6. Price Discrimination

I. Overview

A. Price discrimination is the sale of identical units of a good at different prices.

B. By price discriminating, a firm can capture some of what would be consumer surplus. PS $\uparrow$.

1. In so doing, a monopolist may also increase output, leading to a more efficient outcome, i.e., we may have PS $\uparrow$.

C. Constraints on price discrimination

1. The firm must have market power, otherwise it will just be a price-taker.
2. Arbitrage must somehow be limited; otherwise low price units could be resold and undercut the higher priced units.
3. The firm must somehow be able to detect WTP differences across consumers (or across quantities purchased by a single customer).
4. Legal or moral constraints.

D. The three classical types of price discrimination are methods of coping with the constraints and sorting consumers according to their WTP.

II. A Basic Model (from Varian)

A. A simple quasilinear model helps explain several varieties of price discrimination.

B. Consumers, $i = 1, 2$ have utility $u_i(x) + y$, normalized so that $u_i(0) = 0$. .

1. Think of y as money left over for everything other than x.
2. Consumers are willing to pay up to $u_i(x)$ for x units of the good.
3. Hence i's marginal WTP is $u_i'(x)$.

C. The inverse demand curve for the individual consumer is therefore found by solving the consumer's problem

1. max $u_i(x) + y$
   s.t. $px + y = m$
2. FOC is $p = u_i'(x)$ – the inverse demand curve.
3. Another way to say that i's marginal WTP is $u_i'(x)$.

D. From now on we'll assume that consumer 2 has higher WTP than consumer 1.

1. $u_2'(x) > u_1'(x)$, so by integration,
2. $u_2(x) > u_1(x)$.
3. This implies the **single crossing property**, that the indifference curves of two consumers only cross once.

E. Finally, assume that the monopolist has a cost of $c(x) = cx$.

III. First Degree Price Discrimination

A. The monopolist is able to charge a different price for each unit sold.

1. Sometimes called perfect price discrimination.

B. Suppose the monopolist makes an offer to each buyer of a lump sum payment of r for x units.

$$max_{r \geq 0} r - cx \text{ s.t. } u_i(x) \geq r. \tag{1}$$

C. Constraint holds with equality at optimum.

D. The FOC is $u'(x^*) = c$ .... which is Pareto efficient!

E. So $r^* = u(x^*)$.

1. Note that this x^* is the same level of output as a competitive firm.
2. $u'(x) = p(x) = c$.

F. This lump sum solution is equivalent to charging a different price (for marginal willingness to pay) for each unit of the good.

G. Constraints: all of them are problematic here.

H. Colleges attempt to approximate this for families who apply for financial aid. First degree price discrimination is not necessarily an evil plot by the producer. Given high fixed costs, it may be the only way for the producer to stay in business.

IV. Second Degree Price Discrimination

A. The monopolist charges prices that are not simple per-unit prices.

1. Sometimes called nonlinear pricing.
2. Includes quantity discounts etc.

B. Simplest version: a monopolist offers two different price/quantity *bundles* (r_i, x_i)

1. Bundle i is designed for consumers of type i.
2. The monopolist doesn't know whether a given consumer is type 1 or type 2.
3. The pricing scheme encourages consumers *sort themselves*.

C. In order to get type i consumers to choose the targeted bundle i, the monopolist has to satisfy two types of constraints:

1. Individual Rationality:

$$u_1(x_1) - r_1 \geq 0$$
$$u_2(x_2) - r_2 \geq 0$$

2. Self selection (aka. incentive) constraints:

$$u_1(x_1) - r_1 \geq u_1(x_2) - r_2$$
$$u_2(x_2) - r_2 \geq u_2(x_1) - r_1$$

1. A profit maximizing producer wants to set r_1 and r_2 as high as she can while satisfying the constraints.
2. This fact combined with the single crossing property guarantees that some of the constraints above bind. It turns out (see Varian) that we have

$$r_2 = u_2(x_2) - u_2(x_1) + r_1$$
$$r_1 = u_1(x_1)$$

3. That is, charge the low value consumer his max WTP for the low target bundle, and charge the high value consumer the highest price that doesn't cause him to switch away from the high target bundle.

D. The monopolist's problem

1. The monopolist gets the sum of the profits from the two consumer types.

$$\pi = r_1 - cx_1 + r_2 - cx_2$$

2. All of our hard work above gives us constraints to substitute into this equation:

$$\pi = u_1(x_1) - cx_1 + u_2(x_2) - u_2(x_1) + r_1 - cx_2$$

3. We can maximize this with respect to outputs x_1 and x_2, to obtain the FOCs below.

E. Welfare

1. First, the per unit price charged to the low value consumer is above marginal cost, implying implying a DWL.

$$u_1'(x_1) = p(x_1) = c + u_2'(x_1) - u_1'(x_1)$$

2. Second, our first order conditions tell us that the per unit price charged to the high value consumer is equal to marginal cost. No efficiency loss here.

$$u_2'(x_2) = p(x_1) = c$$

F. Conclusion: To max profit, target a bundle to high value consumer (#2) s.t. price (on last unit) = MC. Find a bundle for low value consumer (#1) that cuts back from the efficient quantity, is (barely) not preferred by #2, but exhausts #1's WTP.

G. Remark. I am unaware of any firm that actually does such calculations to obtain their price/quantity menu. It's hard to estimate the preferences u_i, and estimation errors could throw the calculation way off. Yet this approach gives insight to the menu that firms may settle upon after trial and error.

V. Third Degree Price Discrimination

A. This case is the most realistic, and the calculations parallel the way some firms actually think about it.

B. The monopolist is able to charge different prices to identifiably different groups but not different prices on different units within a group.

1. Think senior citizen discounts.
2. Or last minute shoppers, or domestic vs foreign market.

C. We start with an assumption that the prices posed to a group has no effect on the quantity demand in the other group.

1. The monopolist's problem is

$$\max p_1(x_1)x_1 - cx_1 + p_2(x_2)x_2 - cx_2$$

2. The FOCs from this problem can be written as:

$$p_1(x_1)[1 - \tfrac{1}{|\epsilon_1|}] = c$$
$$p_2(x_2)[1 - \tfrac{1}{|\epsilon_2|}] = c$$

3. Since this implies the left hand side expressions are equal to one another, it implies that $p_1(x_1) > p_2(x_2)$ only when $|\epsilon_1| < |\epsilon_2|$

   a. It turns out (though it is more complicated to show) that the same thing holds if we relax our assumption about the independence of the two discriminatory markets!

D. Welfare

1. Does the ability to price discriminate in the third degree help or hurt social welfare?
2. This depends on the effects on output. Varian shows that it can go either way (or be neutral).

   a. First, the only way welfare can be *improved* is if output increases due to the discrimination.

   b. Second, as long as $(p_1 - c)\Delta x_1 + (p_2 - c)\Delta x_2$ (where the prices are the prices after the discrimination is instituted), welfare has to improve!

   c. Third, if a whole new market is served due to the discrimination, welfare has to improve.

Ex: Third degree price discrimination with linear demand.

VI. Time permitting, also discuss 2-part tariffs, bundling, loyalty programs, ...