CS 400

Graphs: BFS Traversal

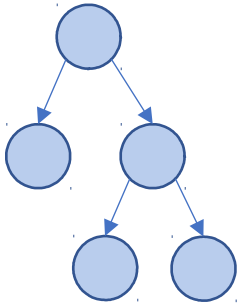
ID: 13-01

Traversal:

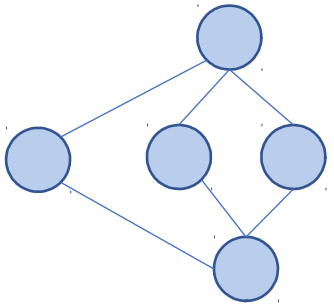
Objective: Visit every vertex and every edge in the graph.

Purpose: Search for interesting sub-structures in the graph.

We've seen traversal before ….but it's different:



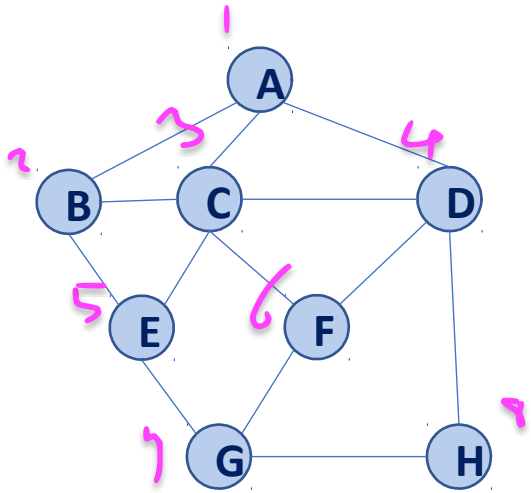
- Ordered
- Obvious Start
- Notion of completeness
  : we know when the traverse is done



- Not ordered
- No obvious start
- No notion of completeness

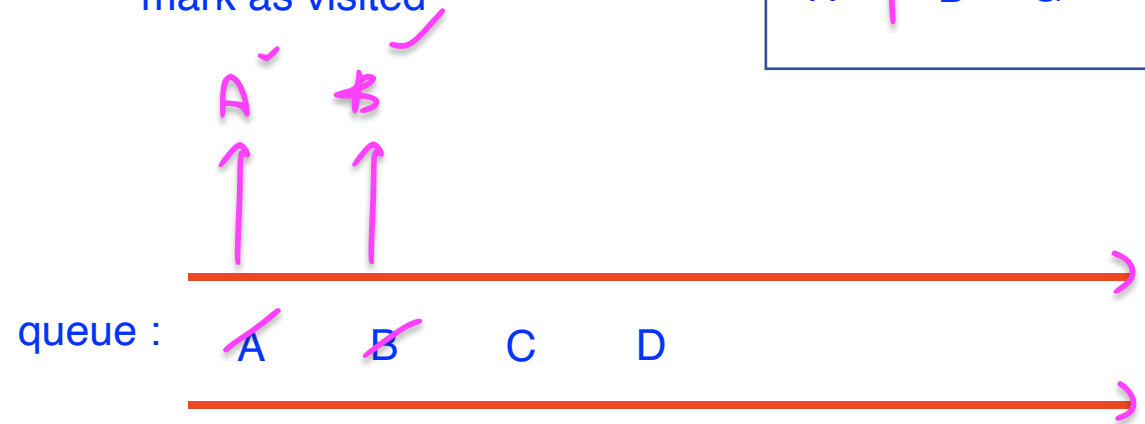
Traversal: BFS

(Breadth-First Search)



A	B	C	D		
B	A	C	E		
C	A	B	D	E	F
D	A	C	F	H	
E	B	C	G		
F	C	D	G		
G	E	F	H		
H	D	G			

mark as visited

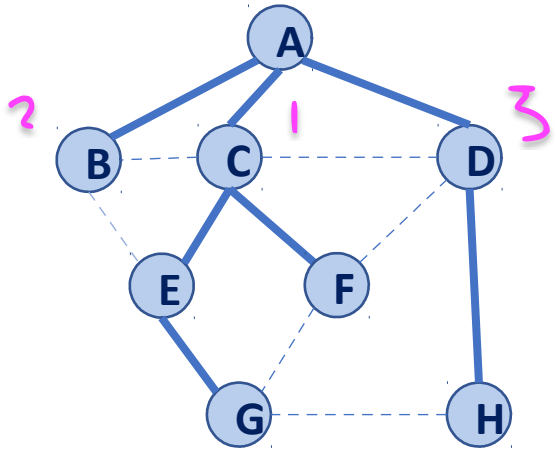


노드를 하나씩 방문하고, 그 다음 노드를 큐에 추가함

=> First node to visit -> all of her children -> all of their children

Traversal: BFS

ordering of incident nodes를 바꿔서



d	p		Adjacent Edges
0	A	**A**	**C B D**
1	A	**B**	**A C E**
1	A	**C**	**B A D E F**
1	A	**D**	**A C F H**
2	C	**E**	**B C G**
2	C	**F**	**C D G**
3	E	**G**	**E F H**
2	D	**H**	**D G**

```
1  BFS(G):
2    Input: Graph, G
3    Output: A labeling of the edges on
4        G as discovery and cross edges
5
6    foreach (Vertex v : G.vertices()):
7      setLabel(v, UNEXPLORED)
8    foreach (Edge e : G.edges()):
9      setLabel(e, UNEXPLORED)
10   foreach (Vertex v : G.vertices()):
11     if getLabel(v) == UNEXPLORED:
12       BFS(G, v)
```

```
14 BFS(G, v):
15   Queue q
16   setLabel(v, VISITED)
17   q.enqueue(v)
18
19   while !q.empty():
20     v = q.dequeue()
21     foreach (Vertex w : G.adjacent(v)):
22       if getLabel(w) == UNEXPLORED:
23         setLabel(v, w, DISCOVERY)
24         setLabel(w, VISITED)
25         q.enqueue(w)
26       elseif getLabel(v, w) == UNEXPLORED:
27         setLabel(v, w, CROSS)
```

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Graphs: BFS Analysis

ID: 13-02

BFS Analysis

Q: Does our implementation handle disjoint graphs? Yes
If so, what code handles this? line 10 - 13

- ***How do we use this to count components?***

add line 13 : component++;

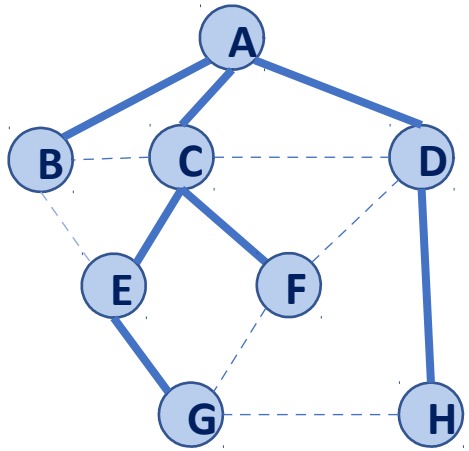
Q: Does our implementation detect a cycle?

- ***How do we update our code to detect a cycle?***

Yes
add line 28 : cycle = true;

Q: What is the running time?

Running time of BFS



d	p	v	Adjacent
0	A	A	C B D
1	A	B	A C E
1	A	C	B A D E F
1	A	D	A C F H
2	C	E	B C G
2	C	F	C D G
3	E	G	E F H
2	D	H	D G

While-loop at **:19**? n

For-loop at **:21**? 2m or n

```
1  BFS(G):
2    Input: Graph, G
3    Output: A labeling of the edges on
4        G as discovery and cross edges
5
6    foreach (Vertex v : G.vertices()):
7      setLabel(v, UNEXPLORED)
8    foreach (Edge e : G.edges()):
9      setLabel(e, UNEXPLORED)
10   foreach (Vertex v : G.vertices()):
11     if getLabel(v) == UNEXPLORED:
12       BFS(G, v)
```

```
14 BFS(G, v):
15   Queue q
16   setLabel(v, VISITED)
17   q.enqueue(v)
18
19   while !q.empty():
20     v = q.dequeue()
21     foreach (Vertex w : G.adjacent(v)):
22       if getLabel(w) == UNEXPLORED:
23         setLabel(v, w, DISCOVERY)
24         setLabel(w, VISITED)
25         q.enqueue(w)
26       elseif getLabel(v, w) == UNEXPLORED:
27         setLabel(v, w, CROSS)
```

BFS Observations

Q: What is a shortest path from **A** to **H**? A - D - H

Q: What is a shortest path from **E** to **H**?

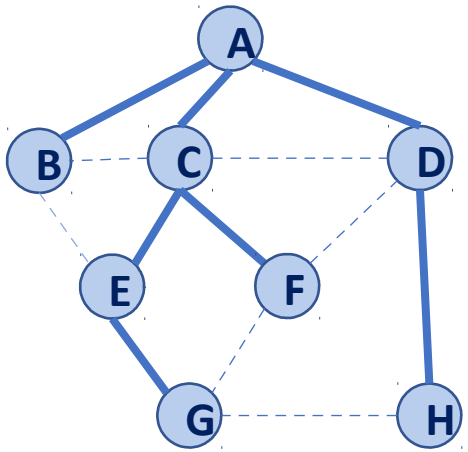
Q: How does a cross edge relate to **d**?

By following a cross edge, we'll never get more than one further than our start

Q: What structure is made from discovery edges?

predecessor

d	p	v	Adjacent
0	A	A	C B D
1	A	B	A C E
1	A	C	B A D E F
1	A	D	A C F H
2	C	E	B C G
2	C	F	C D G
3	E	G	E F H
2	D	H	D G



BFS Observations

Obs. 1: Traversals can be used to count components.

Obs. 2: Traversals can be used to detect cycles.

Obs. 3: In BFS, **d** provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| = 1$$

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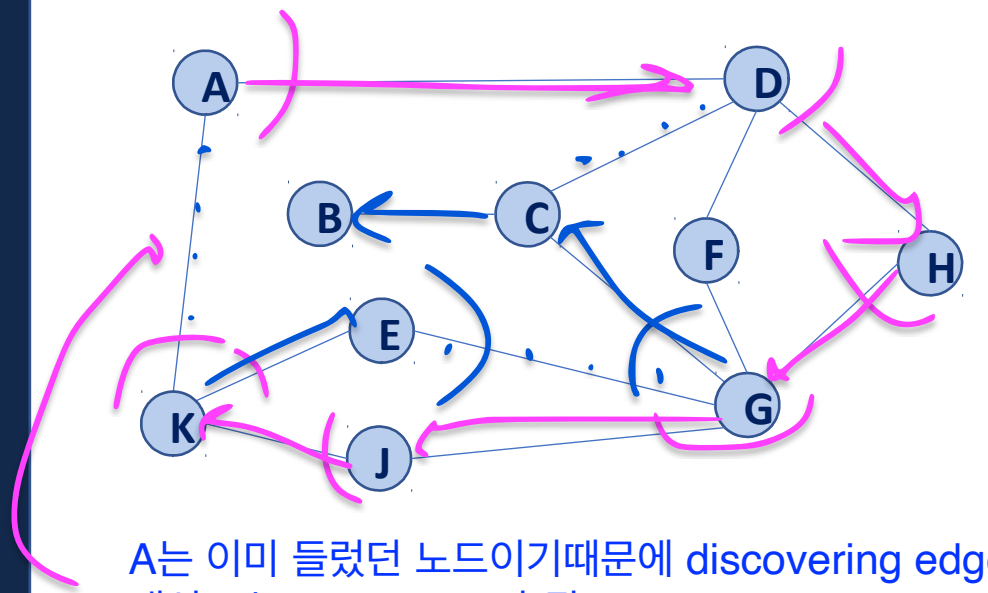
Graphs: DFS Traversal

ID: 13-03

Traversal: DFS

Depth-First Search :
- we wanna go very very deep very quickly.
- maintain our data structure using a stack structure



- A부터 시작. 시계방향으로 원을 그려보자.
- 앗, 아직 안갔던 discovering edge네!
- D로 향함
… 의 반복

A는 이미 들렀던 노드이기때문에 discovering edge가 아님.
대신, k는 back edge가 됨.
- back edge : an edge that's not a discovery edge in a DFS

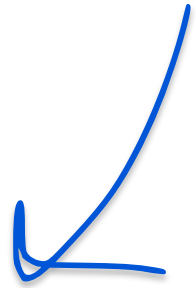
```
1   BFS(G):
2     Input: Graph, G
3     Output: A labeling of the edges on
4         G as discovery and cross edges
5
6     foreach (Vertex v : G.vertices()):
7       setLabel(v, UNEXPLORED)
8     foreach (Edge e : G.edges()):
9       setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12        BFS(G, v)
```

```
14  BFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23          setLabel(v, w, DISCOVERY)
24          setLabel(w, VISITED)
25          q.enqueue(w)
26        elseif getLabel(v, w) == UNEXPLORED:
27          setLabel(v, w, CROSS)
```

DFS(G, W)

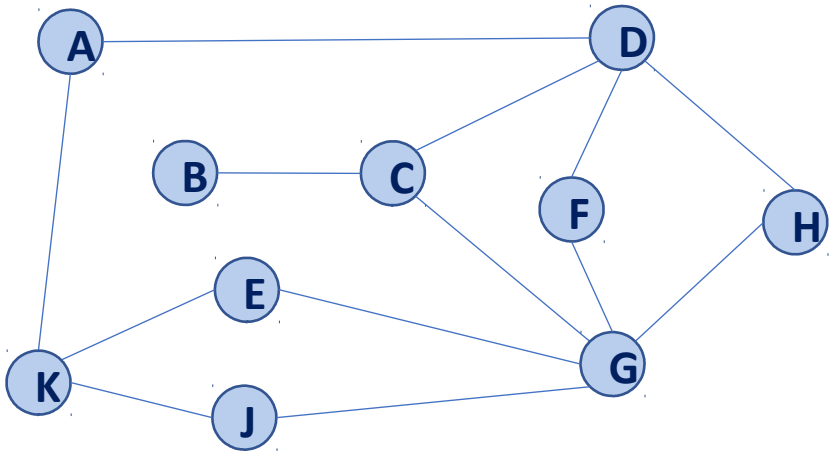
Discovering

```
1  DFS(G):
2    Input: Graph, G
3    Output: A labeling of the edges on
4        G as discovery and back edges
5
6    foreach (Vertex v : G.vertices()):
7      setLabel(v, UNEXPLORED)
8    foreach (Edge e : G.edges()):
9      setLabel(e, UNEXPLORED)
10   foreach (Vertex v : G.vertices()):
11     if getLabel(v) == UNEXPLORED:
12       DFS(G, v)
```



```
14 DFS(G, v):
15   Queue q
16   setLabel(v, VISITED)
17   q.enqueue(v)
18
19   while !q.empty():
20     v = q.dequeue()
21     foreach (Vertex w : G.adjacent(v)):
22       if getLabel(w) == UNEXPLORED:
23         setLabel(v, w, DISCOVERY)
24         setLabel(w, VISITED)
25         DFS(G, w)
26       elseif getLabel(v, w) == UNEXPLORED:
27         setLabel(v, w, BACK)
```

Running time of DFS



Labeling:

- Vertex: 2 * n -> O(n)

- Edge: 2 * m -> O(m)

=> total : O(n + m)

=> BFS와 같은 running time

Queries:

- Vertex: n -> O(n)

- Edge: O(m)

=> we have to visit every single node exactly once,
and we have to visit every single edge exactly once.
So there's no way we can do faster than n plus m time,
so both BFS and DFS traversal are optimal traversal running times for a graph.

=> total : O(n + m)

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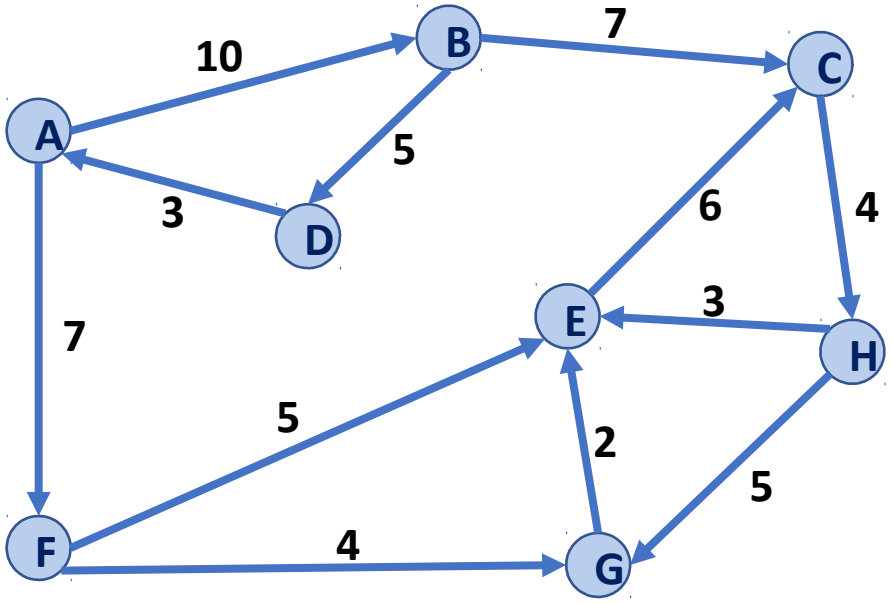
Graphs: Dijkstra's Algorithm

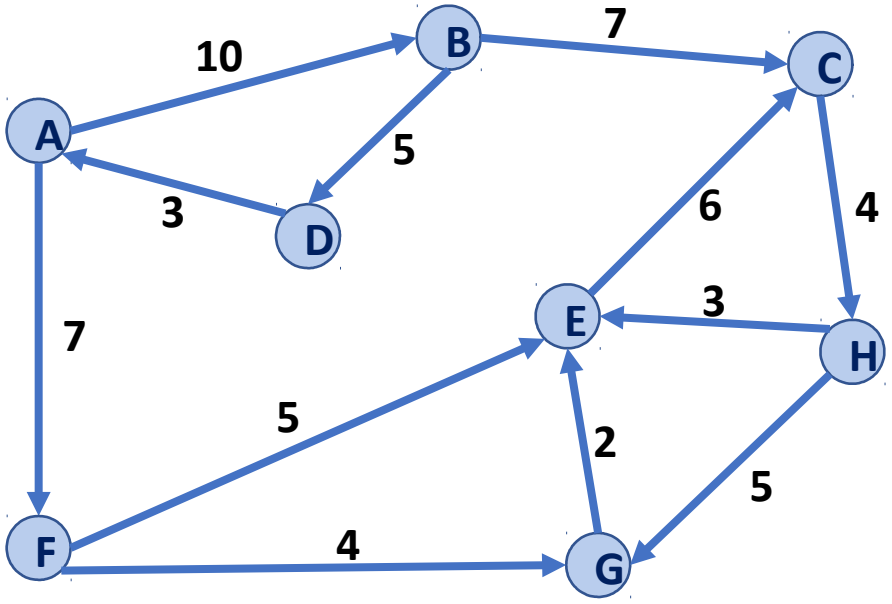
ID: 15-01

Dijkstra's Algorithm (SSSP)




```
     DijkstraSSSP(G, s):
 6     foreach (Vertex v : G):
 7       d[v] = +inf
 8       p[v] = NULL
 9     d[s] = 0
10
11     PriorityQueue Q // min distance, defined by d[v]
12     Q.buildHeap(G.vertices())
13     Graph T          // "labeled set"
14
15     repeat n times:
16       Vertex u = Q.removeMin()
17       T.add(u)
18       foreach (Vertex v : neighbors of u not in T):
19         if _________________ < d[v]:
20           d[v] = ____________________
21           p[v] = m
```

Dijkstra's Algorithm (SSSP)




```
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10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13    Graph T          // "labeled set"
14
15    repeat n times:
16      Vertex u = Q.removeMin()
17      T.add(u)
18      foreach (Vertex v : neighbors of u not in T):
19        if cost(u, v) + d[u] < d[v]:
20          d[v] = cost(u, v) + d[u]
21          p[v] = m
```
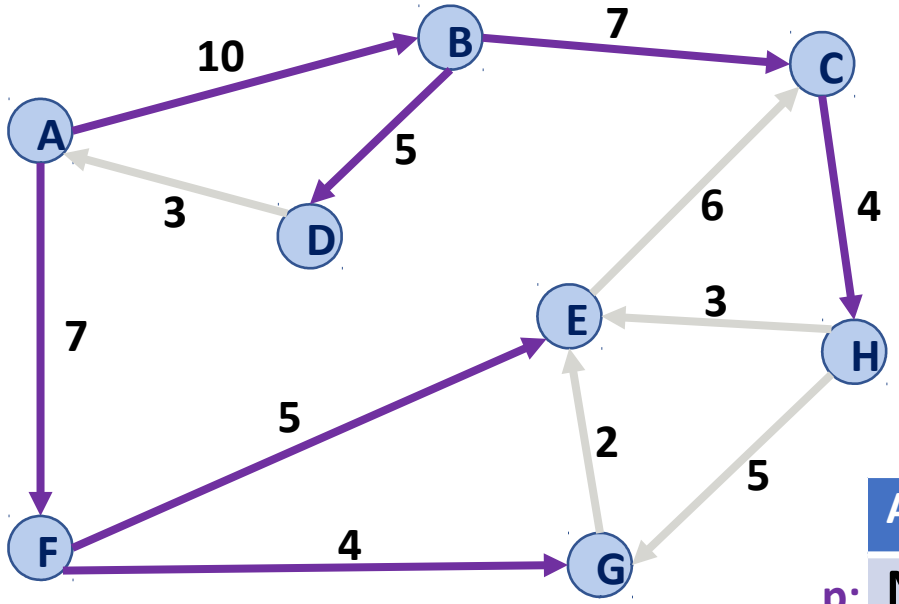

Dijkstra's Algorithm (SSSP)

Dijkstra gives us the shortest path from our path (single source) to **every** connected vertex!



	A	B	C	D	E	F	G	H
p:	NULL	A	B	B	F	A	F	C
d:	0	10	17	15	12	7	11	21

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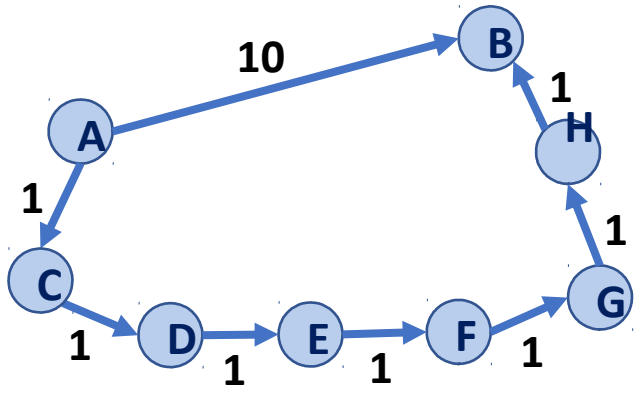
Graphs: Dijkstra’s Edge Cases

ID: 15-02

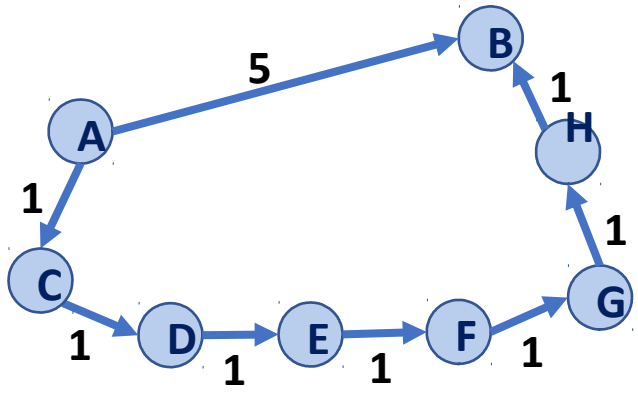
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



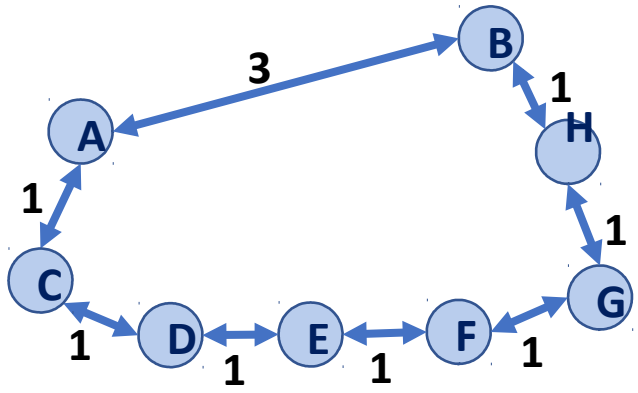
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



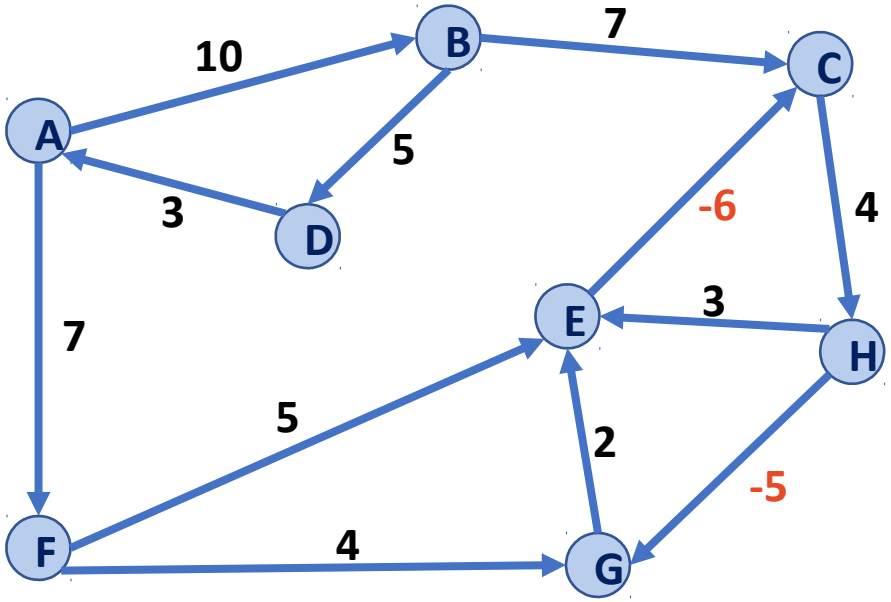
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



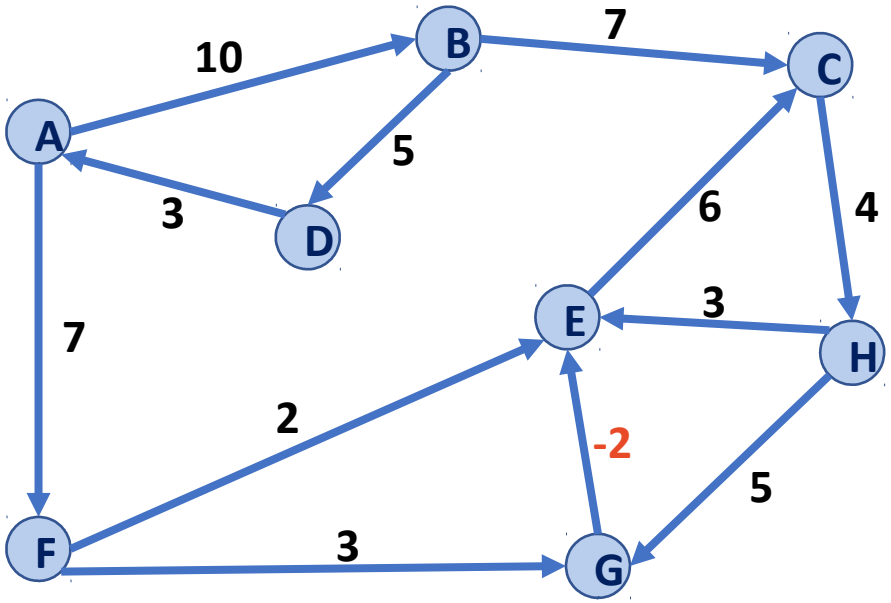
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges, without a negative weight cycle?



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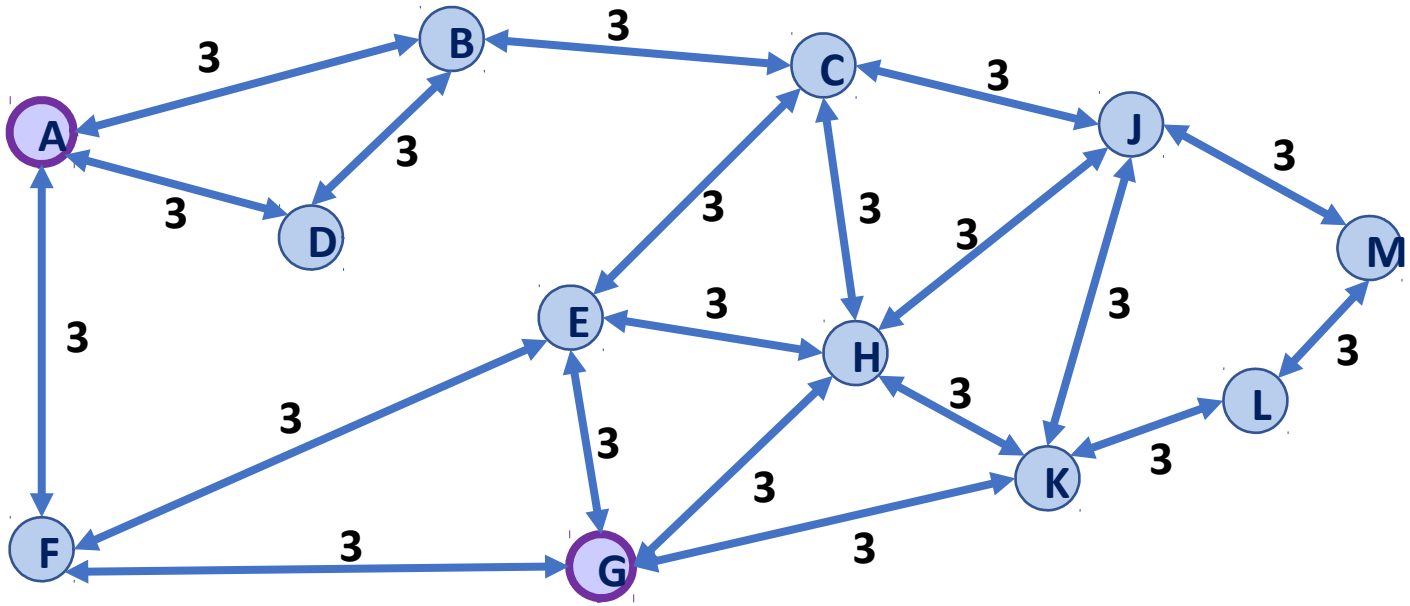
Graphs: Landmark Path Problem

ID: 15-03

Landmark Path Problem

Suppose you want to travel from **A** to **G**.

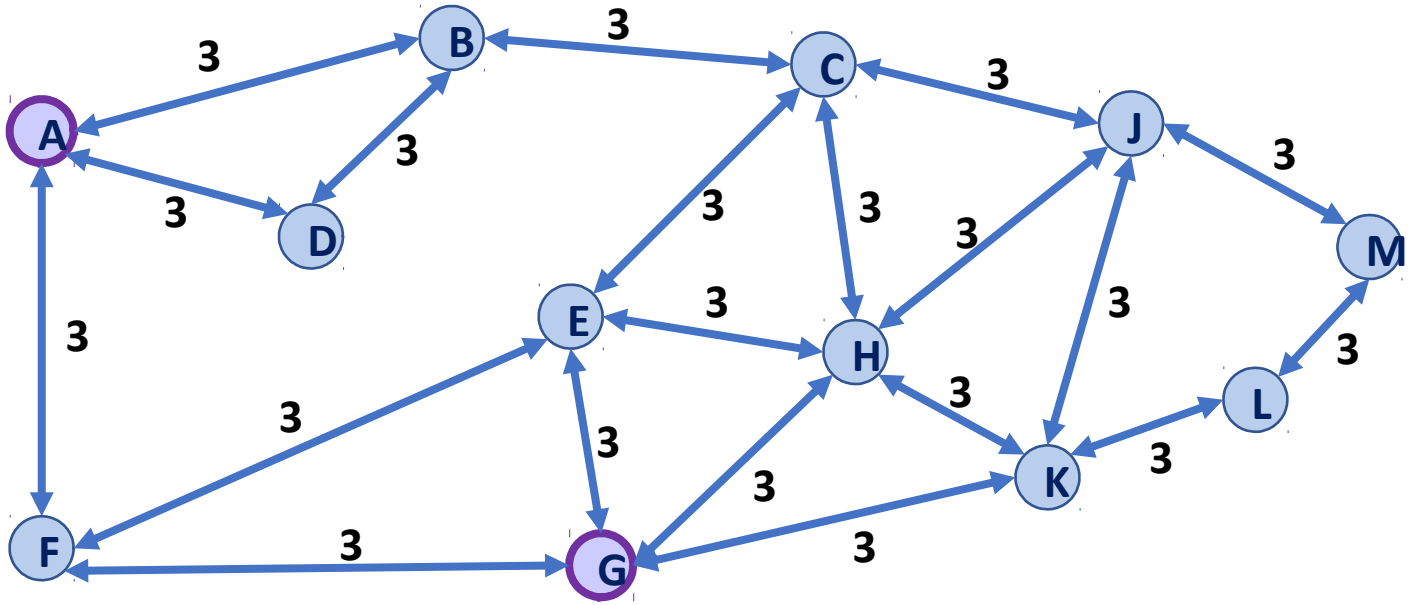
Q1: What is the shortest path from **A** to **G**?



Landmark Path Problem

Suppose you want to travel from **A** to **G**.

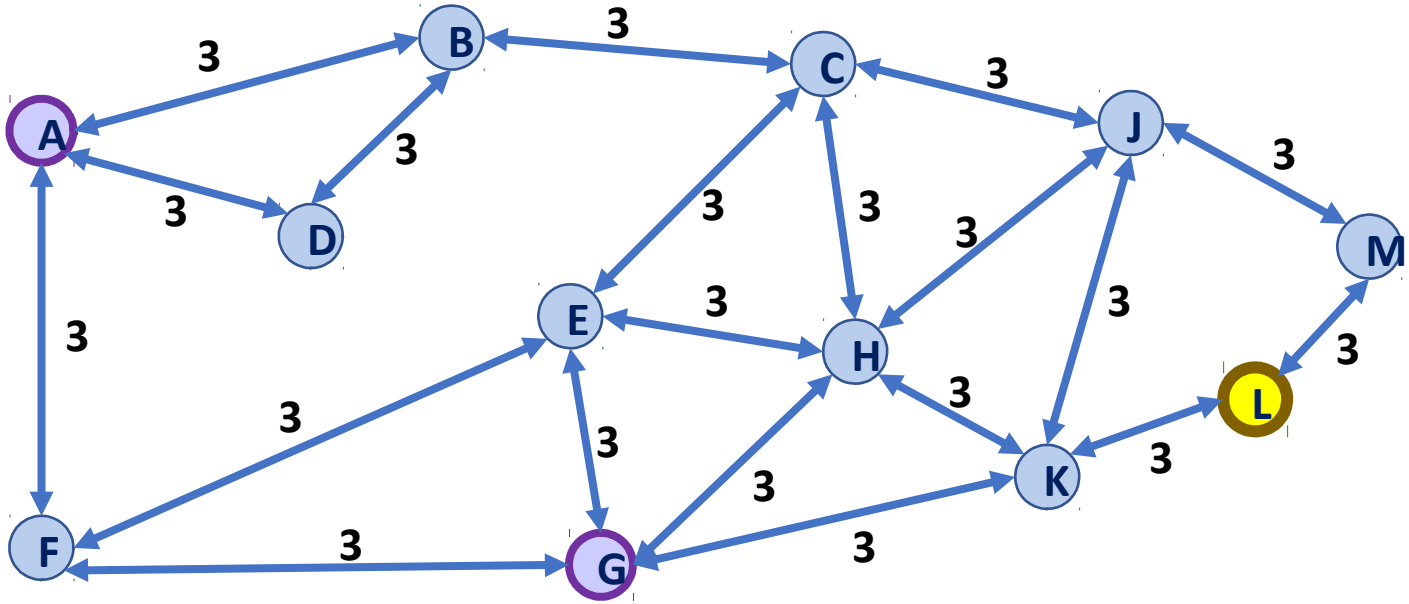
Q2: What is the fastest algorithm to use to find the shortest path?



Landmark Path Problem

In your journey between **A** and **G**, you also want to visit the landmark **L**.

Q3: What is the shortest path from **A** to **G** that visits **L**?



Landmark Path Problem

In your journey between **A** and **G**, you also want to visit the landmark **L**.

Q4: What is the fastest algorithm to find this path?

Q5: What are the specific call(s) to this algorithm?