Separating the Hawks from the Doves: Evidence from Continuous Time Laboratory Games [*]

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Abstract

Human players in our laboratory experiment received flow payoffs over 120 seconds each period from a standard Hawk-Dove bimatrix game played in continuous time. Play converged closely to the symmetric mixed Nash equilibrium under a one-population matching protocol. When the same players were matched in a two-population protocol with the same bimatrix, they showed clear movement towards an asymmetric (and very inequitable) pure Nash equilibrium of the game. These findings support distinctive predictions of evolutionary game theory.

Keywords: Evolutionary dynamics, Hawk-Dove game, game theory, laboratory experiment, continuous time game.

JEL codes: C73, C92, D74

1   Introduction

Evolutionary game theory offers a principled way to select among multiple Nash equilibria. Elaborating on the "mass action" interpretation that Nash first proposed in his dissertation, evolutionary game theory analyzes dynamic processes that may converge to some NE but not to others.

Here we report the first systematic laboratory test of some of the most distinctive predictions of that theory. We focus on the 2x2 bimatrix game Hawk-Dove whose simplicity and rich population dynamics make it an ideal testing ground. Hawk-Dove has two different asymmetric NE in pure strategies as well as one symmetric NE in mixed strategies. Evolutionary game theory predicts that (a) the mixed NE will be selected within a single population (each player interacts with all other players), but (b) one of the pure NE will be selected when row players and column players belong to two disjoint populations (e.g., [8]; [23]). Recently developed laboratory techniques enable us to conduct a direct and stringent test of (a) and (b) and related hypotheses.

Evolutionary dynamics arise from several different processes. Biologists emphasize fecundity and selection: different strategies may lead to different birth and death rates (e.g., [16]). Economists and other social scientists usually focus instead on learning and imitation, which generally produce sign preserving dynamics and in some situations produce replicator dynamics, described below. Thus evolutionary game predictions can be tested in laboratory environments such as ours, in which there are no births and 100% of the human subjects survive.

Even so, such laboratory tests of evolutionary game theory have so far been sparse and often inconclusive. Van Huyck et al. [22] study a 3x3 bimatrix game with a moderately asymmetric Hawk-Dove-like sub-bimatrix. Using a standard design with at most 70 synchronous repetitions, they find no evidence of convergence with a one-population matching protocol and only weak evidence with a two-population protocol. In a study that offers "broad tentative findings," Friedman [9] examines eight different bimatrix games, including one- and two-population Hawk-Dove, under several different information treatments, with 10 to 16 consecutive synchronous repetitions of any given game. Among other things, he finds that mean-matching protocols lead to crisper convergence than random pairwise matching; that

"Kantian" and other behavior inconsistent with standard evolutionary game theory is more likely to emerge when there are fewer than 6 players in a single population; and that convergence to interior NE, when it occurs, cannot be explained by homogeneous mixed strategies. In passing, the paper reports that in almost 7/8 of cases with one-population Hawk-Dove, the strategy profile converges to within two players of the mixed NE but converges within one player of the NE in less than a third of cases. In two-population Hawk-Dove, the profile remains in a 2-player neighborhood of the symmetric mixed NE in 81% of cases, and converges to within one player of the predicted pure NE in only 14% of cases.[1]

From the perspective of evolutionary game theory, a problem with the experiments so far is that they involve a limited number of synchronous repetitions of the stage game, while basic evolutionary models concern asynchronous choices in continuous time. Perhaps more importantly, evolutionary game theory provides long run predictions, which may not emerge clearly even in 100 standard repetitions of the stage game.

To better cope with these problems, the present investigation uses a new software package called ConG (for **Con**tinuous **G**ames). As explained in Section 3 below, ConG enables laboratory subjects to make asynchronous decisions in continuous time, to receive instantaneous feedback, and to alter their decisions as often as they like. Behavior can thus settle down within a minute or two, even after covering the gamut of strategic possibilities. Each period in our experiment lasts 120 seconds, allowing considerable stationary repetition. Our experiment features two runs of 10 periods, one run with one population and the other with two populations. Thus we observe 10 approximations of the "long run" in each treatment within a single cohort of subjects.

To challenge the theoretical prediction, we switch treatments only once, halfway into each session, without informing subjects or altering the payoff matrix or the feedback display. The subtle change from one to two populations (or vice versa) theoretically destabilizes the mixed equilibrium and stabilizes the pure equilibria (or vice versa).

Two other design features are worth highlighting. First, we chose Hawk-Dove parameters

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[1]More distantly related literature uses various sorts of evolutionary models to explain persistent departures from Nash equilibrium in coordination games (e.g., [5]) and from the subgame perfect Nash equilibrium in ultimatum games (e.g., [12]; [11]; [15]). A scattering of papers study Hawk-Dove-like games from non-evolutionary perspectives. For example, Neugebauer et al. [17] show that most players do not employ social preferences in their games, and Duffy and Feltovich [6] examine correlated equilibria. An older literature examined the impact of cheap talk (e.g., [3]) and forward induction (e.g., [4]) in such games.

that yield highly inequitable payoffs in the asymmetric pure Nash equilibria, giving half of the subjects in the two-population protocol a powerful reason to resist convergence to either predicted equilibrium. Second, by randomly reassigning actions to players at the beginning of each 120 second period, we repeatedly perturb the initial conditions, allowing us to address important questions about dynamic adjustment.

Our presentation begins in the next section with a review of relevant theory. It derives the ordinary differential equations specifying standard replicator dynamics [21] for one and two population Hawk-Dove games. It also summarizes the implications of the more general sign-preserving dynamics in one and two populations.[2] The section concludes with a list of predictions that can be tested directly.

The main results are presented in section 4. We observe close convergence to the mixed NE, despite its inefficiency, in the 10 one-population periods in each session. In the 10 two-population periods in the same sessions, we see clear movement towards an asymmetric pure NE, despite its extreme inequity. Sign-preserving dynamics help predict which of the two PNE is selected. The underlying individual behavior reveals the interplay between the rates of best response and the incentives to do so.

Following a concluding discussion, Appendix A details how sign-preserving dynamics can be specified via differential inclusions [1][18], previously known as cone fields [20]. It notes that the better-known alternative formalization, as a system of ODEs drawn from a specified family, is less general and less consistent with the data. The rest of Appendix A contains other mathematical details. Appendix B, available online, collects supplementary data analysis while online Appendix C provides the instructions to laboratory subjects.

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[2]Replicator dynamics are canonical for haploid population genetics and can also arise in social adjustment processes based on imitation (e.g., [2]; [19]). However, many other plausible adjustment processes for humans, including most learning processes, differ markedly from replicator (e.g., [8]; [18]). Sign preserving dynamics capture a very broad set of plausible adjustment processes—essentially those for which current material payoffs, as specified here by the payoff bimatrix, dominate other considerations.

4

2   Theoretical Considerations

Maynard Smith [16] describes a conflict of cost $c > 0$ over a resource of value $v \in (0, c)$. Each player has pure strategy set $\{H, D\}$ and faces the payoff matrix

$$M = \begin{pmatrix} a + \frac{v-c}{2} & a + v \\ a & a + \frac{v}{2} \end{pmatrix},$$

where a is an arbitrary constant. That is, the payoff for choosing H (for Hawk) is $w_H = (1, 0)MS$ and the payoff for D (for Dove) is $w_D = (0, 1)MS$, where the column vector $S = (s, 1 - s)^T$ represents opponents' mixture proportions $s \in [0, 1]$ of H and $1 - s$ of D.

The payoff advantage (for H over D) is $\Delta w(s) = (1, -1)MS = \frac{1}{2}(v - cs)$, with root $s^* = \frac{v}{c} \in (0, 1)$. Hence the symmetric mixed Nash equilibrium of the symmetric game with payoff matrix M (i.e., with bimatrix $[M, M^T]$) is $s^*H + (1 - s^*)D$, yielding per capita payoff $a + \frac{v}{2}(1 - \frac{v}{c})$. This NE is inefficient in that its payoff falls short of $a + \frac{v}{2}$, the per-capita payoff when the conflict profile (H, H) never occurs. The 2 player bimatrix game also has two asymmetric pure Nash equilibria, (H, D) and (D, H), both of which are efficient.

We now apply evolutionary dynamics to the Hawk-Dove payoff matrix M, and derive testable implications. The broadest specification is *sign preserving dynamics*, which impose only the most basic evolutionary principle ("survival of the fittest"): over time, strategies with higher payoff displace strategies with lower payoff (e.g., [23]). A leading special case is *replicator dynamics* [21], which specifies that the growth rate of each strategy is proportional to its relative payoff.

2.1   One population

In a *one-population protocol*, each player in a large population receives the expected payoff when playing a random opponent, or, equivalently, the mean payoff when playing simultaneously the entire population. For the Hawk-Dove game, the state of the system at time t is captured in $s(t) \in [0, 1]$, the fraction of the population currently employing strategy H. Note that the payoff advantage is positive, i.e., $\Delta w(s(t)) = \frac{1}{2}(v - cs(t)) > 0$, if and only if the current state $s(t) < s^* = \frac{v}{c}$. Therefore, for any sign preserving dynamic, $s(t)$ increases

(decreases) whenever it is below (above) s^*. We conclude that for the one-population protocol for this game, the state $s(t)$ converges to s^* from every interior initial state under any continuous time sign preserving dynamic.

Replicator dynamics equate the growth rate $\dot{s}/s$ of Hawk play to the Hawk payoff w_H relative to mean payoff $\bar{w} = sw_H + (1-s)w_D$. Hence they yield the cubic ordinary differential equation

$$\dot{s} = s(w_H - \bar{w}) = s(1-s)\Delta w(s) = \frac{1}{2}s(1-s)(v - cs). \tag{1}$$

Note that the sign of $\Delta w(s)$ determines the sign of the right hand side of (1), so the argument in the previous paragraph tells us that the interior steady state $s^* = \frac{v}{c}$ is stable. The other steady states $s = 0, 1$ of (1) are easily seen to be unstable.

2.2   Two populations

In a *two-population protocol*, there is a large population of row players (population $i = 1$) and a separate large population of column players ($i = 2$). Row players play only column players, and vice-versa.

Let s_i denote the share of Hawks in population $i = 1, 2$, with corresponding column vectors $S^i = (s_i, 1 - s_i)^T$. Using the notation $-i$ (literally, $3 - i$) for the opposing population, the payoff advantage functions now are $\Delta w_i(s_{-i}) = (1, -1)MS^{-i} = \frac{1}{2}(v - cs_{-i})$. Hence replicator dynamics are given by the coupled cubic ODEs

$$\dot{s}_1 = s_1(1 - s_1)\Delta w(s_2) = \frac{1}{2}s_1(1 - s_1)(v - cs_2), \tag{2}$$

$$\dot{s}_2 = s_2(1 - s_2)\Delta w(s_1) = \frac{1}{2}s_2(1 - s_2)(v - cs_1). \tag{3}$$

The phase portrait is shown in Panel A of Figure 1. The (interior) mixed NE now is a saddle point, hence unstable, while both of the pure NE are asymptotically stable. The other corner steady states, at $(s_1, s_2) = (0, 0)$ and $(1, 1)$, are unstable. Per capita payoffs at PNE are a for one population and $a + v$ for the other. Assuming equal population sizes, the PNE are efficient in that $\frac{a + (a + v)}{2(a + v/2)} = 1.0$, but they are inequitable. The standard measure of inequality, the Gini coefficient, is zero at the interior mixed NE. By contrast, the Gini coefficient at
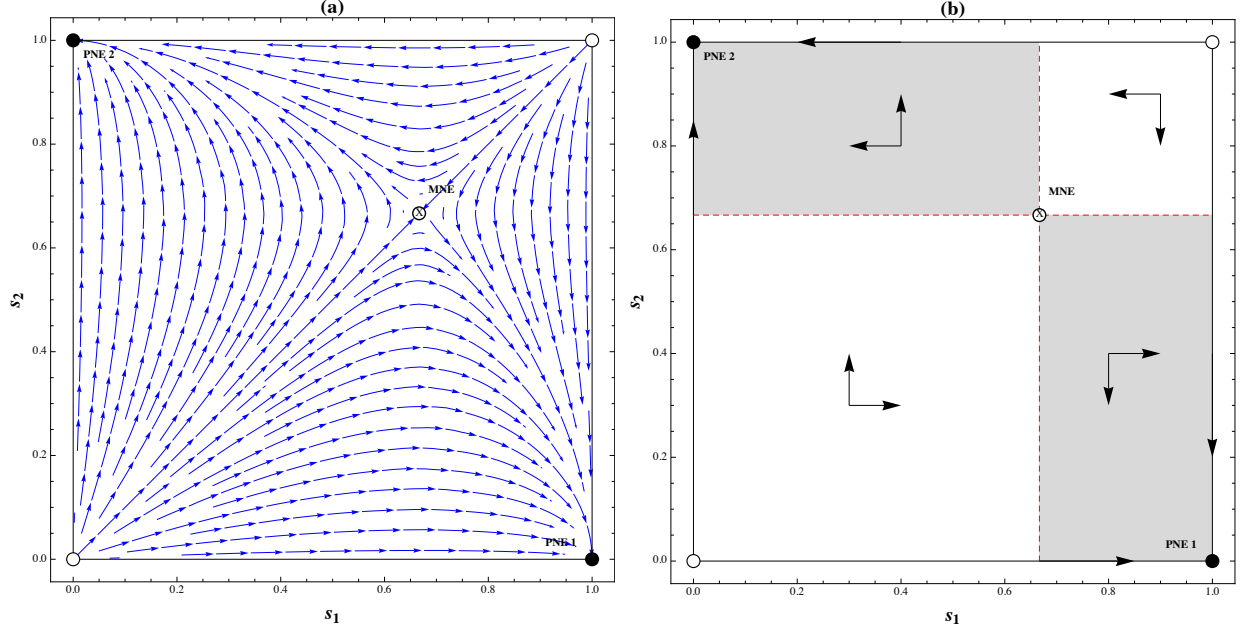
Figure 1: Two population dynamics for Hawk Dove matrix with $a = 3, c = 18$, and $v = 12$. Panel A: phase portrait for replicator dynamics. Panel B: differential inclusions for sign-preserving dynamics; shaded rectangles are absorbing while unshaded rectangles are transient.

either PNE is $\frac{v}{4a+2v}$, which approaches the rather large value of $\frac{1}{2}$ when a is small relative to v: half of the combined population gets very little, while the other half equally divides the spoils. Appendix A verifies the Gini coefficients and uses standard linearization techniques to verify the stability results.

Do the replicator stability results generalize to the broad class of sign preserving dynamics? The answer is affirmative, subject to a few nuances. Here we offer some geometric intuition; see Appendix A.1 for formalities.

Sign preserving dynamics consist of piecewise differentiable trajectories $\{s(t) = (s_1(t), s_2(t)) : t \in [0, \infty)\}$ whose tangent vectors $(\dot{s_1}, \dot{s_2})$ have signs consistent with the component-wise payoff advantage. Panel B of Figure 1 illustrates. The horizontal and vertical lines through $(s_1^*, s_2^*) = (\frac{2}{3}, \frac{2}{3})$ divide the state space into four rectangles. At any point in the open Southwestern rectangle (or on its outer edges, on the horizontal and vertical axes), both Δw_i's are positive, so sign preserving dynamics will produce tangent vectors with both components positive. Hence the fraction of Hawks will increase in both populations, and the trajectory $(s_1(t), s_2(t))$ will move Northeast, until it exits this rectangle.

The Northwestern rectangle is also *transient* in that trajectories that start in its interior or

outer edges will exit, and (given some mild technical conditions presented in the Appendix) will do so in finite time. The other two rectangles are *absorbing*: once a trajectory enters either rectangle, it remains there for all later times. Indeed, as shown in the Appendix, sign preserving dynamics require that any trajectory intersecting one of these rectangles converges asymptotically to its PNE.

The geometry of convergence is slightly different for the replicator dynamic. Its basins of attraction are the triangles separated by the saddle path $s_1 = s_2$. (It may be worth noting that the one-population protocol can be regarded as restriction to this saddle path and, subject to this restriction, the symmetric interior equilibrium is stable in any sign preserving dynamic.) Each of these basins contains one of the absorbing rectangles plus half of each transient rectangle.

2.3 Testable Predictions

Economic applications (including laboratory tests) of the theory must deal with finite populations of human players. Here sampling error and behavioral noise will produce stochastic terms (see Appendix A.2), so the large population, deterministic theory just reviewed should be used to predict central tendencies, not exact behavior.

With this caveat in mind, we obtain the following testable predictions for the Hawk-Dove game played over time in the laboratory with finite human populations.

Prediction 1 . *Under a one-population matching protocol, the average fraction of Hawk play will converge to $s^* = \frac{v}{c}$, while in a two-population protocol with equal population sizes, the overall average fraction will converge to $\bar{s} = \frac{1}{2}$.*

The last part of this prediction is implied by the more stringent

Prediction 2 . *Under a two-population matching protocol, the average fraction of Hawk play will approach $s_i = 1$ in one population and $s_{-i} = 0$ in the other.*

Recall that the (H, H) profile incurs an efficiency loss c, while other profiles are efficient. The asymmetric pure NE therefore are efficient, albeit quite inequitable, while the symmetric interior equilibrium is equitable but inefficient. Hence we have

Prediction 3 . *A two-population matching protocol will lead to greater efficiency and a less equitable distribution of earnings than the one-population protocol.*

Finally, the replicator basins of attraction are triangular and separated by the diagonal $s_1 = s_2$, while convergence in more general sign-preserving dynamics is governed by the absorbing rectangles. These alternative approaches lead respectively to parts (a) and (b) of

Prediction 4 . *Assume $s_{1o} \neq s_{2o}$. The population i that more nearly converges to all-H in a two-population matching protocol is more likely to be the one that (a) has the larger initial fraction s_{io} of H play, or if possible, (b) satisfies the initial condition $s_{io} > \frac{v}{c} > s_{-io}$.*

Other forces, not recognized by the evolutionary theory just reviewed, could also be at work in the laboratory. A smaller-than-predicted fraction of H play could be attributed to risk aversion, since the bimatrix specifies a payoff spread for H ($[a+v] - [a + \frac{v-c}{2}] = \frac{v+c}{2}$) more than twice as wide as for D ($[a + \frac{v}{2}] - [a] = \frac{v}{2}$). Another possibility is that some human subjects might shun the efficient but very inequitable equilibrium payoffs of $(a, a+v)$, and instead seek the efficient egalitarian D-D outcome of $(a + \frac{v}{2}, a + \frac{v}{2})$. In either case, the observed fraction of H play in both matching protocols would be less than predicted, and efficiency would be higher than predicted in the one-population protocol.

On the other hand, we would observe a larger fraction of Hawk play than predicted if subjects engage in negative reciprocity. In particular, the two-population game could be seen as a team contest with the winners gaining the larger prize. The result could be an ongoing war of attrition, or vendetta, with very high and persistent levels of H play in both populations.

3   Experimental Design

To create a stringent test of the predictions, we chose payoff parameters $a = 3$, $v = 12$, and $c = 18$. These yield very unequal payoffs of (3,15) in the pure NE, and rather low efficiency ($\frac{a + \frac{v}{2}(1 - \frac{v}{c})}{a + \frac{v}{2}} = \frac{5}{9}$) in the symmetric mixed NE.

We conducted eight continuous time laboratory sessions with 12 (in one session 10) subjects in each. In each of 20 two minute periods in each session, subjects were independently and randomly assigned an initial action (we will explain later the inferential advantages of this
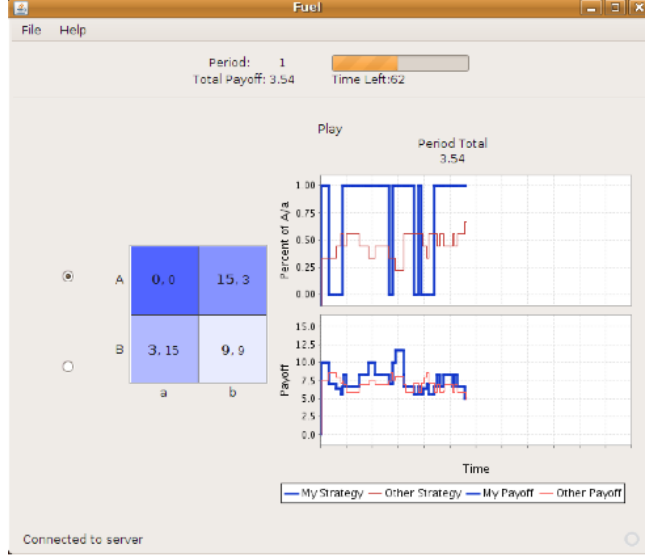
Figure 2: Screenshot of continuous time display. The subject clicks a radio button to choose a strategy, and the matrix row then is shaded blue. Opponents choices are indicated by the column shading. Those choices are graphed over time by a red line in chart on the right; the blue line shows the player's own choice. The lower chart shows the subjects flow payoffs and the opponents average flow payoffs. The top display shows accumulated payoff and time remaining.

randomization) and then could change their actions at any time and as often as they liked.

We used a new software package called ConG;[3] see Figure 2. Every subject is framed as the row player, and chooses between Hawk (labeled A, to maintain a neutral frame) and Dove (labeled B.) The display allows rapid switching between strategies, and at a glance shows the subject's own current flow payoff and accumulated payoff and the average of opponents' choices and payoffs.

Subjects were assigned (without their knowledge) either to population $i = 1$ or to population $i = 2$ at the beginning of the session. We study two treatments, varied within session. In the one population treatment, denoted 1P, each player j is matched each period with each of the $n - 1$ other players. Her instantaneous payoff when n_{-jH} of the other players are choosing A is $w_H = 15 - 15\frac{n_{-jH}}{n-1}$ if she plays A and is $w_D = 9 - 6\frac{n_{-jH}}{n-1}$ if she plays B. In the two population treatment, denoted 2P, each subject j in population $i(j)$ is matched only with the $n/2$ subjects in the other population $-i$, and earns instantaneous payoff $w_H = 15 - 15\frac{n_{-iH}}{n/2}$ for A or $w_D = 9 - 6\frac{n_{-iH}}{n/2}$ for B, where n_{-iH} is the current number of Hawks in the other

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[3]One previous paper reports using ConG. Friedman and Oprea [10] study pairwise play of the prisoners dilemma in continuous time, and find very high rates of cooperation.

population $-i$.

The subject earns $\frac{1}{120}\int_0^{120} w(t)dt$ over any 2 minute period, where $w(t)$ is the instantaneous payoff, w_H or w_D, at second $t \in [0, 120]$, given the subject's current choice and those of the opponents. Appendix A.3 shows that the finite population sizes used in our experiment lead in theory to only small departures from the central tendencies obtained in the large population analysis.

In sessions 1-4 subjects interacted under treatment 1P for the first 10 periods and 2P for the second half. In sessions 5-8 this ordering was reversed.[4] Instructions mentioned the two matching protocols, but subjects were never told which was in force, and the interface offered no obvious clue. Since subjects could not consciously coordinate on structural change, any observed treatment effects must be due to the sort of adaptive forces we want to test.

All sessions were conducted using inexperienced undergraduate subjects at the University of California, Santa Cruz. Instructions, read aloud to subjects, are reproduced in Appendix C. Sessions lasted roughly 75 minutes. Earnings, including a five dollar show up fee, averaged $17.

4   Results

We begin with an overview and tests of the first three predictions. Later subsections examine the fourth prediction and the underlying individual behavior. For tractability, the reported data are taken at one second intervals, so we have a sample of 120 observations each period.

4.1   Within Period Behavior

Panel (a) of Figure 3 plots, second by second, the mean fraction of Hawk play over all 160 periods of data. Black (gray) lines denote 1P (2P) periods, and dotted (solid) lines are from the first five (last five) periods in the treatment. Due to the random assignment of initial actions, all lines start near 0.5, and all rise quickly. The 1P lines reach the predicted MNE

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[4]Supplementary data analysis, available on request, finds that the order of treatments has no qualitative effect on our main results.
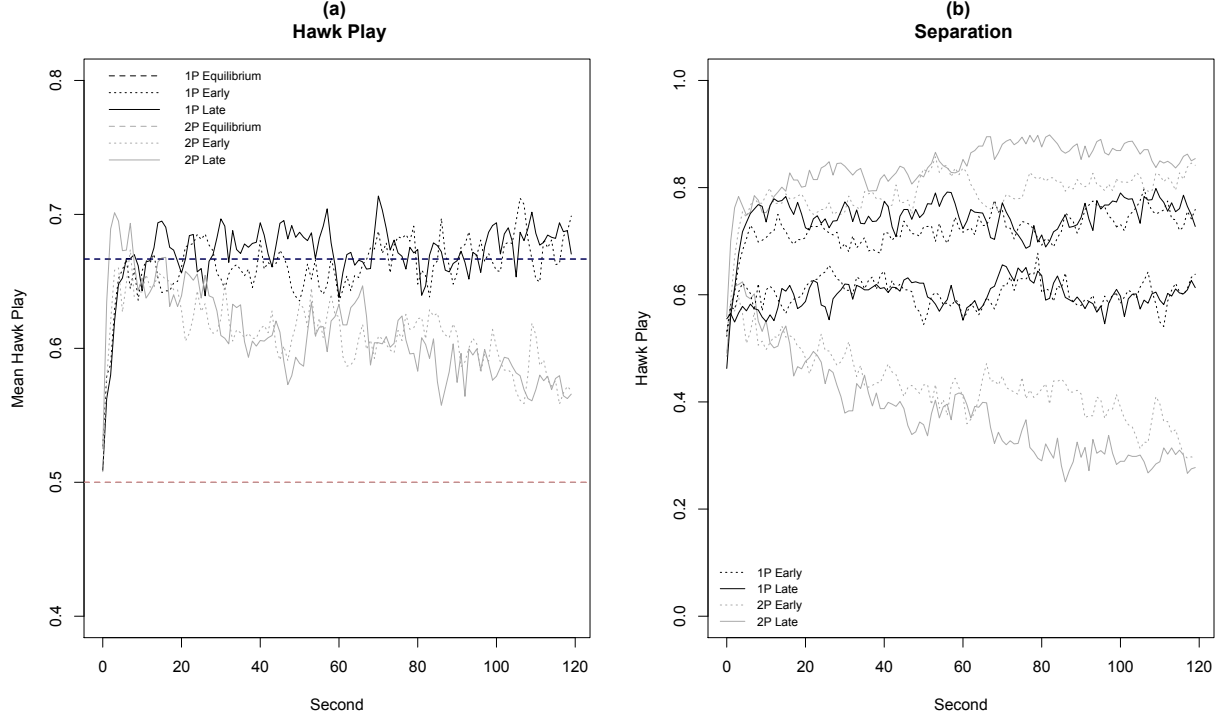
Figure 3: Panel (a) shows mean rates of Hawk play within period. Panel (b) shows mean rates of Hawk play by second within period for the (weakly) more Hawkish and less Hawkish group.

value of 0.67 within a few seconds and remain in that vicinity for the rest of the period.[5] The 2P lines also jump quickly to 0.67 or above, but then trend downward over the rest of the period, eventually covering about half the distance to the PNE level of 0.5.

[Table 1 About Here]

To obtain more quantitative evidence, we estimate

$$s_{Hjkt} = \beta_0 + \beta_1 P2_{jk} + \beta_2 t + \beta_3 P2_{jk} \times t + \epsilon_{jkt} \tag{4}$$

where s_{Hjkt} is the observed fraction of Hawk play in second $t \in \{1, 2, ..., 120\}$ of period k of session j, while $P2$ is an indicator for the two-population treatment and ϵ is assumed to be Gaussian. Standard errors are clustered at the session level to control for within-group

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[5]It should be noted that rates of Hawk play in individual periods vary, especially in early periods. Though play in individual periods does not always converge immediately, the Figure shows that the overall downward bias in Hawk rates evaporates in seconds.

dependence.[6]

Column (1) of Table I collects the coefficient estimates. The entry for t indicates a slight positive trend in the 1P treatment, but the interaction entry indicates that the trend is reversed and much stronger in 2P. The last two lines show the estimated rates of Hawk play at the end of the period. A Wald test ($p < 0.001$) confirms that these final rates differ significantly across treatments.

Thus the evidence supports Prediction 1 and can be summarized as follows.

Result 1 *Average rates of Hawk play quickly and closely approximate the mixed NE value of $\frac{v}{c} = 2/3$ in the 1P treatment, and they move decisively towards pure NE value of 1/2 in the 2P treatment.*

Recall that Prediction 2 is that, in treatment 2P, the rates of H play in the two populations will diverge towards 0 and 1. In 1P, of course, the rates should not differ systematically because here group identity is meaningless. However, Appendix A.4 notes that, given binomial error in the 1P mixed NE, the group with the higher rate should have on average 0.21 more Hawk play.

Panel (b) of Figure 3 gives an overview of the relevant evidence. Using the same coloring and line weighting schemes as before, it displays separate lines for the group with the higher rate and the group with the lower rate of Hawk play that period. The lines in the 1P (black) treatment bracket the mixed NE value of $\frac{2}{3}$ and on average seem separated by the predicted 0.21.

By contrast, the 2P (gray) lines diverge as the Hawkish group on average becomes increasingly Hawkish and the Dovish group gradually becomes more Dovish. This predicted separation is not complete; on average between 1 and 2 players deviate from pure NE in the Dovish population and 1 player deviates in the Hawkish population. Nonetheless, the increasing separation across groups confirms Prediction 2. It is particularly striking in view of the fact that group assignments are constant across the two treatments— the same groups

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[6]A very conservative alternative approach is to treat each session as a single observation and to run Wilcoxon tests on differences (between the two treatments) in differences (between means in first 30 and last 30 seconds), or on final differences (of last 30 second means, between treatments). Both tests, reported in Appendix B, confirm significant treatment effects at the 0.01 level or better for all four variables reported in Table I.

of individual subjects make similar choices in 1P but make divergent choices in 2P, no matter which treatment comes first.

More quantitative evidence comes from a regression similar to (4) above. Now the dependent variable is separation $\Delta s_{jkt} = s_{[1](jk)}(t) - s_{[2](jk)}(t)$, the difference in second t between the observed fraction of Hawk play in the group $[1](jk)$ with the larger average in period k of session j and that observed in the other group $[2](jk)$.

$$\Delta s_{jkt} = \beta_0 + \beta_1 P2_{jk} + \beta_2 t + \beta_3 P2_{jk} \times t + \epsilon_{jkt} \tag{5}$$

Column (2) of Table I reports the coefficient estimates. The intercept measures the initial separation in 1P, and its value of 0.21 is precisely as predicted in a mixed NE. The small and insignificant t coefficient indicates no systematic separation trend in 1P, but the large positive interaction coefficient indicates a very strong trend in 2P. By the end of the period in 1P, on average there is about one extra H player (23 percent of the population of 5 or 6 players) in the more Hawkish population, while in 2P the average final difference is at least three players (60 percent). The difference across treatments is significant at the one percent level. The evidence thus provides support for Prediction 2, and can be summarized as follows.

Result 2 *The difference in Hawk play between the two groups is small and trendless in 1P, while in 2P it increases steadily and becomes quite large and significant.*

Prediction 3 concerns the impact on absolute and relative welfare. Recall that efficiency (Eff = total realized payoff as a fraction of the maximum possible) is predicted to be $\frac{5}{9}$ in 1P and to be 1.0 in 2P. Panel (a) of Figure 4 shows rates of efficiency on average at each second for both treatments. Efficiency starts at 75% (because the H-H encounter rate begins at 25% due to the random assignment of initial actions) but in 1P periods it drops almost immediately to the predicted level and stays there. It shows a similar immediate drop in 2P periods, but then moves decisively higher and approaches 80 percent by the end of later periods (solid gray line).

The usual regression, with efficiency as the dependent variable,

$$\text{Eff}_{jkt} = \beta_0 + \beta_1 P2_{jk} + \beta_2 t + \beta_3 P2_{jk} \times t + \epsilon_{jkt}, \tag{6}$$
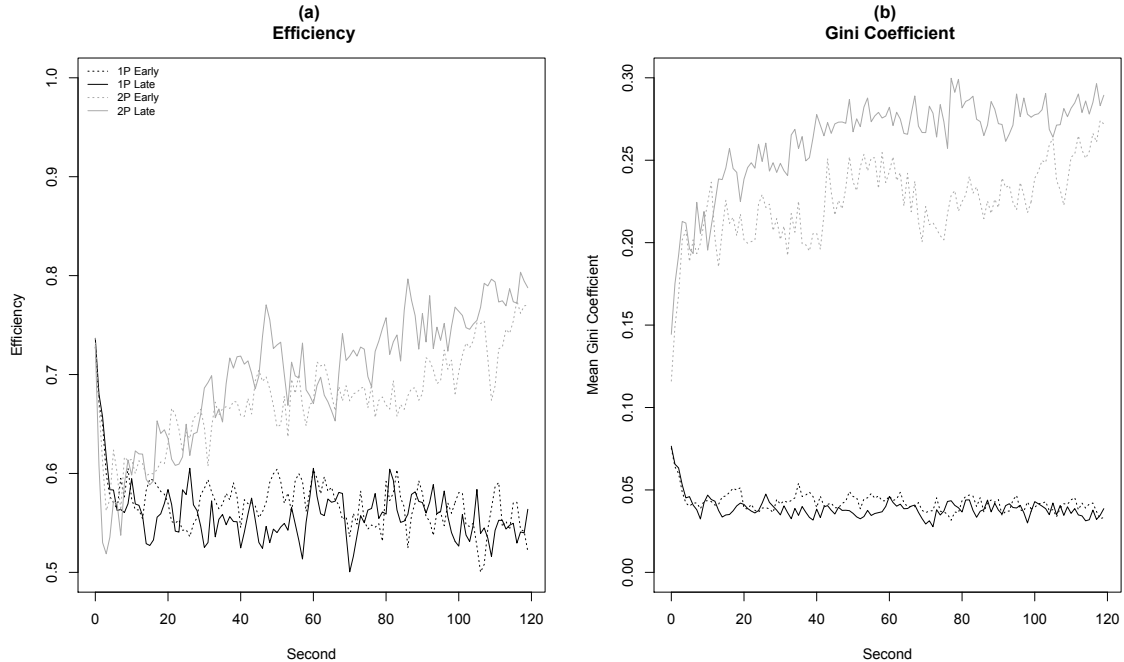
14

Figure 4: Mean efficiency (Panel a) and inequality (Panel b) within period.

yields the coefficient estimates shown in column (3) of Table I. The small but significantly negative t coefficient probably reflects the impact of the strategy initialization; the final estimate of 55% is right on target for P1. Average efficiency in 2P reaches a final value of 77% and is trending strongly upward.

These efficiency gains come at a substantial cost to equity. Panel (b) of Figure 4 reports the Gini coefficient, the standard inequality measure. As predicted, it falls quickly to a very low level in 1P, but increases immediately in 2P and continues to rise modestly over time and across periods, eventually reaching 27.5%, not far below the predicted level of 33.3%.

These impressions are tested in the regression

$$\text{Gini}_{jkt} = \beta_0 + \beta_1 P2_{jk} + \beta_2 t + \beta_3 P2_{jk} \times t + \epsilon_{jkt}, \tag{7}$$

with results shown in the last column of Table I. It confirms that the differences between the 1P and 2P treatments are very large, increasing, significant, and in the predicted direction.

Result 3 *The two-population treatment generates substantially greater average inequity and substantially greater average efficiency than the one-population treatment, and these differ-*

15

ences increase over the course of the period.

4.2   Initial Conditions and Equilibrium Selection

Prediction 4 says that pure Nash equilibrium PNEi is selected in a given 2P period if the initial conditions (s_{1o}, s_{2o}) lie (a) on same side of the diagonal $s_{1o} = s_{2o}$ as PNEi, or at least (b) in the absorbing rectangle containing PNEi.

The prediction concerns the final state, and as its empirical counterpart we chose (s_{1Fjk}, s_{2Fjk}) = average play over the last 10 seconds of period k of session j. As a robustness check, we also looked at the last 20 seconds and found no substantive difference.

To test the prediction we also must choose an empirical measure of the initial condition. Perhaps the most natural choice is the previous period's final state, (s_{1Fjk-1}, s_{2Fjk-1}). That choice, however, leads to a potentially serious endogeneity bias: if one population contains more inherently aggressive players than the other, then an observed association of this initial state with the final state might not be due to the dynamical theories of interest. It also lacks sufficient variation: observed final states are virtually always in an absorbing rectangle (there is only one exception in the 72 relevant periods and that was a near miss), precluding comparisons of initial conditions in transitory vs absorbing rectangles.

To overcome these obstacles to inference, our experiment randomly assigns subjects' strategies at the beginning of each period. To the extent that these random perturbations of the initial state affect the observed final state, we have a real test of the theoretical prediction. Of course, we expect the previous period's final state also to have predictive power, but for the reasons just noted we see no clean way to assess its causal impact.

[Table 2 About Here]

Table II reports the empirical probability that the final state lies in the previous period's absorbing rectangle, in the 58 cases where the random perturbation is not on the diagonal. Robust standard errors are also reported and are clustered at the session level.[7] The table

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[7]The table's standard errors and the hypothesis tests reported in the current section are based on a regression of the dummy variables [Absorbing Rectangle, Same Basin and their interaction] on a dummy variable for returning to last period's basin. Standard errors are robust and clustered at the session level.

shows that when subjects are perturbed into an absorbing rectangle, it matters a great deal which rectangle it is. When it is in the same rectangle as the previous period's final state, then the current state is about 90 percent likely to end up in that same absorbing rectangle. When the perturbation takes the state into the other absorbing rectangle, the final state is about equally likely to end up in either absorbing rectangle. A Wald test indicates that this difference is highly significant ($p < 0.001$). On the other hand, when the initial state is perturbed into a transitory rectangle, the effect essentially disappears ($p = 0.420$): regardless of the basin assigned, the final state is very likely to lie in the same absorbing rectangle as the preceding final state.

Thus the evidence supports Prediction 4(b) and can be summarized as follows.

Result 4 *The initial perturbations have a substantial impact on equilibrium selection when they lie in absorbing rectangles. We detect no impact when they lie in transitory rectangles.*

4.3 Individual Behavior and Best Response

The theory of evolutionary games is concerned with population distributions and has little to say about individual behavior. Nevertheless, our data offer some insight into how individual decision making contributes to aggregate outcomes.

Evolutionary dynamics, of course, assume that players tend to switch to the higher payoff strategy but, as noted at the end of section 2, there may be countervailing forces. Panel (a) of Figure 5 shows when players actually employ the higher payoff strategy. Perhaps surprisingly, only about 60% of them do so in 1P after the first few seconds, and there are no apparent trends within or across periods. In 2P, the best response rate also begins around 50% (as it must, due to random initialization), but it trends upward both within period and across periods, and eventually reaches about 80%.

Panels (b) and (c) of Figure 5 help resolve the puzzle. They show that in 1P the two strategies give essentially the same payoff after the first few seconds, so there is hardly any advantage to best responding. In 2P, however, members of the Hawkish population have a clear incentive to best respond, and the incentive increases over time. This surely helps explain the increase in Hawkish play in this population. Interestingly, the advantage to
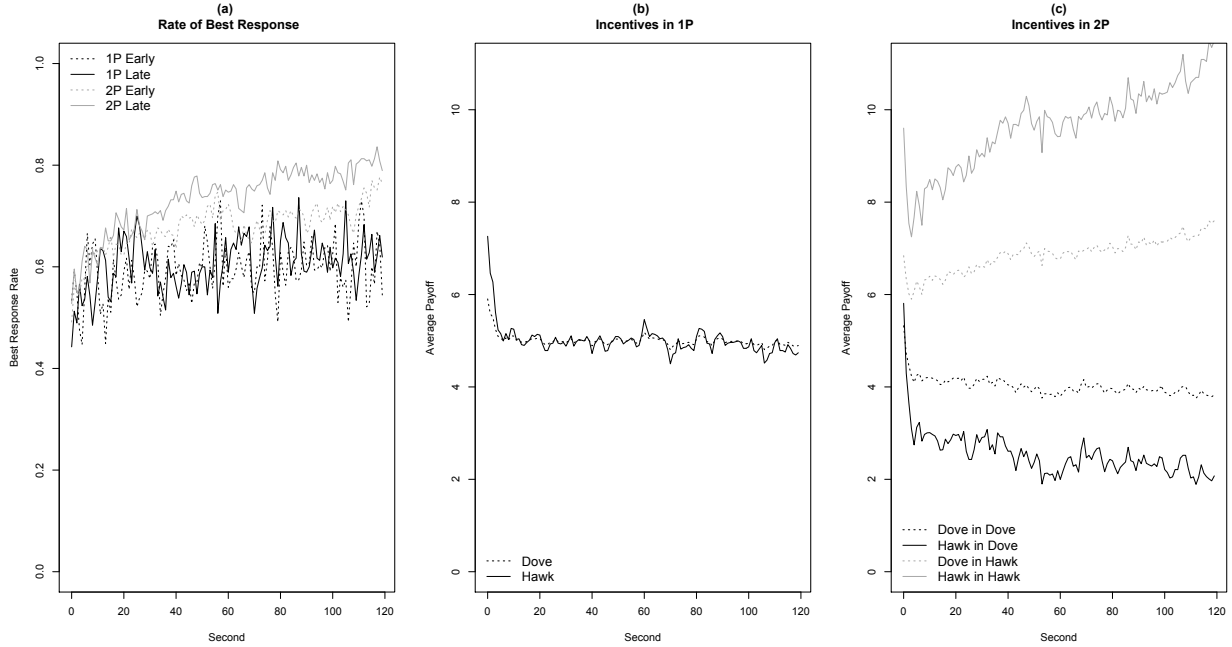
Figure 5: Panel (a) shows best response rates within period. Panel (b) show average payoffs by treatment and strategy in the 1P treatment plotted at each second. Panel (c) classifies the 2P population with more Hawk play as the Hawk population and the other as the Dove population. It then plots payoffs by treatment, action and population tendency (Hawk or Dove) at each second.

playing Dove in the Dovish population is smaller and apparently trendless. This provides a possible self-interest explanation of why we see stronger convergence in the Hawk population than in the Dove population. To summarize,

Result 5 *In the one-population treatment, expected payoffs are nearly equal for the two strategies, and the best response rate is modest and trendless. In the two-population treatment, the incentive to best respond, and the rates of best response, increase on average within and across periods, particularly in the more Hawkish population.*

The data permit other investigations of individual behavior. For example, in 1P periods we get close convergence to $s^* = \frac{2}{3}$, but is it because all individuals over time approximate the same mixed strategy $\frac{2}{3}H + \frac{1}{3}D$, or is it because most individuals specialize, with about twice as many consistently playing H as consistently playing D? Friedman [9] argues that subjects in his discrete time games mostly do specialize, and that near interior NE this can be understood as a variant Harsanyi's [13] notion of purification. Appendix B reports our findings, which also tend to support widespread specialization.

18

Of the countervailing forces mentioned at the end of section 2, we never see excessive D play and thus have no evidence of risk aversion or altruism. On the other hand, there is intriguing (albeit transitory) evidence for vendettas. Figure 3 shows an immediate sharp increase in H play in both treatments. In 1P we interpret this as rapid convergence to the mixed NE. In 2P the H rate trends downwards after about 10 or 15 seconds, as predicted. But the initial sharp increase, which is most pronounced in later 2P periods, is hard to justify as evolutionary dynamics. The observed pattern instead seems consistent with vendetta behavior that evolutionary forces erode away over time.

5 Discussion

Evolutionary game theory, like competitive equilibrium theory, predicts outcomes that are the "result of human action but not ... human design" [7]. However, due to conceptual and technical difficulties, there have been few laboratory tests to date of the distinctive predictions of evolutionary game theory.

In this paper we introduce new laboratory techniques for playing population games in continuous time, and we obtain some striking results. In the one-population treatment, the evolutionary game prediction—that the average fraction of Hawk play will approximate the symmetric mixed NE value of 2/3— turns out to be very accurate after the first few seconds of our two minute periods. In the two-population treatment, the overall rate of Hawk play is substantially lower, though not quite as low as predicted. More importantly, we indeed get divergent rates of Hawk play across the two populations, and the rates move towards the predicted extremes of the asymmetric pure NE, despite its severe inequity. Moreover, we find that the broadest dynamic specification (sign preserving dynamics, formalized as differential inclusions) helps predict which of the two pure NE is selected.

The experiment also sheds some light on matters not emphasized in evolutionary game theory. We see an increasing degree of specialization among strategically identical players, especially in the two-population treatment and among Hawks. The data suggest that evolutionary dynamics arise from an interplay between best response rates and incentives. In the one-population treatment, the payoffs quickly equalize between the two strategies and there is little incentive to best respond. By contrast, in the two-population treatment, the

incentive to best respond increases as the state gets closer to an asymmetric pure NE, and the state gets closer to that NE as more players respond to the incentive.

An intriguing aspect of our experiment is that, even in later two-population periods, players don't immediately jump to where they left off in the previous period. Instead, the rates of Hawk play jump at first, as players perhaps engage in vendettas that gradually fade in the long run (of 120 seconds!), and the "rationality" of pure NE prevails. Are there treatments that prolong the vendetta epoch? Smaller populations, perhaps, or devices that evoke an Us vs Them mentality? Will vendettas disappear altogether in larger populations? Such questions run far beyond the simple evolutionary game theory examined in the present paper, but they now seem ripe for future research.

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Appendix A: Mathematical Details

Sign preserving dynamics as differential inclusions

Here we consider dynamics for the two-population game defined by the HD matrix

$$M = \begin{pmatrix} a + \frac{v-c}{2} & a + v \\ a & a + \frac{v}{2} \end{pmatrix}.$$

A *differential inclusion* on the state space $[0,1]^2$ is defined by a correspondence $V : [0,1]^2 \Rightarrow \Re^2$, $s = (s_1, s_2) \mapsto V(s) \subset \Re^2$. A *solution* to (or *trajectory* of) the differential inclusion V from a given initial condition $s_o = (s_{1o}, s_{2o})$ is a continuous map $T : [0, \infty) \to [0,1]^2, t \mapsto s(t) = (s_1(t), s_2(t))$ such that $s(0) = s_o$ and, except perhaps for t in a measure zero subset of $[0, \infty)$, the tangent vector $\dot{s}(t)$ exists and satisfies $\dot{s}(t) \in V(s(t))$.

For sign preserving dynamics, the set $V(s)$ of allowable tangent vectors is (most of) the orthant defined by the signs of $(\Delta w_1, \Delta w_2)$. To write it out, use the sign function $\text{sgn}(x) = -$ if $x < 0$, $= +$ if $x > 0$, and $= o$ if $x = 0$, and let $\Re_{++} = \{(x,y) \in \Re^2 : x > 0, y > 0\}$ denote the open positive orthant, $\Re_{+-} = \{(x,y) \in \Re^2 : x > 0 > y\}$ denote the Southeast orthant, $\Re_{o-} = \{(x,y) \in \Re^2 : x = 0 > y\}$ denote the line heading South from the origin, etc. Also, let $||\cdot||$ denote the usual Euclidean norm, e.g., $||\Delta w||^2 = \Delta w_1^2 + \Delta w_2^2$. We impose a Lipshitz-Smale commensurability condition via the ε-annulus, $A^\varepsilon(s) = \{(x,y) \in \Re^2 : \varepsilon^{-1}||\Delta w(s)|| \geq ||(x,y)|| \geq \varepsilon||\Delta w(s)||\}$.

Sign preserving dynamics can now be defined as solutions to a differential inclusion V such that, for some small $\varepsilon > 0$,

$$V(s) = A^\varepsilon(s) \cap \Re_{(\text{sgn}(\Delta w_1(s)), \text{sgn}(\Delta w_2(s)))} \tag{8}$$

for all $s \in (0,1)^2$. On the boundaries $s_i = 0$, the set $V(s)$ of tangent vectors is the intersection of the closed half-plane $\dot{s}_i \geq 0$ with the RHS of (8); this prevents the share of Hawks from going negative. Likewise, $V(s)$ on the boundaries $s_i = 1$ intersects the RHS of (8) with the half-plane $\dot{s}_i \leq 0$ to prevent the share from exceeding 1.0.

It is routine to verify that this correspondence V is what in chapter 6A Sandholm [18]

calls good UHC, and therefore has well-behaved solutions. Of course, the solutions are not unique—in some sense, that is the point of using differential inclusions rather than ODEs—but their tangent vectors can vary continuously (or even be constant) on the interior of regions where Δw_1 and Δw_2 have a given signs, e.g., on the transient and the absorbing rectangles in the Hawk-Dove game. The trajectory can have a kink, however, when it crosses regions.

The Lipshitz-Smale annulus implies a positive lower bound on the velocity outside any neighborhood of the equilibrium points (where $||\Delta w|| = 0$) and thus prevents the trajectory from stagnating in the interior of a rectangle or in the relative interior of an edge.

To see that the Southwestern rectangle in the HD game is transitory, recall that $\frac{v}{c} \in (0,1)$ and suppose that an initial state $s_o \in [0,1]^2$ satisfies $\max\{s_{1o}, s_{2o}\} < \frac{v}{c}$. Then both $\Delta w_i = \frac{1}{2}(v - cs_{-i}) > 0$ so sign preserving dynamics allow tangent vectors only in the positive orthant $\Re_{++}$. Hence both s_i increase over time. The Lipshitz-Smale condition insures that the trajectory $(s_1(t), s_2(t))$ will in finite time either (a) exit this rectangle across the vertical boundary $s_1 = \frac{v}{c}, s_2 \in [0, \frac{v}{c})$, or (b) exit this rectangle across the horizontal boundary $s_1 \in [0, \frac{v}{c}), s_2 = \frac{v}{c}$, or (c) enter a (arbitrarily small prespecified) neighborhood of the symmetric mixed NE $(\frac{v}{c}, \frac{v}{c})$.

In case (a), let $\hat{t}$ be the exit time and consider the state at time $\hat{t} + 1$ or (without loss of generality) consider an initial state in the Southeastern rectangle $\{s \in [0,1]^2 : s_1 > \frac{v}{c} > s_2\}$. Here sign preserving dynamics require that $(\dot{s}_1, \dot{s}_2)$ lie in the SE orthant $\Re_{+-}$, so the trajectory heads southeast and can never leave the Southeastern rectangle. Either (i) it hits the outer edge $s_1 = 1, s_2 \in [0, \frac{2}{3})]$ from which it must converge to the pure NE $(1, 0)$ because $\dot{s}_1 = 0$ and $\dot{s}_2 < 0$ on this edge, or (ii) it hits the outer edge $s_1 \in (\frac{v}{c}, 1], s_2 = 0$ from which again it must converge to the pure NE $(1, 0)$, because $\dot{s}_1 > 0 = \dot{s}_2$ on this edge.

In case (b), a parallel argument establishes that the state reaches the the Northwestern rectangle $\{s \in [0,1]^2 : s_2 > \frac{v}{c} > s_1\}$, that no trajectories intersecting this rectangle can leave it, and that all such trajectories converge to the pure NE $(0, 1)$. In case (c) it is possible that the trajectory never intersects either absorbing rectangle. Such a trajectory is a saddle path that converges to the interior NE.

An argument parallel to that in the last three paragraphs establishes that the Northeastern rectangle $\{s \in [0,1]^2 : \min\{s_{1o}, s_{2o}\} > \frac{v}{c}\}$ is also transitory, and that trajectories that begin

there must in finite time exit to one of the absorbing triangles or else be a saddle path trajectory. Thus we have the following result.

Proposition 1 . *Let $\{s(t) : t \in [0, \infty)\}$ be a trajectory with initial condition $s(0) = s_o = (s_{1o}, s_{2o})$ for sign preserving dynamics of the HD game M. Then*

1. *Given any $\epsilon > 0$ there is some $\hat{t} > 0$ such that for all $t \in [\hat{t}, \infty)$ either $s_i(t) > \frac{v}{c} > s_{-i}(t)$ for $i = 1$ or for $i = 2$, or else $||s(t) - (\frac{v}{c}, \frac{v}{c})|| < \epsilon$.*

2. *If $s_{io} > \frac{v}{c} > s_{-io}$ then $s_i(t) > \frac{v}{c} > s_{-i}(t)$ $\forall t \in [0, \infty)$.*

3. *If $s_{io} > \frac{v}{c} > s_{-io}$ then $\lim_{t \to \infty} s_i(t) = 1$ and $\lim_{t \to \infty} s_{-i}(t) = 0$.*

Part 1 says that the open SW and NE rectangles are transient, and part 2 says that the open SE and NW rectangles are absorbing. Part 3 says that all trajectories starting in the open SE (or NW) rectangle converge asymptotically to PNE1 (or to PNE2, respectively).

Four further remarks may be in order.

- Differential inclusions and classes of ordinary differential equations are alternative ways to capture the intuitive idea of sign preserving dynamics. Differential inclusions are more general in that they do not uniquely prescribe behavior at any given state. With any ODE specified within any class, the entire trajectory is determined by the initial condition. By contrast, with a differential inclusion, reinitializing to a given state does not imply a replay of the subsequent trajectory.

- In our HD game, replicator dynamics and sign preserving dynamics offer contrasting predictions of what happens near the unstable corners $(0, 0)$ and $(1, 1)$ of the state space. Replicator dynamics predict very slow adjustment, while sign preserving dynamics suggest adjustment at least as rapid there as anywhere else in the transient rectangles. We didn't test this prediction because our experiment provides almost no relevant data, and also because the contrast arises not from the basic nature of ODEs versus differential inclusions, but rather from their implementation in terms of growth rates (for replicator) versus rates of change (for the differential inclusions).

- Sign preserving dynamics do not always produce such clear predictions, even in 2x2 bimatrix games. For example, in a typical matching pennies type game, sign preserving dynamics predict only that trajectories will spiral clockwise (or counterclockwise, depending on the parametrization). They allow the interior NE to be asymptotically stable (inward spirals), or neutrally stable (closed loops), or unstable (outward spirals). Differential inclusions here even allow self-intersecting trajectories.

- With more than two alternative strategies available to a population, sign preserving dynamics admit many different generalizations. As noted in Weibull [23], Sandholm [18] and elsewhere, these include payoff-monotone dynamics, payoff-positive dynamics, best-response dynamics, and many more. The literature develops these mainly as classes of ODEs, but they could also be developed as differential inclusions.

Stability Analysis

Here we use standard linearization techniques (e.g., [14]) to determine the stability of the HD steady states under replicator dynamics.

Recall that for the given HD matrix, replicator dynamics are given by the following coupled pair of ordinary differential equations:

$$\dot{s_1} = \frac{1}{2}s_1(1-s_1)(v-cs_2) = \frac{1}{2}vs_1 - \frac{1}{2}vs_1^2 - \frac{1}{2}cs_1s_2 + \frac{1}{2}cs_1^2s_2, \tag{9}$$

$$\dot{s_2} = \frac{1}{2}s_2(1-s_2)(v-cs_1) = \frac{1}{2}vs_2 - \frac{1}{2}vs_2^2 - \frac{1}{2}cs_1s_2 + \frac{1}{2}cs_1s_2^2. \tag{10}$$

The Jacobian matrix for this system is:

$$J(s_1,s_2) = \begin{pmatrix} \frac{1}{2}v - vs_1 - \frac{1}{2}cs_2 + cs_1s_2 & -\frac{1}{2}cs_1 + \frac{1}{2}cs_1^2 \\ -\frac{1}{2}cs_2 + \frac{1}{2}cs_2^2 & \frac{1}{2}v - s_2v - \frac{1}{2}cs_1 + cs_1s_2 \end{pmatrix} \tag{11}$$

Evaluating at the steady state PNE1, where $(s_1, s_2) = (1, 0)$, we have

$$J = \begin{pmatrix} -\frac{1}{2}v & 0 \\ 0 & \frac{1}{2}(v-c) \end{pmatrix} \tag{12}$$

Here the eigenvalues are $\lambda_1, \lambda_2 = -\frac{1}{2}v, \frac{1}{2}(v - c) < 0$ and hence we have a sink, i.e., a locally asymptotically stable hyperbolic critical point.

The eigenvalues for the other asymmetric PNE2, where $(s_1, s_2) = (0, 1)$, are similarly seen to be $\lambda_1, \lambda_2 = \frac{1}{2}(v - c), -\frac{1}{2}v < 0$, so PNE2 is another sink.

At the interior MNE steady state $(s_1, s_2) = (\frac{v}{c}, \frac{v}{c})$,

$$J = \begin{pmatrix} 0 & -\frac{1}{2}v + \frac{v^2}{2c} \\ -\frac{1}{2}v + \frac{v^2}{2c} & 0 \end{pmatrix}. \tag{13}$$

The characteristic equation now is $\lambda^2 - \left(-\frac{1}{2}v + \frac{v^2}{2c}\right)^2 = 0$ and the eigenvalues are $\lambda_1, \lambda_2 = \frac{1}{2}v - \frac{v^2}{2c}, -\frac{1}{2}v + \frac{v^2}{2c}$. Since $\lambda_1 > 0 > \lambda_2$ we have a saddle point.

At the corner $(s_1, s_2) = (0, 0)$

$$J = \begin{pmatrix} \frac{1}{2}v & 0 \\ 0 & \frac{1}{2}v \end{pmatrix} \tag{14}$$

Here $\lambda_1 = \lambda_2 = \frac{1}{2}v > 0$ which implies $(s_1, s_2) = (0, 0)$ is a source, i.e., a hyperbolic critical point for which all trajectories starting nearby exit any small neighborhood.

At the opposite corner $(s_1, s_2) = (1, 1)$

$$J = \begin{pmatrix} \frac{1}{2}(c - v) & 0 \\ 0 & \frac{1}{2}(c - v) \end{pmatrix} \tag{15}$$

Therefore $\lambda_1 = \lambda_2 = \frac{1}{2}(c - v) > 0$ so here we have another source.

Finite Populations

Here we show that, with finite populations, the equilibrium states are a bit different than with infinite populations, but the modifications necessary for our experiment turn out to be quite minor. The last paragraph derives the 0.21 approximation for Δs used in Result 2 of the text.

To begin, suppose there is single finite population of n players. Then the fraction of Hawk players is $s_h = \frac{k}{n}$ for some $k \in \{0, 1, ..., n\}$. The protocol is to match each subject with every

other subject, but not with himself. Hence a Hawk player faces $k-1$ Hawks among the $n-1$ matches, and so the relevant state is $s = \frac{s_h n - 1}{n-1}$. The indifference condition for a Hawk thus is $0 = \Delta w(s) = 6 - 9s$ or $\frac{2}{3} = s = \frac{s_h n - 1}{n-1}$. Solving, we see that Hawks are indifferent when

$$s_h^* = \frac{2}{3} + \frac{1}{3n} \tag{16}$$

and that they have no incentive to switch when $s_h \leq s_h^*$.

Similarly, a Dove faces the fraction $\frac{s_d n - 1}{n-1}$ of Doves, where $s_d = \frac{n-k}{n} = 1 - s_h$ is the overall fraction, including himself. Solving the indifference condition $\frac{s_d n - 1}{n-1} = \frac{1}{3}$ yields

$$s_d^* = \frac{1}{3} + \frac{2}{3n} \tag{17}$$

as the state at which Doves are indifferent. Again, Doves are optimizing as long as $s_d \leq s_d^*$, or $s_h = 1 - s_d \geq 1 - s_d^*$.

Thus we have a Nash equilibrium of the finite single population game at any state $s_h \in [1 - s_d^*, s_h^*]$. Of course, in the limit as $n \to \infty$, the NE interval collapses to a single point $s^* = \frac{2}{3}$.

One session in our experiment had $n = 10$ subjects and the others had $n = 12$. In the the 10 player case, equations (16-17) yields the NE interval $[.6, .7]$. Thus any profile with 6 or 7 Hawks is a NE of the 10 player game. In the 12 player case, the NE interval is $[.611, .694]$. The only NE profiles are those with exactly 8 of the 12 players choosing Hawk, since $7/12$ and $9/12$ lie outside the NE interval.

Now consider the case of two finite populations of equal size n. Since each subject is matched with only subjects in the other population, the asymmetric pure NE are unchanged relative to the infinite population case. With $n = 6$ as in most of our two population sessions, the interior NE also remains unchanged, with 4 Hawks in each population. With $n = 5$, the interior NE $(s_1, s_2) = (\frac{2}{3}, \frac{2}{3})$ can not be supported by a pure strategy combination, since $\frac{3}{5} < \frac{2}{3} < \frac{4}{5}$, but it can be supported if at least one player mixes. For example, in NE each population could have 3 Hawk players plus 1 Dove player plus 1 player mixing 1/3 Hawk and 2/3 Dove.

The expected separation in Hawk play between the higher and lower groups in the 1P

treatment is defined as $E|x - y|$, where x and y are two independent draws from a binomial distribution with $p = \frac{2}{3}$ and $n = 6$. By symmetry, $E|x - y|$ is 2 times the Stieltjes integral of $x - y$ over the region where $x > y$. Hence we have $E|x - y| = 2 \int_{x=0}^{1} \int_{y=0}^{x} (x - y) dF(y) dF(x) = 2 \sum_{k=0}^{6} \sum_{j=0}^{k-1} (k - j) p_k p_j$, where $p_i = \frac{6!}{i!(6-i)!} (\frac{2}{3})^i (\frac{1}{3})^{6-i}$ is the binomial probability of drawing exactly i Hawks. Evaluating this sum yields 0.21.

Gini Coefficient

We now calculate the Gini coefficent when two equal-size populations play a PNE. Recall that the Gini coefficient for the MNE is zero since all subjects receive the same payout.

The Gini coefficent is defined as $\frac{A}{A+B}$ where is A is defined as the area between the diagonal and the Lorenz curve and B is defined as the area below the Lorenz Curve. Since $A + B = \frac{1}{2}$, we can write that Gini coefficent is simply $2A$.

In the PNE, one population will receive a per capita payoff of $a + v$ (Hawk) while the other population will receive just a (Dove). The sum is $2a + v$, so one population evenly splits $\frac{a}{2a+v}$ and the other evenly splits $\frac{a+v}{2a+v}$ of the total payoff. Hence the Lorenz curve when the two populations have equal mass consists of the line segments connecting $(0,0)$ to $(\frac{1}{2}, \frac{a}{2a+v})$ and $(\frac{1}{2}, \frac{a}{2a+v})$ to $(1,1)$. The area of the corresponding triangle is $A = \frac{1}{2}(\frac{1}{2} - \frac{a}{2a+v}) = \frac{v}{4(2a+v)}$, and so the Gini coefficient at either PNE is $2A = \frac{v}{2(2a+v)}$. For the parameters used in the experiment, the Gini coefficient is $\frac{12}{2(2*3+12)} = \frac{1}{3}$.

Table I: Regression results for behavior within period.

	(1) Hawk	(2) Separation	(3) Efficiency	(4) Gini
Intercept	65.023***	21.701***	59.305***	4.45***
	(0.900)	(0.871)	(1.818)	(0.295)
$P2$	0.453	11.350***	2.421	16.76***
	(0.301)	(1.864)	(1.923)	(1.489)
t	0.027***	0.013	-0.031***	-0.007***
	(0.003)	(0.008)	(0.007)	(0.001)
$P2 \times t$	-0.086***	0.214***	0.165***	0.06***
	(0.004)	(0.035)	(0.032)	(0.005)
	At t=120 seconds			
1P	69.06***	23.26***	54.62***	3.67***
2P	57.79***	60.27***	76.80***	27.51***

1. In all cases, standard errors are robust and clustered at the session level.
2. Estimates and standard errors are multiplied by 100 for legibility and can be interpreted as percentages.

Table II: Fractions of observations n in which the final state $s_{F,n}$ is in the same absorbing rectangle as $s_{F,n-1}$, as a function of the random initialization $\tilde{s}_o$ in the current period.

	Different Basin	Same Basin
Absorbing Rectangle	9/20=0.45	17/19=0.89
	(0.099)	(0.068)
Transitory Rectangle	10/11=0.91	6/8=0.75
	(0.099)	(0.154)

1. Rows refer to observations in which $\tilde{s}_o$ lies in an Absorbing or Transitory rectangle, and columns refer to observations in which $\tilde{s}_o$ lies in the Same or Different basin of attraction as last period's final state $s_{F,n-1}$.

2. Standard errors are in parentheses and are calculated using estimates from a dummy variable regression with standard errors clustered at the session level.