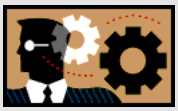
Managerial Economics & Business Strategy

Baye Chapters 4-5

Edited by DF 1/07



Overview

I. Consumer Behavior
- Indifference Curve Analysis
- Consumer Preference Ordering

II. Constraints
- The Budget Constraint
- Changes in Income
- Changes in Prices

III. Consumer Optimum

IV. Generating Demand Curves
- Individual Demand
- Market Demand

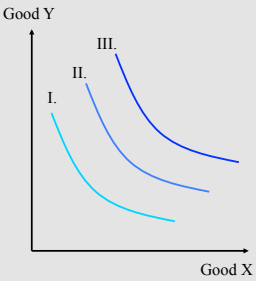
Consumer Behavior

- Consumer Opportunities
  - The possible goods and services consumer can afford to consume.
- Consumer Preferences
  - The goods and services consumers actually consume.
- Given the choice between 2 bundles of goods a consumer either
  - Prefers bundle A to bundle B: $A \succ B$, or $U(A)>U(B)$
  - Prefers bundle B to bundle A: $A \prec B$, or $U(A)<U(B)$
  - Is indifferent between the two: $A \sim B$, or $U(A)=U(B)$

Indifference Curve Analysis

Indifference Curve
- A curve that defines the combinations of 2 or more goods that give a consumer the same level of satisfaction.
- Represented by U(X,Y), whose partial derivatives are denoted U_X, U_Y

Marginal Rate of Substitution
- The rate at which a consumer is willing to substitute one good for another and maintain the same satisfaction level.
- MRS = U_X/U_Y

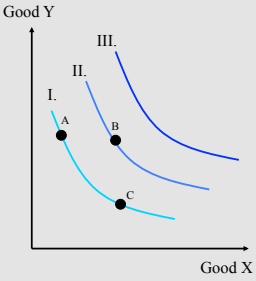


Consumer Preference Ordering Properties

- Complete—everything can be compared
- Monotone—More is Better
- Diminishing Marginal Rate of Substitution
- Transitive

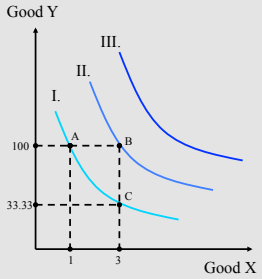
Complete Preferences

- Completeness Property
  - Consumer is capable of expressing preferences (or indifference) between all possible bundles. ("I don't know" is NOT an option!)
    - If the only bundles available to a consumer are A, B, and C, then the consumer
      - is indifferent between A and C (they are on the same indifference curve).
      - will prefer B to A.
      - will prefer B to C.



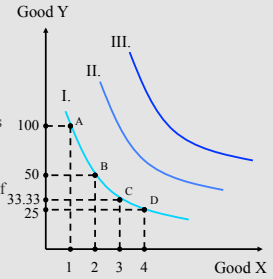
More Is Better!

- More Is Better Property
  - Bundles that have at least as much of every good and more of some good are preferred to other bundles.
    - Bundle B is preferred to A since B contains at least as much of good Y and strictly more of good X.
    - Bundle B is also preferred to C since B contains at least as much of good X and strictly more of good Y.
    - More generally, all bundles on IC_{III} are preferred to bundles on IC_{II} or IC_{I}. And all bundles on IC_{II} are preferred to IC_{I}.



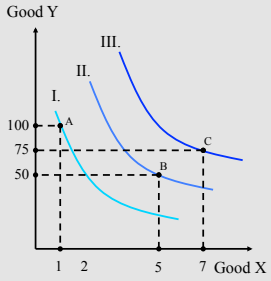
Diminishing Marginal Rate of Substitution

- Marginal Rate of Substitution
  - The amount of good Y the consumer is willing to give up to maintain the same satisfaction level decreases as more of good X is acquired.
  - The rate at which a consumer is willing to substitute one good for another and maintain the same satisfaction level.
- To go from consumption bundle A to B the consumer must give up 50 units of Y to get one additional unit of X.
- To go from consumption bundle B to C the consumer must give up 16.67 units of Y to get one additional unit of X.
- To go from consumption bundle C to D the consumer must give up only 8.33 units of Y to get one additional unit of X.



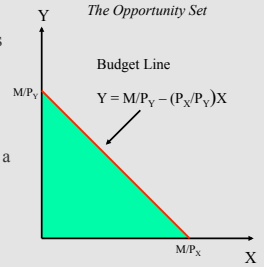
Consistent Bundle Orderings

- Transitivity Property
  - For the three bundles A, B, and C, the transitivity property implies that if $C \succ B$ and $B \succ A$, then $C \succ A$.
  - Transitive preferences along with the more-is-better property imply that
    - indifference curves will not intersect.
    - the consumer will not get caught in a perpetual cycle of indecision.



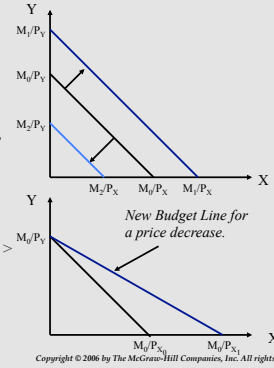
The Budget Constraint

- Opportunity Set
  - The set of consumption bundles that are affordable.
    - $P_x X + P_y Y \leq M$.
- Budget Line
  - The bundles of goods that exhaust a consumers income.
    - $P_x X + P_y Y = M$.
- Market Rate of Substitution
  - The slope of the budget line
    - $-P_x / P_y$



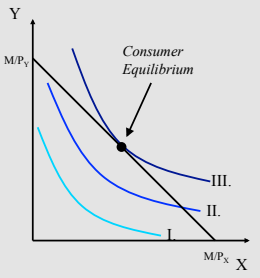
Changes in the Budget Line

- Changes in Income
  - Increases lead to a parallel, outward shift in the budget line ($M_1 > M_0$).
  - Decreases lead to a parallel, downward shift ($M_2 < M_0$).
- Changes in Price
  - A decreases in the price of good X rotates the budget line counter-clockwise ($P_{X_0} > P_{X_1}$).
  - An increases rotates the budget line clockwise (not shown).



Consumer Optimum

- The optimal bundle is an affordable bundle that yields the highest level of satisfaction.
  - Consumer equilibrium occurs at a point where $MRS = P_X / P_Y$.
  - Equivalently, the slope of the indifference curve equals the budget line.
  - Or else the optimum is at a corner

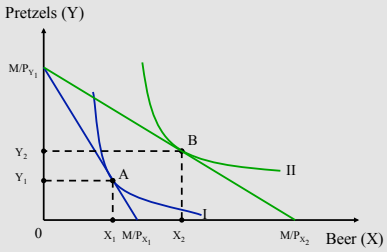


Price Changes and Consumer Equilibrium

- Substitute Goods
  - An increase (decrease) in the price of good X leads to an increase (decrease) in the consumption of good Y.
    - Examples:
      - Coke and Pepsi.
      - Verizon Wireless or T-Mobile.
- Complementary Goods
  - An increase (decrease) in the price of good X leads to a decrease (increase) in the consumption of good Y.
    - Examples:
      - DVDs and DVD players.
      - Computer CPUs and monitors.

Complementary Goods

When the price of good X falls and the consumption of Y rises, then X and Y are complementary goods. ($P_{X_1} > P_{X_2}$)



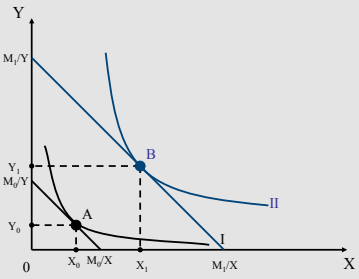
Income Changes and Consumer Equilibrium

- Normal Goods
  - Good X is a normal good if an increase (decrease) in income leads to an increase (decrease) in its consumption.
- Inferior Goods
  - Good X is an inferior good if an increase (decrease) in income leads to a decrease (increase) in its consumption.

Normal Goods

An increase in income increases the consumption of normal goods.

($M_0 < M_1$).



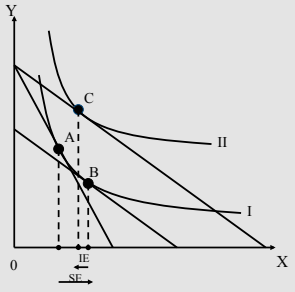
Decomposing the Income and Substitution Effects

Initially, bundle A is consumed. A decrease in the price of good X expands the consumer's opportunity set.

The substitution effect (SE) causes the consumer to move from bundle A to B.

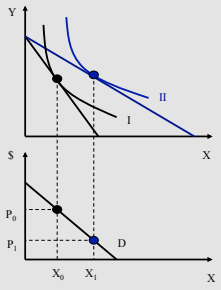
A higher "real income" allows the consumer to achieve a higher indifference curve.

The movement from bundle B to C represents the income effect (IE). The new equilibrium is achieved at point C.



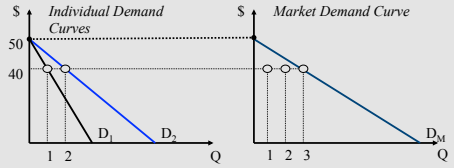
Individual Demand Curve

- An individual's demand curve is derived from each new equilibrium point found on the indifference curve as the price of good X is varied.



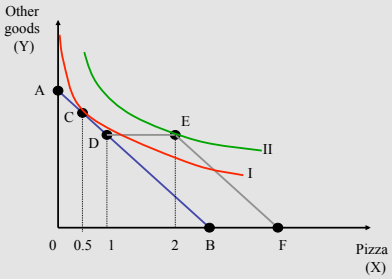
Market Demand

- The market demand curve is the horizontal summation of individual demand curves.
- It indicates the total quantity all consumers would purchase at each price point.



A Classic Marketing Application

A buy-one, get-one free pizza deal.



Conclusion

- Indifference curve properties reveal information about consumers' preferences between bundles of goods.
  - Completeness.
  - More is better.
  - Diminishing marginal rate of substitution.
  - Transitivity.
- Indifference curves along with price changes determine individuals' demand curves.
- Market demand is the horizontal summation of individuals' demands.

Production and Cost: Overview

I. Production Analysis
- Total Product, Marginal Product, Average Product
- Isoquants
- Isocosts
- Cost Minimization

II. Cost Analysis
- Total Cost, Variable Cost, Fixed Costs
- Cubic Cost Function
- Cost Relations

III. Multi-Product Cost Functions

IV. Learning Curve

Production Analysis

- Production Function
  - $Q = F(K,L)$
  - The maximum amount of output that can be produced with K units of capital and L units of labor.
- Short-Run vs. Long-Run Decisions
- Fixed vs. Variable Inputs

Total Product

- Cobb-Douglas Production Function
- Example: $Q = F(K,L) = K^{.5} L^{.5}$
  - K is fixed at 16 units.
  - Short run production function:
    $Q = (16)^{.5} L^{.5} = 4 L^{.5}$
  - Production when 100 units of labor are used?
    $Q = 4 (100)^{.5} = 4(10) = 40$ units

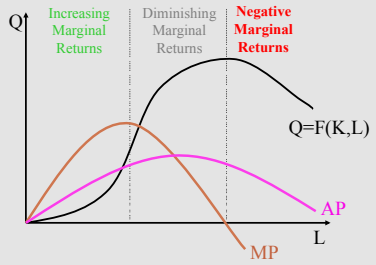
Marginal Productivity Measures

- Marginal Product of Labor: $MP_L = dQ/dL$
  - Measures the output produced by the last worker.
  - Slope of the short-run production function (with respect to labor).
- Marginal Product of Capital: $MP_K = dQ/dK$
  - Measures the output produced by the last unit of capital.
  - When capital is allowed to vary in the short run, MP_K is the slope of the production function (with respect to capital).

Average Productivity Measures

- Average Product of Labor
  - $AP_L = Q/L$.
  - Measures the output of an "average" worker.
  - Example: $Q = F(K,L) = K^{.5} L^{.5}$
    - If the inputs are K = 16 and L = 16, then the average product of labor is $AP_L = [(16)^{0.5}(16)^{0.5}]/16 = 1$.
- Average Product of Capital
  - $AP_K = Q/K$.
  - Measures the output of an "average" unit of capital.
  - Example: $Q = F(K,L) = K^{.5} L^{.5}$
    - If the inputs are K = 16 and L = 16, then the average product of labor is $AP_L = [(16)^{0.5}(16)^{0.5}]/16 = 1$.

Increasing, Diminishing and Negative Marginal Returns



Guiding the Production Process

- Producing on the production function
  - Aligning incentives to induce maximum sustainable worker effort.
- Employing the right level of inputs
  - When labor or capital vary in the short run, to maximize profit a manager will hire
    - labor until the value of marginal product of labor equals the wage: $VMP_L = w$, where $VMP_L = P \times MP_L$.
    - capital until the value of marginal product of capital equals the rental rate: $VMP_K = r$, where $VMP_K = P \times MP_K$.

Isoquant

- The combinations of inputs (K, L) that yield the producer the same level of output.
- The shape of an isoquant reflects the ease with which a producer can substitute among inputs while maintaining the same level of output.

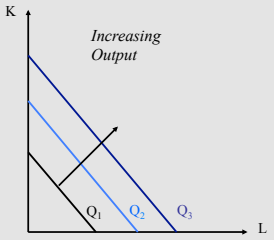
Marginal Rate of Technical Substitution (MRTS)

- The rate at which two inputs are substituted while maintaining the same output level.

$$MRTS_{KL} = \frac{MP_L}{MP_K}$$

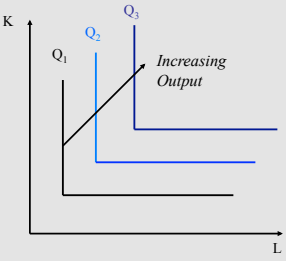
Linear Isoquants

- Capital and labor are perfect substitutes
  - $Q = aK + bL$
  - $MRTS_{KL} = b/a$
  - Linear isoquants imply that inputs are substituted at a constant rate, independent of the input levels employed.



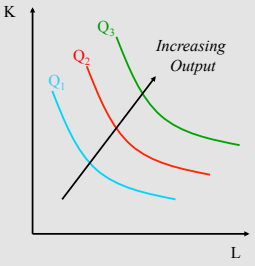
Leontief Isoquants

- Capital and labor are perfect complements.
- Capital and labor are used in fixed-proportions.
- $Q = \min \{bK, cL\}$
- Since capital and labor are consumed in fixed proportions there is no input substitution along isoquants (hence, no $MRTS_{KL}$).



Cobb-Douglas Isoquants

- Inputs are not perfectly substitutable.
- Diminishing marginal rate of technical substitution.
  - As less of one input is used in the production process, increasingly more of the other input must be employed to produce the same output level.
- $Q = K^a L^b$
- $MRTS_{KL} = MP_L/MP_K$



Isocost

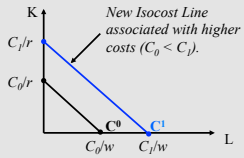
- The combinations of inputs that produce a given level of output at the same cost:

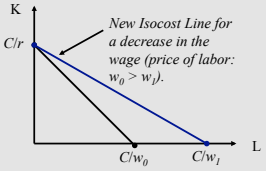
$$wL + rK = C$$

- Rearranging,

$$K = (1/r)C - (w/r)L$$

- For given input prices, isocosts farther from the origin are associated with higher costs.
- Changes in input prices change the slope of the isocost line.





Cost Minimization

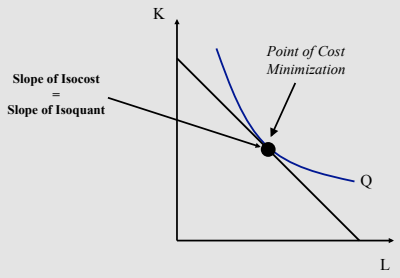
- Marginal product per dollar spent should be equal for all inputs:

$$\frac{MP_L}{w} = \frac{MP_K}{r} \Leftrightarrow \frac{MP_L}{MP_K} = \frac{w}{r}$$

- But, this is just

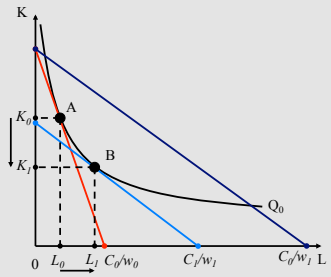
$$MRTS_{KL} = \frac{w}{r}$$

Cost Minimization



Optimal Input Substitution

- A firm initially produces Q_0 by employing the combination of inputs represented by point A at a cost of C_0.
- Suppose w_0 falls to w_1.
  - The isocost curve rotates counterclockwise; which represents the same cost level prior to the wage change.
  - To produce the same level of output, Q_0, the firm will produce on a lower isocost line (C_1) at a point B.
  - The slope of the new isocost line represents the lower wage relative to the rental rate of capital.



Cost Analysis

- Types of Costs
  - Fixed costs (FC)
  - Variable costs (VC)
  - Total costs (TC)
  - Sunk costs

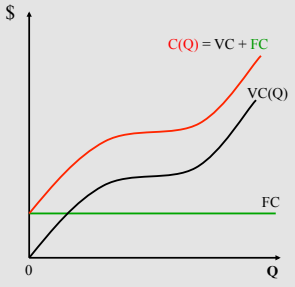
Total and Variable Costs

C(Q): Minimum total cost of producing alternative levels of output:

$C(Q) = VC(Q) + FC$

VC(Q): Costs that vary with output.

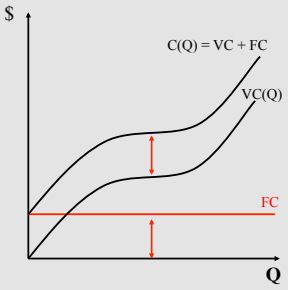
FC: Costs that do not vary with output.



Fixed and Sunk Costs

FC: Costs that do not change as output changes.

Sunk Cost: A cost that is forever lost after it has been paid.



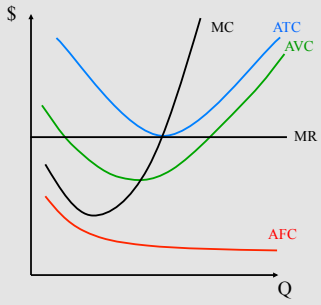
Some Definitions

Average Total Cost
$ATC = AVC + AFC$
$ATC = C(Q)/Q$

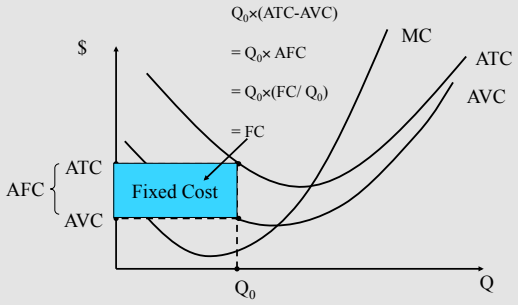
Average Variable Cost
$AVC = VC(Q)/Q$

Average Fixed Cost
$AFC = FC/Q$

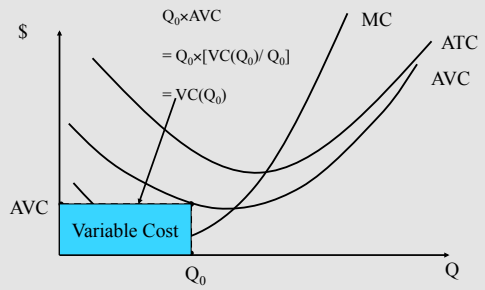
Marginal Cost
$MC = \Delta C/\Delta Q$



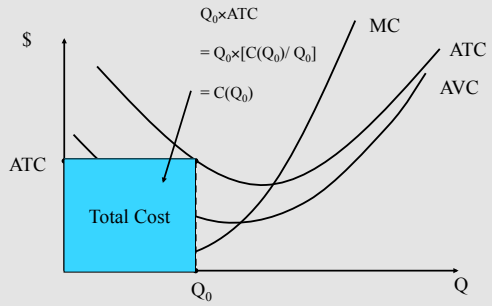
Fixed Cost



Variable Cost



Total Cost



Cubic Cost Function

- $C(Q) = f + a\,Q + b\,Q^2 + cQ^3$
- Marginal Cost?
  - Memorize:

    $MC(Q) = a + 2bQ + 3cQ^2$
  - Calculus:

    $dC/dQ = a + 2bQ + 3cQ^2$

An Example

- Total Cost: $C(Q) = 10 + Q + Q^2$
- Variable cost function:

  $VC(Q) = Q + Q^2$
- Variable cost of producing 2 units:

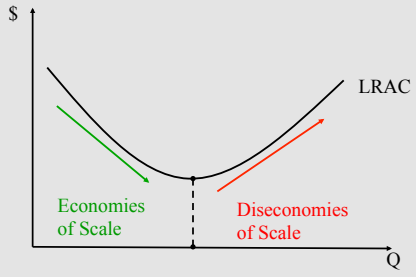
  $VC(2) = 2 + (2)^2 = 6$
- Fixed costs:

  $FC = 10$
- Marginal cost function:

  $MC(Q) = 1 + 2Q$
- Marginal cost of producing 2 units:

  $MC(2) = 1 + 2(2) = 5$

Economies of Scale



Multi-Product Cost Function

- $C(Q_1, Q_2)$: Cost of jointly producing two outputs.
- General function form:

$$C(Q_1, Q_2) = f + aQ_1Q_2 + bQ_1^2 + cQ_2^2$$

Economies of Scope

- $C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)$.
  - It is cheaper to produce the two outputs jointly instead of separately.
- Example:
  - It is cheaper for Time-Warner to produce Internet connections and Instant Messaging services jointly than separately.

Cost Complementarity

- The marginal cost of producing good 1 declines as more of good two is produced:

$$\Delta MC_1(Q_1,Q_2)/\Delta Q_2 < 0.$$

- Example:
  - Cow hides and steaks.

Quadratic Multi-Product Cost Function

- $C(Q_1, Q_2) = f + aQ_1Q_2 + (Q_1)^2 + (Q_2)^2$
- $MC_1(Q_1, Q_2) = aQ_2 + 2Q_1$
- $MC_2(Q_1, Q_2) = aQ_1 + 2Q_2$
- Cost complementarity: $a < 0$
- Economies of scope: $f > aQ_1Q_2$

  $C(Q_1, 0) + C(0, Q_2) = f + (Q_1)^2 + f + (Q_2)^2$

  $C(Q_1, Q_2) = f + aQ_1Q_2 + (Q_1)^2 + (Q_2)^2$

  $f > aQ_1Q_2$: Joint production is cheaper

A Numerical Example:

- $C(Q_1, Q_2) = 90 - 2Q_1Q_2 + (Q_1)^2 + (Q_2)^2$
- Cost Complementarity?

  Yes, since $a = -2 < 0$

  $MC_1(Q_1, Q_2) = -2Q_2 + 2Q_1$
- Economies of Scope?

  Yes, since $90 > -2Q_1Q_2$

Learning Curve

- Cost declines with *accumulated* output A
- $A = \Sigma Qs$, s=0 to t.
- Idea: efficiency improves with experience due to individual learning and better team coordination.
- Original examples: aircraft and ship building in WWII.
- Recent examples: microprocessors, fuel cells
- ln MC = a – b ln A is usual functional form
- The incremental cost decreases b% when accumulated output increases 1%

Conclusion

- To maximize profits (minimize costs) managers must use inputs such that the value of marginal of each input reflects price the firm must pay to employ the input.
- The optimal mix of inputs is achieved when the $MRTS_{KL} = (w/r)$.
- Cost functions are the foundation for helping to determine profit-maximizing behavior in future chapters.