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① $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Multiply row1 by 2 and substract from row 2

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Multiply row1 by 3 and subtract from row 3

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Multiply row1 by 6 and subtract from row 4

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

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Swap ~~so~~ R2 and R3

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Multiply R_3 by $-4/3$ and + R_2

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Multiply R_2 by $-1/4$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

X R4 by -1 & add to R_2

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -9 & 17/4 \end{bmatrix}$$

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$\times R_3$ by $-1/3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -9 & 17/4 \end{bmatrix}$$

$\times R_4$ by 3 & add to R_3

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

$\therefore$ Non-zero rows = 4

$$\boxed{\rho(A) = 4}$$

(2) Let the standard bases for symmetric 2x2 matrix be

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

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& the standard bases of P_2 be :

$$B' = \{1, x, x^2\}$$

$$[T]_B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

~~After~~ Apply row-operations

$$[T]_B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(T) = 2$

& Nullity $= 1$

(3)

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

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$$\therefore \left| \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} \right|$$

$$= (2-\lambda)^2 - 1$$

$$= \lambda^2 - 4\lambda + 3$$

$$= (\lambda-1)(\lambda-3) = 0$$

$$\therefore \boxed{\lambda = 1, 3}$$

$$\therefore \boxed{\text{Eigen values} = \lambda = 1, 3}$$

For $\lambda_1 = 1$

$$(A - \lambda_1 I) v_1 = \left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) v_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$v_1 = 0$

$$\therefore v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda_2 = 3$

$$(A - \lambda_2 I) v_2 = \left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) v_3$$

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$$= \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} v_2 = 0$$

$$\therefore v_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

So,

for

A^{-1}

their eigen values $= \frac{1}{1}$ & $\frac{1}{3}$ $\Rightarrow \boxed{1 \ \& \ \frac{1}{3}}$

and eigen vectors are same as A

for

$A + 4I$ & $\lambda + 4 = 1 + 4$ and $3 + 4$

$\boxed{= 5 \text{ and } 7}$

and same eigen vectors as A

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(4)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

$x(0) = y(0) = z(0) = 0$

Iteration - 1

$$x^{(1)} = \frac{7.85 + 0.1y^{(0)} + 0.2z^{(0)}}{3} = \frac{7.85}{3} = 2.6167$$

$$y^{(1)} = \frac{-19.3 - (0.1)x^{(1)} + 0.3z^{(0)}}{7} = -2.7571$$

$$z^{(1)} = \frac{71.4 - 0.3x^{(1)} + 0.2y^{(1)}}{10} = 7.14$$

Iteration 2

$$x^{(2)} = \frac{7.85 + 0.1y^{(1)} + 0.2z^{(1)}}{3} = 2.8098$$

$$y^{(2)} = \frac{-19.3 - 0.1x^{(2)} + 0.3z^{(1)}}{7} = \cancel{2.8098}$$

-2.9832

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$$z^{(2)} = \frac{71.4 - 0.3x^{(2)} + 0.2y^{(2)}}{10} = 7.013$$

Iteration 3

$$x^{(3)} = \frac{7.85 + 0.1y^{(2)} + 0.2z^{(2)}}{3} = 2.8271$$

$$y^{(3)} = \frac{-19.3 - 0.1x^{(3)} + 0.3z^{(2)}}{7} = -2.9961$$

$$z^{(3)} = \frac{71.4 - 0.3x^{(3)} + 0.2y^{(3)}}{10} = 7.003$$

5. A system is considered consistent if it has atleast one solution, a common solution satisfying all the equations

If a system of equation does not have a common solution, then it is said to be inconsistent

Now,

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$$x + 3y + 2z = 0$$
$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

Let,

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to 2R_1 - R_2$; $R_3 \to 3R_1 - R_3$; $R_4 \to R_4 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_2 \longleftrightarrow R_4$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 14 & 2 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & -7 & -1 & 0 \end{array}\right]$$

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Multiply R_3 by -1 and add to R_2
Multiply R_3 by 1 and add to R_4

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 28 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$
$\rho(A:B) = 2$

$\rho(A) = \rho(A:B) \Rightarrow$ consistent system.
&
dimension $>$ $\rho(A)$
infinitely many solution

let
$z = t$, so
$y = 0$
& $x = -3t$
are the parametric solution.

(6) To define linear transformation:

i) Additivity

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let $u = a_1 + b_1 x + c_1 x^2$
$v = a_2 + b_2 x + c_2 x^2$

$T(u+v) = T((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2)$

$= ((a_1+a_2)+1) + ((b_1+b_2)+1)x + ((c_1+c_2)+1)x^2$

$T(u) + T(v) = (a_1+1) + (b_1+1)x + (c_1+1)x^2 +$
$(a_2+1) + (b_2+1)x + (c_2+1)x^2$

$= ((a_1+1) + (a_2+1)) + ((b_1+1) + (b_2+1))x +$
$((c_1+1) + (c_2+1))x^2$

$= ((a_1+a_2)+1) + ((b_1+b_2)+1)x + ((c_1+c_2)+1)x^2$

$\therefore T(u+v) = T(u) + T(v)$

ii) <u>Homogeneity</u>

let $u = a + bx + cx^2$
& α be any scalar

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$$T(cu) = T(da + dbx + dcx^2)$$
$$= ((da+1) + (db+1)x + (dc+1)x^2)$$

$$dT(u) = d((a+1) + (b+1)x + (c+1)x^2)$$
$$= (da+d) + (db+d)x + (dc+d)x^2$$

$$\therefore \boxed{T(du) \neq dT(u)}$$

$\therefore$ $T : P_2 \rightarrow P_2$ is not a linear transformation as homogenity is not satisfied.

(7)

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 3 \\ 3 & 0 & 0 \end{bmatrix}$$

(7) Forming the equations

$$a + 3b - 2c = 0$$
$$2a + b + c = 0$$
$$3a + 0b + 3c = 0$$

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Let

$$A:B = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 3 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

~~So~~ A:B only has trivial solution ($a = b = c = 0$)

$\therefore$ ~~S is~~ vectors in S are lineary Independent

Since we have 3 linearly independent vectors in V3(R), so S spans the entire space.

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(8) $3x - 6y + 2z = 23$
$-4x + y - z = -15$
$x - 3y + 7z = 16$

$x_0 = y_0 = z_0 = 1$

~~Iteration-1~~

$\cancel{x_1 = \frac{23 + 6y_0 + 2z_0}{3} = \frac{23 + 6(1) + 2(1)}{3} = 8}$

$\cancel{y_1 = \frac{-15 + 4x_0 + z_0}{-6} =}$

Iteration -1

$x^{(1)} = \frac{1}{3}(23 - (-6 \times 1) - (2 \times 1)) = 6$

$y^{(1)} = \frac{1}{1}(-15 - (-4 \times 1) - (-1 \times 1)) = -10$

$z^{(1)} = \frac{1}{7}(16 - (1 \times 1) - (-3 \times 1)) = 2$

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Iteration - 2

$$x^{(2)} = \frac{1}{3}(23 - (-6(-10))) - (2 \times 2) = 7$$

$$y^{(2)} = (-15 - (-24) - (-1 \times 2)) = 1$$

$$z^{(2)} = \frac{1}{7}(16 - (1 \times 6) - (-3 \times (-10))) = 1$$

Iteration -3

$$x^{(3)} = \frac{1}{3}(23 - (-6 \times 1) - (2 \times 1)) = 6$$

$$y^{(3)} = 1(-15 - (-4 \times 7) - (-1 \times 1)) = -12$$

$$z^{(3)} = \frac{1}{7}(16 - (1 \times 7) - (-3 \times 1)) = 1$$

$$\therefore \boxed{x \approx 6 \,;\ y \approx -12 \,;\ z \approx 1}$$

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(9) One application of matrix operations in image processing is convolution, where a small matrix is applied to each pixel in the image to perform operations like blurring, sharpening or edge detection.

For eg. A blur filter kernel could be used to blur an image by averaging pixel values in the neighbourhood. This process is applied to every pixel.

Let's say we have a grayscale image represented by matrix of pixel values.

We want to apply simple 3X3 blur filter to image.

So,

$$\text{Blur Kernel} = \frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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(10) Linear transformation play a crucial role in computer vision particular in operations like rotating 2D images. These transformations involve applying a linear function to every pixel in the image resulting in a modified version of the original. For rotating images, affine transformations are commonly used because they maintain straight lines and parallelism. This is ~~ach~~ achieved by multiplying the co-ordinates of each pixel by the rotating matrix, which encodes the rotation angle and other parameters like translation and scaling.

By systematically applying this transformation to every pixel in the image, a new image is generated where each pixel is positioned according to the desired rotation angle. This process enables seamless rotation of images in various computer vision tasks, including image processing, object detection and pattern recognition.