al., 2023), LOGIGLUE sets a new standard by offering a uniquely formatted, extensive training set. It encompasses an array of reasoning types and provides a well-organized, varied categorization of datasets. We compare with each benchmark in Table 2. Details discussion is given in Appendix B.

4 Experiments and Results

We mainly design two sets of experiments. The first focus on fine-tuning a relative small LM for logical reasoning tasks to evaluate the impact of various fine-tuning techniques. The second experiment tests pretrained language models using the leave-out testing set, aiming to assess and compare their performance across different reasoning types.

4.1 Fine-tuning Performance

We selected Flan-T5-large (Chowdhery et al., 2022) as our base model for training due to two pivotal reasons. Firstly, Flan-T5 stands as an instruction-fine-tuned iteration of T5, exhibiting enhanced performance when compared to its peers. Secondly, Flan-T5’s manageable size renders it to be trainable on a machine that is conducive to an academic setting.

Single Task Fine-Tuning. We fine-tune Flan-T5-large on each dataset and report average accuracy in Table 3. The model exhibits superior performance on synthetic data compared to hand-crafted alternatives as demonstrated by the observation that the top 5 performances are predominantly synthetic. This trend holds while considering the ANLI dataset, which, despite having a more substantial training set than its synthetic counterparts, yielded inferior results. This suggests that there might be some heuristic patterns exhibited in the synthetic datasets that the model can easily learn rather than learning the underlying logic.

Multitask Fine-tuning. We fine-tune the Flan-T5 Large on all in-domain datasets utilizing a weighted sampling technique to accommodate for the unbalanced size of the training datasets. While the complexity of both multi-task and single-task models is comparable, the multi-task model demonstrates a significantly higher average performance. As indicated in Table 3, the LogiT5 model surpasses each single-task model by a substantial margin, ranging between 30-60%. Multi-task training presents a distinct advantage over single-task models, particularly in scenarios involving low-resource data as shown in Table 4. This is supported by research (Parmar et al., 2022; Luo et al., 2022). Specifically, in cases of limited training data (i.e., limited to 1/2 K training samples, first block results in Table 4), LogiT5 tends to outperform single-task models, likely due to its ability to transfer skills or knowledge acquired from other datasets to enhance performance in low-resource

Model	Dataset Creation Approach	Test Split of Corresp. Dataset	LogiGLUE Avg. In-Domain Perf.
Single-Task Model			
Flan-T5	Synthetic		
+ AbductionRule-Animal		100.00	50.88
+ Rulebert-Union		99.69	34.61
+ ProofWriter		99.42	23.42
+ CLUTTR-Robust		97.22	50.17
+ LogicNLI		82.60	31.49
+ α NLI	Crowd Sourcing Academic Challenges	78.26	44.00
+ α ARCT		72.31	45.02
+ FOLIO		66.66	46.36
+ ANLI		61.16	21.03
+ LogiQA		37.94	43.32
Multi-Task Model			
LogiT5	-	-	80.88

Table 3: Performance of fine-tuned Flan-T5 (large) on each single dataset, followed by average performance over LogiGLUE IID Testing datasets.

Dataset	Training Size	Single-Task Model	LogiT5
α ARCT	2420	72.31	77.22
FOLIO	1004	66.66	74.02
α NLI	169,654	78.26	76.37
ANLI	162,865	61.16	59.53

Table 4: Performance of Single Task and LogiT5 model: the first block is low resource domain and the second block is high resource.

domains. This is evident in LogiT5’s enhanced performance, showing improvements of 5% and 8% in the α ARCT and FOLIO tasks, respectively. In contrast, for tasks with ample training data, such as the α NLI and ANLI datasets (second block results in Table 4), multi-task training does not offer additional benefits. This could be attributed to the fact that a large training set already provides a robust learning environment for the model, making the added value of multi-task training negligible. This trend highlights a significant constraint of multi-task training in situations where individual datasets are already extensive, limiting its effectiveness.

Fine-tuned LogiT5 on Single Dataset. Here, we further fine-tune LogiT5 on each dataset. However, upon analyzing the performance, we did not observe notable advantages from this additional fine-tuning even though small margin gains are achieved, which only yield 0.4% average improvement. The results of each dataset is given in Table 8 in Appendix. This suggests that LogiT5 has likely already learned the majority of knowledge from these tasks.

CoT Distillation Fine-tuning. As shown by previous work (Namgyu Ho, 2022), distill the chain-of-thoughts from a large model to a small student model boosting the performance of the student model. We apply such CoT finetuning strategy and conduct experiments on LogiQA, identified as the most challenging task, by distilling the CoT from LLama-chat-7B to Flan-T5 Large. Initially, we generated a single answer for each question, retaining only the samples where the predicted answer was correct, resulting in approximately 3K valid samples (**Type 1**). Alternatively, we created 10 answers for each question and preserved the samples with at least one correct predicted answer, which generated a unique set of 6K questions (**Type 2**). It is worth noting that some questions offered multiple correct reasoning paths. In such cases, we either opted for a singular path or utilized all available paths, the latter approach amassing a total of 15K training samples (**Type 3**). We trained the Flan-T5 model utilizing datasets consisting of 3K, 6K, and 15K samples, derived from the generated CoT, with the results delineated in Table 5. Our findings indicate that the training with 3K and 6K samples did not enhance the CoT’s fine-tuning efficacy. However, an increased dataset size of 15K samples facilitated a 4% improvement in performance, suggesting that CoT distillation becomes more beneficial with a larger volume of data.

Data	Type 0	Type 1	Type 2	Type 3
Acc	0.37	0.38	0.37	0.41

Table 5: Performance of Flan-T5 trained with CoT fine-tuning on the LogiQA dataset. Type 0 is without using CoT 7K data, Type 1 is with CoT 3K data, Type 2: CoT 6K data, Type 3: CoT 15K.

4.2 Evaluations of Pretrained LM and LogiT5 on Leave-out Testing Set

Type of Reasoning	Dataset	GPT4	GPT3.5	LLaMA-Chat-7B	Flan-T5	LogiT5
Deductive	bAbi-deductive	97.39	87.80	69.10	38.90	65.00
	Bird-Electricity	64.61	49.24	50.94	42.12	66.32
	NatLang	52.46	50.94	48.80	57.00	70.24
	Winologic	88.39	45.53	41.96	66.07	60.71
	WaNLI	65.20	45.02	31.50	49.90	61.90
	Bigbech-deduction	84.38	40.00	43.00	46.92	33.84
	Rulebert-Union	62.40	47.20	65.00	24.80	61.90
	prOntoQA	86.00	28.00	60.20	6.50	33.50
	Avg.	73.59	49.21	48.05	41.52	56.67
Inductive	CLUTTR-Systematic	9.00	8.11	8.60	15.75	100
	Bigbech-logical-args	72.30	40.63	37.50	50.00	40.63
	bAbi-inductive	1.30	5.40	30.80	56.49	12.90
	Avg.	31.55	18.04	25.63	40.74	51.17
Abductive	AbductionRule	98.76	68.20	52.47	3.76	96.10
Mixed	ReClor	76.00	61.00	41.00	42.00	47.00
	LogiQA 2.0	89.95	63.83	61.36	49.30	45.20
	Avg.	82.98	62.42	51.18	45.65	46.10

Table 6: Performance of Different Language Models on Leave-out Testing Set.

We examine three large language models: GPT-4, GPT-3.5-turbo, and LLaMA-2-chat (7B). We also examine Flan-T5 large and LogiT5, which are relative smaller language models (less than 1B parameters). We randomly sample 20% of the testing sets for evaluation due to the inference cost of GPT-4 and GPT-3.5-turbo. We provide the results on the full set using the other models in Table 6 in Appendix C.4. Our evaluation is based on zero-shot capabilities using chain-of-thought prompting. The prompt is given in the Appendix C.3.

Comparison Across Different Models. Table 6 clearly shows that GPT-4 outperforms other models in overall performance. Remarkably, LLaMA-2-Chat, despite having fewer parameters than GPT-3.5, manages to achieve comparable results. In addition, we dissect why GPT-4 and GPT-3 performs worse on some datasets. In our error analysis, we observed that for some tasks, such as CLUTTR and bAbi (inductive reasoning), GPT-3 frequently says that no relationship or answer could not be determined. Sometimes, there is a flaw in the model’s generated reasoning chain, leading to wrong answers. Take bAbi as an example, the given input is: Context: Wolves are afraid of mice. Winona is a cat. Mice are afraid of sheep. Sheep are afraid of wolves. Jessica is a sheep. Cats are afraid of wolves. Emily is a mouse. Gertrude is a wolf. Question: What is Gertrude afraid of? The ground truth is mouse, however the GPT-3 response is “Based on the context, Gertrude is a wolf. From statement 4, we know that sheep are afraid of wolves. Therefore, Gertrude is afraid of sheep”. To answer the question, we should know what the wolf is afraid of, rather than what is afraid of wolves. This highlights that the model makes an error in understanding the direction of the relationship.

Comparison Across Different Type of Reasoning. Table 6 reveals a consistent pattern in the performance of all three LLMs across reasoning types, ranking from strongest to weakest as abductive, deductive, and inductive reasoning. As discussed earlier, abductive reasoning involves inferring the most plausible cause behind an observation, a task at which these LLMs excel due to their extensive training on vast text corpora. Conversely, inductive reasoning requires the extrapolation of patterns from broad observations, a more challenging task for LLMs approached in a zero-shot manner due to insufficient exposure to varied instances required for pattern recognition. Incorporating few-shot in-context learning could potentially enhance inductive reasoning capabilities.

Compare Fine-tuned Model with Pretrained Models. When we assess the performances of LogiT5 and Flan-T5 as shown in Table 6, it is evident that LogiT5 generally outperforms Flan-T5, indicating that fine-tuning offers significant benefits to smaller LMs. In comparison to other pretrained large language models (LLMs) like GPT-3 and LLaMA, LogiT5 is comparable or even surpasses them in some datasets, particularly in tasks requiring inductive and abductive reasoning. However, in datasets involving mixed reasoning, LLMs significantly outperform LogiT5. This may arise because datasets like ReClor and LogiQA, both of which are from academic challenges and publicly accessible, therefore, might have been included in the LLMs’ training corpus, allowing these models to recall relevant information.

5 Conclusion

In this study, we concentrate our efforts on a crucial area of research: logical reasoning over natural language. Initially, we offer a survey to provide a thorough comprehension of this domain, emphasizing the role of large language models in addressing this demanding task. Following this, we assemble a benchmark for logical reasoning named LOGIGLUE, set to be publicly available to aid forthcoming research. Finally, utilizing LOGIGLUE, we fine-tune a language model and assess LLMs’ logical reasoning capability.

Limitation

Our survey, while comprehensive, focuses on the primary trends in enhancing the logical reasoning capabilities of Large Language Models (LLMs), given the rapid advancements in this field. While it may not encompass every paper on logical reasoning evaluation or methodology, it provides a robust overview of significant developments. Regarding the LOGIGLUE benchmark, it captures the most widely studied datasets at the time of publication. While recent datasets like LogiBench is not included initially, we plan to continuously integrate new logical reasoning datasets into our open-source benchmark for both training and testing, ensuring its relevance and utility in the evolving landscape of LLM research.

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Dataset	License
Abductive NLI	Apache-2.0 license
CLUTTR	Attribution-NonCommercial 4.0 International
Abduction Rules	MIT License
Adversarial NLI	Attribution-NonCommercial 4.0 International
LogiNLI	Creative Commons Attribution 4.0 International License
proofWriter	Creative Commons Attribution 4.0 International License
RuleBERT	Creative Commons Attribution 4.0 International License
Folio	CC-BY-SA-4.0 license
LogiQA	CC BY-NC-SA 4.0 DEED
baBi	BSD License
Electric-bird	Apache-2.0 license
Winologic	Creative Commons Attribution 4.0 International License
Wanli	Creative Commons Attribution 4.0 International License
BigBench	Apache-2.0 license
ProntoQA	Apache-2.0 license

Table 7: Dataset License.

A Dataset Licence

All datasets included in LogiGLUE benchmark are covered under licenses: Apache-2.0, MIT License, Attribution-NonCommercial 4.0 International (CC BY-NC 4.0), and BSD License. Each dataset license is given in Table 7. These licenses generally permit the reproduction of the datasets for non-commercial purposes. LogiGLUE will be public available under the license of Attribution-NonCommercial 4.0 International license.

B Compare with Existing Logical Reasoning Benchmark

GLoRe (Teng et al., 2023) offers only an evaluation dataset, lacking associated training data. In contrast, our dataset includes training sets formatted in a distinct and innovative style. Unlike LogiTorch (Helwe et al., 2022), which does not specify the reasoning types for its datasets, LogiGLUE categorizes each dataset into either deductive/inductive/abductive reasoning, also encompassing more datasets compared to LogiTorch. While LogiBench (Anonymous, 2023) is an extensive dataset, it is limited to deductive reasoning and solely consists of synthetically generated data. Furthermore, CROW (Ismayilzada et al., 2023) primarily concentrates on commonsense reasoning and is exclusively designed for evaluative purposes.

C Additional Experiments Results

C.1 Detailed Fine-tuning Results

In Table 8, we provide the detailed performance of each model that trained with single-task, multi-task and multi-task with single task training methods on each in-domain datasets.

C.2 CoT Distillation Fine-tuning

With the CoT fine-tuning, we observed that the fine-tuning takes longer time and a larger learning rate in the beginning is helpful. Thus, instead of using $1e-4$ as the learning rate, we trained the model with 40 epochs and a learning rate of $3e-4$ we use. We do see that the model performance increase when the number of epochs increase.

C.3 Zero-shot Evaluation with CoT Prompting Format

In the zero-shot evaluation of Llama, GPT-3 and GPT-4, we add a “Only based on the context, let’s think step by step.” sentence in the end to prompt models to generate reasoning. Take Winologic dataset as an example, the input to the model is following:

Dataset	Single-Task (Flan-T5-large)	Multi-Task (LogiT5)	Single Task (LogiT5)
α ARCT 2019	72.31	77.22	76.74
α NLI 2019	78.26	76.37	78.46
CLUTTR-Robust 2019	97.22	96.53	97.22
AbductionRule-Animal 2019	100	100	100
ANLI 2020	61.16	59.53	60.53
LogiQA 2021	37.94	38.56	39.94
LogicNLI 2021a	82.60	88.40	88.65
ProofWriter 2021	99.42	98.85	99.55
Rulebert-Union 2021	99.69	99.36	99.70
FOLIO 2022	66.66	74.02	72.06
Average	79.52	80.88	81.28

Table 8: Three training strategies for models and the performance on In-domain Testing Dataset.

Context: I can’t cut that tree down with that axe; it is too thick. Question: The it is more likely to refer to the tree than the axe because when the person is using the axe to cut down the tree, if the axe is too thick, the person might not be successful. Options: - False - True Please select the correct option. **Only based on the context, let’s think step by step.**

C.4 Full Evaluation On Out-of-Domain Datasets

In Table 9, we present comprehensive results for all out-of-domain datasets. Due to the financial constraints, we’ve excluded the complete GPT-4 and GPT-3.5-turbo outcomes on the full set. However, our primary results of GPT-4 and GPT-3.5-turbo tested on a randomly selected 20% of the test set are presented in the main paper, which can be taken as an approximation of full-set performance. It is worth to mentioning that GPT-3.5 and LLama-2 produce free-form text, making evaluation challenging, while Flan-T5’s structured output, trained on instruction fine-tuned data, simplifies this process. LLama-2 initially showed poor results, but upon using ConceptNET Speer et al. (2017) to account for synonyms, its performance improved. The babi dataset revealed LLama’s tendency to rely on pre-trained knowledge, ignoring input text, a phenomenon also noted in Varshney et al. (2023). Our analysis also includes a comparison of reasoning processes with and without Chain of Thought (CoT) prompting. While CoT offers a modest advantage, the impact is not markedly significant. We attribute this to LLaMA’s inherent tendency to generate reasoning chains, even without explicit step-by-step reasoning prompts.

Dataset	Flan-T5-large	LogiT5	LLAMA-2-7b-chat	LLAMA-2-7b-chat-CoT
bAbi(induction) 2015a	59.44	13.12	33.46	29.04
bAbi(deduction) 2015a	40.18	65.84	58.76	64.84
CLUTTR-Systematic 2019	15.43	100	38.82	30.62
AbductionRule-person 2019	00.00	95.97	31.31	54.91
ReClor 2020	40.20	47.00	39.80	40.20
Bird-Electricity 2021	41.12	65.29	50.00	38.78
NatLang 2021	56.03	70.24	49.52	50.29
Winologic 2021	68.51	61.39	48.93	49.28
WaNLI 2022	49.88	62.22	32.80	32.32
Rulebert-Union 2021	24.92	62.34	50.02	52.26
BigBench(logical-deduction) 2022	46.31	30.92	22.38	25.61
BigBench(logical args) 2022	43.75	40.62	40.62	31.25
LogiQA 2.0 2023a	52.66	23.13	50.74	50.15
PrOntoQA-OOD 2023	6.50	33.50	60.20	66.5
Average	38.70	55.11	43.38	44.00

Table 9: Performance on Leave-out Testing Datasets.