PS-3

Leen Alrawas

September 22, 2023

1 Abstract

In this problem set, we study the error in calculating the derivatives and how it is affected by increasing the approximation accuracy. In addition, we compare for loops to a package in Numpy that operates on matrices. Lastly, using random numbers we simulate the nuclear decay of two reactions using two different approaches. Code scripts are available on `https://github.com/Leen-Alrawas/phys-ga2000/tree/main/ps-3`.

2 Solutions

1. In this differentiation problem, the error reached its minimum at $\sigma = 10^{-8}$. Mathematically, as σ approaches zero, the approximation error should be decreased. However, computationally there is also a trade-off with round-off errors caused by losing precision doing calculations on a computer. This will be more significant when trying to add numbers with very different orders or subtracting numbers with similar orders which is what happens in this example.

2. The efficiency using "for loops" for matrix multiplication has degraded at a rate of N^3 as expected where N is the matrix dimension. However, using the `dot()` command from Numpy was considerably faster and was not hugely affected by increasing the matrix dimension. See Figure 1.

3. The decay chain for Bi-213 was simulated (Figure 2). As expected, the number of Bi-213 decreases over time and the number of the more stable isotope Bi-209 increases. The probability that the decay of Bi-213 was to Tl-209 is 2.09%, which is a low probability. If the Bi-213 decay did not yield a Tl-209 atom, then it produced a Pb-209 atom. In addition, Tl-209 half time is low making it an unstable isotope so it decays again to Pb-209, which explains the low number of Tl-209 atoms at all time steps. The maximum number of Tl-209 atoms can be recorded during the simulation and it was in the range of [10,20] atoms.

4. Using the nonuniform probability distribution of the nuclear decay, the decay of 1000 Tl-208 to Pb-208 was simulated (Figure 3). The times of the decay of individual atoms were first generated using the transformation method. The 1000 atoms took about 2250 seconds to fully decay to lead atoms.

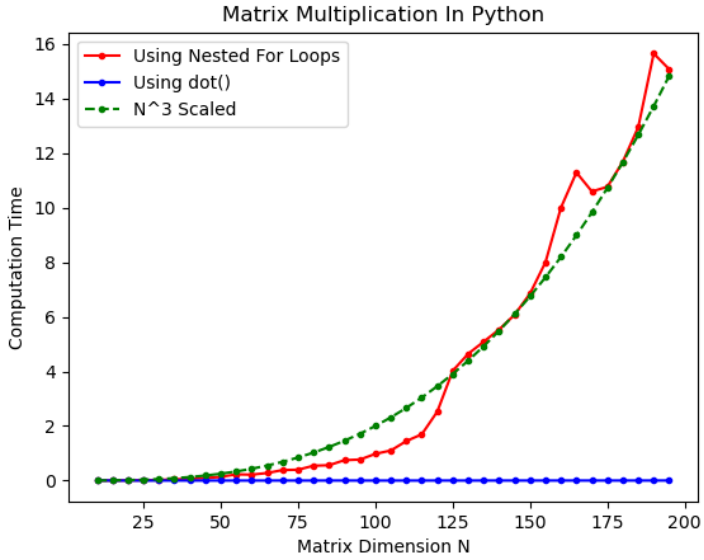

Figure 1: Time comparison between two implementations for finding the product of two matrices: one uses "for looping", and another uses the command `dot()` from Numpy.

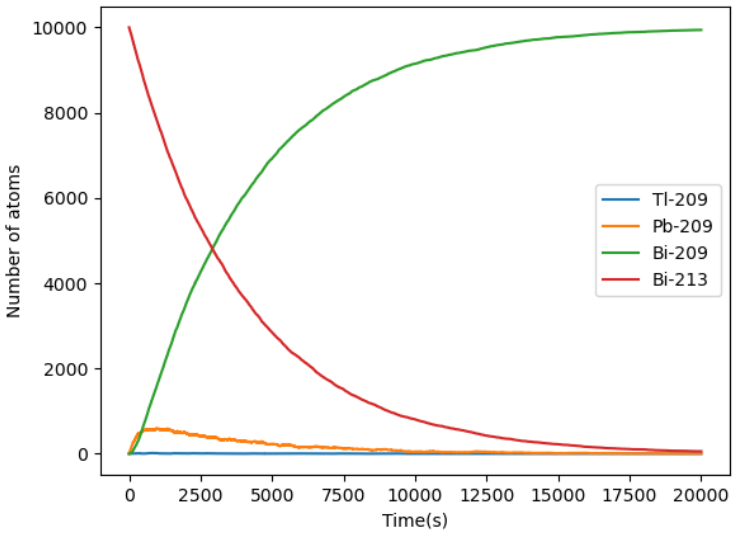

Figure 2: The nuclear decay of Bi-213 over a time interval of 20,000 seconds. The chain reaction includes the possibilities of Bi-213 decaying to either Pb-209 or Tl-209, then all atoms end decaying to Bi-209.

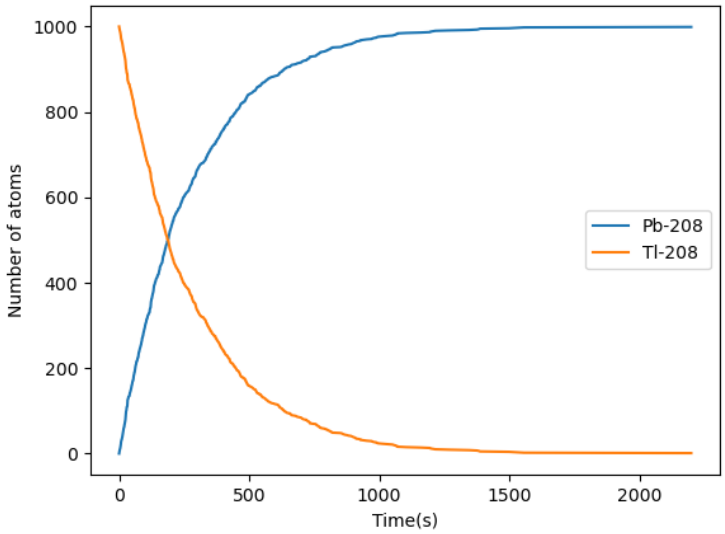

Figure 3: The nuclear decay of 1000 atoms of Tl-208 to Pb-208.