

A SUPPLEMENTARY MATERIALS - PROOFS

A.1 Proof of Proposition 3.9

Proposition 3.9[Threshold] There exist a threshold $p_{th} \leq 0.5$, such that $B_r(p) > B_p(p) > B_g(p)$ for any $p < p_{th}$.

PROOF. First, we prove that there exists a unique intersection point of $B_p(p)$ and $B_g(p)$ within $(0, 1)$. (i) We establish the existence by $\lim_{p \rightarrow 0^+} B_p(p) > \lim_{p \rightarrow 0^+} B_g(p)$ and $\lim_{p \rightarrow 1^-} B_g(p) > \lim_{p \rightarrow 1^-} B_p(p)$. (ii) We prove uniqueness. When $0 < p < 1$, $B_p(p)$ is monotonically decreasing. When $D > 2$, $0 < p \leq 0.5$, $B_g(p)$ is monotonically decreasing. When $D > 2$, $0.5 < p < 1$, $B_g(p)$ is monotonically increasing. When $D = 2$, $B_g(p)$ is a constant function. Due to (i) and (ii), $B_p(p)$ and $B_g(p)$ have a unique intersection point within $(0, 1)$. Let p_{pg} be the x-coordinate of the intersection point of $B_p(p)$ and $B_g(p)$.

Similarly, we can prove that if $B_r(0.5) \leq B_p(0.5)$, then $B_p(p)$ and $B_r(p)$ have a unique intersection point within $(0, 0.5]$. Let p_{pr} be the x-coordinate of the intersection point of $B_p(p)$ and $B_r(p)$.

Second, we prove that $p_{th} = \min\{p_{pg}, p_{pr}, 0.5\}$ by the graph of $B_p(p)$, $B_g(p)$ and $B_r(p)$. $B_p(p)$ is a quadratic function opening upwards defined on $(0, 1)$, and the axis of symmetry of $B_p(p)$ is larger than 1. when $D > 2$, $B_g(p)$ is a quadratic function opening upwards defined on $(0, 1)$, and the axis of symmetry of $B_g(p)$ is 0.5; when $D = 2$, $B_g(p)$ is a constant function defined on $(0, 1)$. $B_r(p)$ is a function composed of a linear term p and a reciprocal term $1/p$, defined on $(0, 0.5]$. Let $B'_p(p)$, $B'_g(p)$, and $B'_r(p)$ denote the derivative of $B_p(p)$, $B_g(p)$, and $B_r(p)$, respectively. When $0 < p \leq 0.5$, $0 \geq B'_g(p) > B'_p(p) > B'_r(p)$. When $0.5 < p < 1$, $B'_g(p) > 0 > B'_p(p)$. After drawing the graph of $B_p(p)$, $B_g(p)$ and $B_r(p)$, we can prove the following two cases. (i) $p_{pg} \leq 0.5$. If $B_r(0.5) \geq B_p(0.5)$, then $p_{th} = p_{pg}$; if $B_r(0.5) < B_p(0.5)$, then $p_{th} = \min\{p_{pg}, p_{pr}\}$. (ii) $p_{pg} > 0.5$. If $B_r(0.5) \geq B_p(0.5)$, then $p_{th} = 0.5$; if $B_r(0.5) < B_p(0.5)$, then $p_{th} = p_{pr}$. Thus, $B_r(p) > B_p(p) > B_g(p)$ for $p < p_{th}$. \square

A.2 Proof of Lemma 4.1

Lemma 4.1 Processing capacity vector f^* is the optimal solution to problem (27) only if $\sum_k C_k(f_k^*) = \bar{C}$.

PROOF. If $\sum_k C_k(f_k^*) < \bar{C}$, then there always exists a set of small value ϵ_k for k , $\epsilon = \{\epsilon_k, k \in \mathcal{K}\}$, such that $\sum_k C_k(f_k^* + \epsilon_k) \leq \bar{C}$ and $\phi(f^* + \epsilon) < \phi(f^*)$. The later inequality leads to the fact that $B(R\phi(f^* + \epsilon)) < B(R\phi(f^*))$. This implies a contradiction, so f^* cannot be the optimal solution to problem (27). \square

A.3 Proof of Lemma 4.2

Lemma 4.2 Let $\phi_k(f_k) = \frac{S_k D_k E}{f_k} + T_k^{UL} + T_k^{DL}$. Processing capacity vector f^* is the optimal solution only if $\phi_k(f_k^*)$ is identical for $k \in \mathcal{K}$.

PROOF. Suppose f^* is the optimal solution and $\phi_k(f_k^*)$ is not identical. Then, there exists an k^- such that $\phi_{k^-}(f_{k^-}^*)$ is the smallest, and another k^+ such that $\phi_{k^+}(f_{k^+}^*)$ is the largest. Let $f'_k < f_k^*$, $\phi_k(f'_k) < \phi_{k^+}(f_{k^+}^*)$ and $f' = (f'_1, \dots, f'_{k-1}, f'_k, f_{k+1}^*, \dots, f_K^*)$. We can show that $\phi(f') = \phi(f^*)$, according to Eq. (24). Thus, $B(R\phi(f')) = B(R\phi(f^*))$. Meanwhile, since $f'_k < f_k^*$ and $C_k(f'_k) < C_k(f_k^*)$, $C(f') < C(f^*) \leq \bar{C}$. According to Lemma 4.1, f' cannot be the optimal solution to problem (27). Further, since f' and f^* lead to the same objective value, i.e., $B(R\phi(f')) = B(R\phi(f^*))$, f^* cannot be the optimal solution. This leads to a contradiction. Therefore, Lemma 4.2 must be true. \square

A.4 Proof of Proposition 4.3

Proposition 4.3 If τ_2^* is the optimal solution to problem (28), then $f_k^* \triangleq \phi_k^{-1}(\frac{\tau_2^*}{R})$ for $k \in \mathcal{K}$ optimizes problem (27).

PROOF. Suppose τ_2^* is the optimal solution to (28), but $f_k^* \triangleq \phi_k^{-1}(\frac{\tau_2^*}{R})$ is not the optimal solution to (27), where $\phi_k^{-1}(\cdot)$ is the inverse function of $\phi_k(\cdot)$. We aim to show contradiction under the two possible cases.

Case 1. There exists a k such that f_k^* is not a feasible solution to problem (27). This cannot be the case, because $\sum_k C_k(\phi_k^{-1}(\frac{\tau_2^*}{R})) = \bar{C}$. That is, $\sum_k C_k(f_k^*) = \bar{C}$ satisfies the constraint.

Case 2. There exists a k such that f_k^* is feasible but cannot minimize the objective function. Then, there exists another f' such that $B(R\phi(f')) < B(R\phi(f^*))$. And $\sum_k C_k(f'_k) = \bar{C}$ according to Lemma 4.1. Thus, there exists a $\tau'_2 = R\phi(f')$ such that $B(\tau'_2) < B(\tau_2^*)$ and $\sum_k C_k(\phi_k^{-1}(\frac{\tau'_2}{R})) = \bar{C}$. Thus, τ_2^* cannot be the optimal solution. This leads to a contradiction. Therefore, Proposition 4.3 must be true. \square