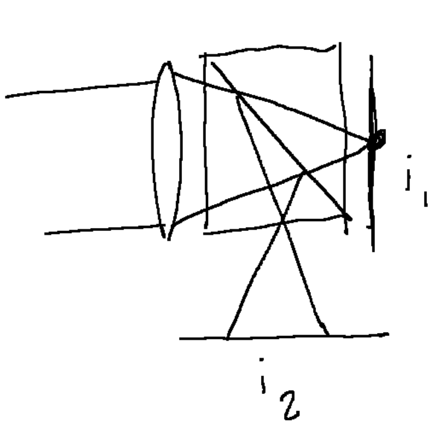
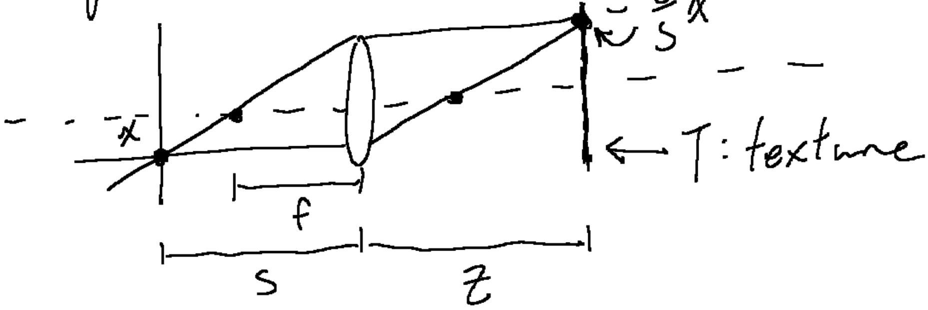


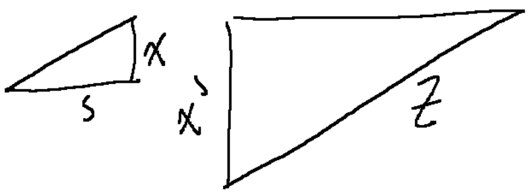
Camera that is focused on 2 points in the scene. This works since the Gaussian lens law states that the point of focus depends on i

$$I(x;s) = k(x;s) * P(x;s)$$

$P(x;s) = T\left(-\frac{z}{s}x\right)$ where $T(x)$ is brightness at position x on the texture



Similar Triangles derivation



$$-\frac{z}{s} = \frac{x'}{x}$$

$$-x\frac{z}{s} = x'$$

$k(x;s) = \frac{1}{\sigma^2}\exp\left(\frac{-\|x\|^2}{2\sigma^2}\right)$ x is a vector so we use norm(x)

$\sigma = A\left(\frac{1}{z} - \rho\right)s + A$ where ρ is optical power and A is stdev of the aperture code

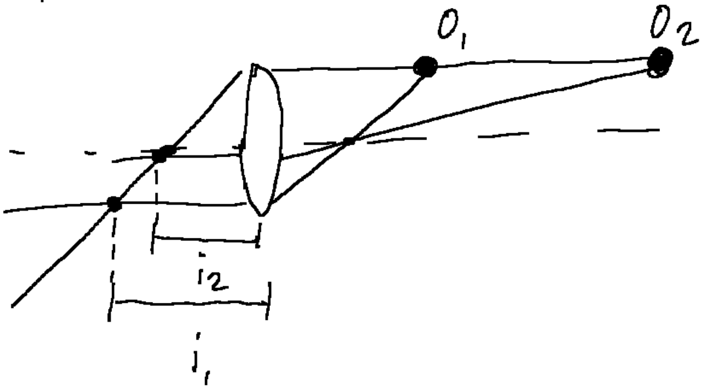
σ is the defocus level

defocus lvl

$$\sigma(s) = A\left(\frac{1}{z} - \rho\right)s + A$$

$$\rho = \frac{1}{f} \qquad \frac{1}{f} = \frac{1}{i} + \frac{1}{z} \qquad -\frac{1}{i} = \frac{1}{z} - \rho$$

$$\sigma(s) = -\frac{A}{i}s + A$$



if $i = s$ then $\sigma(s) = 0$ since sensor is on image plane

if $i = A$ $\quad \sigma(s) = -s + A$

then $s = 0 \Rightarrow \sigma(s) = A$ which makes sense since thus like right against the mask

---

Since the magnification of the image is different for the two sensors (if $s_2 > s_1$ then s_2 has a smaller image) therefore we register the imgs to a consensus location c

$\tilde{I}(x; s) = I\left(\frac{s}{c}x\right)$ where $\tilde{I}$ is the scaled img

$\rightarrow$

$\tilde{I}(x;s) = I(\frac{s}{c}x)$

$\tilde{I}(x;s) = \tilde{k}(x;s) * P(x;c)$

Then scaled psf

$\tilde{k}(x;s) = \left(\frac{s}{c}\right)^2 k\left(\frac{s}{c}x\right)$

$\left(\frac{s}{c}\right)^2$ makes sense since the intensity changes quadratically for image scaling

$$\tilde{k}_s(x;s) = \underbrace{-\frac{c^2 \sigma A}{s^3} \nabla^2}\tilde{k}(x;s)$$

$P(x;c)$ is pinhole img at conversion distance c and doesn't depend on s

$$\tilde{I}_s(x;s) = \tilde{k}_s(x;s) * P(x;c)$$

$$= \ldots \tilde{k}(x;s) * P(x;c)$$

$$= -\frac{c^2 \sigma A}{s^3} \nabla^2 \tilde{I}(x;s)$$

$$\sigma = A\left(\frac{1}{z} - \rho\right)s + A \longrightarrow z = \frac{a}{b + \frac{\tilde{I}_s(x;s)}{\nabla^2 \tilde{I}(x;s)}}$$

$a = -A^2 \quad b = -A^2\left(\frac{1}{f} - \frac{1}{s}\right)$

$\longrightarrow$

Using setup we can capture imgs: $I_1 = I(x; s_1)$

$I_2 = I(x; s_2)$

then we approx. img derivs.

$$\tilde{I}_{s,\text{approx.}} = I_1(Rx+t) - I_2(x)$$

Matrix $R \in \mathbb{R}^{2\times 2}$ and vector $t \in \mathbb{R}^{2\times 1}$ describe a homography that aligns images I_1 and I_2 including rescaling

$$\nabla^2 \tilde{I}_{\text{approx.}} = \frac{1}{2} \nabla^2 (I_1(Rx+t) + I_2(x))$$

→ The consensus location c is s_2 so I_1 is scaled to s_2 to produce a img compensated for magnification but with the same psf

→ This averages the two imgs for higher acc approx.

Recall:

$\tilde{I}(x; s) = I(\frac{s}{c} x)$ this changes only the magnification and preserves the defocus blur. Intuitively, scaling up a blurry image doesn't make it less blurry

→

Confidence / Sensitivity Analysis

$$C = \tilde{I}_s^2$$

predicts degeneracy due to lack of texture

$$SNR(\tilde{I}_s) = \frac{\tilde{I}_s}{|\tilde{I}_s - \tilde{I}_{s,approx}|}$$

$$SNR(\nabla^2 \tilde{I}) = \frac{\tilde{I}_s}{|\nabla^2 \tilde{I} - \nabla^2 \tilde{I}_{approx}|}$$

$$SNR_{dB} = 10 \log_{10}(SNR)$$

Implementation

Reduce non-uniform background lighting by use of a $K \times K$ box filter

$$I_i^{bck} = I_i - \frac{1}{K^2} B \star I_i \qquad i = 1, 2$$

Also gaussian blur I_i due to camera sensor noise

Compute homography between two sensors via SIFT keypoints.

$$S \equiv H, e$$

Scaled PSF Derivation

$$\tilde{k}(x;s) = \frac{s^2}{c^2} k\left(\frac{s}{c}x; s\right)$$

$$\tilde{k}_s = \frac{2s}{c^2} k\left(\frac{s}{c}x; s\right) + \frac{s^2}{c^2} k_s\left(\frac{s}{c}x; s\right)$$

$$k(x;s) = \frac{1}{\sigma^2}\exp\left(\frac{\|x\|^2}{2\sigma^2}\right)$$

$$\tilde{k}_s = \frac{2s}{c^2\sigma^2}\exp\left(\frac{\|\frac{s}{c}x\|^2}{2\sigma^2}\right) + \frac{s^2}{c^2} k_s\left(\frac{s}{c}x; s\right)$$

$$\downarrow$$

$$\frac{\frac{s^2}{c^2}\|x\|^2}{2\sigma^2}$$

$$k_s\left(\frac{s}{c}x; s\right) = \frac{1}{\sigma^2}\,\frac{\frac{2s}{c^2}\|x\|^2}{2\sigma^2}\exp\left(\frac{\|\frac{s}{c}x\|^2}{2\sigma^2}\right)$$

$$= \frac{s\|x\|^2}{\sigma^4 c^2}\exp\left(\frac{s^2\|x\|^2}{2\sigma^2 c^2}\right)$$

$$\boxed{\psi = \frac{s^2\|x\|^2}{2\sigma^2 c^2}} \qquad k_s\left(\frac{s}{c}x\right) = \frac{2}{s}\psi\exp(\psi)$$

$$\tilde{k}_s = \frac{2\cancel{s}}{c^2}\cdot\frac{2}{\cancel{s}}\psi\exp(\psi) + \frac{s^3\|x\|^2}{c^4\sigma^4}\exp(\psi)$$

$$= \frac{4}{c^2}\psi\exp(\psi) + \frac{2s}{c^2\sigma^2}\psi\exp(\psi)$$

$$= \frac{2}{c^2}\left(2 + \frac{s}{\sigma^2}\right)\psi\exp(\psi)$$