Assignment 2

11510257　彭福

Exe.1

Solution：

a) 1) $(2n)^2 = 4n^2$, so it is slower a factor of 4.

2) $(n+1)^2 = n^2 + 2n + 1$, so it is slower an additive $2n+1$.

b) 1) $(3n)^2 = 9n^2$, so it is slower a factor of 8.

2) $(n+1)^3 = n^3 + 3n^2 + 3n + 1$, so it is slower an additive $3n^2+3n+1$.

c) 1) $100(2n)^2 = 400n^2$, so it is slower a factor of 4.

2) $100(n+1)^2 = 100n^2 + 200n + 100$, so it is slower a additive $200n+100$.

d) 1) $2n*\log(2n)$, so it is slower an additive $2n*\log2$.

2) $(n+1)*\log(n+1)$, so it is slower an additive $\log(n+1) + n\,[\log(n+1) - \log n]$.

e) 1) $2^{2n} = (2^n)^2$, so it is slower the square of the previous running time.

2) 2^{n+1}, so it is slower a factor of 2.

Exe.5

Solution：

According to the question, assume that $f(n) \leq \mathrm{C} * g(n)$,where C is a constant.

a) Ture.

Proof:

$$\begin{aligned} \log_2 f(n) &\leq \log_2(C * g(n)) \\ &= \log_2 C + \log_2 g(n) \\ &= O(\log_2 g(n)) \end{aligned}$$

b) False.

Counterexample:

$$\begin{aligned} 2^{f(n)} &\leq 2^{C*g(n)} \\ &= (2^{\mathrm{g(n)}})^C \end{aligned}$$

c) True.

Proof:

$$\begin{aligned} f(n)^2 &\leq \left(C * g(\mathrm{n})\right)^2 \\ &= \mathrm{C}^2 * g(n)^2 \\ &= \mathrm{O}(g(n)^2) \end{aligned}$$