Solutions to Recitation 2

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This is my own solution to the Recitation 2 of 18.014, which includes Theorem I.9–10 and Exercise 4 from Apostol's *Calculus* (1: 18–19).

The cancellation law (Theorem I.7) implies the uniqueness of the inverse if it exists and thus we use the notation a^{-1} introduced in the Theorem I.8 freely without any ambiguity. Furthermore, if $a \neq 0$, then $a^{-1} \neq 0$. If otherwise, $a \cdot a^{-1} = a \cdot 0 = 0 \neq 1$.

1. If $a \neq 0$, then $b/a = b \cdot a^{-1}$.

   We have

$$
\begin{aligned}
a \cdot (b/a) &= b && \text{by definition of division,} \\
&= 1 \cdot b && \text{by the identity axiom,} \\
&= (aa^{-1}) \cdot b && \text{by the inverse axiom,} \\
&= a \cdot (a^{-1}b) && \text{by associativity,} \\
&= a \cdot (ba^{-1}) && \text{by commutativity.}
\end{aligned}
$$

   We apply the cancellation law (Theorem I.7) to the above and obtain $b/a = b \cdot a^{-1}$. $\square$

2. If $a \neq 0$, then $(a^{-1})^{-1} = a$.

   Since $a \neq 0$, $a^{-1} \neq 0$. And we have

$$
\begin{aligned}
a^{-1} \cdot (a^{-1})^{-1} &= 1 && \text{by definition of reciprocal,} \\
&= a \cdot a^{-1} && \text{by the inverse axiom,} \\
&= a^{-1} \cdot a && \text{by commutativity.}
\end{aligned}
$$

   By the cancellation law, we obtain $(a^{-1})^{-1} = a$. $\square$

3. Zero has no reciprocal.

   Suppose zero has a reciprocal and let us denote it as 0^{-1}. Then

$$
\begin{aligned}
0 &= 0 \cdot 0^{-1} && \text{by Theorem I.6,} \\
&= 1 && \text{by definition of reciprocal.}
\end{aligned}
$$

   But by the identity axoim, $0 \neq 1$. This is contradictory. $\square$