The area of a circle can be given by

$$
\iint\limits_{x^2+y^2 \leqslant r^2} dx\, dy = \int_{-r}^{r} \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} dy\, dx
$$

$$
= \int_{-r}^{r} 2\sqrt{r^2 - x^2}\, dx
$$

$$
= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\sqrt{r^2 - (r \sin t)^2}\, d(r \sin t)
$$

$$
= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2r^2 \cos^2 t\, dt
$$

$$
= r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos 2t)\, dt
$$

$$
= r^2 \left(t + \frac{\sin 2t}{2} \right) \Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}
$$

$$
= \pi r^2.
$$