Solutions to Recitation 1

Lei Zhao

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This is my own solution to the recitation 1 of 18.014, which includes exercise 10, 15, and 18 from Apostol's *Calculus* (1: 16).

Problem 1

Question: Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Answer: Let $X = A \cap (B \cup C)$ and $Y = (A \cap B) \cup (A \cap C)$. If $x \in X$, then $x \in A$ and $x \in B \cup C$, which means $x \in B$ or $x \in C$. If $x \in B$, then $x \in A \cap B$, which implies $x \in Y$. Similarly if $x \in C$, we can also deduce $x \in Y$. In both cases, we have $x \in Y$; hence, $X \subseteq Y$. Conversely, if $x \in Y$, then $x \in A \cap B$ or $x \in A \cap C$. If $x \in A \cap B$, then $x \in A$ and $x \in B$, which implies $x \in B \cup C$. Thus $x \in X$. By the same fasion if $x \in A \cap C$, $x \in X$. In both cases, we have $x \in X$. This is to say $Y \subseteq X$. Therefore, $X = Y$. $\square$

Let $X = A \cup (B \cap C)$ and $Y = (A \cup B) \cap (A \cup C)$. If $x \in X$, then $x \in A$ or $x \in B \cap C$. If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$, which is the same as $x \in Y$. If $x \in B \cap C$, then $x \in B$ and $x \in C$, which further implies that $x \in B \cup A$ and $x \in C \cup A$; hence $x \in Y$. In both cases, we have $x \in Y$; thus $X \subseteq Y$. Conversely, if $x \in Y$, then $x \in A \cup B$ and $x \in A \cup C$. If $x \in A$, then $x \in X$. If $x \notin A$, it must be the case that $x \in B$ and $x \in C$. This is to say $x \in B \cap C$, which implies $x \in X$. In both cases, we have $x \in X$; hence $Y \subseteq X$. Therefore, $X = Y$. $\square$

Problem 2

Question: Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

Answer: If $x \in A \cup B$, then $x \in A$ or $x \in B$. If $x \in A$, then by $A \subseteq C$ we have $x \in C$. Similarly, if $x \in B$, then by $B \subseteq C$ we have $x \in C$. Hence, $A \cup B \subseteq C$. $\square$

Problem 3

Question: Prove $A - (B \cap C) = (A - B) \cup (A - C)$.

Answer: Let $X = A - (B \cap C)$ and $Y = (A - B) \cup (A - C)$. If $x \in X$, then $x \in A$ and $x \notin B \cap C$, which means it is not the case that $x \in B$ and $x \in C$. This is the same as saying $x \notin B$ or $x \notin C$. If $x \notin B$, then $x \in A - B$, which implies $x \in Y$. And if $x \notin C$, then $x \in A - C$, which also implies $x \in Y$. Thus $X \subseteq Y$. Conversely, if $x \in Y$, then $x \in A - B$ or $x \in A - C$. If $x \in A - B$, then $x \in A$ and $x \notin B$, which implies $x \notin B \cap C$. Thus $x \in X$. Similarly if $x \in A - C$, we deduces that $x \in X$. This means $Y \subseteq X$. Therefore, $X = Y$. $\square$