PS-7

Leen Alrawas

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Abstract

In this problem set we implement different minimization algorithms and apply them to two problems. The first one implies finding the minimum of a function using Brent's minimization method. In the second problem, we fit some data into the logistic model by minimizing the negative likelihood function. Code scripts are available at `https://github.com/Leen-Alrawas/phys-ga2000/tree/main/ps-7`.

1 Methods and Results

1. We implement Brent's one-dimensional minimization method that uses quadratic optimization up to a certain point until its conditions break. The conditions are if the parabolic step falls outside the bracketing interval or the parabolic step is greater than the step before the last. If these conditions are met, the method reverts to the golden section method. As an example, take $f(x) = (x - 0.3)^2 e^x$. The graph of $f(x)$ is shown in Fig(1). I chose $(a = -1, b = 0.5, c = 1)$ as the starting point. The implementation was compared to the `scipy` built-in Brent's function using the same maximal error of 10^{-16}. A snapshot of the results is shown in Fig(2).

2. the data is modeled using the logistic regression (1). The optimal parameters (β_0, β_1) along with the errors and their covariance matrix are shown in Fig(3).

$$p(x) = \frac{1}{1 + e^{-\beta_0 - \beta_1 x}} \tag{1}$$

The method consists of minimizing the negative logarithmic likelihood function. To avoid zero input in the logarithmic function, a value of the machine precision error was added to the function argument. The model gives a good representation of the data (Fig 4). In general, the logistic regression model is used for scenarios where the outcomes are binary. In this case, the outcomes considered of 1 representing yes to the answer to the question and 0 representing no, where the varying parameter is the person's age. It is reasonable to see a positive correlation between age and the possibility of answering with a 'yes'.

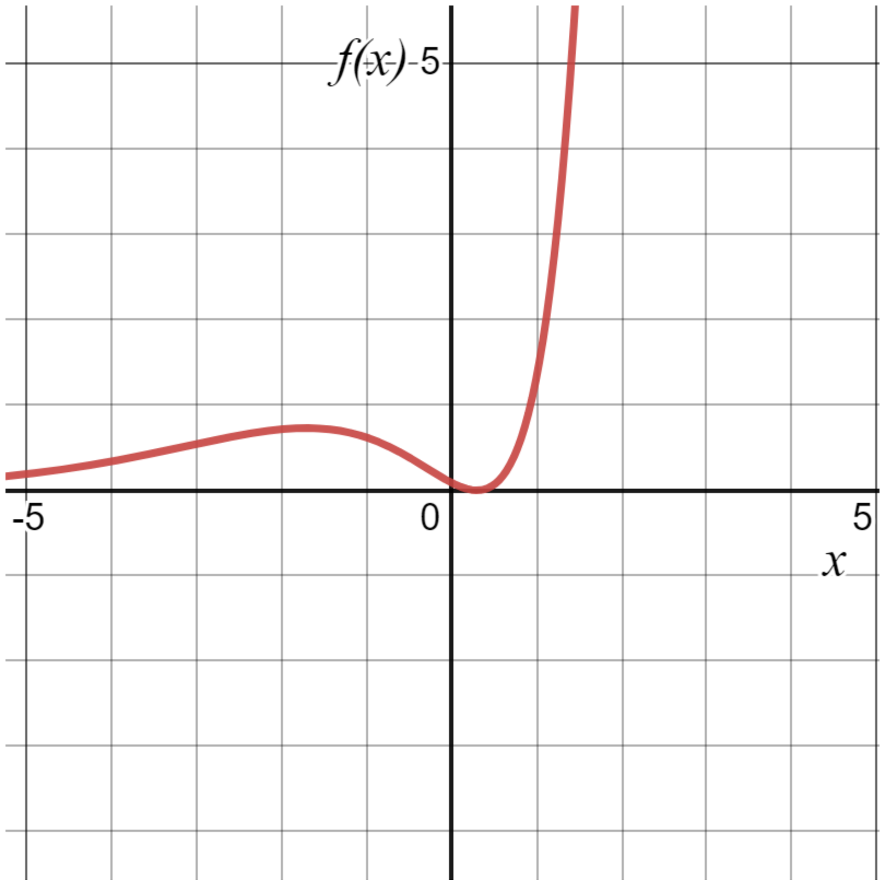
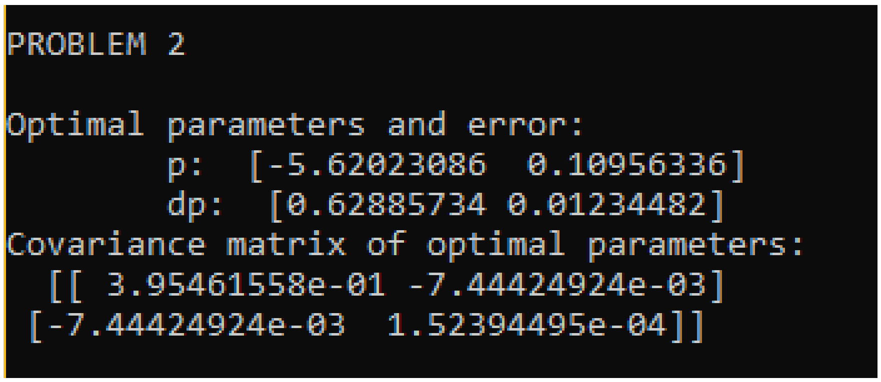


Figure 1: $f(x) = (x - 0.3)^2 e^x$. The function has a minimum point at $x = 0.3$.

```
PROBLEM 1

Optimal solution - Scipy: 0.2999999999999925
Optimal solution - Brent's: 0.29999999999999116
difference = 1.3322676295501878e-15
```

Figure 2: Brent's one dimensional minimization of $f(x)$.



```
PROBLEM 2

Optimal parameters and error:
        p:  [-5.62023086  0.10956336]
        dp:  [0.62885734 0.01234482]
Covariance matrix of optimal parameters:
  [[ 3.95461558e-01 -7.44424924e-03]
 [-7.44424924e-03  1.52394495e-04]]
```

Figure 3: Logistic model parameters calculation.

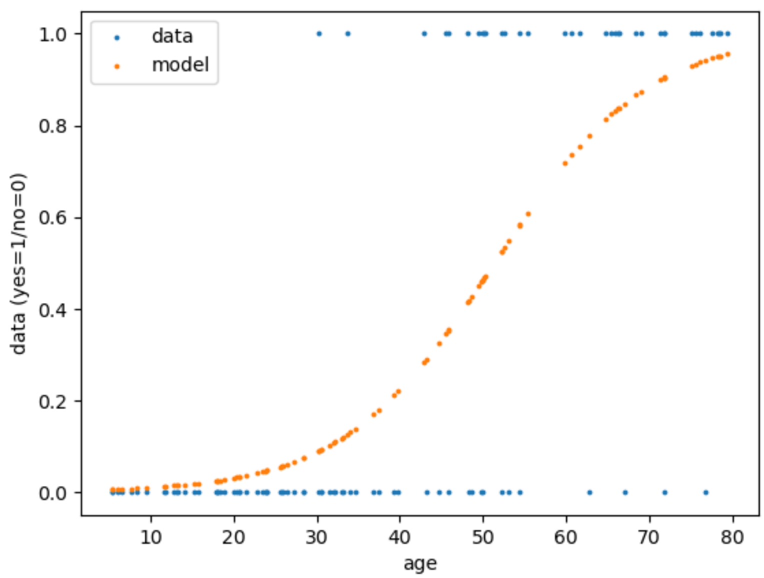


Figure 4: Logistic model vs the data.