Formulas for Econ 200, Fall 2016, UCSC

Equations for Competitive Markets

Linear Demand: $q_d = a - bp$ **Linear Supply**: $q_s = x + yp$
Log-linear demand: $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$ **Log-Linear Supply**: $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$
Total Surplus=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost
Total Cost=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost
Quantity Tax (tax per unit): $p_d = p_s + t$; **Value Tax** (tax on percentage spent): $p_d = (1+t)p_s$
Price Elasticity of Demand: $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p}\frac{p}{D}$; If $|\varepsilon| > 1$ then curve is elastic.
Tax Incidence Formula: $p_s(t) = p^* - \frac{t|D'|}{S'+|D'|}$; $p_d = p^* + \frac{tS'}{S'+|D'|}$; If ε_d is constant: $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d|+\varepsilon_s}$

Equations for Consumer Choice and Demand

Marginal Utility: $MU_i = \frac{\partial u}{\partial x_i}$; **Marginal Rate of Substitution**: $MRS_{ji} = \frac{MU_i}{MU_j}$ and at interior optimum $= \frac{p_i}{p_j}$
Perfect Substitutes: $u(x_1, x_2) = x_1 + cx_2$; **Cobb-Douglas**: $u(x_1, x_2) = \ln(x_1) + c\ln(x_2)$
CES Utility: $u(x_1, x_2) = \frac{1}{\rho}\ln(x_1^\rho + x_2^\rho)$; $\rho \in (-\infty, 1]$; **Quasilinear**: $u(x_0, x_1) = x_0 + g(x_1)$
Marshallian Demand: $\mathbf{x}^* = (x_1^*(\mathbf{p}, m), x_2^*(\mathbf{p}, m), ...)$ is the solution to $\max_{\mathbf{x}\geq 0} u(\mathbf{x})$ s.t. $m - \mathbf{p}\cdot\mathbf{x}$. The Lagrangian is $\mathcal{L} = u(\mathbf{x}) + \lambda(m - \mathbf{p}\cdot\mathbf{x})$. The FOCs can be written $MU_i = \lambda p_i$ or $MRS_{ji} = \frac{p_i}{p_j}$.
The solutions $x_i^*(\mathbf{p}, m)$ are homogeneous degree 0.
Hicksian Demand: $h_i^*(\mathbf{p}, u_0) : \min_x \mathbf{p}\cdot\mathbf{x}$ s.t. $u(\mathbf{x}) \geq u_0$
Roy's Identity: $x_i^*(\mathbf{p}, m) = -\frac{\partial v}{\partial p_i}/\frac{\partial v}{\partial m}$;
Slutsky Equation: $\frac{\partial x_i(\mathbf{p},m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p},v(\mathbf{p},m))}{\partial p_i} - \frac{\partial x_i^*}{\partial m}x_i^*(\mathbf{p}, m)$; (**Elasticity Form**): $\varepsilon_i = \varepsilon_i^h - s_i\varepsilon_m$ for $s_i = \frac{p_i x_i}{m}$.
Demand Elasticity identity for product i: $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j\neq i}\varepsilon_{ij} = 0$

Equations for Cost and Technology

Technical Rate of Substitution: $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j} < 0$;
MC: $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$, and $\int MC = VC$.
Factor Prices: $\mathbf{w} = (w_1, w_2, ..., w_n)$; **Production Function**: $y = f(x_1, y_2)$
Cost Function with two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$
$= \min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2$ s.t. $y = f(x_1, x_2)$
Shepard's Lemma conditional factor demand: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w},y)}{\partial w_i}$
Learning Curve: The typical specification is for $Y_t = \Sigma_{s\leq t} y_s$, AC falls proportionately, $\ln AC_t = AC_0 - b\ln Y_t$

Equations for Competitive Firms

SR Profit Maximization: $\max_{y, x_v \geq 0} \pi = \max_{y\geq 0}[\max_{x_v \geq 0} R(y) - w_v x_v - w_f x_f$ s.t. $y = f(x_v, \bar{x}_f)] = \max_{y\geq 0}[R(y) - c(y)]$
Revenue if firm is competitive: $R(y) = py = pf(x_v, \bar{x}_f)$ **FOC of *un*conditional factor demand**: $p\frac{\partial f(x_v,\bar{x}_f)}{\partial x_v} = w_v$
Hotelling's Lemma, Supply: $y^*(p, \mathbf{w}) = \frac{\partial \pi(p,\mathbf{w})}{\partial p}$; unconditional factor demands: $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p,\mathbf{w})}{\partial w_i}$
Shutdown Condition (Competitive Firms): $-F > py - c_v(y) - F \implies AVC = \frac{c_v(y)}{y} > p$

Equations for Monopolies

FOC for a monopolist: $p(y) + p'(y)y = c'(y)$ which can be rewritten as $p = \frac{1}{1+\frac{1}{\varepsilon}} MC$; valid if $\varepsilon < -1$.

Passing Along Costs: $\frac{\partial p}{\partial c} = \frac{1}{2 + yp''(y)/p'(y)}$.

Risky Choice. Given a lottery with monetary outcomes $m_1, ..., m_n$ and corresponding probabilities $p_1, ..., p_n$, its **expected value** is $Em = \sum_i p_i m_i$ and its **variance** is Var $m = \sigma_m^2 = E(m - Em)^2 = \sum_i p_i (m_i - Em)^2$.
Given **Bernoulli function** $u(m)$ — so $u' > 0$ and, if the person is risk-averse, $u'' < 0$ —
the **certainty equivalent** m^{CE} to the lottery solves $u(m^{CE}) = Eu(m) = \sum_i p_i u(m_i)$.
The **coefficient of absolute risk aversion** is $a(m) = -u''(m)/u'(m)$ and
the **coefficient of relative risk aversion** is $r(m) = ma(m)$.
The **risk premium** is RP = $Em - m^{CE}$. It is also given by the second term of the Taylor expansion of u around Em.

Decision Theory. Probability Identities: $P(A|B) = \frac{P(A \cap B)}{P(B)}$; $P(A \cap B) = P(B \cap A)$; Given probability sets A,B,C,D:
$P(A) = P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D)$

Bayes Theorem: $p(s|m) = \frac{p(m|s)}{\sum_{t \in S} p(m|t)p(t)} p(s)$ or $\frac{p(m|s)}{p(t|m)} = \left[\frac{p(m|s)}{p(m|t)}\right] \left[\frac{p(s)}{p(t)}\right]$ or ln[posterior odds] $=$ ln[likelihood ratio]$+$ ln[prior odds]. Note: $s =$ state, $m =$ message

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$$

Cournot. Given $D^{-1}(Y) = a - bY$, where $Y = y_i + Y_{-i} = \sum_{j=1}^{n} y_j$. Here $BR_i(Y_{-i}) = \underset{y_i}{\text{argmax}}\ \pi_i = P(Y)y_i - c(y_i)$
$\implies P(Y) + P'(Y)y_i - MC_i(y_i) = 0$.
To solve for the Nash equilbrium, we want to find where the Best Response functions intersect.

$\Rightarrow NE_{Cournot} : Y^* = \frac{N}{(N+1)b}(a - c) \implies y_i^* = \frac{Y^*}{N} = \frac{a-c}{(N+1)b}$

Bertrand. For firms with homogeneous goods, $p = MC$ if equal MC, and $p =$ second lowest MC $-$ one tick if MC's differ.

Stackelberg. Leader solves $\max \pi_L(y_L, BR_F(y_L)) = D^{-1}(y_L + BR_F(y_L))y_L - c(y_L)$

Kinked Demand Curve & Sticky Prices. MR has discontinuous drop at $D(\bar{p})$ where $\bar{p}$ is the established price. Firms will match $p < \bar{p}$ and will not match $p > \bar{p}$. Profit maximization $\implies$ not changing quantity (or price) as long as $MC_{low} < MC < MC_{high}$ where MC_{low} is $MR_{match} \cap D(\bar{p})$, and MC_{high} is $MR_{no-match} \cap D(\bar{p})$.

Conjectural Variations. If $p(Y) = p(y_1 + y_{-1})$, then firm 1's **FOC** is $c'(y_1) = p(Y) + y_1 p'(Y)[1 + \nu]$. The conjectural variation $\nu = \frac{dy_{-1}}{dy_1}$ is 0 for Cournot, is -.5 for Stackelberg leader (in linear duopoly), is -1.0 in Bertrand, and $\nu = \frac{y_{-1}}{y_1}$ in collusion/Cartel.

Hotelling location models. *Duopoly case on* $[0, 1]$: Firm i's BR to location choice $z_j < .5$ is $z_i = z_j + \epsilon$, and to $z_j > .5$ is $z_i = z_j - \epsilon$. Unique NE will be back to back at $z = .5$. **Delivered Price** for firm j at location z is $p_j(z) = p + t|z - z_j|$.

Monopolistic Competition Solve standard monopoly problem $MR = MC$ and $p = D^{-1}(q^*)$ Determine whether economic profits are > 0 or < 0. In LR equilibrium $\pi = 0$ since $LRAC = LRAR$.

Public Goods & Externalities. Let $C(Y) = Y$. Assume agents $i = 1, \ldots, n$ have $u_i(m, Y) = m + g_i(Y)$. WTP for each agent is $g_i'(Y)$. Efficient quantity Y^o maximizes $B(Y) - C(Y) = \sum_{i=1}^n g_i(Y) - Y$. The FOC is $1 = B'(Y) = \sum_{i=1}^n g_i'(Y)$.
External cost $e(x)$ is subtracted from the usual TS in efficiency calculations. **Pigouvian tax** is $t = e'(x^o)$.