How to Convert a Context-Free Grammar to Greibach Normal Form

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Objective

This mini-tutorial will answer these questions:

1. What is Greibach Normal Form?

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2. What are the benefits of having a grammar in Greibach Normal Form?

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This mini-tutorial will answer these questions:

1. What is Greibach Normal Form?
2. What are the benefits of having a grammar in Greibach Normal Form?
3. What algorithm can be used to convert a context-free grammar to Greibach Normal Form?

What is Greibach Normal Form?

A context-free grammar is in Greibach Normal Form if the right-hand side of each rule has one terminal followed by zero or more non-terminals:

$$A \rightarrow aX$$

zero or more non-terminal symbols

one terminal symbol

Example of a grammar in Greibach Normal Form

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

Every right-hand side consists of exactly one terminal followed by zero or more non-terminals.

Example of a grammar not in Greibach Normal Form

terminal at end is not allowed

$S \to aBc$

$B \to b$

Not in Greibach Normal Form

What are the benefits of Greibach Normal Form?

Deriving a string P from a grammar that is in Greibach Normal Form takes one step per symbol.

$$\#derivation\ steps\ =\ |P|$$

Example derivation

Grammar in Greibach Normal Form

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

Derive this string: $aababb$

$$S \rightarrow aB \rightarrow aaBB \rightarrow aabSB \rightarrow aabaBB \rightarrow aabab \rightarrow aababb$$

$$|aababb| = 6$$

$$\#derivation\ steps = 6$$

Contrast to a grammar that's not in Greibach Normal Form

Grammar not in Greibach Normal Form

$$S \to aA$$
$$A \to B$$
$$B \to C$$
$$C \to a$$

Derive this string: aa

$$S \to aA \to aB \to aC \to aa$$

$$|aa| = 2$$

$$\#derivation\ steps = 4$$

Greibach Normal Form is a Prefix Notation

Grammar in Greibach Normal Form

$Expr \rightarrow +AB$

$A \rightarrow 5$

$B \rightarrow *CD$

$C \rightarrow 2$

$D \rightarrow 3$

Derive this string: $+5*23$

$$Expr \rightarrow +AB \rightarrow +5B \rightarrow +5*CD \rightarrow +5*2D \rightarrow +5*23$$

$$|+5*23| = 5$$

$$\#derivation\ steps = 5$$

Benefits of Greibach Normal Form

- Strings can be quickly parsed.
- Expressions can be efficiently evaluated.

Every grammar has an equivalent grammar in Greibach normal form

To every ε-free context-free grammar we can find an equivalent grammar in Greibach normal form.

Grammar and its equivalent Greibach Normal Form grammar

$S \to aBc$
$B \to b$

convert

$S \to aBC$
$B \to b$
$C \to c$

Not Greibach Normal Form

Greibach Normal Form

Grammar and its equivalent Greibach Normal Form grammar

$S \rightarrow Bc$
$B \rightarrow b$

convert

$S \rightarrow bC$
$C \rightarrow c$

Not Greibach Normal Form

Greibach Normal Form

Algorithm

- We have seen a couple simple examples of converting grammars to Greibach Normal Form.
- They didn’t reveal a systematic approach to doing the conversion.
- The following slides show a systematic approach (i.e., algorithm) for doing the conversion.

But first …

Before we examine the algorithm, we need to understand two concepts:

1. Chomsky Normal Form
2. Left-recursive rules

Chomsky Normal Form

- We will see that the algorithm requires the grammar be converted to Chomsky Normal Form.
- A context-free grammar is in *Chomsky Normal Form* if each rule has one of these forms:
  1. $X \rightarrow a$
  2. $X \rightarrow YZ$
- That is, the right-hand side is either a single terminal or two non-terminals.

Left-recursive rules

- The algorithm requires that the grammar have no left-recursive rules.
- This is a left-recursive rule: $A_k \rightarrow A_k\alpha \mid \beta$
- The alternative (β) allows a derivation to “break out of” the recursion.
- Every left-recursive rule must have an alternative (β) that allows breaking out of the recursion.

Algorithm to eliminate left-recursion

- Let's see how to eliminate the left-recursion in this rule: $A_k \rightarrow A_k\alpha \mid \beta$
- The rule generates this language: $\beta\alpha^n$, $n \geq 1$
- To see this, look at a few derivations:

$$A_k \rightarrow A_k\alpha \rightarrow \beta\alpha$$
$$A_k \rightarrow A_k\alpha \rightarrow A_k\alpha\alpha \rightarrow \beta\alpha\alpha$$
$$A_k \rightarrow A_k\alpha \rightarrow A_k\alpha\alpha \rightarrow A_k\alpha\alpha\alpha \rightarrow \beta\alpha\alpha\alpha$$

Eliminate left-recursion

- We want to eliminate the left-recursion in this rule: $A_k \to A_k\alpha \mid \beta$
- And we know the rule produces $\beta\alpha^n$
- We can easily generate α^n with this rule:

  $$A_{n+1} \to \alpha A_{n+1} \mid \alpha$$

(Assume the grammar has "n" rules. So A_{n+1} is a new rule that we just created.)

How to eliminate left-recursion

- The language $\beta\alpha^n$ can, as we've seen, be generated using a left-recursive rule:

  $$A_k \to A_k\alpha \mid \beta$$

- But the language can also be generated using these rules:

  $$\begin{aligned} A_k &\to \beta A_{n+1} \\ A_{n+1} &\to \alpha A_{n+1} \mid \alpha \end{aligned}$$

- With those two rules we have eliminated the left recursion.

Multiple left-recursive alternatives

- Of course, A_k may have multiple alternatives that are left-recursive:

  $A_k \rightarrow A_k\alpha_1 \mid A_k\alpha_2 \mid \ldots \mid A_k\alpha_r$
- And A_k may have multiple other alternatives:

  $A_k \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_s$
- So A_k may generate:
  - β_1 followed by a string X: $\beta_1 X$ $X=\{\alpha_1,\ldots,\alpha_r\}^+$

  …
  - β_s followed by a string X: $\beta_s \alpha$ $X=\{\alpha_1,\ldots,\alpha_r\}^+$

Rule to generate $\{\alpha_1,\ldots, \alpha_r\}^+$

We need rules to generate a string X:

$$A_{n+1} \rightarrow \alpha_i \qquad i = 1..r$$

$$A_{n+1} \rightarrow \alpha_i A_{n+1} \qquad i = 1..r$$

Rule to generate $\{\alpha_1,\ldots,\alpha_r\}^+$

And we need a rule to generate $\beta_1 A_{n+1},\ldots,\beta_s A_{n+1}$:

$A_k \to \beta_i A_{n+1}$ $\qquad i = 1..s$

Beautiful definition of how to eliminate left-recursion

- Replace this left-recursive rule:

  $A_k \rightarrow A_k\alpha_1 \mid A_k\alpha_2 \mid \ldots \mid A_k\alpha_r \mid \beta_1 \mid \beta_2 \mid \ldots \mid \beta_s$

- By these rules:

  $A_k \rightarrow \beta_i A_{n+1}$ $\quad i = 1..s$

  $A_{n+1} \rightarrow \alpha_i$ $\quad i = 1..r$

  $A_{n+1} \rightarrow \alpha_i A_{n+1}$ $\quad i = 1..r$

Here's the algorithm to convert a grammar to Greibach Normal Form

- First, convert the grammar rules to Chomsky Normal Form. After doing so all the rules are in one of these forms:
  1. $A_i \rightarrow a$, where "a" is a terminal symbol
  2. $A_i \rightarrow A_jA_k$

  The first form is in Greibach Normal Form, the second isn't.
- Then, order the rules followed by substitution:
  - Convert the rules to ascending order: each rule is then of the form: $A_i \rightarrow A_{i+m}X$
  - After ordering, the highest rule will be in Greibach Normal Form: $A_n \rightarrow aX$. The next-to-highest rule will depend on the highest-rule: $A_{n-1} \rightarrow A_nY$. Substitute A_n with its rhs: $A_{n-1} \rightarrow aXY$. Now that rule is in Greibach Normal Form. Continue down the rules, doing the substitution.

rhs = right-hand side

Apply the Algorithm to this Grammar

Convert this grammar, G, to Greibach Normal Form:

$$S \rightarrow Ab$$
$$A \rightarrow aS$$
$$A \rightarrow a$$

First, what language does G generate?

Answer: $L(G) = a^n b^n$

Let's derive a couple sentences to convince ourselves:

$$S \rightarrow Ab \rightarrow ab$$
$$S \rightarrow Ab \rightarrow aSb \rightarrow aAbb \rightarrow aabb$$

Step 1

Change the names of the non-terminal symbols to A_i

$$S \rightarrow Ab$$
$$A \rightarrow aS$$
$$A \rightarrow a$$

Change S to A_1, A to A_2

$$A_1 \rightarrow A_2b$$
$$A_2 \rightarrow aA_1$$
$$A_2 \rightarrow a$$

Step 2

Convert the grammar to Chomsky Normal Form.

$$A_1 \rightarrow A_2 b$$
$$A_2 \rightarrow aA_1$$
$$A_2 \rightarrow a$$

$$A_1 \rightarrow A_2 A_3$$
$$A_2 \rightarrow A_4 A_1$$
$$A_2 \rightarrow a$$
$$A_3 \rightarrow b$$
$$A_4 \rightarrow a$$

Chomsky Normal Form

Step 3

Modify the rules so that the non-terminals are in ascending order. By “ascending order” we mean:

$$\text{if } A_i \rightarrow A_jX \text{ is a rule, then } i < j$$

k^{th} rule not in ascending order

Suppose the first k-1 rules are in ascending order but the k^{th} rule is not. Thus, $A_k \rightarrow A_jX$ is a rule and $k \geq j$.

2 cases

- We want to put this rule $A_k \rightarrow A_jX$ into ascending order.
- We must deal with two cases:
  1. k > j
  2. k = j (left-recursive rule)

Case 1: k > j

$A_k \rightarrow A_j X$

$\uparrow$

Replace this with the rhs of the rule(s) for A_j. If the resulting rule(s) is still not in ascending order, continue replacing until it is in ascending order.

Example

These rules are in ascending order

$$\begin{aligned} &\dots \\ &A_7 \rightarrow A_{10}U \\ &A_7 \rightarrow A_8X \\ &A_8 \rightarrow A_{15}Y \\ &A_9 \rightarrow A_7Z \end{aligned}$$

But this is not in ascending order

Replace this with the rhs of A_7

$$\begin{aligned} &\dots \\ &A_7 \rightarrow A_{10}U \\ &A_7 \rightarrow A_8X \\ &A_8 \rightarrow A_{15}Y \\ &A_9 \rightarrow A_{10}UZ \\ &A_9 \rightarrow A_8XZ \end{aligned}$$

Now replace this with the rhs of A_8. Repeat until A_9 is in ascending order.

Beautiful definition of how to modify $A_k \to A_jX$, $k > j$

- Let A_j be this rule:
  $A_j \to Y_1 \mid \ldots \mid Y_m$
- Replace the rule $A_k \to A_jX$ by these rules:
  $A_k \to Y_iX \quad i = 0..m$
- Each Y_i begins with either a terminal symbol or some A_m where $j < m$.
- Recursively repeat the substitution for each Y_i that begins with A_m and $k > m$.

Avoid getting into a loop!

…

$A_7 \to A_7U$

$A_7 \to A_8X$

$A_8 \to A_{15}Y$

$A_9 \to A_7Z$

Suppose we replace A_7 with the rhs of this rule:

We then have this: $A_9 \to A_7UZ$

We're stuck in a loop!

Process A_7 before A_9

…

$A_7 \rightarrow A_7U$ ⟵ This rule is not in ascending order. Before processing A_9 we must process $A_1 – A_8$ (put them in ascending order). Earlier we showed how to eliminate left-recursion.

$A_7 \rightarrow A_8X$

$A_8 \rightarrow A_{15}Y$

$A_9 \rightarrow A_7Z$

Worst Case: k-1 substitutions

$A_1 \to A_2X_1$
$A_2 \to A_3X_2$
$A_3 \to A_4X_3$
$A_4 \to A_5X_4$
$A_5 \to A_6X_4$
$A_6 \to A_7X_5$
$A_7 \to A_8X_6$
$A_8 \to A_9X_7$
$A_9 \to A_1X_8$
$\uparrow$

Replace this with the rhs of A_1. Now we have $A_9 \to A_2X_1$ so replace A_2 with the rhs of A_2. Now we have $A_9 \to A_3X_2X_1$ so replace A_3 … and so forth.
In the worst case, for rule A_k we will need to make k-1 substitutions.

Apply Step 3 to our Grammar

Modify the rules with k > j:

$$\left.\begin{array}{l} A_1 \rightarrow A_2 A_3 \\ A_2 \rightarrow A_4 A_1 \end{array}\right\} \text{Already in order: } 1 < 2 \text{ and } 2 < 4$$

$$A_2 \rightarrow a$$

$$A_3 \rightarrow b$$

$$A_4 \rightarrow a$$

Case 2: k = j

- The grammar may have some left-recursive rules; that is, rules like this: $A_k \rightarrow A_kX$
- We want to eliminate the left-recursion.
- See the earlier slides for how to eliminate left recursion.

Apply Case 1 and Case 2 processing to A_1, then A_2, then …

- The previous slides for Step 3 might be a bit misleading. They seem to say: “First process all rules where $k > j$ and then process all rules where $k = j$.” That is incorrect.
- Start at A_1 and ensure it is in ascending order. Only after you’ve put A_1 in ascending order do you process A_2. And so forth.

Eliminate left recursion in our Grammar

Replace left-recursive rules:

$$\left.\begin{array}{l} A_1 \rightarrow A_2 A_3 \\ A_2 \rightarrow A_4 A_1 \end{array}\right\} \text{No left-recursive rules}$$

$$A_2 \rightarrow a$$

$$A_3 \rightarrow b$$

$$A_4 \rightarrow a$$

Step 4

- Let A_n be the highest order variable (non-terminal).
- Then the rhs of A_n must be a terminal symbol (otherwise the rhs would be a non-terminal symbol, $A_n \rightarrow A_{n+1}X$ and A_{n+1} would be the highest order variable).
- The leftmost symbol on the rhs of any rule for A_{n-1} must be either A_n or a terminal symbol. If it is A_n then replace A_n with the rhs of the A_n rule(s). Repeat for A_{n-2}, A_{n-3}, …, A_1. After doing this we end up with rules whose rhs starts with a terminal symbol.

Beautiful definition of how to modify $A_{n-1} \rightarrow A_nX$

- Let A_n be this rule:
  $A_n \rightarrow a_1Y_1 \mid \ldots \mid a_mY_m$
- Let A_{n-1} be this rule:
  $A_{n-1} \rightarrow A_nX$
- Replace the rule $A_{n-1} \rightarrow A_nX$ by these rules:
  $A_{n-1} \rightarrow a_iY_iX \qquad i = 0..m$

Apply Step 4 to our Grammar

Replace left-most non-terminals, working from A_4 to A_1:

$$
\begin{aligned}
&A_1 \to A_2A_3 \\
&\mathbf{A_2 \to A_4\,A_1} \\
&A_2 \to a \\
&A_3 \to b \\
&A_4 \to a
\end{aligned}
$$

Replace A_4 by a

$$
\begin{aligned}
&\mathbf{A_1 \to A_2A_3} \\
&A_2 \to aA_1 \\
&A_2 \to a \\
&A_3 \to b \\
&A_4 \to a
\end{aligned}
$$

Replace A_2 by aA_1

Replace A_2 by a

$$
\begin{aligned}
&A_1 \to aA_1A_3 \\
&A_1 \to aA_3 \\
&A_2 \to aA_1 \\
&A_2 \to a \\
&A_3 \to b \\
&A_4 \to a
\end{aligned}
$$

Step 5

Change symbol names back to their original names.

$$A_1 \rightarrow aA_1A_3$$
$$A_1 \rightarrow aA_3$$
$$A_2 \rightarrow aA_1$$
$$A_2 \rightarrow a$$
$$A_3 \rightarrow b$$
$$A_4 \rightarrow a$$

Change A_1 to S, A_2 to A

$$S \rightarrow aSA_3$$
$$S \rightarrow aA_3$$
$$A \rightarrow aS$$
$$A \rightarrow a$$
$$A_3 \rightarrow b$$
$$A_4 \rightarrow a$$

Grammar is now in Greibach Normal Form

$S \rightarrow Ab$
$A \rightarrow aS$
$A \rightarrow a$

Not in Greibach Normal Form
$L(G) = a^n b^n$

$S \rightarrow aSA_3$
$S \rightarrow aA_3$
$A \rightarrow aS$
$A \rightarrow a$
$A_3 \rightarrow b$
$A_4 \rightarrow a$

Greibach Normal Form
$L(G) = a^n b^n$

Unused rules

$S \rightarrow aSA_3$
$S \rightarrow aA_3$
$A \rightarrow aS$
$A \rightarrow a$
$A_3 \rightarrow b$
$A_4 \rightarrow a$

Cannot reach these rules from the start symbol, S. So remove them.

Grammar without unused rules

$$S \rightarrow aSA_3$$
$$S \rightarrow aA_3$$
$$A_3 \rightarrow b$$

Still in Greibach Normal Form

Verify the grammar generates $a^n b^n$

$$S \rightarrow aSA_3$$
$$S \rightarrow aA_3$$
$$A_3 \rightarrow b$$

Let's do a couple derivations to convince ourselves that it generates $a^n b^n$

$$S \rightarrow aA_3 \rightarrow ab$$
$$S \rightarrow aSA_3 \rightarrow aaA_3A_3 \rightarrow aabA_3 \rightarrow aabb$$

Recap of the steps

- Step 0: Determine the language that the grammar generates
- Step 1: Change the non-terminal names to A_i
- Step 2: Convert the grammar to Chomsky normal form
- Step 3a: Modify the rules $A_k \rightarrow A_jX$, $k > j$ so that the leftmost non-terminal A_j, $k \leq j$
- Step 3b: Eliminate left-recursion
- Step 4: If the leftmost symbol on the rhs of rule A_{n-1} is A_n, replace that A_n with the rhs of the A_n rules. Then do Step 4 for A_{n-2}. Repeat until at A_1.
- Step 5: Change the non-terminal names back to their original names.
- Step 6: Remove unused rules.
- Step 7: Verify that the new grammar generates the same language as the original language.

Minimize work

Step 1 says:

Change the non-terminal names to A_i

That is just another way of saying that we are to give the symbol names an ordering. Choose the ordering wisely as it can impact the amount of work needed to do the conversion.

Good ordering for this grammar?

$$S \rightarrow AB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

Change S to A_1
A to A_2,
B to A_3

$$A_1 \rightarrow A_2A_3$$
$$A_2 \rightarrow a$$
$$A_3 \rightarrow b$$

Change S to A_3
A to A_2,
B to A_1

$$A_3 \rightarrow A_2A_1$$
$$A_2 \rightarrow a$$
$$A_1 \rightarrow b$$

Compare the orderings

$$\left.A_1 \rightarrow A_2A_3\right\}$$ This rule is in ascending order

$$A_2 \rightarrow a$$

$$A_3 \rightarrow b$$

$$\left.A_3 \rightarrow A_2A_1\right\}$$ This rule is not in ascending order so we will have to work to get it into ascending order

$$A_2 \rightarrow a$$

$$A_1 \rightarrow b$$

Lesson Learned

To minimize the amount of work needed to convert a grammar to Greibach Normal Form, assign a lower number to X and a higher number to Y if there is a rule $X \rightarrow Y$. For example, change X to A_1 and Y to A_2.

Convert this grammar

For the following grammar find an equivalent grammar in Greibach Normal Form :

$S \rightarrow S + A$ $\quad A \rightarrow AB$ $\quad B \rightarrow (S)$ $\quad B \rightarrow b$

$S \rightarrow A$ $\quad A \rightarrow B$ $\quad B \rightarrow a$ $\quad B \rightarrow c$

Step 0

- Determine the language that the grammar generates

$S \rightarrow S + A \qquad A \rightarrow AB \qquad B \rightarrow (S) \qquad B \rightarrow b$

$S \rightarrow A \qquad A \rightarrow B \qquad B \rightarrow a \qquad B \rightarrow c$

- The grammar generates the language containing simple arithmetic expressions with addition and multiplication (in the guise of concatenation)

Derive **a + bc + b**

$S \rightarrow S + A$ $A \rightarrow AB$ $B \rightarrow (S)$ $B \rightarrow b$
$S \rightarrow A$ $A \rightarrow B$ $B \rightarrow a$ $B \rightarrow c$

a + bc + b is generated by:

$S \rightarrow S + A$
$\rightarrow S + b$
$\rightarrow S + A + b$
$\rightarrow S + AB + b$
$\rightarrow S + Ac + b$
$\rightarrow S + Bc + b$
$\rightarrow S + bc + b$
$\rightarrow A + bc + b$
$\rightarrow B + bc + b$
$\rightarrow a + bc + b$

Step 1

Change the non-terminal names to A_i

$S \rightarrow S + A$ $\quad A \rightarrow AB$ $\quad B \rightarrow (S)$ $\quad B \rightarrow b$

$S \rightarrow A$ $\quad A \rightarrow B$ $\quad B \rightarrow a$ $\quad B \rightarrow c$

Change S to A_1,
A to A_2, B to A_3

$A_1 \rightarrow A_1 + A_2$ $\quad A_2 \rightarrow A_2A_3$ $\quad A_3 \rightarrow (A_1)$ $\quad A_3 \rightarrow b$

$A_1 \rightarrow A_2$ $\quad A_2 \rightarrow A_3$ $\quad A_3 \rightarrow a$ $\quad A_3 \rightarrow c$

Step 2

Convert the grammar to Chomsky normal form

See next slide →

Convert the grammar to Chomsky normal form:

$A_1 \rightarrow A_1 + A_2$ $\quad A_2 \rightarrow A_2A_3$ $\quad A_3 \rightarrow (A_1)$ $\quad A_3 \rightarrow b$

$A_1 \rightarrow A_2$ $\quad A_2 \rightarrow A_3$ $\quad A_3 \rightarrow a$ $\quad A_3 \rightarrow c$

$$\Downarrow$$

$A_1 \rightarrow A_1A_4A_2$ $\quad A_2 \rightarrow A_2A_3$ $\quad A_3 \rightarrow A_5A_1A_6$ $\quad A_3 \rightarrow b$

$A_1 \rightarrow A_2$ $\quad A_2 \rightarrow A_3$ $\quad A_3 \rightarrow a$ $\quad A_3 \rightarrow c$

$A_4 \rightarrow +$ $\quad A_5 \rightarrow ($

$A_6 \rightarrow)$

$$\Downarrow$$

$A_1 \rightarrow A_1A_7$ $\quad A_2 \rightarrow A_2A_3$ $\quad A_3 \rightarrow A_5A_8$ $\quad A_3 \rightarrow b$

$A_1 \rightarrow A_2$ $\quad A_2 \rightarrow A_3$ $\quad A_3 \rightarrow a$ $\quad A_3 \rightarrow c$

$A_4 \rightarrow +$ $\quad A_5 \rightarrow ($

$A_7 \rightarrow A_4A_2$ $\quad A_6 \rightarrow)$

$A_8 \rightarrow A_1A_6$

Convert the grammar to Chomsky normal form:

$A_1 \to A_1A_7$
$\cancel{A_1 \to A_2}$
$A_4 \to +$
$A_7 \to A_4A_2$

$A_2 \to A_2A_3$
$\cancel{A_2 \to A_3}$

$A_3 \to A_5A_8$
$\cancel{A_3 \to a}$
$A_5 \to ($
$A_6 \to)$
$A_8 \to A_1A_6$

$A_3 \to b$
$\cancel{A_3 \to c}$

$A_1 \to A_1A_7$
$A_1 \to A_2$
$A_4 \to +$
$A_7 \to A_4\ A_2$

$A_2 \to A_2A_3$
$A_2 \to A_5A_8$
$A_2 \to a$
$A_2 \to b$

$A_3 \to A_5A_8$
$A_3 \to a$
$A_5 \to ($
$A_6 \to)$
$A_8 \to A_1A_6$

$A_3 \to b$
$A_3 \to c$

Continued →

$A_1 \rightarrow A_1A_7$ $A_2 \rightarrow A_2A_3$ $A_3 \rightarrow A_5A_8$ $A_3 \rightarrow b$
$A_1 \rightarrow A_2$ $A_2 \rightarrow A_5A_8$ $A_3 \rightarrow a$ $A_3 \rightarrow c$
$A_4 \rightarrow +$ $A_2 \rightarrow a$ $A_5 \rightarrow ($
$A_7 \rightarrow A_4 A_2$ $A_2 \rightarrow b$ $A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_1 \rightarrow A_1A_7$ $A_2 \rightarrow A_2A_3$ $A_3 \rightarrow A_5A_8$ $A_3 \rightarrow b$
$A_1 \rightarrow A_2A_3$ $A_2 \rightarrow A_5A_8$ $A_3 \rightarrow a$ $A_3 \rightarrow c$
$A_1 \rightarrow A_5A_8$ $A_2 \rightarrow a$ $A_5 \rightarrow ($
$A_1 \rightarrow a$ $A_2 \rightarrow b$ $A_6 \rightarrow)$
$A_1 \rightarrow b$ $A_8 \rightarrow A_1A_6$
$A_4 \rightarrow +$
$A_7 \rightarrow A_4A_2$

Now it's in Chomsky Normal Form

Step 3

Process the rules, from lowest to highest, putting them into ascending order

$A_1 \rightarrow A_1A_7$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow A_4A_2$

$A_2 \rightarrow A_2A_3$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_1A_7$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow A_4\ A_2$

$A_2 \rightarrow A_2A_3$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$\mathbf{A_1 \rightarrow A_2A_3A_9}$
$\mathbf{A_1 \rightarrow A_5A_8\ A_9}$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow A_4\ A_2$
$\mathbf{A_9 \rightarrow A_7}$
$\mathbf{A_9 \rightarrow A_7A_9}$

$A_2 \rightarrow A_2A_3$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \to A_2A_3A_9$
$A_1 \to A_5A_8A_9$
$A_1 \to A_2A_3$
$A_1 \to A_5A_8$
$A_1 \to a$
$A_1 \to b$
$A_4 \to +$
$A_7 \to A_4A_2$
$A_9 \to A_7$
$A_9 \to A_7A_9$

$A_2 \to A_2A_3$
$A_2 \to A_5A_8$
$A_2 \to a$
$A_2 \to b$

$A_3 \to A_5A_8$
$A_3 \to a$
$A_5 \to ($
$A_6 \to)$
$A_8 \to A_1A_6$

$A_3 \to b$
$A_3 \to c$

$A_1 \to A_2A_3A_9$
$A_1 \to A_5A_8A_9$
$A_1 \to A_2A_3$
$A_1 \to A_5A_8$
$A_1 \to a$
$A_1 \to b$
$A_4 \to +$
$A_7 \to A_4A_2$
$A_9 \to A_7$
$A_9 \to A_7A_9$

$\mathbf{A_2 \to A_5A_8A_{10}}$
$\mathbf{A_2 \to aA_{10}}$
$\mathbf{A_2 \to bA_{10}}$
$A_2 \to A_5A_8$
$A_2 \to a$
$A_2 \to b$
$\mathbf{A_{10} \to A_3}$
$\mathbf{A_{10} \to A_3A_{10}}$

$A_3 \to A_5A_8$
$A_3 \to a$
$A_5 \to ($
$A_6 \to)$
$A_8 \to A_1A_6$

$A_3 \to b$
$A_3 \to c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow A_4A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$\mathbf{A_7 \rightarrow +A_2}$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow A_1A_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$\mathbf{A_8 \rightarrow A_2A_3A_6}$
$\mathbf{A_8 \rightarrow A_5A_8A_6}$
$\mathbf{A_8 \rightarrow A_2A_6}$
$\mathbf{A_8 \rightarrow A_5A_6}$
$\mathbf{A_8 \rightarrow aA_6}$
$\mathbf{A_8 \rightarrow bA_6}$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$\mathbf{A_8 \rightarrow A_5A_8A_{10}A_3A_6}$
$\mathbf{A_8 \rightarrow aA_{10}A_3A_6}$
$\mathbf{A_8 \rightarrow bA_{10}A_3A_6}$
$\mathbf{A_8 \rightarrow A_5A_8A_3A_6}$
$\mathbf{A_8 \rightarrow aA_3A_6}$
$\mathbf{A_8 \rightarrow bA_3A_6}$
$A_8 \rightarrow A_5A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow A_5A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$\mathbf{A_8 \rightarrow (A_8A_{10}A_3A_6}$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow A_5A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$
$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow A_5A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow A_5A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$\mathbf{A_8 \rightarrow (A_8A_3A_6}$
$A_8 \rightarrow aA_3A_6$
$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow A_5A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow A_5A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$
$A_8 \rightarrow bA_3A_6$
$\mathbf{A_8 \rightarrow (A_8A_6}$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow A_5A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow A_7$
$A_9 \rightarrow A_7 A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$
$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$\mathbf{A_8 \rightarrow (A_6}$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$\mathbf{A_9 \rightarrow +A_2}$
$A_9 \rightarrow A_7A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$
$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$
$\mathbf{A_9 \rightarrow A_7A_9}$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow A_3$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$
$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \to A_2A_3A_9$
$A_1 \to A_5A_8A_9$
$A_1 \to A_2A_3$
$A_1 \to A_5A_8$
$A_1 \to a$
$A_1 \to b$
$A_4 \to +$
$A_7 \to +A_2$
$A_9 \to +A_2$

$A_9 \to +A_2A_9$

$A_2 \to A_5A_8A_{10}$
$A_2 \to aA_{10}$
$A_2 \to bA_{10}$
$A_2 \to A_5A_8$
$A_2 \to a$
$A_2 \to b$
$\mathbf{A_{10} \to A_5A_8}$
$\mathbf{A_{10} \to a}$
$\mathbf{A_{10} \to b}$

$\mathbf{A_{10} \to c}$
$A_{10} \to A_3A_{10}$

$A_3 \to A_5A_8$
$A_3 \to a$
$A_5 \to ($
$A_6 \to)$
$A_8 \to (A_8A_{10}A_3A_6$
$A_8 \to aA_{10}A_3A_6$
$A_8 \to bA_{10}A_3A_6$
$A_8 \to (A_8A_3A_6$
$A_8 \to aA_3A_6$

$A_8 \to bA_3A_6$
$A_8 \to (A_8A_6$
$A_8 \to A_2A_6$
$A_8 \to (A_6$
$A_8 \to aA_6$
$A_8 \to bA_6$

$A_3 \to b$
$A_3 \to c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$\mathbf{A_{10} \rightarrow (A_8}$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$A_{10} \rightarrow A_3A_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow (A_8$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$\mathbf{A_{10} \rightarrow A_5A_8A_{10}}$
$\mathbf{A_{10} \rightarrow aA_{10}}$
$\mathbf{A_{10} \rightarrow bA_{10}}$
$\mathbf{A_{10} \rightarrow cA_{10}}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow (A_8$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$\mathbf{A_{10} \rightarrow (A_8A_{10}}$
$A_{10} \rightarrow aA_{10}$
$A_{10} \rightarrow bA_{10}$
$A_{10} \rightarrow cA_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

Done! Every rule is in ascending order. We started at A_1 and worked our way to A_{10}. W

Step 4

Process the rules, from A_{10} to A_1, putting them into Greibach Normal Form

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow (A_8$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$A_{10} \rightarrow (A_8A_{10}$
$A_{10} \rightarrow aA_{10}$
$A_{10} \rightarrow bA_{10}$
$A_{10} \rightarrow cA_{10}$

$A_3 \rightarrow A_5A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

The rules A_{10} - A_4 are already in Greibach Normal Form so our starting point is A_3.

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$A_2 \rightarrow A_5A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow (A_8$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$A_{10} \rightarrow (A_8A_{10}$
$A_{10} \rightarrow aA_{10}$
$A_{10} \rightarrow bA_{10}$
$A_{10} \rightarrow cA_{10}$

$\mathbf{A_3 \rightarrow (A_8}$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$\mathbf{A_2 \rightarrow (A_8A_{10}}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$A_2 \rightarrow A_5A_8$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow (A_8$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$A_{10} \rightarrow (A_8A_{10}$
$A_{10} \rightarrow aA_{10}$
$A_{10} \rightarrow bA_{10}$
$A_{10} \rightarrow cA_{10}$

$A_3 \rightarrow (A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \rightarrow A_2A_3A_9$
$A_1 \rightarrow A_5A_8A_9$
$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_5A_8$
$A_1 \rightarrow a$
$A_1 \rightarrow b$
$A_4 \rightarrow +$
$A_7 \rightarrow +A_2$
$A_9 \rightarrow +A_2$

$A_9 \rightarrow +A_2A_9$

$A_2 \rightarrow (A_8A_{10}$
$A_2 \rightarrow aA_{10}$
$A_2 \rightarrow bA_{10}$
$\mathbf{A_2 \rightarrow (A_8}$
$A_2 \rightarrow a$
$A_2 \rightarrow b$
$A_{10} \rightarrow (A_8$
$A_{10} \rightarrow a$
$A_{10} \rightarrow b$

$A_{10} \rightarrow c$
$A_{10} \rightarrow (A_8A_{10}$
$A_{10} \rightarrow aA_{10}$
$A_{10} \rightarrow bA_{10}$
$A_{10} \rightarrow cA_{10}$

$A_3 \rightarrow (A_8$
$A_3 \rightarrow a$
$A_5 \rightarrow ($
$A_6 \rightarrow)$
$A_8 \rightarrow (A_8A_{10}A_3A_6$
$A_8 \rightarrow aA_{10}A_3A_6$
$A_8 \rightarrow bA_{10}A_3A_6$
$A_8 \rightarrow (A_8A_3A_6$
$A_8 \rightarrow aA_3A_6$

$A_8 \rightarrow bA_3A_6$
$A_8 \rightarrow (A_8A_6$
$A_8 \rightarrow A_2A_6$
$A_8 \rightarrow (A_6$
$A_8 \rightarrow aA_6$
$A_8 \rightarrow bA_6$

$A_3 \rightarrow b$
$A_3 \rightarrow c$

$A_1 \to \mathbf{(A_8A_{10}A_3A_9}$ $A_2 \to (A_8A_{10}$ $A_3 \to (A_8$ $A_3 \to b$

$A_1 \to \mathbf{aA_{10}\,A_3A_9}$ $A_2 \to aA_{10}$ $A_3 \to a$ $A_3 \to c$

$A_1 \to \mathbf{bA_{10}\,A_3A_9}$ $A_2 \to bA_{10}$ $A_5 \to ($

$A_1 \to \mathbf{(A_8A_3A_9}$ $A_2 \to (A_8$ $A_6 \to)$

$A_1 \to \mathbf{aA_3A_9}$ $A_2 \to a$ $A_8 \to (A_8A_{10}A_3A_6$

$A_1 \to \mathbf{bA_3A_9}$ $A_2 \to b$ $A_8 \to aA_{10}A_3A_6$

$A_1 \to A_5A_8A_9$ $A_{10} \to (A_8$ $A_8 \to bA_{10}A_3A_6$

$A_1 \to A_2A_3$ $A_{10} \to a$ $A_8 \to (A_8A_3A_6$

$A_1 \to A_5A_8$ $A_{10} \to b$ $A_8 \to aA_3A_6$

$A_1 \to a$ $A_{10} \to c$ $A_8 \to bA_3A_6$

$A_1 \to b$ $A_{10} \to (A_8A_{10}$ $A_8 \to (A_8A_6$

$A_4 \to +$ $A_{10} \to aA_{10}$ $A_8 \to A_2A_6$

$A_7 \to +A_2$ $A_{10} \to bA_{10}$ $A_8 \to (A_6$

$A_9 \to +A_2$ $A_{10} \to cA_{10}$ $A_8 \to aA_6$

$A_9 \to +A_2A_9$ $A_8 \to bA_6$

$A_1 \rightarrow (A_8A_{10}A_3A_9$ $A_2 \rightarrow (A_8A_{10}$ $A_3 \rightarrow (A_8$ $A_3 \rightarrow b$

$A_1 \rightarrow aA_{10}A_3A_9$ $A_2 \rightarrow aA_{10}$ $A_3 \rightarrow a$ $A_3 \rightarrow c$

$A_1 \rightarrow bA_{10}A_3A_9$ $A_2 \rightarrow bA_{10}$ $A_5 \rightarrow ($

$A_1 \rightarrow (A_8A_3A_9$ $A_2 \rightarrow (A_8$ $A_6 \rightarrow)$

$A_1 \rightarrow aA_3A_9$ $A_2 \rightarrow a$ $A_8 \rightarrow (A_8A_{10}A_3A_6$

$A_1 \rightarrow bA_3A_9$ $A_2 \rightarrow b$ $A_8 \rightarrow aA_{10}A_3A_6$

$A_1 \rightarrow \mathbf{(A_8A_9}$ $A_{10} \rightarrow (A_8$ $A_8 \rightarrow bA_{10}A_3A_6$

$A_1 \rightarrow A_2A_3$ $A_{10} \rightarrow a$ $A_8 \rightarrow (A_8A_3A_6$

$A_1 \rightarrow A_5A_8$ $A_{10} \rightarrow b$ $A_8 \rightarrow aA_3A_6$

$A_1 \rightarrow a$ $A_{10} \rightarrow c$ $A_8 \rightarrow bA_3A_6$

$A_1 \rightarrow b$ $A_{10} \rightarrow (A_8A_{10}$ $A_8 \rightarrow (A_8A_6$

$A_4 \rightarrow +$ $A_{10} \rightarrow aA_{10}$ $A_8 \rightarrow A_2A_6$

$A_7 \rightarrow +A_2$ $A_{10} \rightarrow bA_{10}$ $A_8 \rightarrow (A_6$

$A_9 \rightarrow +A_2$ $A_{10} \rightarrow cA_{10}$ $A_8 \rightarrow aA_6$

$A_9 \rightarrow +A_2A_9$ $A_8 \rightarrow bA_6$

$A_1 \to (A_8A_{10}A_3A_9$ $A_2 \to (A_8A_{10}$ $A_3 \to (A_8$ $A_3 \to b$

$A_1 \to aA_{10}A_3A_9$ $A_2 \to aA_{10}$ $A_3 \to a$ $A_3 \to c$

$A_1 \to bA_{10}A_3A_9$ $A_2 \to bA_{10}$ $A_5 \to ($

$A_1 \to (A_8A_3A_9$ $A_2 \to (A_8$ $A_6 \to)$

$A_1 \to aA_3A_9$ $A_2 \to a$ $A_8 \to (A_8A_{10}A_3A_6$

$A_1 \to bA_3A_9$ $A_2 \to b$ $A_8 \to aA_{10}A_3A_6$

$A_1 \to (A_8A_9$ $A_{10} \to (A_8$ $A_8 \to bA_{10}A_3A_6$

$A_1 \to \mathbf{(A_8A_{10}A_3}$ $A_{10} \to a$ $A_8 \to (A_8A_3A_6$

$A_1 \to \mathbf{aA_{10}A_3}$ $A_{10} \to b$ $A_8 \to aA_3A_6$

$A_1 \to \mathbf{bA_{10}A_3}$ $A_{10} \to c$ $A_8 \to bA_3A_6$

$A_1 \to \mathbf{(A_8A_3}$ $A_{10} \to (A_8A_{10}$ $A_8 \to (A_8A_6$

$A_1 \to \mathbf{aA_3}$ $A_{10} \to aA_{10}$ $A_8 \to A_2A_6$

$A_1 \to \mathbf{bA_3}$ $A_{10} \to bA_{10}$ $A_8 \to (A_6$

$A_1 \to A_5A_8$ $A_{10} \to cA_{10}$ $A_8 \to aA_6$

$A_1 \to a$ $A_8 \to bA_6$

$A_1 \to b$

$A_4 \to +$

$A_7 \to +A_2$

$A_9 \to +A_2$

$A_9 \to +A_2A_9$

$A_1 \to (A_8A_{10}A_3A_9$ $\quad A_2 \to (A_8A_{10}$ $\quad A_3 \to (A_8$ $\quad A_3 \to b$

$A_1 \to aA_{10}A_3A_9$ $\quad A_2 \to aA_{10}$ $\quad A_3 \to a$ $\quad A_3 \to c$

$A_1 \to bA_{10}A_3A_9$ $\quad A_2 \to bA_{10}$ $\quad A_5 \to ($

$A_1 \to (A_8A_3A_9$ $\quad A_2 \to (A_8$ $\quad A_6 \to)$

$A_1 \to aA_3A_9$ $\quad A_2 \to a$ $\quad A_8 \to (A_8A_{10}A_3A_6$

$A_1 \to bA_3A_9$ $\quad A_2 \to b$ $\quad A_8 \to aA_{10}A_3A_6$

$A_1 \to (A_8A_9$ $\quad A_{10} \to (A_8$ $\quad A_8 \to bA_{10}A_3A_6$

$A_1 \to (A_8A_{10}A_3$ $\quad A_{10} \to a$ $\quad A_8 \to (A_8A_3A_6$

$A_1 \to aA_{10}A_3$ $\quad A_{10} \to b$ $\quad A_8 \to aA_3A_6$

$A_1 \to bA_{10}A_3$ $\quad A_{10} \to c$ $\quad A_8 \to bA_3A_6$

$A_1 \to (A_8A_3$ $\quad A_{10} \to (A_8A_{10}$ $\quad A_8 \to (A_8A_6$

$A_1 \to aA_3$ $\quad A_{10} \to aA_{10}$ $\quad A_8 \to A_2A_6$

$A_1 \to bA_3$ $\quad A_{10} \to bA_{10}$ $\quad A_8 \to (A_6$

$\mathbf{A_1 \to (A_8}$ $\quad A_{10} \to cA_{10}$ $\quad A_8 \to aA_6$

$A_1 \to a$ $\quad A_8 \to bA_6$

$A_1 \to b$

All the rules are now in Greibach Normal Form!

$A_4 \to +$

$A_7 \to +A_2$

$A_9 \to +A_2$

$A_9 \to +A_2A_9$