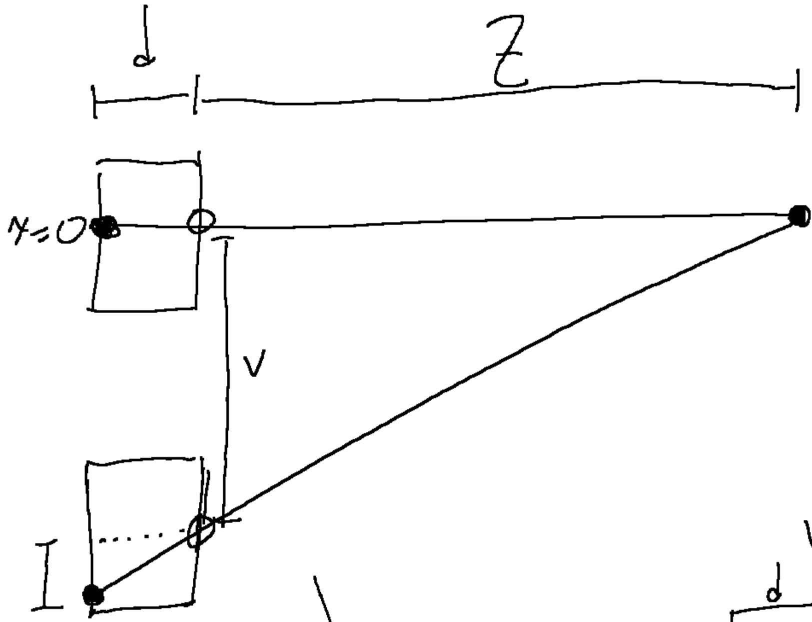
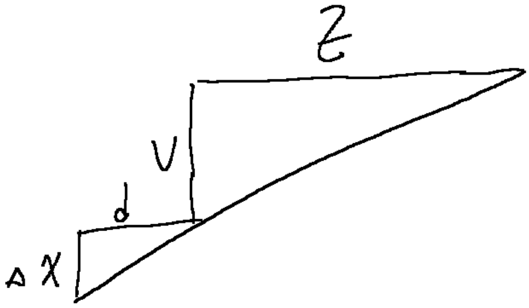




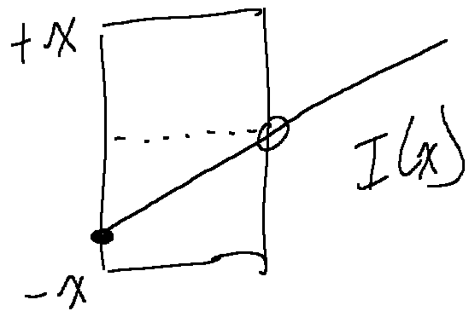
$$\Delta x = \frac{vd}{z}$$

similar triangles

$$\frac{\Delta x}{d} = \frac{v}{z}$$

$$\Delta x = \frac{dv}{z}$$

$$f(x; v) = I\left(x - \frac{vd}{z}\right)$$



$I(x)$: intensity of pixel on sensor

$$I_v(x) = \left.\frac{\partial f(x;v)}{\partial v}\right|_{v=0} \rightarrow \frac{\partial\, I\left(x - \frac{vd}{z}\right)}{\partial v}$$

eval at $v=0$

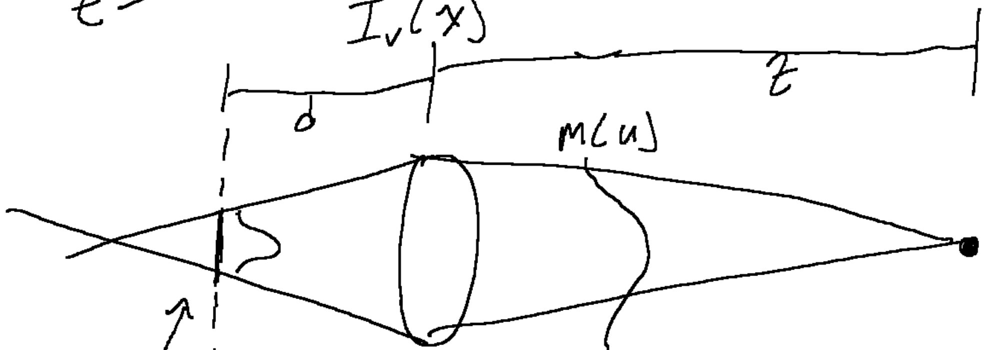
$$\rightarrow -\frac{d}{z} I'\left(x - \frac{vd}{z}\right)\Big|_{v=0} \rightarrow -\frac{d}{z} I'(x) = I_v(x)$$

$$I_x(x) = \left.\frac{\partial I(x - \frac{vd}{z})}{\partial x}\right|_{v=0} = I'(x)$$

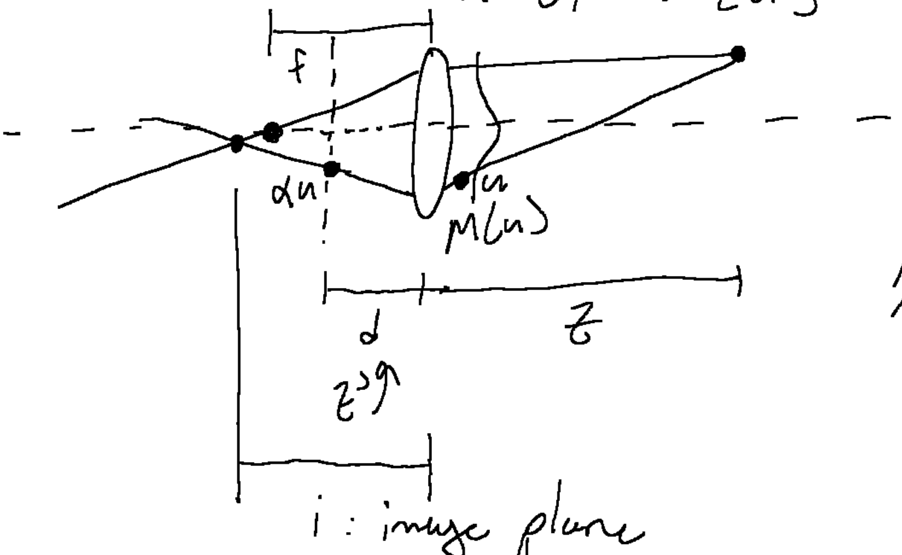
$$I_v(x) = -\frac{d}{z} I_x(x) \leftarrow \text{Eqn 4}$$

$$-\frac{I_v(x)}{d\, I_x(x)} = \frac{1}{z}$$

$$z = -\frac{d\, I_x(x)}{I_v(x)} \leftarrow$$ to find this is the problem of stereo matching



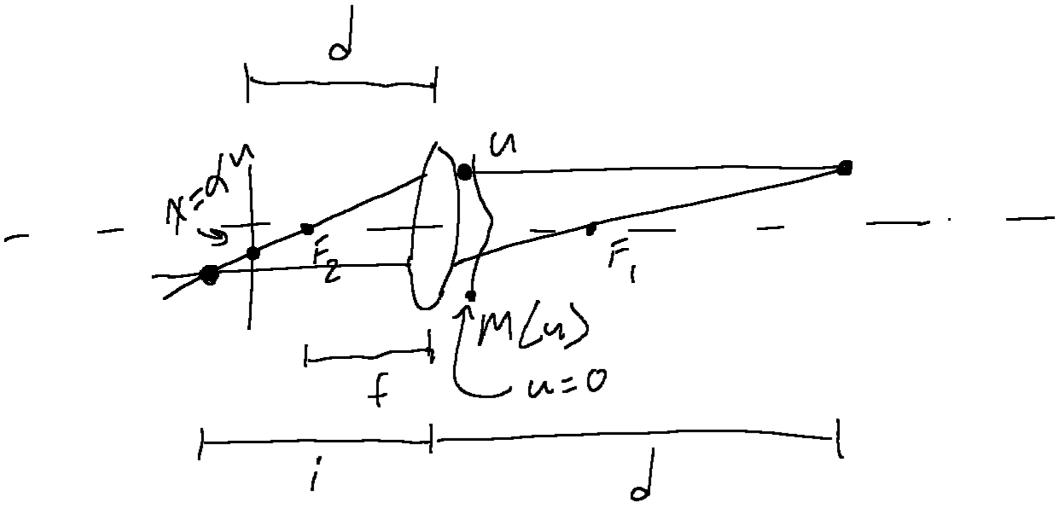
defocus blur will be a scaled and dilated version of M(u)



$$\frac{1}{f} = \frac{1}{d} + \frac{1}{z}$$

$$x = \alpha u$$

$$x(z') =$$



$$\frac{1}{f} = \frac{1}{z} + \frac{1}{i} \qquad x = \alpha u$$

$x(0) = u$

$x(i) = 0$

$$x(z') = -\frac{u}{i} z' + u = -u\left(\frac{z'}{i} - 1\right)$$

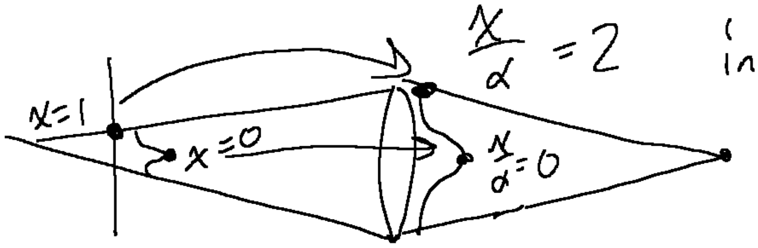
z' = distance from lens

$$x(d) \to \alpha u = -u\left(\frac{d}{i} - 1\right)$$

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{z} \qquad \Rightarrow \alpha = 1 - \frac{d}{i}$$

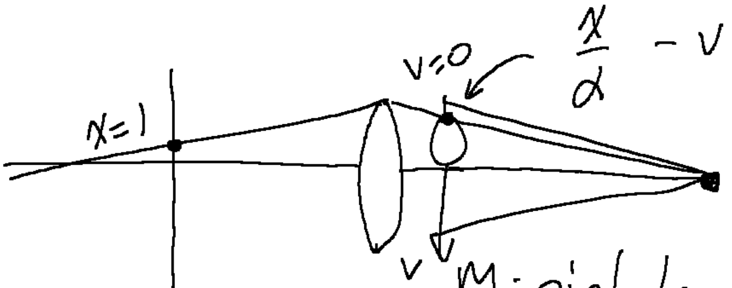
$$\alpha = 1 - d\left(\frac{1}{f} - \frac{1}{z}\right)$$

$$\boxed{\alpha = 1 - \frac{d}{f} + \frac{d}{z}}$$

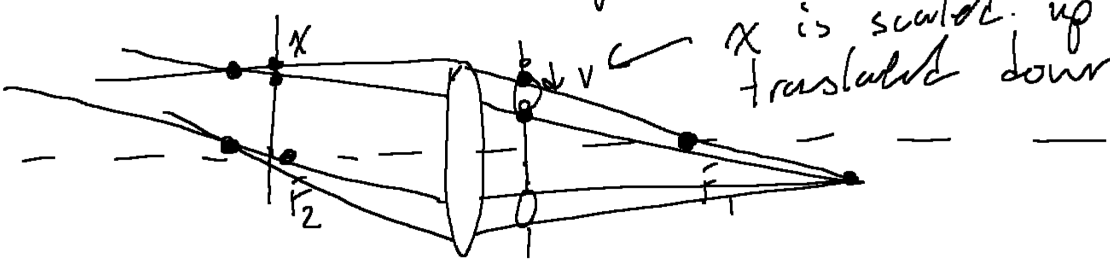


intensity is scaled up by 2x as well

$$f(x;v) = \frac{1}{\alpha} M\left(\frac{x}{\alpha} - v\right)$$



M: pinhole



x is scaled up then translated down

$$I_v(x) = \partial f(x;v)\big|_{v=0}$$

$$= \frac{\partial\left(\frac{1}{\alpha} M\left(\frac{x}{\alpha} - v\right)\right)}{\partial v} = \boxed{-\frac{1}{\alpha} M'\left(\frac{x}{\alpha}\right)}$$

$M'(\cdot)$ is the derivative of $M(\cdot)$ with respect to its <u>argument</u>

Has the same general form as $I(x) = \frac{1}{\alpha} M\left(\frac{x}{\alpha}\right)$

Then spatial derivative $I_x(x) = \frac{\partial}{\partial x} f(x, v)|_{v=0}$
$= \frac{1}{\alpha^2} M'\left(\frac{x}{\alpha}\right)$

$\rightarrow I_x(x) = -\frac{1}{\alpha} I_v(x)$

thus

$$\alpha = -\frac{I_v(x)}{I_x(x)}$$

This is bad since there are singularities when $I_x(x) = 0$

$$\alpha = 1 - \frac{d}{f} + \frac{d}{z}$$

so we can find z if we can estimate the two derivatives

We instead formulate as an optimization problem

$\alpha I_x + I_v(x) = 0$ so total squared error

is $E(\alpha) = \sum_{x \in P} (I_v(x) + \alpha I_x(x))^2$

where P is a set of image patches

α is a spatially varying variable estimated by patches of the image used to estimate z

$$E(\alpha) = \sum_{x \in P} (I_v(x) + \alpha I_x(x))^2$$

$$\frac{dE}{d\alpha} = \sum_{x \in P} 2(I_v(x) + \alpha I_x(x)) \cdot I_x(x) = 0$$

$\frac{dE}{d\alpha}$ is finding the minimum for the $E(\alpha)$ func.

$$\boxed{\alpha = -\frac{\sum_{x \in P} I_v(x) I_x(x)}{\sum_{x \in P} I_x^2(x)}}$$

This is the least squares soln for α in patch P. Singularity still exist when $I_x(x) = 0$ for the whole patch (means that the patch is textureless.

Remember that this is still $\underline{1D}$

To prevent singularity we add a very small constant σ^2 to the denominator

This is ok since $I_x^2(x)$ is large in textured patches and the term allows $\alpha = 0$ in textureless patches

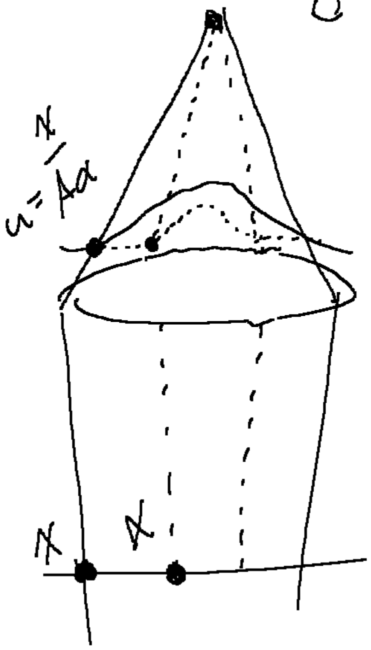
This is easily extended to 2d just say $v = (u, w)$ and I_x, I_y

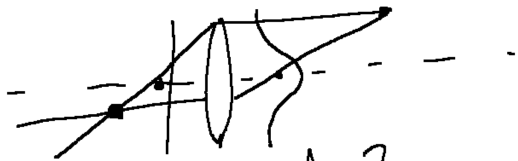
Optimal Aperture Size Diffy

"Range from defocus"

$$f(x;A) = \frac{1}{A\alpha} M\left(\frac{x}{A\alpha}\right)$$

By aperture size A they mean scaling the mask $M(u)$ up and down. So scaling up would let more light in.





A=2

A=1

less light intensity and smaller mask projection

the smaller the aperture the smaller the middle clear area and therefore the smaller the projected mask

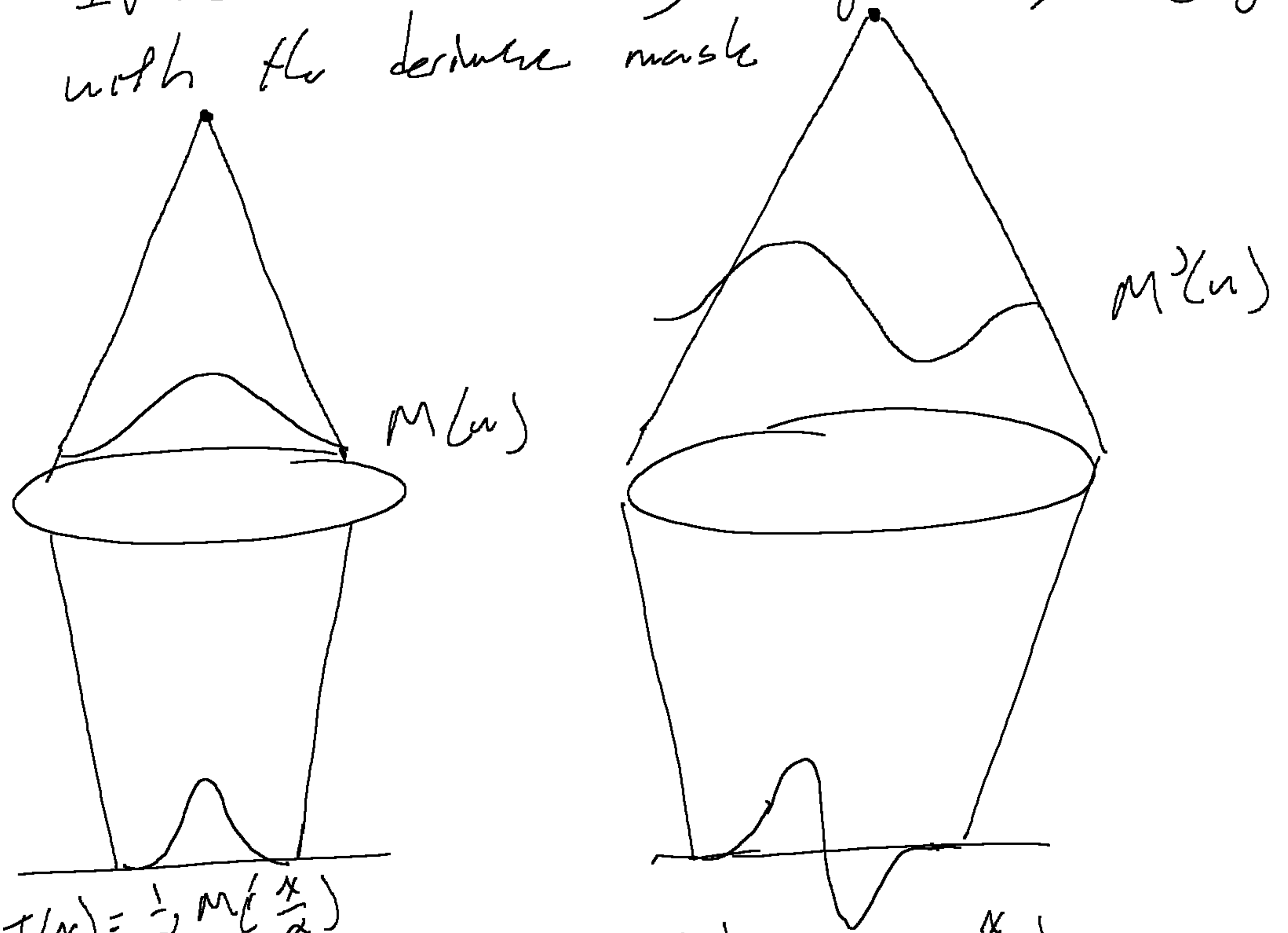
$$I_A(x) = \frac{\partial f}{\partial A} = \frac{\partial \frac{1}{\alpha} A^{-1} \cdot M\left(\frac{x}{\alpha} A^{-1}\right)}{\partial A}$$

$$= -\frac{1}{\alpha} A^{-2} M\left(\frac{x}{\alpha} A^{-1}\right) - \frac{1}{\alpha} A^{-1} \frac{x}{\alpha} A^{-2} M'\left(\frac{x}{\alpha} A^{-1}\right)$$

$$= \frac{-1}{\alpha} M\left(\frac{x}{\alpha}\right) - \frac{x}{\alpha^2} M'\left(\frac{x}{\alpha}\right)$$

$$= \boxed{-\frac{1}{\alpha}\left(M\left(\frac{x}{\alpha}\right) + \frac{1}{\alpha} M'\left(\frac{x}{\alpha}\right)\right)}$$

$I_v(x)$ can be directly computed by imaging with the derivative mask



$I(x) = \frac{1}{\alpha} M\left(\frac{x}{\alpha}\right)$

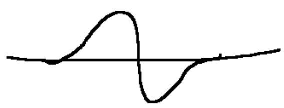
$-I_v(x) = \frac{1}{\alpha} M'\left(\frac{x}{\alpha}\right)$

↳ viewpoint

NOT aperture

$\downarrow \frac{d}{dx} \downarrow$

$I(x) = \frac{1}{\alpha^2} M'\left(\frac{x}{\alpha}\right)$



$\longrightarrow \div \longrightarrow \alpha$

If a Gaussian mask is used:

$$M(u;A) = \frac{1}{A}\exp\left(-\frac{u^2}{2A^2}\right)$$

then $I_A(x) = \frac{1}{\alpha} M''\left(\frac{x}{\alpha}\right)$

which can be related to the 2nd spatial derivative

$$I_{xx}(x) = \frac{1}{\alpha^3} M''\left(\frac{x}{\alpha}\right) = \frac{1}{\alpha^2} I_A(x)$$

So least squares can be used.

<u>Note</u>: $I_A(x)$ is always found directly by imaging through the appropriate mask

Can now use least squares and expand to 2D easily