Contents

Chapter 9: Imperfect Competition

A firm has market power when it can influence the market price. We begin with the deadweight loss due to monopoly relative to the competitive outcome. We then present the two classic (static) models of imperfect competition due to Bertrand and Cournot. Under Bertrand competition, firms simultaneously choose price to maximize profit, while under Cournot competition they simultaneously choose quantity. We show that the two otherwise identical models reach very different conclusions, and consider ways to reconcile them.

Other topics in oligopoly are treated even more briefly using game theory. We briefly mention product differentiation, apply the Folk Theorem to obtain tacit collusion in on-going Bertrand competition, and consider competition with spatially differentiated goods.

1 Market power

Let demand $x(p)$ be a smooth decreasing function.

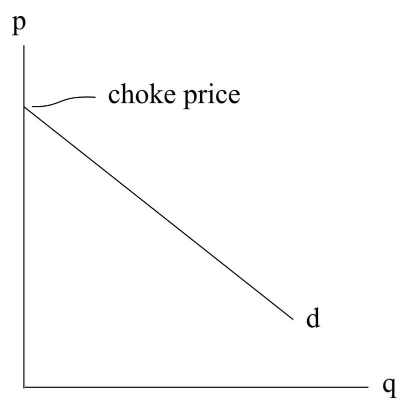

Figure 1: Inverse demand.

Inverse demand $p(.) = x^{-1}(.)$, aka WTP or demand price, satisfies the usual assumptions:

- $p(q)$ twice differentiable,
- $p'(q) < 0$, and

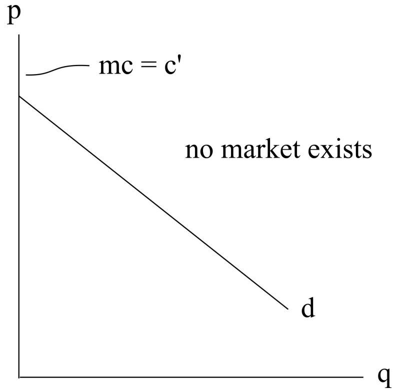

Figure 2: Industry not viable because MC always exceeds demand price.

- $p(0) < \infty$ (choke price exists).

Costs are such that:

- $c(q)$ is twice differentiable,
- $c' > 0$, and
- $c''(q) \geq 0$, so marginal cost is positive and non-decreasing.

Also assume

- $0 \leq c'(0) \leq p(0)$, that is, the industry may be viable.
- $c'(\infty) \geq p(\infty)$, that is, marginal costs eventually rise above WTP.

Then:

1. $\exists\ q_o$ such that

$$p(q_o) = c'(q_o), \tag{1}$$

   - i.e. there is a social optimum.
   - Proof: Define marginal surplus $z(q) = p(q) - c'(q)$. Current assumptions imply:
     - $z' < 0, z(0) > 0, z(\infty) < 0$.

  - Hence the intermediate value theorem tells us that there is a unique q_o such that $0 = z(q_o) = p(q_o) - c'(q_o)$

- Total surplus is $S(q) = \int_0^q z(q)dq$. Use the Fundamental Theorem of Calculus to see that it is maximized at q_o.

2. The standard monopolist problem

$$\max_{q \geq 0} \pi = q\,[p(q)] - c(q) \tag{2}$$

has solution q_m.

- $0 < q_m < q_o$, as shown below.
- This can be shown by comparing the first-order conditions:
- The monopolist's FOC is

$$q_m\,[p'(q_m)] + p(q_m) \leq c'(q_m), \text{ with equality if } q_m > 0. \tag{3}$$

The first term is the price effect, which is negative since $p'(q) < 0$, and the second term is the quantity effect. Marginal revenue is the sum of these two effects.

- Equation (3) is the same as (1) except for the negative first term.
- Thus when $q > 0$ we can write q_m as the solution to $z(q) = -qp'(q) > 0$.
- Again applying the intermediate value theorem to z, we conclude that indeed $q_m < q_o$
- The deadweight loss (DWL) due to monopoly is found by integrating the area between demand and marginal cost between q_m and q_o.

$$DWL = \int_{q_m}^{q_o} z(q)dq > 0 \tag{4}$$

- With perfect price discrimination the profit maximizing output is q_o, so in this case $DWL = CS = 0$.

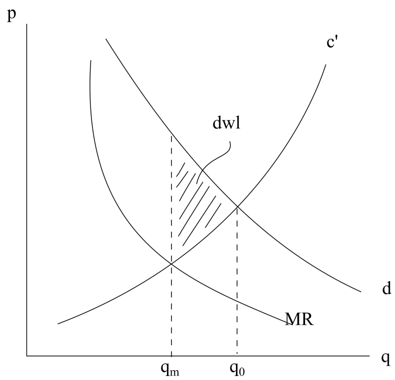

Figure 3: DWL > 0 is the area between q_m and q_o, above marginal cost c' and below inverse demand $p(q)$.

2 Static Bertrand (1883) model

- Consider a duopoly with two firms j and k. Market demand $x(p)$ satisfies the previous assumptions.
- Production function is the same for both firms and is linearly homogenous. That is, we have constant returns to scale (CRS), which implies constant marginal cost equal to average cost in the long-run: $AC = MC = c$.
- Assume $x(c) \in (0, \infty)$ so that the industry is viable.
- Output is homogenous (i.e., identical goods i.e., no product differentiation).
- Consider the following simultaneous move game: firms announce price p_i and p_j.
- Sales are

$$x_j(p_j, p_i) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k. \end{cases} \tag{5}$$

  - This means we have a winner-take-all market where the firm with the lower price gets all the sales. They split the market if prices are equal.

- Another key assumption is that production costs are only incurred for actual sales (i.e. firms produce to order).
  - Profit is then:

$$\pi_j(p_j, p_k) = x_j(p_j, p_k)\left[p_j - c\right] \tag{6}$$

- Problem: x_j is discontinuous at $p_j = p_k$, so the FOC not much help.
- Since this is a simultaneous move game, we can look for the Bertrand Nash equilibrium as a simultaneous best-response.
- The best-response functions are in price space since these are the choice variables.
- To avoid technicalities, we will treat price space as discrete with finely spaced grid points, e.g., prices are to nearest penny (0.01).

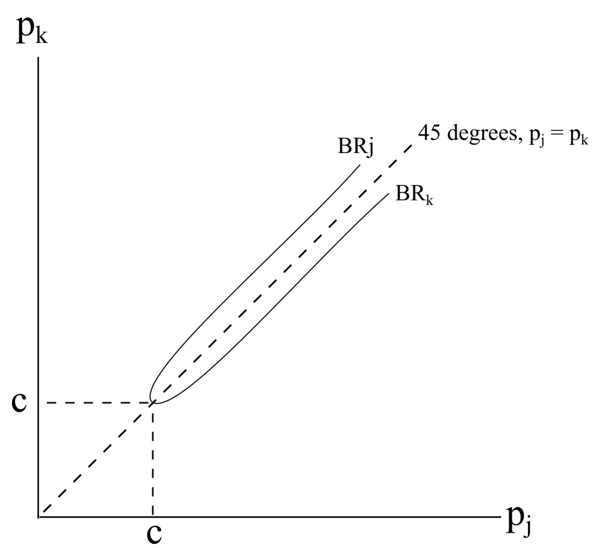

Figure 4: Bertrand Nash equilibrium and best-response functions.

-
  - Suppose $c = 1$. If $p_j = 6$, the best response by k is $p_k = 5.99$.
  - With p_j on the horizontal axis, the best-response function for k lies just below the 45° line ($p_j = p_k$) for all $p_j > c$.

- If $p_j \leq c$ then a selection from k's BR correspondence is $B_k(p_j) = c$.
- Similarly, for player j the best-response function lies just above the the 45° line. The unique NE:

$$p_j = p_k = c, \tag{7}$$

$$\pi_j = \pi_k = 0 \tag{8}$$

- Thus, with only two firms we obtain the perfectly competitive outcome in this winner-take-all, produce to order framework.
- This same NE is obtained if there are more than 2 identical firms.
- Suppose firm k has a higher marginal cost: $c_k > c_j + 2\epsilon$.
  * The best-response of firm j is to charge epsilon less than c_k and take the entire market:

$$x\left(c_k - \epsilon\right)\left[c_k - \epsilon - c_j\right] \approx (c_k - c_j)x(c_k) \tag{9}$$

  * An example. Let $P = 100 - Q$ be market demand, with marginal costs $c_k = 12 > c_j = 10$ and $\epsilon = 1$.
  * Then $\pi = \frac{1}{2}(88)(2) = 88$ if $p_j = p_k = c_k$, and $\pi = 89(1) = 89$ if $p_j = c_k - \epsilon$.

Now let's relax the assumption of homogenous goods and look at product differentiation.

- Given the pitiful Bertrand-NE profits with homogeneous goods, firms have an incentive to differentiate their products.
- Now consumers will switch to their less preferred variety only if the price difference is large enough.
- Both BR functions have *positive slope*: $\frac{\partial p_k^{br}}{\partial p_j} = \frac{\partial p_j^{br}}{\partial p_k} > 0$ for $p_i > c$, $i = j, k$. This property is called "strategic complements." Here it means that when firm k raises price, firm j has an incentive to increase its own price.

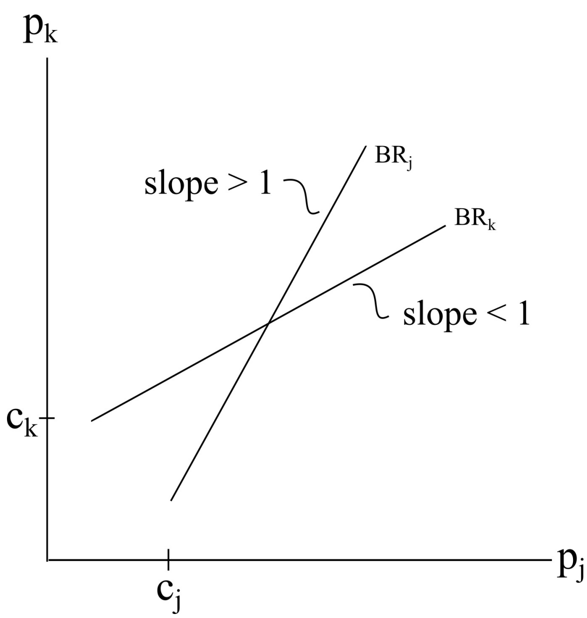

Figure 5: Differentiated products Bertrand Nash equilibrium and best-response functions.

- With p_j on the horizontal axis the best-response for j now has a slope that is greater than one.
- Typically in these models, when firm k increases price by one unit, firm j has an incentive increase their price by less than one unit: $0 < \frac{\partial p_k^{br}}{\partial p_j} < 1$.

- The differentiated good Bertrand model is a workhorse in applications. Be sure to try a homework problem to become familiar with it.

3 Static Cournot (1838) model

- The model is identical to Bertrand, other than the assumption that firms simultaneously choose quantity.
  - MCWG example: farmers pick a perishable crop, take it to market where (via Walrasian auctioneer) the market clearing price is given by demand: $P(q_j+q_k)$.

- A key difference is that costs are incurred for all units produced, and are not "produce to order" like in Bertrand.

- Firms maximize profit, taking the other firms output as given: $\frac{\partial q_k}{\partial q_j} = \frac{\partial q_j}{\partial qk} = 0$. Another way to think about this is that the market price is not in the firms information set when they are choosing quantity. However, firms do know the cost when they make their quantity choice. By contrast, under Bertrand firms' do not know their quantity and hence realized costs when choosing their price since their quantity depends on the other firm's price.

$$\max_{q_j \geq 0} P(q_j + q_k)q_j - cq_j \tag{10}$$

- The first-order condition is:

$$P'(q_j + q_k)q_j + P(q_j + q_k) = c \tag{11}$$

- Equation (11) implies that the BR functions are negatively sloped, a property known as "strategic substitutes."

- Implicit in Equation (11) is that the firm recognizes that $\frac{\partial P}{\partial q_j} \neq 0$, but assumes $\frac{\partial q_k}{\partial q_j} = 0$.

  - That is, they recognize that changes in their own output will change market price, but not that changes in their output will induce the other firm to change output.

  - Since the BR functions are negatively sloped, $\frac{\partial q_j^{br}}{\partial q_k} < 0$ and $\frac{\partial q_k^{br}}{\partial q_j} < 0$, the firm overestimates the fall in price from an increase in their own output due to the price effect: $P'(q_j + q_k)q_j$.

– Note that the assumption $\frac{\partial q_k}{\partial q_j} = 0$ is inconsistent with the reaction function slope. We briefly address this possibility below where j can have a conjecture or belief about how k will respond to changes in j's output.

Using the FOC one can show that the Cournot equilibrium price is in between p_m and $p_o = c$.

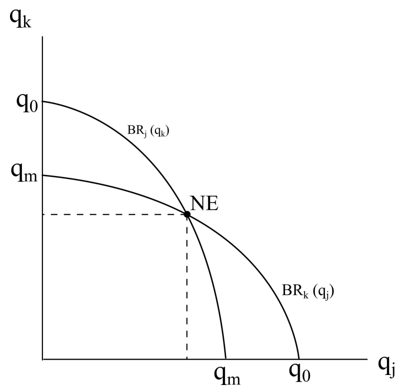

Figure 6: Cournot Nash equilibrium and best response functions.

Say something around here about Stackelberg.

4 Reconciling Cournot and Bertrand

- Bertrand would seem the better assumption.
  – Most firms choose price (direct approach to model evaluation: validity of the assumptions), but gets the wrong result.
- Cournot is opposite.
  – Cournot gets the right answer: $\pi > 0$, but for the wrong reason (indirect approach: valid conclusions).
- How can we reconcile this apparent disconnect?

- Think of quantity as a long-run choice of capacity and price as a short-run competition, given these capacity choices.

- If we have Bertrand competition with capacity constraints that are common knowledge then: $p_j = p_k = c$, $\pi_j = \pi_k = 0$ is not a NE.
  - If $p_k > c$ then firm j can not supply the entire market.
  - This implies $\pi_k > 0$. This was noted by Edgeworth in 1897.

- Kreps and Scheinkman (1983) *Bell Journal of Economics* (now *RAND Journal*) show that under certain conditions (namely that high value demands get satisfied first when low price firm has demand greater than its capacity) that the unique SPNE of the price-competition game with capacity constraints is the Cournot outcome.

5 Repeated Bertrand game

- Let's return to the identical good, equal marginal cost setup. In the 1-shot NE, price is equal to marginal cost and economic profit is zero.

- Consider the infinite horizon ($T = \infty$) repeated Bertrand game and the strategy

$$s_j = \begin{cases} \text{set } p_{j,0} = p_m, \\ \text{set } p_{j,t} = p_m \text{ if } p_{k,t-1} \geq p_m \\ \text{set } p_{j,t} = c \text{ otherwise.} \end{cases} \tag{12}$$

- This is a Bertrand trigger strategy. If it is a best-response to itself, then we have a model of tacit collusion.
  - No communication need take place, but the monopoly outcome is obtained.
  - Any deviation triggers an immediate price war.

- Note that the 1-shot NE can replace c in a more complicated setup.

- So when is this strategy a BR to itself? Let's find conditions on the discount factor d that support the collusive outcome $(p_j = p_m, p_k = p_m)$ as an NE of the repeated game.

- Write out the payoff streams.

  - For example, suppose

$$\begin{aligned} \pi_j(p_m, p_m) &= 2 = \pi_k(p_m, p_m) \\ \pi_j(c, c) &= 0 = \pi_k(c, c). \end{aligned} \tag{13}$$

  - Also, suppose that a player would earn 4 if they chose $p_m - \epsilon$ and the other player chose p_m (if $p_m - \epsilon$ is the 1-shot best-response to p_m that captures the entire market)

  - If both players choose the trigger strategy in (12) then the present value of the payoff stream is

$$PV_d(2, 2d, 2d^2, ...) = 2\sum_{t=0}^{\infty} d^t = \frac{2}{1-d}. \tag{14}$$

  - If a player were to deviate then present value of the payoff stream is

$$PV_d(4, 0d, 0d^2, ...) = 4. \tag{15}$$

  - Thus tacit collusion is sustainable if

$$\begin{aligned} \frac{2}{1-d} &> 4 \\ 2 &> 4(1-d) \\ d &> \frac{1}{2} \end{aligned} \tag{16}$$

  or in terms of the critical discount rate, assuming that the continuation probability is $q = 1$,

$$\begin{aligned} d &\equiv \frac{1}{1+r} > \frac{1}{2} \\ r &< 1. \end{aligned}$$

6 Conjectural Variations

Recall that, for homogeneous goods with inverse demand $p(Y) = p(y_1 + y_{-1})$, firm 1's problem can be written as $\max_{[y_1 \geq 0]} y_1 p(y_1 + y_{-1}) - c(y_1)$. The first order condition fully written out is

$$\begin{aligned} c'(y_1) &= p(Y) + y_1 p'(Y)[\frac{dy_1}{dy_1} + \frac{dy_{-1}}{dy_1}] && (17) \\ &= p(Y) + y_1 p'(Y)[1 + \nu], && (18) \end{aligned}$$

where $\nu = \frac{dy_{-1}}{dy_1}$ is firm 1's *conjectural variation* — her belief about how a change in her output y_1 will affect the total output y_{-1} of all rivals.

- $\nu = 0$ is the Cournot conjectural variation. She takes as given her rivals' output level, and (incorrectly!) assumes that she can't affect it.

- $\nu = -0.5$ is the Stackelberg leader's conjectural variation in the simple linear duopoly. More generally, it can be the slope of other firms' summed reaction functions.

- $\nu = -1.0$ is the competitive or Bertrand conjectural variation. It ensures that price = MC, and says that the firm believes that other firms will replace any units it withholds from the market.

- $\nu = y_{-1}/y_1$ is the collusion conjectural variation – other firms will maintain their current share.

- “consistent” conjectural variations equate ν to the actual comparative statics of the model for each firm (Bresnahan, 1981).

Economic theorists no longer find it fashionable to write down arbitrary expressions for ν, and the idea of consistent conjectures never got much empirical support. But it might be a helpful way to think about applications. See for example the McGinty (2016) treatment of greenhouse gas abatement treaties.

7 Spatial Competition: Hotelling location models.

Let us now take a deeper look at imperfect substitutes. So far, we have taken as given substitution elasticities in utility functions and demand functions. We also noted (from the differentiated good Bertrand model) that firms tend to be more profitable when their products are less substitutable. How can we model that dimension of competition?

Hotelling (1929) apparently was the first to take up that challenge. His “Main Street” model used a spatial metaphor to describe the substitutability among products. Think of the producers choosing the products’ characteristics (e.g., fuel economy, acceleration, and seating capacity of cars) in order to fill niches of the market that are relatively undersupplied. That is, firms choose location in the space of characteristics.

Hotelling considered a very special characteristic space: location within the interval $[0, 1]$. This could be taken literally as an address on a small town’s Main Street, or metaphorically as in characteristic space.

We begin with a duopoly where firms choose location but not price; for simplicity we assume that price is fixed at $p_1 = p_2 = p > c$, where $c = c_1 = c_2$ is the constant marginal cost faced by both firms.

- Label the firms so that the location choices satisfy $z_1 \leq z_2 \in [0, 1]$.

- For simplicity, assume that consumers' preferred locations are uniformly distributed along $[0, 1]$.

- Also, for simplicity, assume linear transportation (or transformation) cost $t > 0$. Thus the delivered price at location z for firm j is $p_j(z) = p + t|z - z_j|$. The assumption is that consumers buy at the lowest delivered price.

- Under current simplifications, this means that firm 1 gets all customers in $[0, \hat{z})$ and firm 2 gets those in $(\hat{z}, 1]$, where $\hat{z}$ solves $p_1(z) = p_2(z)$. In other words, the market shares are $\hat{z}$ and $1 - \hat{z}$, where the customer at $\hat{z} = 0.5(z_1 + z_2)$ faces the same delivered price from both firms.

- Since $p > c$, firms maximize profit by maximizing market share.

- What is firm i's BR to location choice z_j of firm j? If $z_j < 0.5$, it is to locate a tiny bit to the right, at $z_j + \epsilon$. If $z_j > 0.5$, it is to locate a tiny bit to the left, at $z_j - \epsilon$. This is how i maximizes market share.

- So the unique NE of this Hotelling location game is for both firms to locate back-to-back at $z = 0.5$.

This simple game is sometimes used to explain (in part) why firms in a similar line of business tend to locate next door to eachother, and why political parties used to adopt very similar platforms.

There are many, many extensions of the model. Expanding the duopoly location problem above to triopoly yields no NE in pure strategies. With 4 firms, the pure strategy NE all have 2 firms back to back at $z = .25$ and the other 2 at $z = .75$.

What if firms first commit to specific locations and then pick prices? Figure 7 illustrates how the shares are determined from an arbitrary set of locations and prices for 4 firms. The analysis is a bit tricky because the endpoints of the line segment play a special

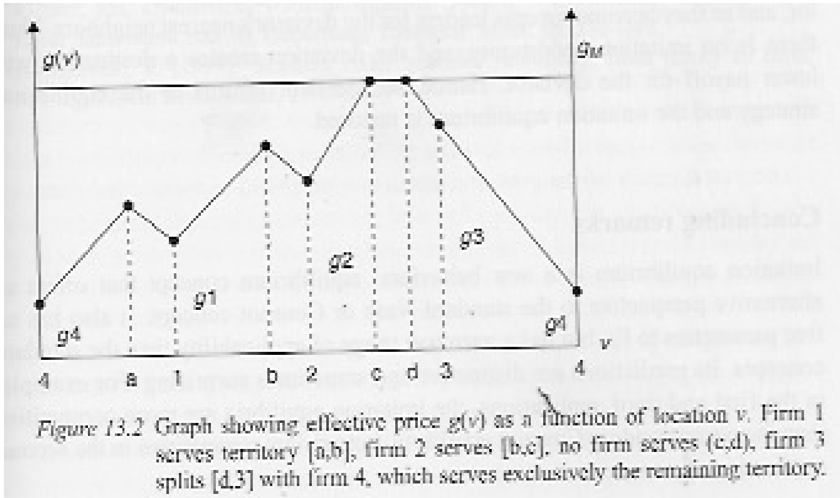

Figure 7: From Selten chapter in Friedman and Cassar (2004). Here $gi = p_i(z) - c$, and i is location z_i (denoted v in the Figure).

role. To make the location space more homogeneous (the technical word is 'isotropic'), one can join the endpoints to make a circle, and this is assumed in the Figure. It can be shown in this case that the unique NE in pure strategies is for firms to space themselves equally around the circle (maximum differentiation) and to all charge the price $p_i = c + t$.

Other variants of the Hotelling location game consider two dimensional locations, on a rectangle or (to make it isotropic) a torus and various numbers of firms. One can also consider nonlinear transportation (or transformation) costs. There seems to be room for applied work here, but I've not seen much published recently. There are ongoing laboratory experiments by UCSC PhD Curtis Kephart.

8 Behavioral considerations

To be added later

9 Further Reading

Dixit, A. (1979). "A Model of Duopoly Suggesting a Theory of Entry Barriers," *Bell Journal of Economics*, 10 (1): 20-32.

Bresnahan "Duopoly Models with Consistent Conjectures," *American Economic Review*, vol. 71 pp. 934-945, 1981).

Bresnahan, T. and Reiss, P. (1991). "Entry and Competition in Concentrated Markets," *The Journal of Political Economy*, 99 (5): 977-1009.

Eaton, J. and G. Grossman (1986). "Optimal Trade and Industrial Policy under Oligopoly," *The Quarterly Journal of Economics*, 101(2) 383-406.

Kreps and Scheinkman (1983) *Bell Journal of Economics complete reference if included*