

Assignment 4

11510257 彭福

Exe.2

Solution:

a) True.

Proof:

Squaring each edge cost does not affect the order of edge during the execution of algorithm, so T must still be a minimum spanning tree for this different costs.

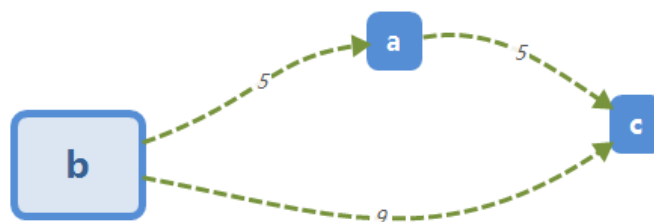
$$\text{i.e. } a < b < \dots < z \rightarrow a^2 < b^2 < \dots < z^2$$

b) False.

Counterexample :

Before squaring, $b \rightarrow c$ is the shortest path.

After squaring, $b \rightarrow a \rightarrow c$ is the shortest path.



Exe.8

Solution :

Suppose that G has two different MST \mathbf{P} and \mathbf{Q} .

Assume that we find a minimum weighted edge \mathbf{e} from \mathbf{P} which is not included in \mathbf{Q} . And we add \mathbf{e} to \mathbf{Q} to become

\mathbf{I} . ($\mathbf{Q} + \mathbf{e} = \mathbf{I}$) it is easy to know that \mathbf{I} must has a circuit that at least has one edge \mathbf{f} whose weight is larger than \mathbf{e} .

After removing \mathbf{f} , we get another connected graph \mathbf{J} whose weight is smaller than \mathbf{H} , which is a contradiction.

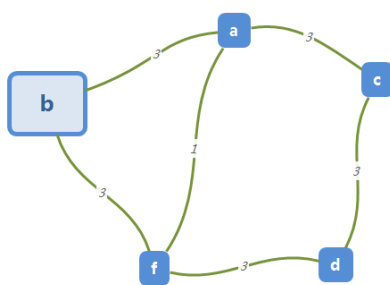
So G has a unique minimum spanning tree.

Exe.22

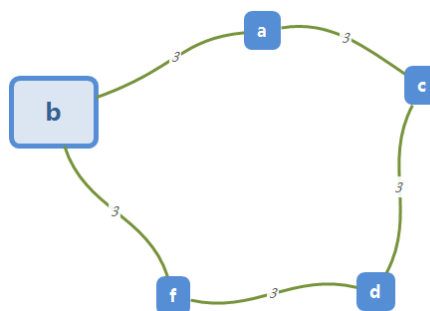
Solution:

False.

Counterexample:



Graph G



Spanning tree T

every edge $e \in$ some MST, but H is NOT a MST.