IMPACT OF DATA TRANSFORMATIONS ON MODEL ACCURACY: HOMOGRAPHY AND GAUSSIAN NOISE

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Abstract

This report evaluates the robustness of various machine learning models, including Logistic Regression, SVM, MLP, and CNN, under two types of data transformations: Homography and Gaussian Noise. CNN demonstrates the highest accuracy under Homography transformations, showing resilience to severe perspective distortions. For Gaussian Noise, all models experience significant performance drops, with SVM using a Polynomial Kernel exhibiting greater robustness compared to other models. These findings highlight the trade-offs between model complexity and robustness to data transformations.

1 Problem Setting

This study examines how six machine learning models—Logistic Regression, FDA, SVM (RBF and Polynomial), MLP, and CNN—handle data distortions such as Homography transformations and Gaussian Noise. These transformations test the robustness of models under challenging conditions, offering insights into their strengths and limitations. The goal is to guide model selection for tasks involving imperfect data.

2 Dataset and Data Split

2.1 Dataset Overview

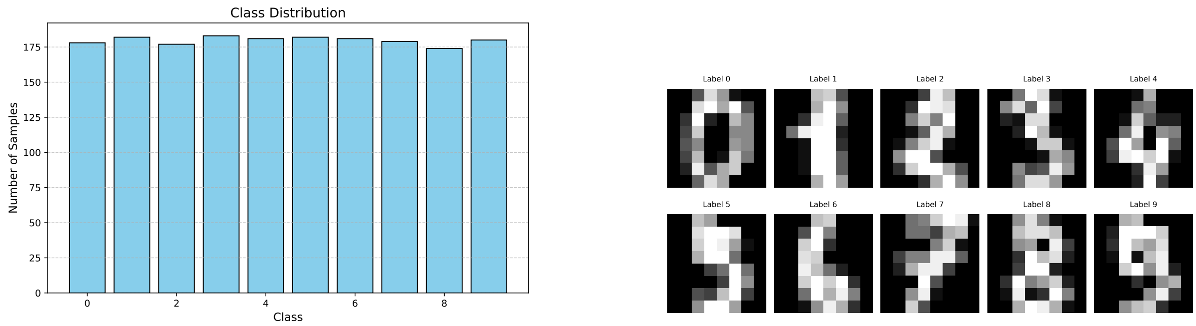
The dataset used for this study contains 1,797 samples, each represented as an 8×8 grayscale image, resulting in 64 pixel values per sample. This dataset consists of numerical data derived from handwritten digits. The dataset is uniformly distributed across 10 unique classes, representing digits from 0 to 9. This uniform distribution ensures that each class has roughly the same number of samples, which contributes to a balanced training process.

2.2 Train and Test Data Split

The dataset is split into training and test sets. The training set contains 1,437 samples, while the test set consists of 360 samples. The test set is reserved for evaluating the generalization performance of the trained model.

2.3 Train and Validation Split with K-Fold Cross-Validation

To assess model performance and mitigate overfitting, 5-fold cross-validation was applied to the training set. In this approach, the training data (1,437 samples) is divided into five subsets. For each fold, one subset is used for validation while the remaining four subsets are used for training.



(a) Class Distribution Visualization

(b) Sample MNIST Images

Figure 1: Dataset Visualizations: Class distribution and example MNIST images.

3 Model Training and Validation

3.1 Model Overview

The following machine learning models were used in this study to evaluate their robustness against data transformations. Each model was trained with appropriate hyperparameters and settings to ensure fair evaluation.

3.1.1 Logistic Regression

Logistic Regression, designed for binary classification, was adapted to the multi-class task using the one-vs-rest strategy. The Limited-memory BFGS (LBFGS) solver was used for efficient optimization across multiple binary classifiers.

3.1.2 Fisher Discriminant Analysis (FDA)

Fisher Discriminant Analysis (FDA) is a linear model that finds a closed-form solution to project data onto a lower-dimensional space, maximizing class separability. It was employed to assess its effectiveness in handling linearly separable datasets.

3.1.3 Support Vector Machine (SVM)

Two variants of SVM were evaluated in this study. The first used a Gaussian kernel, a non-linear classifier effective for capturing complex decision boundaries. The second employed a polynomial kernel of degree 3, enabling it to model interactions between features.

3.1.4 Multilayer Perceptron (MLP)

The MLP model is a feedforward neural network with three hidden layers containing 128, 64, and 32 units, respectively, each using the ReLU activation function. The output layer consists of 10 units with a softmax activation function for multi-class classification. The model was trained using the Adam optimizer with a learning rate of 0.001 and L2 regularization ($\alpha = 10^{-4}$) to prevent overfitting, with the maximum number of optimization iterations set to 1500.

3.1.5 Convolutional Neural Network (CNN)

The CNN used in this study is designed for image data and features the following architecture:

1. Two convolutional layers, each with 64 filters, a 3×3 kernel, and ReLU activation. Batch normalization is applied to stabilize training.

2. A max-pooling layer with a 2×2 window for down-sampling.

3. Two additional convolutional layers, each with 128 filters, a 3×3 kernel, and ReLU activation.

4. A second max-pooling layer with the same configuration as the first.

5. A fully connected layer with 256 units, followed by the output layer with 10 units, representing the classification classes.

The model was optimized using the Adam optimizer and trained for 20 epochs with a batch size of 32.

3.2 Validation Accuracy for Each Model

The table below presents the average validation accuracy for each model. Overall, all models demonstrated high accuracy, reflecting their ability to effectively classify the dataset. Non-linear models and neural networks slightly outperformed linear models, but the differences in performance were not substantial.

Table 1: Average Validation Accuracy for Each Model

Model	**Average Validation Accuracy**
Logistic Regression	0.9617
Fisher Discriminant Analysis (FDA)	0.9478
SVM (Gaussian Kernel)	0.9861
SVM (Polynomial Kernel)	0.9882
Multilayer Perceptron (MLP)	0.9763
Convolutional Neural Network (CNN)	0.9792

4 Experimental Setup

4.1 Homography Transformation

Homography transformations were applied to simulate perspective distortions by randomly shifting the coordinates of the image corners. The maximum offset (*max_offset*) used to define the range of these random shifts was tested at three levels: 1.5, 2.0, and 2.5 pixels. These progressively severe transformations were designed to evaluate the models' ability to handle perspective changes.

4.2 Gaussian Noise

Gaussian noise was added to the images to simulate noisy conditions, commonly encountered in practical scenarios like low-light environments. Noise levels were defined by the standard deviation (*sigma*) of the Gaussian distribution, with three levels tested: 0.2, 0.5, and 0.7. These levels introduced increasing degrees of image degradation to test the robustness of the models.

4.3 Transformation Visualization

To provide a qualitative insight into the applied transformations, Figure 2 illustrates the impact of Homography (max_offset = 2.5) and Gaussian Noise (sigma = 0.7) on sample images from the dataset. These examples represent the most challenging transformations applied in this study.

5 Experiment Results

5.1 Baseline Accuracy

The baseline accuracy represents the performance of each model on the unaltered MNIST dataset. Table 2 summarizes the accuracy for all models, showing that CNN and SVM with a Polynomial Kernel achieved the highest accuracy. Linear models, such as Logistic Regression and FDA, performed well but were slightly outperformed by non-linear models and neural networks.

Analysis: CNN achieved the highest baseline accuracy due to its ability to capture spatial hierarchies in image data, while SVM with a Polynomial Kernel excelled at modeling complex feature interactions. Linear models like FDA, while effective for linearly separable data, showed slightly lower accuracy due to their inability to handle non-linear relationships effectively.

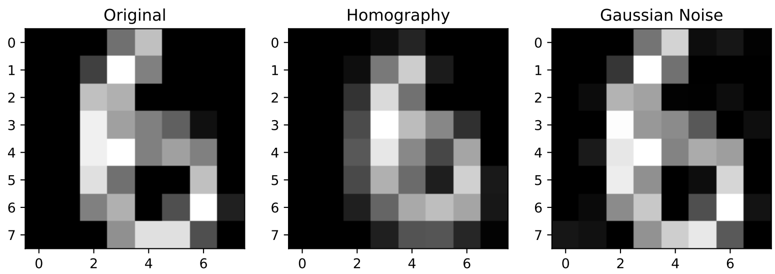


Figure 2: Visualization of Homography Transformation (max_offset = 2.5) and Gaussian Noise (sigma = 0.7).

Table 2: Baseline Accuracy for Each Model

Model	**Baseline Accuracy**
Logistic Regression	0.9667
Fisher Discriminant Analysis (FDA)	0.9444
SVM (Gaussian Kernel)	0.9861
SVM (Polynomial Kernel)	0.9917
Multilayer Perceptron (MLP)	0.9778
Convolutional Neural Network (CNN)	0.9944

5.2 Homography Transformation

For transformations with $max\_offset = 1.5$, the accuracy of most models remained consistent with baseline results. As such, detailed results for this case are omitted, and the focus is on higher severity levels ($max\_offset = 2$ and 2.5).

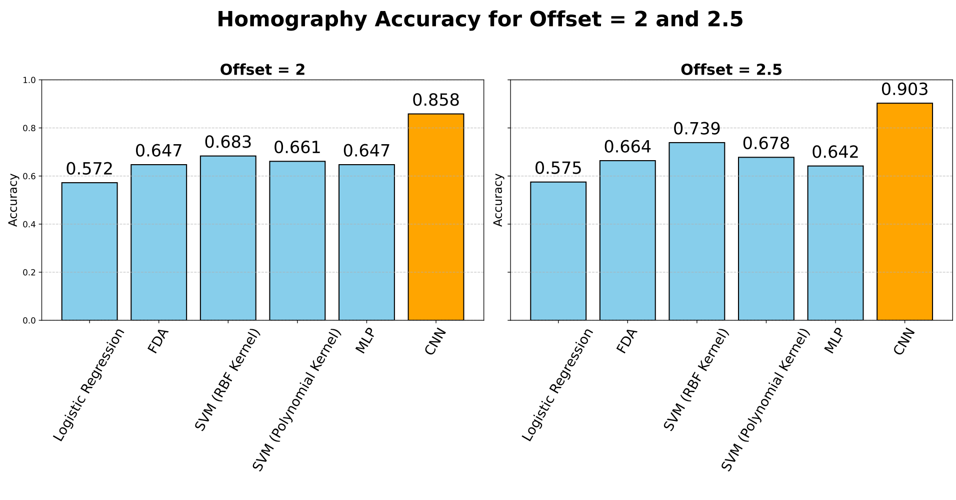


Figure 3: Accuracy under homography transformations with different max_offset values.

Analysis: CNN displayed remarkable resilience to homography transformations, maintaining accuracy above 85% even at $max\_offset = 2.5$, leveraging its spatial invariance capabilities. SVM with a Gaussian Kernel also exhibited robust performance, benefiting from its non-linear transformation modeling capacity. In contrast, Logistic Regression and MLP saw significant accuracy drops with increasing transformation severity due to their simpler architectures. FDA, reliant on linear assumptions, performed poorly under severe transformations.

5.3 Gaussian Noise Transformation

For noise levels with $sigma = 0.2$, most models performed similarly to their baseline accuracy, indicating minimal impact. Results for this case are not presented, and the analysis emphasizes higher severity levels ($sigma = 0.5$ and 0.7).

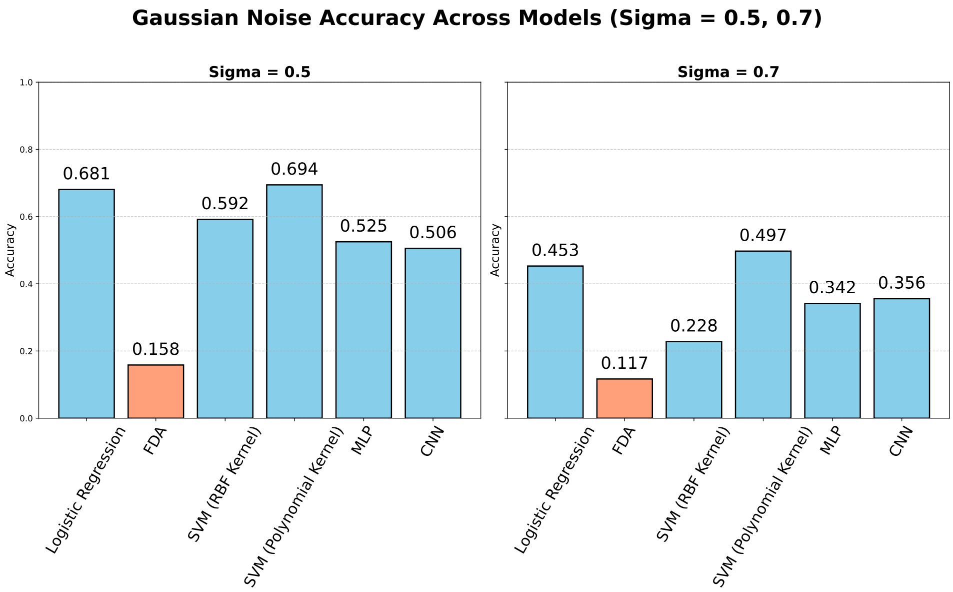


Figure 4: Accuracy under Gaussian noise transformations with different sigma values.

Analysis: SVM with a Polynomial Kernel demonstrated the highest tolerance to noise, particularly at $sigma = 0.5$, while CNN showed moderate robustness but experienced significant degradation at $sigma = 0.7$. Logistic Regression performed reasonably at low noise levels but deteriorated at higher noise levels, reflecting its sensitivity to input variability. FDA exhibited the sharpest accuracy decline, underscoring its vulnerability to noisy environments due to its reliance on linear discrimination. MLP, while better than FDA and Logistic Regression, struggled with higher noise levels, indicating limitations in its learned representations for handling extreme variability.

5.4 Summary

Overall, CNN emerged as the most robust model under homography transformations due to its ability to extract spatially invariant features. For Gaussian noise, SVM with a Polynomial Kernel showed superior performance, suggesting its robustness in noisy conditions. Linear models like Logistic Regression and FDA, though computationally efficient, were unable to handle significant distortions or noise effectively. While MLP performed better than linear models in non-linear scenarios, its robustness remains inferior to CNN and SVM under challenging conditions.

6 Conclusion

This study assessed the robustness of six machine learning models—Logistic Regression, FDA, SVM with Gaussian and Polynomial kernels, MLP, and CNN— under homography and Gaussian noise transformations. CNN excelled in handling perspective distortions, while SVM with a Polynomial Kernel showed strong tolerance to noise. Linear models like Logistic Regression and FDA, though efficient, struggled with significant distortions, and MLP, while better in non-linear tasks, was less robust than CNN and SVM. These findings highlight the need for careful model selection based on robustness to specific data distortions, with future work extending to more diverse datasets and transformations.