Econ 204B Midterm Answer-key

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1 . (a) Compute the posterior probabilities of success (A) and failure (B) following a good (a) or bad (b) beta version. Also compute the message probabilities p(a) and p(b).

We first compute the joint probability between A,B and a as:

$$P(A,a) = P(a|A)P(A) = \frac{9}{10} \cdot \frac{3}{4} = \frac{9}{40}$$
$$P(B,a) = P(a|B)P(B) = \frac{3}{10} \cdot \frac{3}{4} = \frac{9}{40}$$

then the joint probability between A,B and b can be computed as:

$$P(A,b) = P(b|A)P(A) = [1 - P(a|A)]\,P(A) = \frac{1}{40}$$
$$P(B,b) = P(b|B)P(B) = [1 - P(a|A)]\,P(A) = \frac{21}{40}$$

hence combine those results we get:

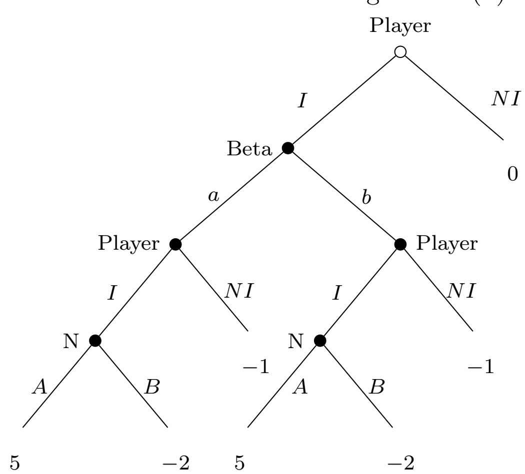
$$p(a) = P(A,a) + P(B,a) = \frac{9}{20}$$
$$P(b) = P(A,b) + P(B,a) = \frac{11}{20}$$

using the formula of conditional probability:

$$P(A|a) = \frac{P(A,a)}{P(a)} = \frac{9/40}{9/20} = \frac{1}{2}$$
$$P(A|b) = \frac{P(A,b)}{P(b)} = \frac{1/40}{11/20} = \frac{1}{22}$$
$$P(B|a) = \frac{P(B,a)}{P(a)} = \frac{9/40}{9/20} = \frac{1}{2}$$
$$P(B|b) = \frac{P(B,b)}{P(b)} = \frac{21/40}{11/20} = \frac{21}{22}$$

(b) Draw your decision tree and solve it. Write out your optimal plan.

Figure 1: 1(b)



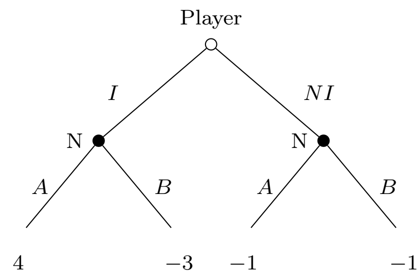
The tree of this problem looks like the following (Figure 1).

First we can compute $\mathbf{E}(V|a) = (-2)P(B|a) + (7-2)P(A|a) = \frac{3}{2}$, and similarly $\mathbf{E}(V|b) = -1$. Therefore, the total expected value is $\mathbf{E}V_{beta} = P(a) \cdot \frac{3}{2} + P(b) \cdot (-1) = \frac{1}{8}$. Hence my optimal plan will be always invest at the first step. And invest at the second step if a, do not invest at second if b.

(c) Now suppose that a beta version will not be available after all, and you have to decide now whether or not to invest 2; the overall probabilities and gross payoffs are unaffected. Draw and solve the revised decision tree.

The decision tree is showed in Figure 2.

Figure 2: 1(c)



Notice that $\mathbf{E}(I) = 5 \cdot \frac{1}{4} - 2 \cdot \frac{3}{4} = -\frac{1}{4} < \mathbf{E}(NI) = 0$, this suggests we will chose not invest in this case, hence we have $\mathbf{E}V_{no\,beta} = 0$.

(d) What is the information value of the beta version?

The answer is following by taking the difference between the expected payoff in (b) and (c):

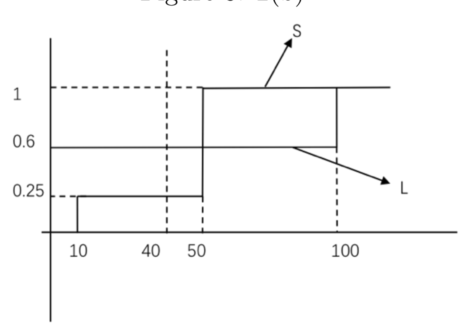
$$V = \mathbf{E}V_{beta} - \mathbf{E}V_{no\,beta} = \frac{1}{8} - 0 = \frac{1}{8}$$

2. (a) What is the mean and variance of this lottery?

$$\mu(L) = 100 \times 0.4 + 0 \times 0.6 = 40$$
$$Var(L) = 0.4 \times (100 - 40)^2 + 0.6 \times (0 - 40)^2 = 2400$$

(b) Write down a lottery with the same possible payoffs that first-order stochastically dominates L, and another lottery (with two possible payoffs, not necessarily the same as in L) that second-order stochastically dominates L.

Figure 3: 2(b)



Let's denote F be FOSD, and S be SOSD. Then

$$F = (100, 0.5; 0, 0.5)\,(or\, any\, lottery\, that\, also\, pays\, 100\, with\, probability > 0.4)$$

is a FOSD lottery. And consider also

$$S = (10, \frac{1}{4}; 50, \frac{3}{4})\,(or\, \mu_S = \mu_L = 40\, and\, integral\, inequality\, can\, be\, seen\, like\, Figure\, 3)$$

(c) What is the most that a person with Bernoulli function u(x) Vx would pay to play lottery L? What is her risk premium?

$$\mathbf{E}u_L = 0.4 \cdot \sqrt{100} + 0.6 \cdot \sqrt{0} = 4$$

since$u(CE) = \mathbf{E}u_L$, then we have $CE = 16$.

$$\mathbf{E}V_L = 0.4 \cdot 100 + 0.6 \cdot 0 = 40$$

therefore $\pi = \mathbf{E}V_L - CE = 40 - 16 = 24$.

(d) Compute this person's coefficients of absolute and of relative risk aversion at the mean of L. Which coefficient (if either) has the same value at x=100?

$$ARA = -\frac{u^{''}(x)}{u^{'}(x)} = -\frac{-\frac{1}{4}x^{-3/2}}{\frac{1}{2}x^{-1/2}} = \frac{1}{2x} \Rightarrow ARA(40) = \frac{1}{2 \times 40} = \frac{1}{80},\ ARA(100) = \frac{1}{200}$$

$$RRA = --\frac{u^{''}(x)}{u^{'}(x)} \cdot x = ARA \cdot x = \frac{1}{2} \Rightarrow RRA(40) = RRA(100) = \frac{1}{2}$$

The coefficient of relative risk aversion has the same value at $x = 100$.

3 (a) Does either player have a dominant strategy?

No player has a dominant strategy here.

(b) Does either player have a dominated strategy?

The dominated strategy for player 1is A, which is dominated by B.

The dominated strategy for player 2is a, which is dominated by $(0.9b + 0.1c)$.

(c) Which strategy profiles survive iterated deletion of strictly dominated strategies? Explain each deletion very briefly.

By the result from (b), we can using IDDS to delete A&a, the new matrix is

		player 2	
		b	c
player 1	B	$\underline{2}, \underline{3}$	$4, 1$
	C	$0, 4$	$\underline{6}, \underline{5}$

hence, the strategies are (B, C) for player 1 and (b, c) for player 2.

(d) Find all Nash equilibria (NE) in pure strategies, if any.

From part (c), the NE are (B, b) and (C, c).

(e) Find all Nash equilibria (NE) in mixed strategies, if any.

We set $Prob(b) = p$ and $Prob(B) = q$, then we have the following

$$2p + 4(1-p) = 0 \cdot p + 6(1-p) \Rightarrow p = \tfrac{1}{2}$$
$$3q + 4(1-q) = 1 \cdot q + 5(1-q) \Rightarrow q = \tfrac{1}{3}$$

		player 2		
		p	1-p	
		b	c	
player 1	B	$\underline{2}, \underline{3}$	$4, 1$	q
	C	$0, 4$	$\underline{6}, \underline{5}$	1-q

so, mixed NE strategy is $(x = \frac{1}{3}B + \frac{2}{3}C,\ y = \frac{1}{2}b + \frac{1}{2}c)$.

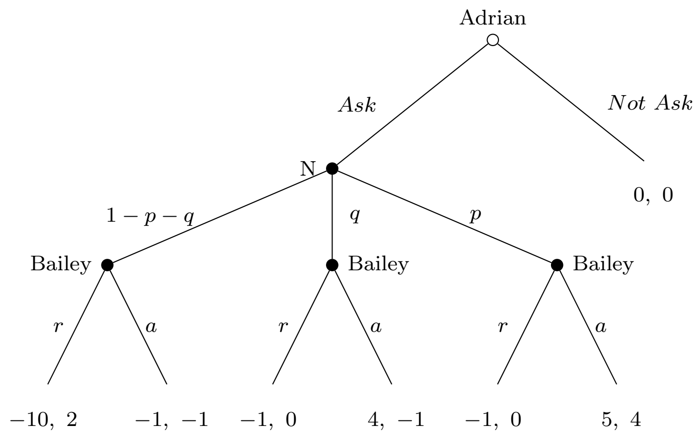
(f) Find all payoff dominant NE and all risk dominant NE, if any.

The payoff dominant NE is (C, c). The risk dominant NE is (B, b).

4. (a) Draw the EFG for this strategic situation.

One way to draw the tree is

Figure 4: 4(a)



where $p = p(L, N)$, $q = (D, N)$, $1 - p - q = (D, M)$.

(b) Write down the corresponding NFG.

		Bailey			
		rrr	rra	rar	raa
Adrian	Ask	9p+9q-10, 2-2p-2q	15+9q-10, 2p-2q+2	9p+14q-10, 2-2p-3q	10-5p-6q, 2+2p-3q
	Not ask	0, 0	0, 0	0, 0	0, 0

		Bailey			
		arr	ara	aar	aaa
Adrian	Ask	9p+9q-10, 2-2p-2q	15+9q-10, 2p-2q+2	9p+14q-10, 2-2p-3q	10-5p-6q, 2+2p-3q
	Not ask	0, 0	0, 0	0, 0	0, 0

(c) Find all pure strategy NE of the NFG.

Base on NFG we wrote above, we can find NE of NFG given specific value of p and q. For example $p = q = \frac{1}{3}$, then the solution will be like part (e).

(d) Find all subgames of the EFG.

There are 3 proper subgames beginning with Bailey's move, and given specific values of p and q we can compute the expected payoffs for Adrian (ask and not ask). Then by comparing those values, we can derive the solution for that subgames.

(e) Find a (Bayesian) NE that is subgame perfect when p =1/3 and q=1/3.

If $p = \frac{1}{3}\, q = \frac{1}{3}$, for Adrian (form tree)

$$\mathbf{E}V(Ask) = -\frac{10}{3} - \frac{1}{3} + \frac{5}{3} = -2$$
$$\mathbf{E}V(Not\, ask) = 0$$

So, he will choose not ask. For Bailey, for NFG in part (b), her strategy will be rra. Combining these the Bayesian SPNE is (NA, rra).

(f) Find the necessary conditions on p and q for there to be a Bayesian Nash Equilibrium in which Adrian asks Bailey on a date.

$$\mathbf{E}V(Ask) = 10p - 10q - 1 - q + 5p$$
$$\mathbf{E}V(Not\, ask) = 0$$

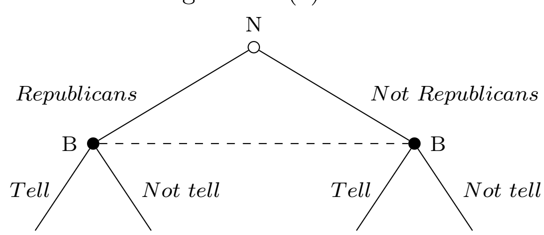
If $\mathbf{E}V(Ask) \geq \mathbf{E}V(Not\, ask)$, Adrian will choose to ask, so

$$15p + 9q - 10 \geq 0 \Longleftrightarrow 15p + 9q \geq 10$$

which gives us the necessary condition we want.

5. (a) Sketch a game form to represent the following situation: Player B chooses whether or not to tell Player A a joke that is unkind to Republicans. B is uncertain whether or not player A is a Republican.

Figure 5: 5(a)



(b) Now sketch a game form in which, after hearing the joke, A is uncertain whether or not B realizes that A indeed is a Republican.

Figure 6: 5(b)