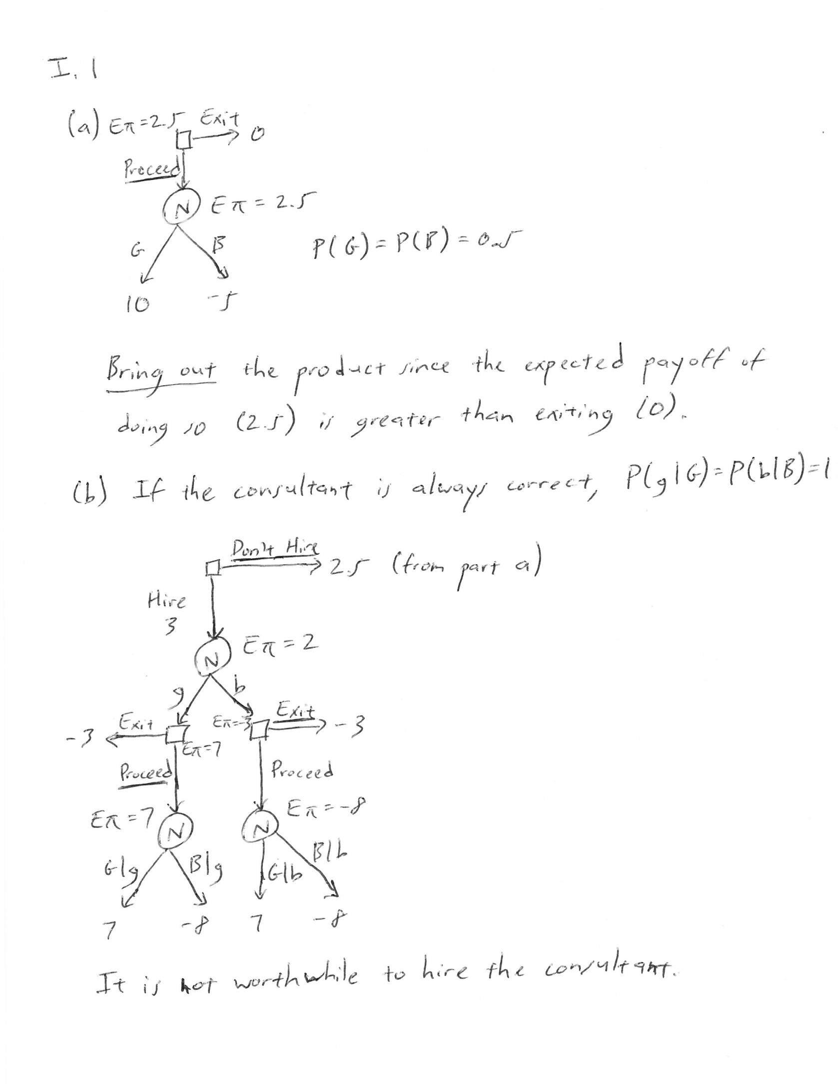
Practice Problems on Decision Theory

Econ 200

I. Problems similar to ones you have seen.


I. 1

(a) $E\pi = 2.5$ Exit → 0

$P(G) = P(B) = 0.5$

Bring out the product since the expected payoff of doing so (2.5) is greater than exiting (0).

(b) If the consultant is always correct, $P(g|G) = P(b|B) = 1$:

It is not worthwhile to hire the consultant.

Figure 1:

2. Test marketing is useful but not completely reliable; witness the "New Coke" debacle of the late 1980s. If a product initially believed to have a 60% chance of success has a positive test market, what is the updated success probability if test marketing is 80% reliable? 90% reliable? (Here reliability refers to both type I and type II errors.)

Solution: Denote the possible outcomes as $\{S, F\}$ where S is outcome in which the product is successful and F is the outcome in which the product fails. Test marketing can indicate that the product will likely to be successful (s) or fail (f). To find the updated success probabilities $\Pr(S|s)$ and $\Pr(S|f)$, we apply Bayes Theorem:

$$\Pr(S|s) = \frac{\Pr(s|S)\Pr(S)}{\Pr(s)} = \frac{\Pr(s|S)\Pr(S)}{\Pr(s|S)\Pr(S) + \Pr(s|F)\Pr(F)}$$
$$\Pr(S|f) = \frac{\Pr(f|S)\Pr(S)}{\Pr(f)} = \frac{\Pr(f|S)\Pr(S)}{\Pr(f|S)\Pr(S) + \Pr(f|F)\Pr(F)}$$

Let's first consider the case where test marketing is 80% reliable. In this case, we are given the following information:

$$\begin{aligned}
&\Pr(S) = 0.6 \\
&\Pr(F) = 1 - \Pr(S) = 0.4 \\
&\Pr(s|S) = 0.8 \\
&\Pr(f|F) = 0.8 \\
&\Pr(f|S) = 1 - \Pr(s|S) = 0.2 \\
&\Pr(s|F) = 1 - \Pr(f|F) = 0.2
\end{aligned}$$

Plugging these numbers in, we calculate that $\Pr(S|s) = 0.857$ and $\Pr(S|f) = 0.273$. Repeat this process to find the probabilities when test marketing is 90% reliable.

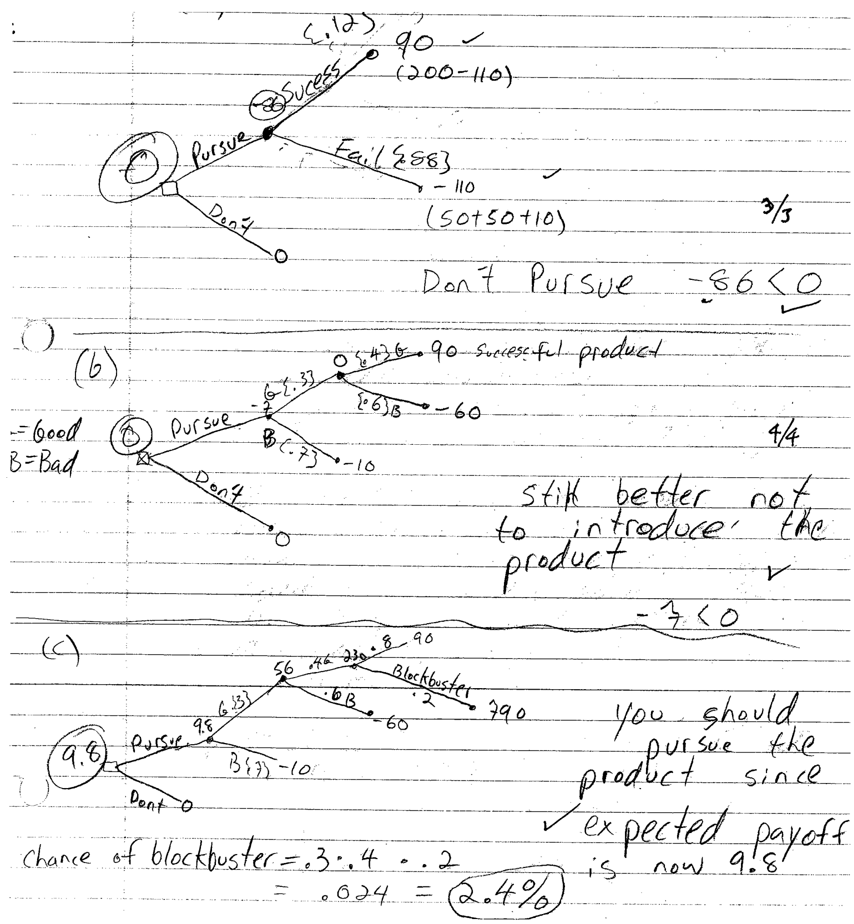


Figure 2: Solution to question 3.