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Proof

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|  | |
| For any arbitrary in , we can always find such that is also in . | |
|  | |
| Therefore, is in and . | |
|  | |
| Therefore, . | |
| Combine the fact , we know . | ∎ |

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Proof

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| Therefore, is unique to and . | ∎ |

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Proof

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|  | |
|  | (1) |
|  | (2) |
| Let be a skolem function for the existential quantifier in in (2). | |
| Let . | |
|  | |
|  | (3) |
| Combine (2) and (3), we get . | |
| Similarly, we can use the same approach to prove . | ∎ |

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|  |



Proof

Let and . And suppose and .

is a lower bound for .

is a greatest lower bound for .

because infimum is unique.

This contradicts with the supposition .

It is impossible for to be less than .

Let and .

Let and .

Therefore, .

|  |  |
| --- | --- |
| Similarly, we can do the same thing for and | ∎ |

1. , both sets are bounded.

|  |  |
| --- | --- |
|  | True |
|  | True |
|  | False |
|  | False |

Proof

Let and and .

(1)

is arbitrary, so we have

(2)

Combine (1) and (2), is a least upper bound for .

because supremum is unique.

Similarly, .

Let and .

∎