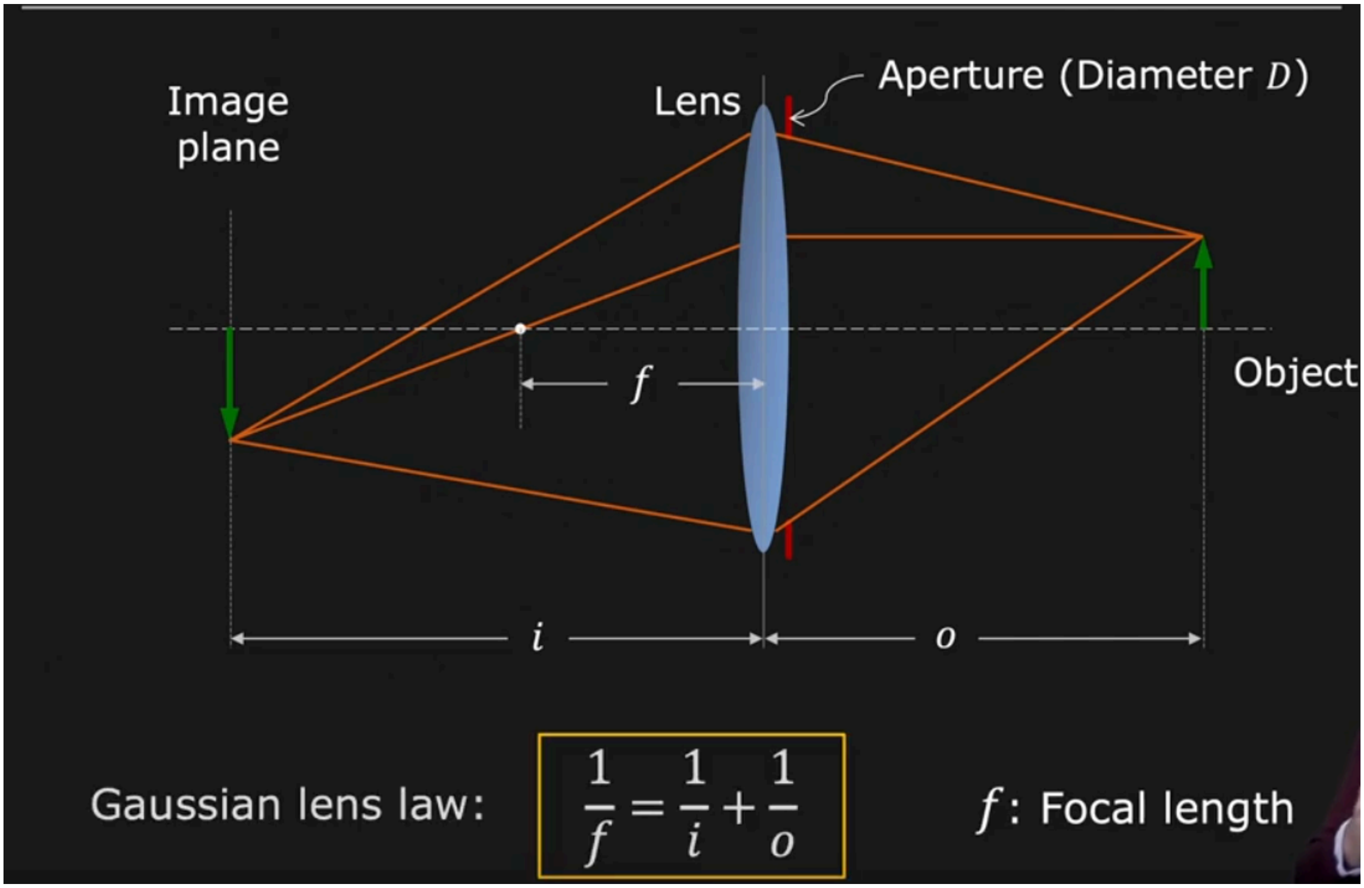
Gaussian Lens Law

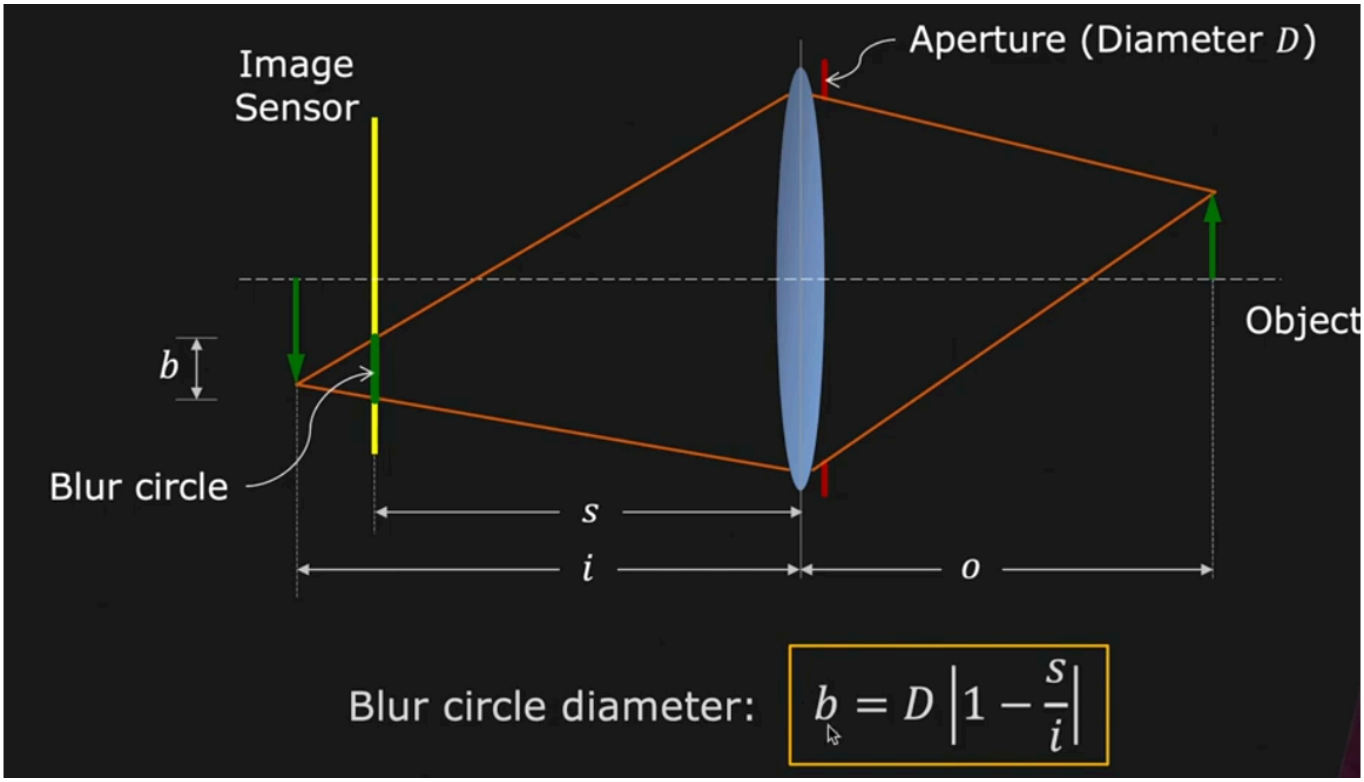


Focal Length: measures how strongly the system converges or diverges light; inverse of optical power

Optical power / focal length relation
$P = 1/f$

If the image sensor moves forward (i decreases), then light forms a blur circle - a circle since the aperture is a circle (will always have the same shape as the aperture)

Size of blur circle depends on the distance of the image sensor from the image plane



Smaller aperture size D, smaller blur circle.

Point Spread Function

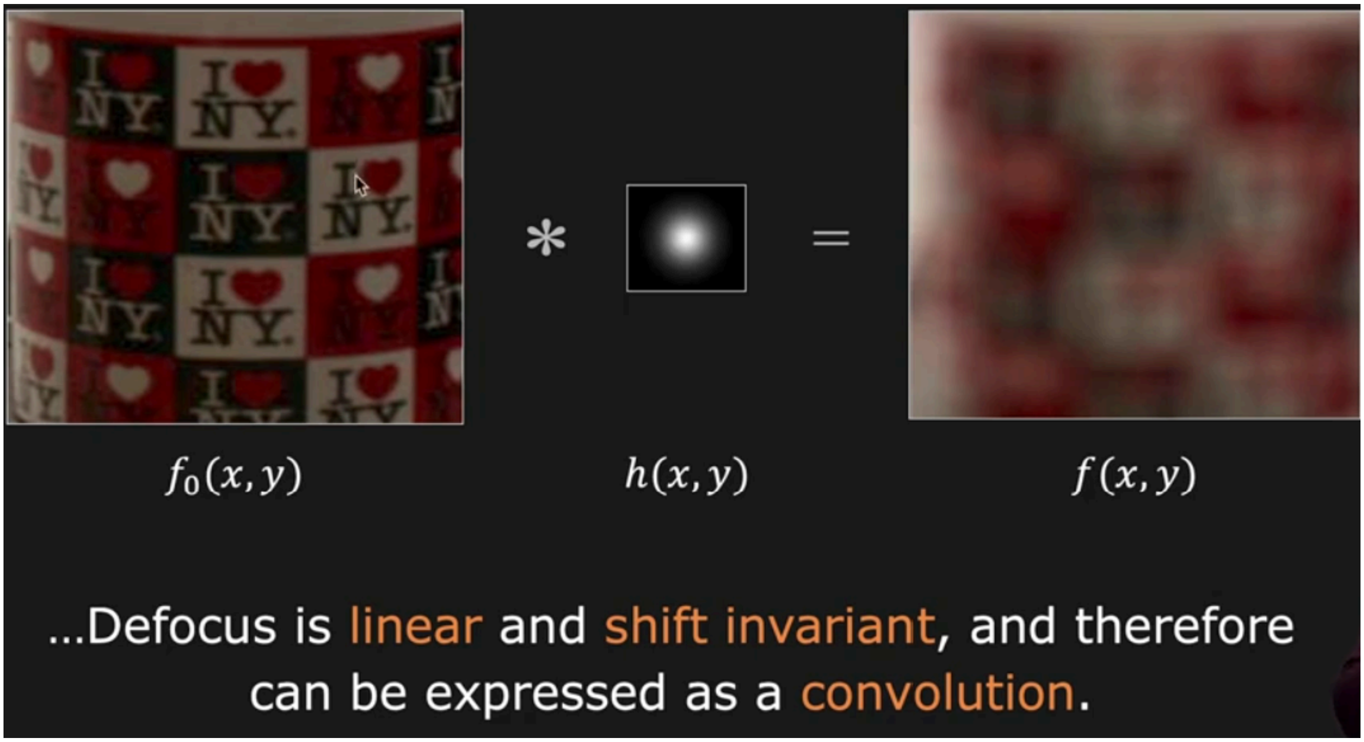
PSD defn: the response of a camera system to a point source (an impulse)

Pillbox PSF: ideal camera systems would have a uniform blur circle with sharp edges

Diffraction at aperture (bending of light at edge of aperture), lens aberrations, image sensor (active area of pixel) results in gaussian psf

Gaussian PSF: PSF is assumed to be a gaussian generally
An approximation is : $\sigma = \frac{b}{2}$

The focused image convolved with the PSF (assuming focused image is constant depth) gives you the captured image



Shift Invariance: Blur effect (defocus) is the same at each point in image (not true in real life since blur is often worse at edges)

Linearity of Defocus: The defocus operator is a linear transformation

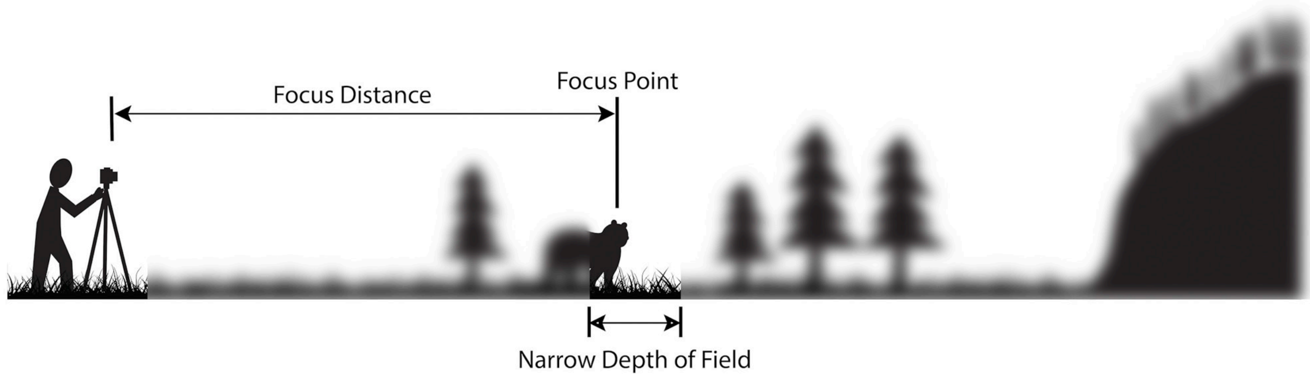
1. Df(Image A + Image B) = Df(Image A) + Df(Image B)
2. Df(c *Image A)* = *c* Df(Image A)

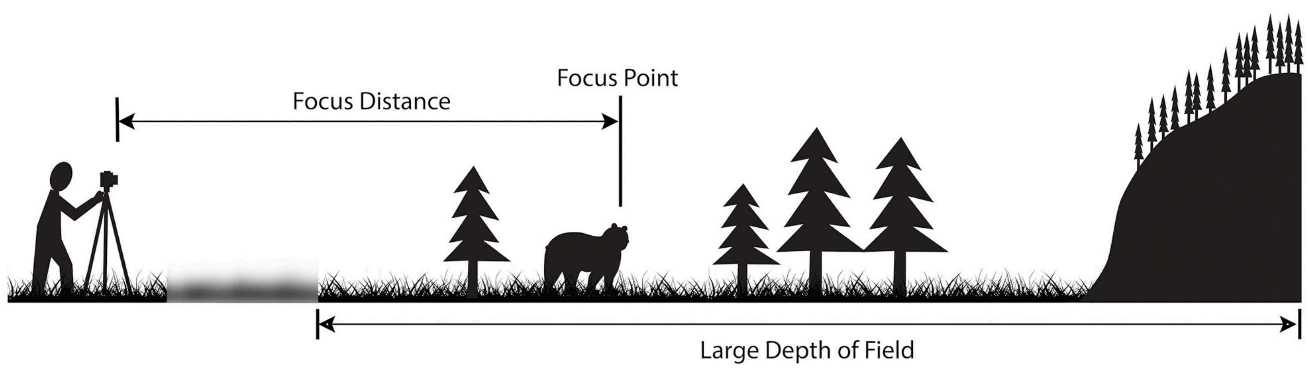
This doesnt work if the focused image has different depths since the defocus function would change for each pixel

1. for practical purposes - we split the image into smaller region and assume the depth within each region is constant so we can still use this model

Depth From Focus

Depth of Field: Distance between farthest two objects that are still relatively sharp





Img Src: https://photographylife.com/what-is-depth-of-field

Sweep the plane of focus through the scene, take a stack of image (focal stack), for each image, find the image in the stack that the point is best focused

1. change plane of focus until point is clear then you can estimate depth
2. different points in the image come in and out of focus at different focal planes / sensor locations

If you know the sensor location (s) then you can find the distance from the lens to the object using the gaussian lens law

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{o}$$

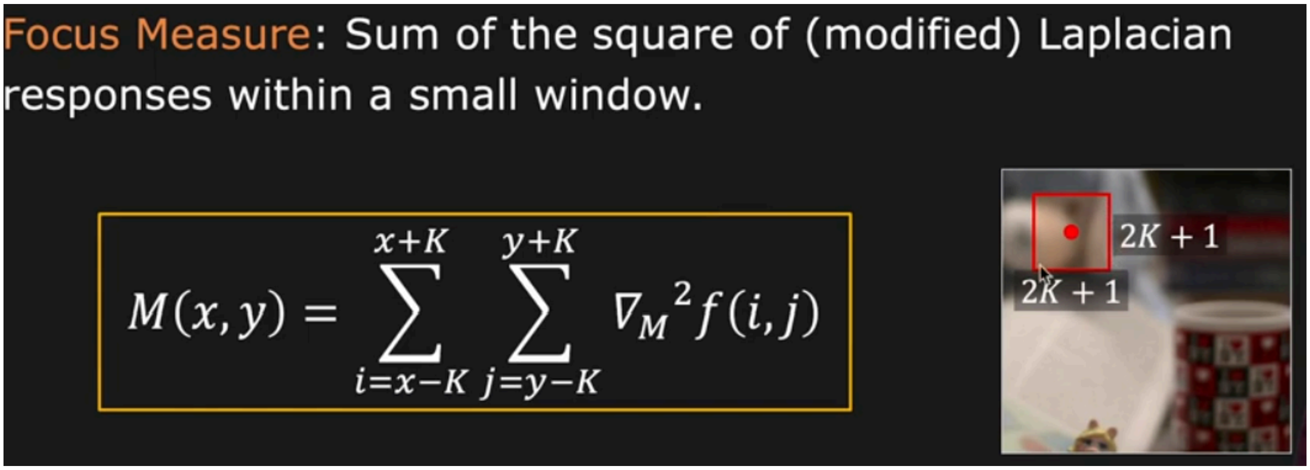
Where we know s and f (focal length)

How do we find the best focused image:

1. defocus attenuates high frequencies so we can just use a high pass filter to measure the amt of high frequency content within each patch

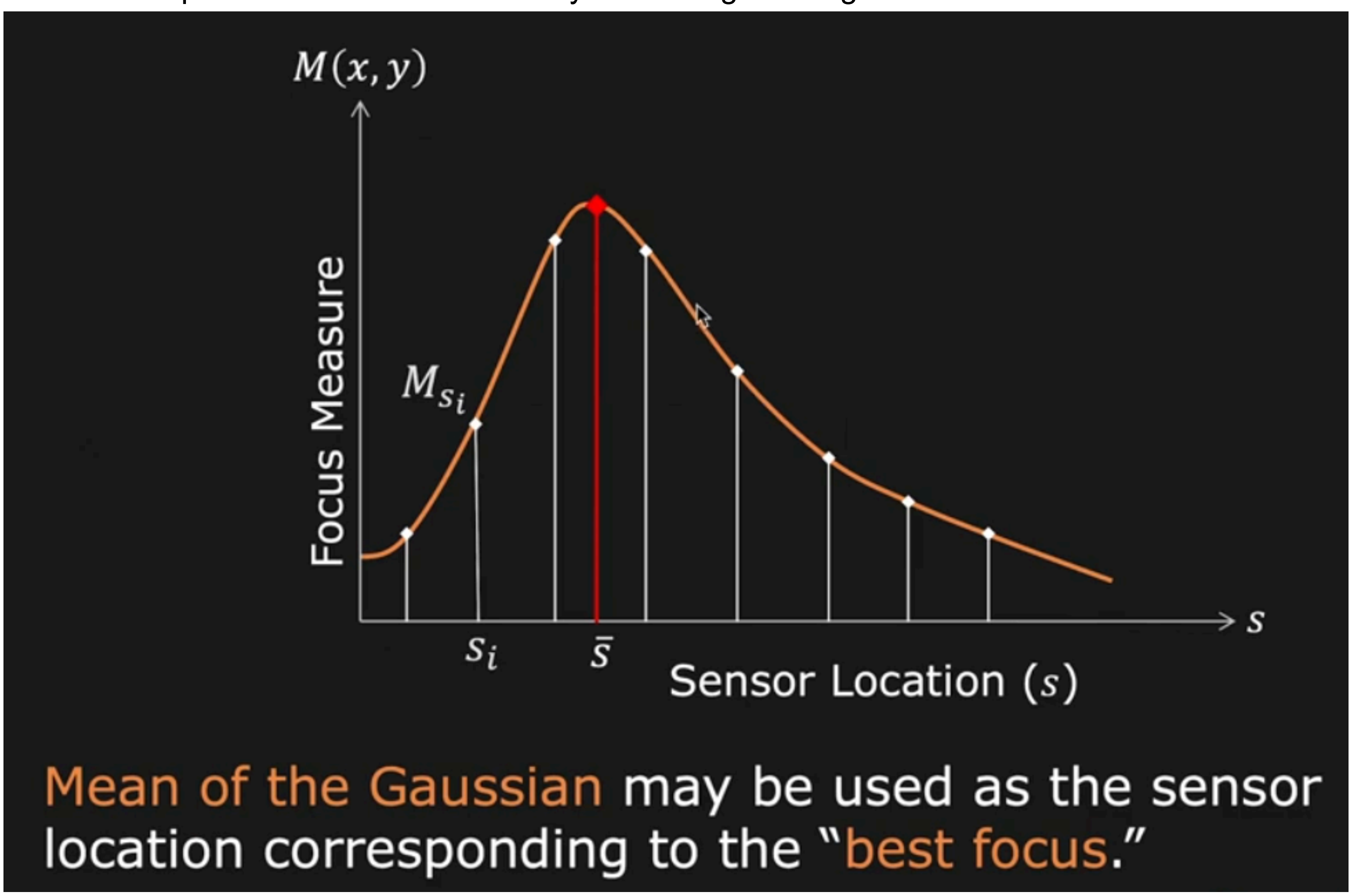
   For example, use: $\nabla_M{}^2 f = \left|\frac{\partial^2 f}{\partial x^2}\right| + \left|\frac{\partial^2 f}{\partial y^2}\right|$ (Similar to Laplacian)
2.

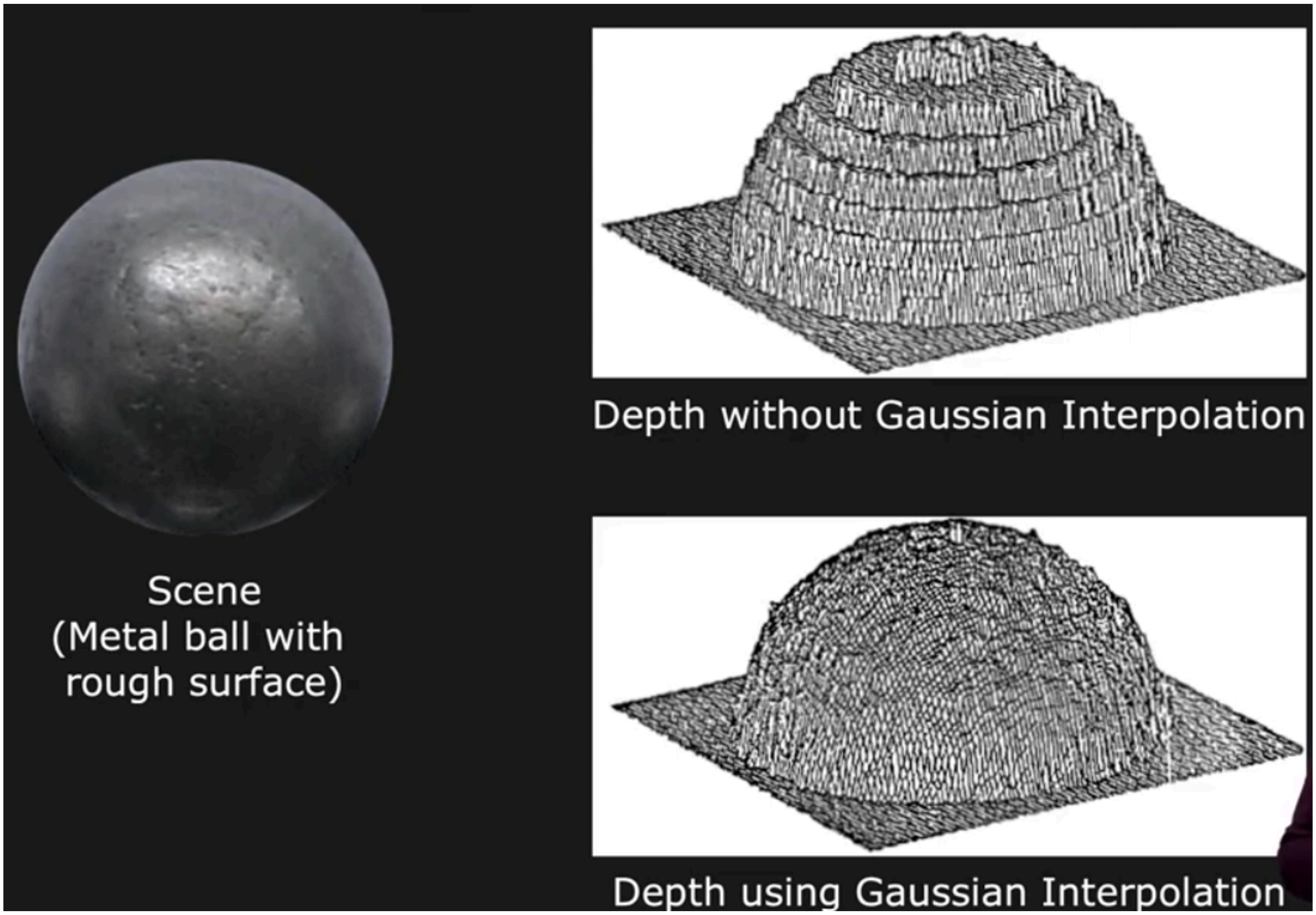
1. This measures the amount of change in the image
2. Known as the modified laplacian

3. 

    1. Compute the modified laplacian for each pixel in each window
    2. M(x,y) is the focus measure for the point (x,y)
    3. high frequencies = higher focus measure

We can interpolate the sensor location by estimating it as a gaussian-like function





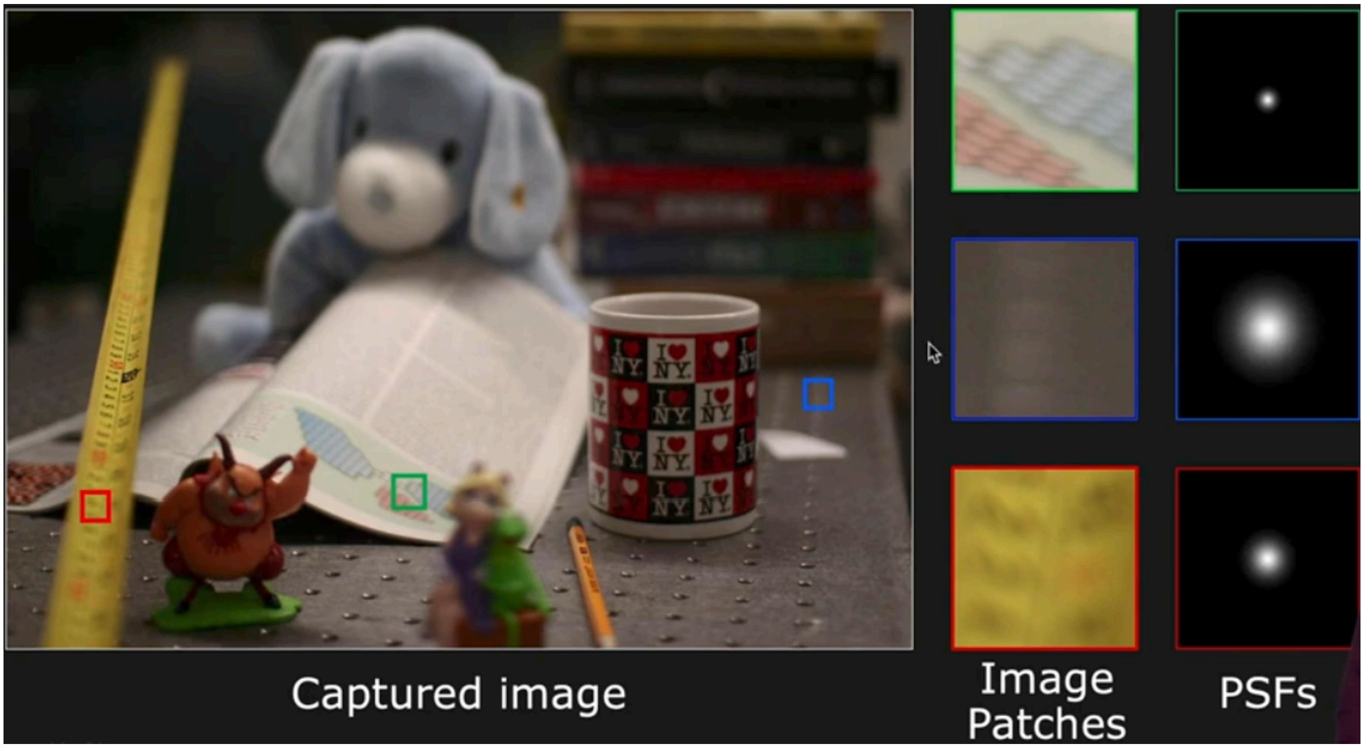
Use case in real life:

1. microscope - motorize stage to move sample around and produce focal stack
    1. can reconstruct very small structures using depth from focus

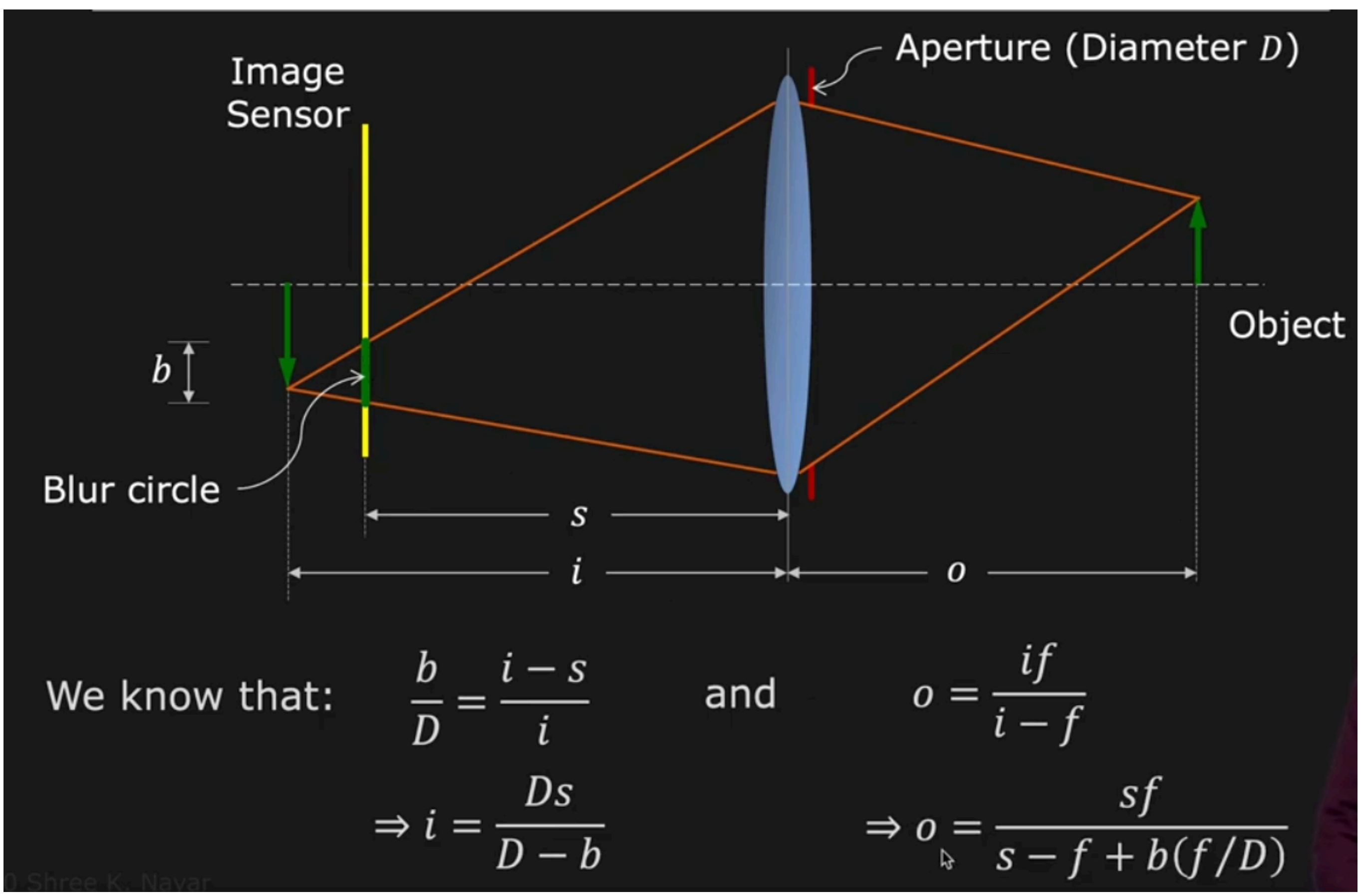
Depth from Defocus

Downside of depth from focus is that its not fast enough for real time

Given an image, the depth of any point can be computed by measuring how defocused it is.

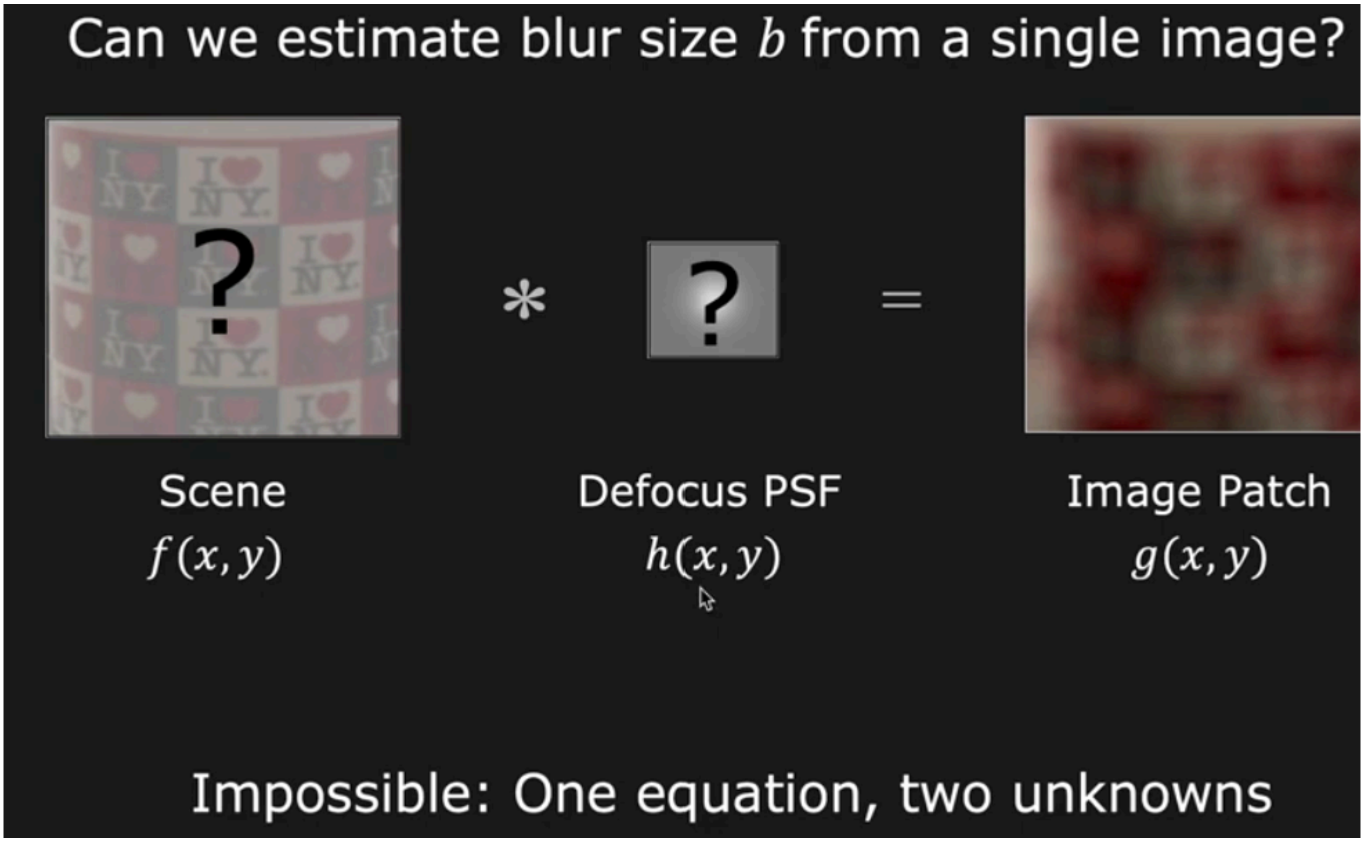


1. some points have smaller or larger PSFs
2. so we want to estimate the PSF for every corresponding point

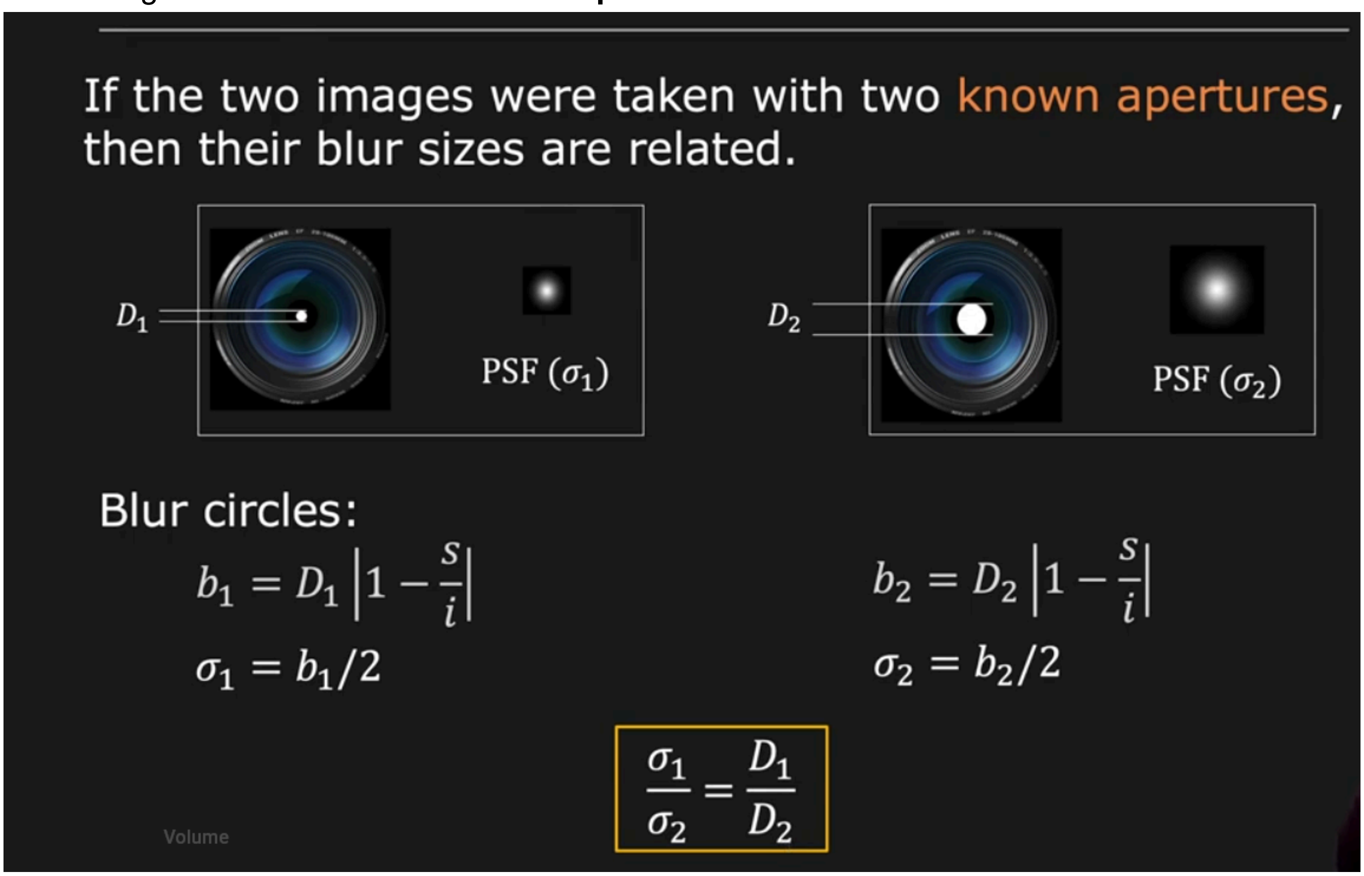


In this case, we don't know i and o since these are interrelated. We know s, D, and f. If we can estimate the diameter of the blur circle b, then we can determine i and o

F-number: $\frac{f}{D}$



If two images are taken with two **known apertures** then their blur sizes are related.



This applies to all points in the scene.

Find (σ_1, f) using: $\frac{\partial E}{\partial \sigma_1} = 0$ and $\frac{\partial E}{\partial f} = 0$

Using σ_1 compute size of blur circle: $b_1 = 2\sigma_1$

Object distance (depth):

$$\boxed{o = \frac{s_1 f}{s_1 - f + b_1 (f/D)}}$$