

Assignment 1

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1. Solution:

False. Counterexample:

M1	W2	W1
M2	W1	W2

W1	M1	M2
W2	M2	M1

We can easily see that we have a stable matching: $(M1, W2), (M2, W1)$, which is a counterexample.

2. Solution:

True. Explanation: because GS algorithm make sure that if m likes w most and w likes m most then (m, w) must be a couple in every stable matching S .

3.Solution:

Algorithm:

Initially all $s \in S$ and $t \in T$ are free

While there is a show s or t do not have a time slot

 Choose such a show s or t ,name it as p

 Let q is the slot in which p has the highest rating

 If q is free then

(p, q) is settled

 Else q is booked by p'

 If p' has higher rating than p

p still does not have a time slot

 Else p has higher rating than p'

(p, q) is settled

p' lost this time slot

 End if

 End if

End while

Return (S,T)

8.Solution:

Assume we have three men m_1 to m_3 and three women w_1 to w_3 . (column w_3' pretends to prefer m_3 to m_1)

M1	M2	M3	W1	W2	W3	W3'
W3	W1	W3	M1	M1	M2	M2
W1	W3	W1	M2	M2	M1	M3
W2	W2	W2	M3	M3	M3	M1

One possibility of GS algorithm with the true preference list of w_3 : (m_1, w_3) , (m_2, w_1) , (m_3, w_2) . Thus pairing w_3 with m_1 , who is her second choice.

Then we consider execution of the GS algorithm when w_3 pretends she prefers m_3 to m_1 .

Thus m_1 proposes to w_3 , m_2 to w_1 , then m_3 to w_3 . She accepts the proposal, leaving m_1 alone. Then m_1 proposes to w_1 which causes w_1 to leave her current partner m_2 , who consequently proposes to w_3 . Finally, the algorithm pairs up m_3 and w_2 . As we see, w_3 ends up with the man m_2 , who is her true favorite. Thus we conclude that by falsely

switching order of her preferences, a woman may be able to get a more desirable partner in the GS algorithm.