

SPREADING EFFECTS IN THE TERM STRUCTURE  
OF FOREIGN EXCHANGE RATES

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# ABSTRACT

## SPREADING EFFECTS IN THE TERM STRUCTURE OF FOREIGN EXCHANGE RATES

The roles of speculators, arbitrageurs, trader and hedgers in the formation of spot and forward foreign exchange rates are analyzed by means of an explicit partial equilibrium, two currency model with a range of forward maturities. The principal theoretical finding is that there will generally be a time-varying discrepancy between a given forward rate and the market expectation of the corresponding future spot rate. Moreover, this discrepancy does not bear a systematic relationship to risk characteristics of the currencies. Rather, it is attributable to the fact that currencies are storable goods and hence term structure constraints will "spread" the effects of market expectations across maturities. The spreading effect is computed under certain parametric specifications for market participants, and is illustrated numerically in some examples of a single speculator with rational expectations.

## § Introduction

What is today's "market expectation" of the U.S. dollar spot price of the U.K. Pound Sterling for a date six months hence? Academics have a ready answer: that expectation is simply today's six month forward exchange rate, possibly adjusted by some "premium" based on the risk characteristics of exchange rates. The logic behind this answer seems unsailable: any discrepancy between the market expectation and the forward rate represents an opportunity for speculative profit, which in equilibrium should be commensurate with the speculative risk.

Nevertheless, exchange dealers and other participants have long been sceptical of this approach. They routinely price forward exchange by adding a "swap rate" based on current Eurocurrency interest rate differentials to the prevailing spot exchange rate<sup>1</sup> (i.e., by using the Interest Rate Parity Theorem). From this perspective, it seems natural to regard today's forward rates as possessing no more predictive power for future spot rates than today's spot rates. Consequently, such practitioners often entertain expectations regarding a future spot rate that are not systematically related to the corresponding current forward rate or to the customary determinants of a "risk premium."

The two views are not irreconcilable. One way to resolve the controversy, on terms rather more favorable to academics than practitioners, is to imagine that all asset prices in the world are simultaneously determined by the market-clearing condition that optimal portfolio demands equal existing asset supplies, as in the Capital Asset Pricing Model. Under appropriate assumptions (e.g., an exogenous stationary normal random process generating asset returns, no market segmentation, etc.), one can derive an equilibrium forward rate that satisfies interest rate parity and at the same

time is the risk-adjusted expectation of the corresponding future spot rate.

Unfortunately, there seems to be little consensus among theoreticians on precisely which risk characteristics are relevant.<sup>2</sup> Perhaps more damaging to this approach has been the general empirical failure of the forward rate (whether or not adjusted for a risk premium) as a predictor of the subsequent spot rate.<sup>3</sup>

In this paper I suggest a theoretical reconciliation on terms more favorable to practitioners. In order to address the controversy properly, I require a model of exchange rates for a range of maturities (i.e., a foreign exchange rate term structure) in which "market expectations" can be expressed in terms of individual participants' beliefs. These requirements in themselves entail fairly complicated notation, so I keep the model as simple as possible in other respects. Classifying the roles of market participants as (i) traders (i.e., primarily exporters and importers), (ii) speculators (including hedgers), and (iii) (covered interest) arbitrageurs, I construct a partial equilibrium model for the home currency (parochially denoted \$) price of foreign currency (£).

Particularly in its emphasis on the arbitrage possibilities arising from the storability of currencies (here via bank deposits), my model resembles some recent independently conceived theoretical work on commodities markets, such as Newbery and Stiglitz (1982) and Turnovsky (1983). It also in some respects resembles the not-so-recent Modern Theory of exchange rates developed by Tsiang (1959) and others, as expounded by Grubel (1966) and Kenen (1964), for instance. A secondary goal of this paper is to introduce a flexible partial equilibrium approach to foreign exchange rate determination that incorporates many of the conveniences of MT without some of its logical difficulties.

I begin in the next section with a verbal sketch of my argument that the storability of currencies (and covered interest arbitrage) generally induces "blurring effects," discrepancies between forward and corresponding expected future spot rates that cannot usefully be regarded as risk premia. The following section briefly presents my partial equilibrium model, explicitly deriving the foreign exchange rate term structure from given participant beliefs and risk preferences, interest rate term structures and trade demand schedules in a parametric example. Numerical examples illustrate the model's compatibility with Rational Expectations constraints. The final section briefly discusses the general applicability of the analysis and its implications for empirical work.

## §2. A Verbal Presentation.

An agent who participates in foreign exchange markets for purposes other than speculation or arbitrage is referred to as a trader. My specification of trader behavior is very rudimentary: their aggregate net demand for spot £ is negatively related to the current spot exchange rate  $p(t)$  -- measured in \$ per £ -- and to some unspecified function  $e$  of all other "external" factors, such as macroeconomic variables or official intervention. Traders as such do not transact in the forward markets.

Speculators are risk-averse agents who take uncovered positions in forward £ with the intention of profitably reversing these positions at a later date. Specifically, given the information currently available to him, each speculator is assumed to form beliefs about the distribution of future

exchange rates and to compute his expected-utility maximizing position for each of a range of forward maturities. His current demand for spot and forward £ is then the difference between this optimal position and the actual position he brings to the current period. This demand involves transactions that were not expected on the basis of previous information as well as previously scheduled transactions such as offsetting a forward contract at maturity). Hedging activity can be subsumed in this specification by including scheduled £-denominated cash flows (arising, say, from importing) in the actual position. I write here as if agents denominate their wealth in \$; one can show that it makes essentially no difference if some or all denominate wealth in £.

The final set of agents are covered interest arbitrageurs, who take advantage of price discrepancies to make essentially riskless profits. In particular, arbitrageurs transact in spot and forward exchange markets and \$ and £ (Euro) deposit markets so as to maintain covered interest parity.<sup>4</sup>

It should be emphasized at this point that despite my casual references to "agents," I am really classifying roles of market participants. An appendix available on request shows that the market impact of any rational agent can be described in terms of his impact via one or more roles. For instance, an "investor" who buys £ assets spot with the intention of reselling later can be regarded as a speculator-plus-arbitrageur, who, as speculator, buys the £ forward (from himself as arbitrageur), and as arbitrageur, borrows \$ (the opportunity cost of the £ assets), buys £ spot and sells £ forward (to himself, as speculator); the artificial contracts disappear upon aggregation.<sup>5</sup>

Suppose that the term structure of \$-£ exchange rates is determined by market clearing, and so at equilibrium the net demands over all agent (-roles) sum to zero at every maturity. The key observation is that if new

information (regarding the realization of  $e$  for some future date,<sup>6</sup> say) causes some or all speculators to revise their beliefs and consequently to alter their optimal positions (at current prices) for that date, price adjustments will occur not only for forward contracts maturing at the given date. Because of covered interest arbitrage, interest rate parity will force price adjustments at every forward maturity, including spot. Hence arbitrage "spreads" the effects of market expectations across forward maturities.

This argument for "spreading effects" is based on the idea that speculation exerts a force on prices that is weak relative to the force exerted by arbitrage.<sup>6A</sup> Assuming risk neutrality or perfect foresight and unconstrained access to financial markets, one would expect both forces to be irresistible,<sup>7</sup> and in a sense, indistinguishable. However, in the more realistic case that speculation on future exchange rates (and interest rates) is quite limited due to uncertainty and risk aversion, and/or borrowing constraints, while riskless arbitrage is relatively unconstrained, one no longer can presume that each forward exchange is driven to "the" risk-adjusted expectation of the corresponding future spot rate. Although speculative forces act in this direction, the stronger force of arbitrage ensures an alignment of forward rates according to interest rate differentials, and so discrepancies generally remain. In general one can say only that the term structure of exchange rates will shift up or down so as to minimize some average of these discrepancies across maturities.

These discrepancies between forward rates and expectations (possibly risk-adjusted) for corresponding future spot rates will typically differ in sign across maturities at a given time, and may well change sign for a given maturity across time as will be illustrated in the next section, Figure 1. They do not depend in any stable

fashion on the risk characteristics of the exchange rate, and therefore cannot usefully be regarded as risk premia. Consequently, sceptical market practitioners may well be right in discounting the predictive power of a forward rate. On the other hand, my argument implies that there are no remaining opportunities for abnormal speculative profits in equilibrium, a conclusion consistent with longstanding academic tradition.

### §3. A Partial Equilibrium Model.

Let  $T$  be the set of trader(-roles), and  $G_k^*(t)$  represent the desired contract of trader  $k \in T$  at time  $t$ , measured in  $\mathcal{L}$ . That is,  $G_k^*(t) = 1$  means  $k$  wishes to purchase  $1\mathcal{L}$  for  $p(t)\$$ , while  $G_k^*(t) = -1$  means he wishes to sell  $1\mathcal{L}$ . My rudimentary specification for trader activity is  $\sum_{k \in T} G_k^*(t) = z(p(t), e(t))$ . A simple parametric example of the aggregate net demand function  $z$  is linear with additive "shocks;" viz.,  $z(p, e) = (\bar{p} - p) d + e$ , where  $\bar{p} > 0$  is the normal (or balanced-trade for  $e=0$ ) level of the spot rate  $p(t)$ , the slope parameter  $d > 0$  so the standard Marshall-Lerner condition holds,<sup>8</sup> and  $e(t) \begin{matrix} > \\ < \end{matrix} 0$  summarizes all the variables that can shift trader demand. I make no special assumptions here about the distribution of  $e$ . Note that in the absence of other agent-roles, the spot exchange rate in this parametric specification would be  $p(t) = \bar{p} + e(t)/d$ .

Speculators, indexed by  $i \in S$ , can transact in forward markets of maturity date  $s > t$ ; such contracts are denoted  $G_i(t, s)$ , and their price (i.e., the forward exchange rate) by  $p(t, s)$ . Thus, if speculator  $i$  holds his contract to maturity and settles at the then prevailing spot rate  $p(s)$ , his \$ profit or loss will be  $G_i(t, s)[p(s) - p(t, s)]$ . It will often be convenient to refer to a speculator's net position as of date  $t$  for a particular maturity  $s$ ,  $g_i(t, s) = \sum_{u \leq t} G_i(u, s)$ . That is,  $i$ 's net position  $g_i(t, s)$  is the sum of



all contracts made at past dates  $u \leq t$  which mature on the given date  $s$ .

Speculators are assumed to choose current transactions according to the following optimization procedure. Given available (public and private) information at time  $t$ , speculator  $i$  forms subjective probability distributions regarding the future path of the Foreign Exchange rate term structure. He then finds a set of net positions  $g_i^*(t,s)$ ,  $s > t$ , that maximizes expected utility-of-wealth, given his risk preferences and current wealth.

Reasonable specifications and standard approximations lead to the following linear decision rule:

$$(1) \quad g_i^*(t,s) = (E_{it} p(s) - p(t,s)) R_i(t,s)/V_{it}p(s),$$

where  $E_{it}$  and  $V_{it}$  are respectively the expectation and variance operators with respect to speculator  $i$ 's subjective probability distributions at time  $t$ , and  $1/R_i$  is his coefficient of absolute risk aversion, evaluated at a wealth level that incorporates gains and losses to date  $t$  on contracts maturing at date  $s$ . I will sometimes abbreviate  $i$ 's "willingness to speculate" parameter  $R_i(t,s)/V_{it}p(s) \geq 0$  by  $a_{its}$ . Finally, then, I can express speculator  $i$ 's net demand for maturity  $s$  forward contracts at date  $t$  by

$$(2) \quad G_i^*(t,s) = g_i^*(t,s) - g_i^*(t-1,s) = a_{its}(E_{it}p(s) - p(t,s)) - g_i^*(t-1,s).$$

Note that a speculator's desired position is always zero whenever the forward rate  $p(t,s)$  equals the expected future spot rate  $E_{it}(p(s))$ . Since this will always be the case when  $s = t$  (i.e., for today's spot rate), one concludes that under specification (1), speculators desire zero spot posi-

tions, so  $G_i^*(t) = -g_i(t-1, t)$  and hence according to (2) speculators always offset forward contracts at or before maturity.

It is instructive to compute the equilibrium term structure in the absence of arbitrageurs for our parametric example. In this case, the equilibrium spot exchange rate will satisfy the equation<sup>9</sup>

$$0 = \sum_{i \in T} G_i^*(t) + \sum_{i \in S} G_i^*(t) = (\bar{p} - p(t))d + e(t) - g(t-1, t),$$

whose solution is clearly

$$(3) \quad p(t) = \bar{p} + (e(t) - g(t-1, t))/d.$$

That is, the spot rate is just as in the traders-only case except to the extent that the "shock"  $e(t)$  has been anticipated and offset<sup>10</sup> by speculators at past dates  $u < t$ .

The equilibrium forward rate  $p(t, s)$ ,  $s > t$ , likewise is obtained from market-clearing:

$$0 = \sum_{i \in T \cup S} G_i^*(t, s) = \sum_{i \in S} G_i^*(t, s) = \sum_{i \in S} g_i^*(t, s) - \sum_{i \in S} g_i(t-1, s).$$

The last term is zero (because  $\sum_{i \in S} G_i^*(u, s) = 0$  for all  $u < t$ , and  $g_i(t-1, s) = \sum_{u < t} G_i^*(u, s)$ ), so the equilibrium condition reduces to:

$$0 = \sum_{i \in S} g_i^*(t, s) = \sum_{i \in S} a_{its} (E_{it} p(s) - p(t, s)).$$

Define  $w_{ts} = \sum a_{its}$  and  $w_{its} = a_{its}/w_{ts}$ , and solve the last equation to obtain

$$(4) \quad p(t,s) = \sum_{i \in S} w_{its} E_{it} p(s),$$

where the weights  $w_{its}$  satisfy  $0 \leq w_{its} \leq 1$  and  $\sum_{i \in S} w_{its} = 1$ . Thus, in this case, forward rates are determined entirely by speculators; in fact (4) asserts that each forward rate is a certain weighted average of speculators' current expectations of the future spot rates, and therefore may be regarded as the "market expectation."

The weight of each speculator reflects his willingness to speculate ( $a_{its}$ ) relative to the aggregate willingness ( $w_{ts}$ ). Recall that each agent's willingness to speculate is the product of his risk acceptance  $R_i$  and the precision of his forecast  $1/\text{Var}_{it}$ . In general, then, a speculator's weight will vary according to the maturity  $s$ , principally because his forecast precision may drop off with increasing  $s$  at a different rate than his fellow speculators. For instance, a speculator with a relatively large  $R$  and rapid decline with  $s$  in  $\text{Var}_{it}^{-1} p(s)$  can be regarded as a "near-term specialist:" his influence in terms of market share and price impact will be felt primarily in near term (smaller  $s$ ) forward markets.

One can easily incorporate hedging activity into this analysis. Due perhaps to exporting or importing, an agent  $i$  may schedule at date  $t$  a  $\pm$  flow of  $X_i(t,s)$  to be received (or paid, if  $X_i < 0$ ) at date  $s > t$ .

It turns out that the only effect of consequence is that the new optimal forward contract  $\hat{G}_i$  is shifted from  $G_i^*$  (as given in equation (2)), by  $X_i$ , viz,  $\hat{G}_i^*(t,s) = G_i^*(t,s) - X_i(t,s)$ . In words, a speculator will first offset the newly scheduled  $\pm$  payment (hedge 100%) and then choose his speculative contracts essentially as before.<sup>11</sup>

The modifications to the equilibrium price equations are then straight-

forward. Defining  $x_i(t) = x_i(t-1, t) = \sum_{u < t} X_i(u, t)$  and  $x(t) = \sum_{i \in S} x_i(t)$ , one may regard  $\hat{e}(t) = e(t) - x(t)$  as the current "unhedgable shock" and  $X(t, s) = \sum_{i \in S} X_i(t, s)$  as aggregate current hedging for date  $s$ . One obtains<sup>12</sup> in the same manner as before:

$$(3') \quad p(t) = \bar{p} + (\hat{e}(t) - g(t-1, t))/d, \text{ and}$$

and

$$(4') \quad p(t, s) = \sum_{i \in S} w_{its} E_{it} p(s) - X(t, s)/w_{ts}.$$

Equation (3') suggests that the "shock"  $e(t)$  will be offset by previously scheduled commitments  $x(t)$ , as well as by previous speculation. Equation (4') incorporates a Hicks-Keynes sort of risk premium  $-X(t, s)/w_{ts}$  that shifts the forward rate away from the "market expectation." The sign of this premium is the opposite of that for (aggregate) net hedging; its magnitude is directly proportional to that of net hedging, and inversely proportional to aggregate willingness to speculate.

Despite their intuitive appeal, equations (3)-(4') do not represent the model's predictions because they disregard covered interest arbitrage, which unifies the exchange rate term structure with interest rate structures. Specifically, let  $r_{\$}(t, s)$  be the nominal Eurodollar interest rate ("LIBOR") prevailing at date  $t$  for deposits or loans<sup>13</sup> maturing at date  $s$ , expressed as a continuously compound per period rate. Similarly, let  $r_{\pounds}(t, s)$  represent the corresponding Eurosterling rate, and let  $r(t, s) = r_{\$}(t, s) - r_{\pounds}(t, s)$  represent the interest rate differential. Then, the term structures are unified by:

$$(5) \quad p(t, s) = p(t) e^{(s-t)r(t, s)}, \text{ for all } t \text{ and } s \geq t.$$

As is well known, the interest rate parity relationship (5) will always hold under the assumptions: (1) bid-ask spreads and default risks are negligible, (2) arbitrageurs can borrow and lend unlimited quantities of \$ and £ at the (possibly endogenous) rates  $r_{\$}$  and  $r_{£}$ , and (3) arbitrageurs can buy and sell £ for \$ spot and forward in unlimited quantities at the (possibly endogenous) rates  $p(t)$  and  $p(t,s)$ .

We characterize arbitrageurs by the assumption that their foreign exchange transactions are always covered and hence riskless. The paradigmatic scheme is buying (or selling) a given forward contract and offsetting spot, but the covering could be a bit more complicated, involving forward contracts for different maturities, together with appropriate \$ and £ borrowing and lending. In any case, one can readily verify that arbitrageur  $j$ 's net forward transactions  $G_j(t, \cdot)$  will be riskless if and only if they are supported by appropriate borrowing and lending and satisfy:

$$(6) \quad \sum_{s \geq t} e^{-(s-t)r_{£}} G_j(t,s) = 0, \quad \text{for all } j \in A,$$

where  $A$  is the set of arbitrageurs. In words, the (£) discounted present value of each arbitrageur's FX contracts ( $j$ 's "open position") must be zero. The careful reader may observe that (5) and (6) together imply that arbitrage, while riskless, is also profitless. This is, of course, due to neglect of bid-ask spread; presumably arbitrageurs do earn a normal return in the real world.

For present purposes, the key observation is that spot and forward rates may no longer be regarded as separately determined when arbitrage is possible. Specifically, net speculative demand is no longer zero for any particular maturity; it may be offset by net arbitrage activity, which is

constrained by (6). The upshot for our parametric example is contained in the following proposition, which employs the following special notation.

$$\hat{g}(t) = \sum_{i \in S} \sum_{s > t} e^{-(s-t)r_t} \hat{g}_i(t-1, s) = (t) \text{ present value of aggregate old speculative positions;}$$

$$W_t = \sum_{i \in S} \sum_{s > t} e^{-(s-t)(r-r_t)} a_{its} = \text{a discounted value of aggregate willingness to speculate.}$$

Proposition 1: If speculators chose their contracts according to (1), traders follow the linear rule  $z$ , and arbitrageurs obey (6) while enforcing (5), then for each date  $t$  and maturity  $s$ , equilibrium exchange rates are:

$$(3^*) \quad p(t) = \bar{p} \left( \frac{d}{d+W_t} \right) + (\hat{e}(t) - \hat{g}(t)) / (d+W_t) + \sum_{i \in S} \hat{W}_{it} \hat{E}_{it} p, \quad \text{and}$$

$$(4^*) \quad p(t, s) = p(t) e^{(s-t)r(t, s)}$$

Note: The "weights"  $\hat{W}_{it}$ , defined below, are non-negative, but do not in general sum to unity. The "discounted average expected FX rates"  $\hat{E}_{it} p$  are also defined below.

Proof: The forward rate equation (4\*) is just assumption (5), so only (3\*) must be established. The equilibrium condition is that desired spot contracts sum to zero, i.e.,

$$(7) \quad 0 = \sum_{k \in T} G_k^*(t) + \sum_{j \in A} G_j^*(t) + \sum_{i \in S} G_i^*(t).$$

The first term is still  $(\bar{p} - p(t))d + e(t)$ , and (6) implies that  $G_j^*(t) = - \sum_{s > t} e^{-(s-t)r_t} G_j^*(t, s)$  for each  $j \in A$ . But all forward contracts must have counterparties, so for each  $t$  and  $s > t$ ,  $\sum_{j \in A} G_j^*(t, s) = - \sum_{i \in S} G_i^*(t, s)$ .

These last two equations imply that the second term of (7) is

$$\sum_{i \in S} \sum_{s > t} e^{-(s-t)r_t} G_i^*(t, s) = \sum_{i \in S} \sum_{s > t} e^{-(s-t)r_t} \{a_{its} (E_{it} p(s) - p(t)) e^{(s-t)r_t} - g_i(t-1, s) - X_i(t, s)\}.$$

As previously noted, the third term in (7) is  $-\sum_{i \in S} \hat{g}_i(t-1, t)$ . Putting these expressions together and rearranging slightly, one obtains

$$(7') \quad 0 = d(\bar{p} - p(t)) + (\hat{e}(t) - \hat{g}(t)) + \sum_{i \in S} \sum_{s > t} \hat{a}_{its} (E_{it} p(s) e^{-(s-t)r_t} - p(t)),$$

where  $\hat{a}_{its} = a_{its} e^{(s-t)(r_t - r_t)}$ . Set  $W_{it} = \sum_{s > t} a_{its} e^{-(s-t)r_t}$ ,

and define

$$\hat{E}_{it} p = \sum_{s > t} a_{its} e^{-(s-t)r_t} E_{it} p(s) / W_{it}.$$

Then the equilibrium condition (7') yields

$$dp(t) + W_t p(t) = d\bar{p} + (\hat{e}(t) - \hat{g}(t)) + \sum_{i \in S} W_{it} \hat{E}_{it} p. \quad \text{Defining } \hat{W}_{it} = W_{it} / (d + W_t) \text{ and solving yields (3*).} \quad \text{QED.}$$

The thrust of this proposition is that, because of arbitrage, the spot rate is affected by all current speculation. Therefore any new information that prompts revisions in speculators' expectations of future spot prices  $E_{it} p(s)$  will cause corresponding changes in the current spot rate. In fact, by (4\*), the entire term structure of exchange rates will shift in response to new information. The strength of the response depends on the "willingness to speculate" parameters  $a_{its}$ , attenuated by the time horizon  $(s-t)$  for the information via the discount factors. Equation (3\*) expresses these responses in terms of the individual speculators' influences: each will tend to move the spot rate towards his "discounted average" expectation  $\hat{E}_{it} p$  (so his beliefs regarding a particular  $p(s)$  matter only inso-

far as they affect this average); I's "opinion"  $\hat{E}_{it} p$  will be incorporated by the market to the extent his discounted willingness to speculate  $W_{it}$  is significant relative to the aggregate willingness  $W_t$ , augmented by traders' aggregate responsiveness  $d$  to spot prices.

To the extent that aggregate current willingness to speculate  $W_t$  is small (and past speculation  $g(t)$  has also been small), (3\*) approximates the trader-only equilibrium spot rate (3). To the extent these factors are large relative to  $d$ , the spot rate (indeed, the entire term structure) is a creature of market expectations. In either case, specific forward rates according to (4\*) evidently are not tied any more closely to expectations concerning the corresponding future spot rates than they are to expectations concerning spot rates which will prevail at other dates. Thus we obtain have a new explanation of why a forward rate might not be a particularly good predictor of its corresponding future spot rate: arbitrage "spreads" the market impact of expectations across maturities.

Note that one can express the last term in (7') as

$$\sum_{s>t} (\bar{E}_t p(s) - p(t,s)) w_{ts} e^{(s+t)r_t(t,s)} \quad \text{where } \bar{E}_t \text{ is the "market expectation" defined by the RHS of equation (4).}$$

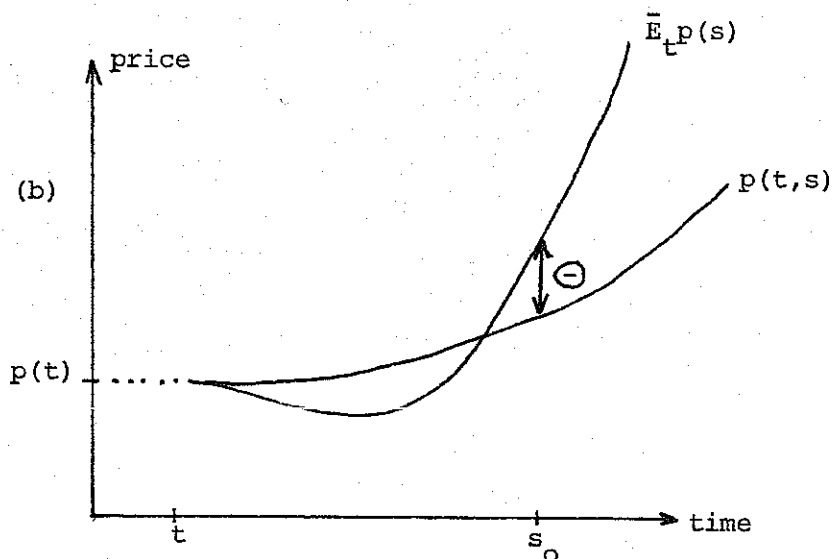
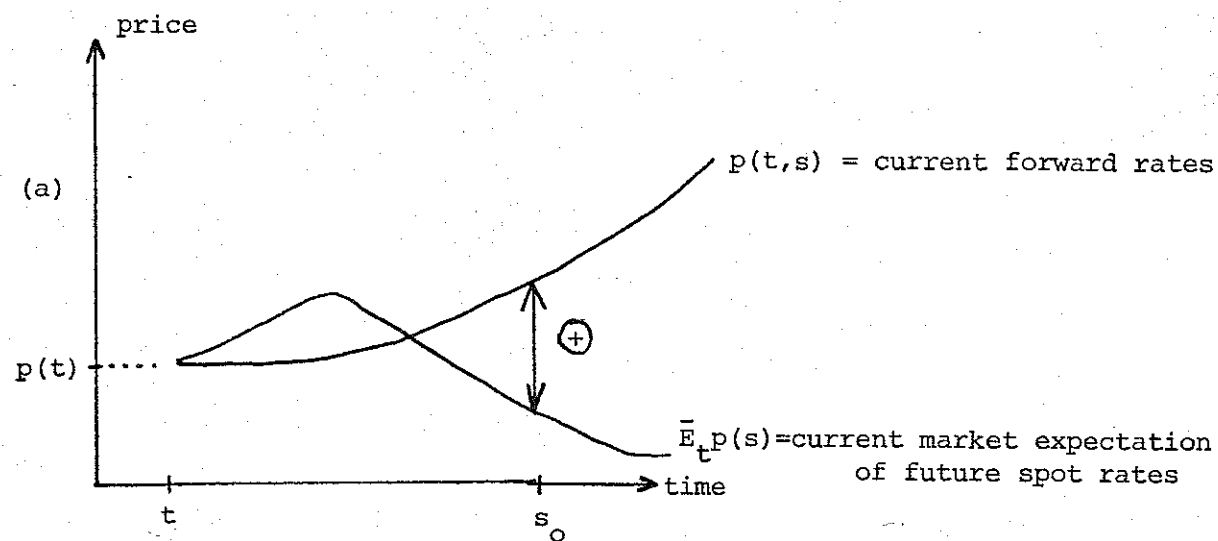
If  $p(t)$  is at its "normal" level  $\bar{p}$  and  $\hat{e}(t) - \hat{g}(t) = 0$  (i.e., any "shock" is offset), then this term must vanish, so the indicated weighted average of the "discrepancies"  $\bar{E}_{ts} p(s) - p(t,s)$  is zero. Hence the discrepancies in general are of different signs at different maturities.



Figure 1 illustrates this point. In panel (a),  $p(t, s_0)$ , the current (thrust) forward rate at maturity ( $s_0 t$ ), is an upward-biased (+) estimate of the current market expectation for the time  $s_0$  spot rate,  $\bar{E}_t p(s)$ . The bias is negative at short maturities, and is zero on weighted average, the short maturities being more heavily weighted. The configuration in panel (b) is also possible: positive discrepancies at short maturities and negative at long. Note the difficulty in interpreting the discrepancy at a specific maturity (e.g.,  $(t-s_0)$ , denoted by the vertical arrow) as a risk-premium: it can change size and even sign without any change in risk characteristics.

Given a relatively greater aggregate willingness to speculate near term, it follows from arguments of this section that the beliefs of speculators regarding near-term developments are more important than "long-run" beliefs in determining the current level of the FX rate term structure.

Figure 1: Spreading Effects



Thus, for example, the one-month forward rate should be a relatively good indicator of the "market expectation" of the spot rate one month hence, whereas the six-month forward rate should be a relatively poor indicator of the "expected" spot rate six months hence.

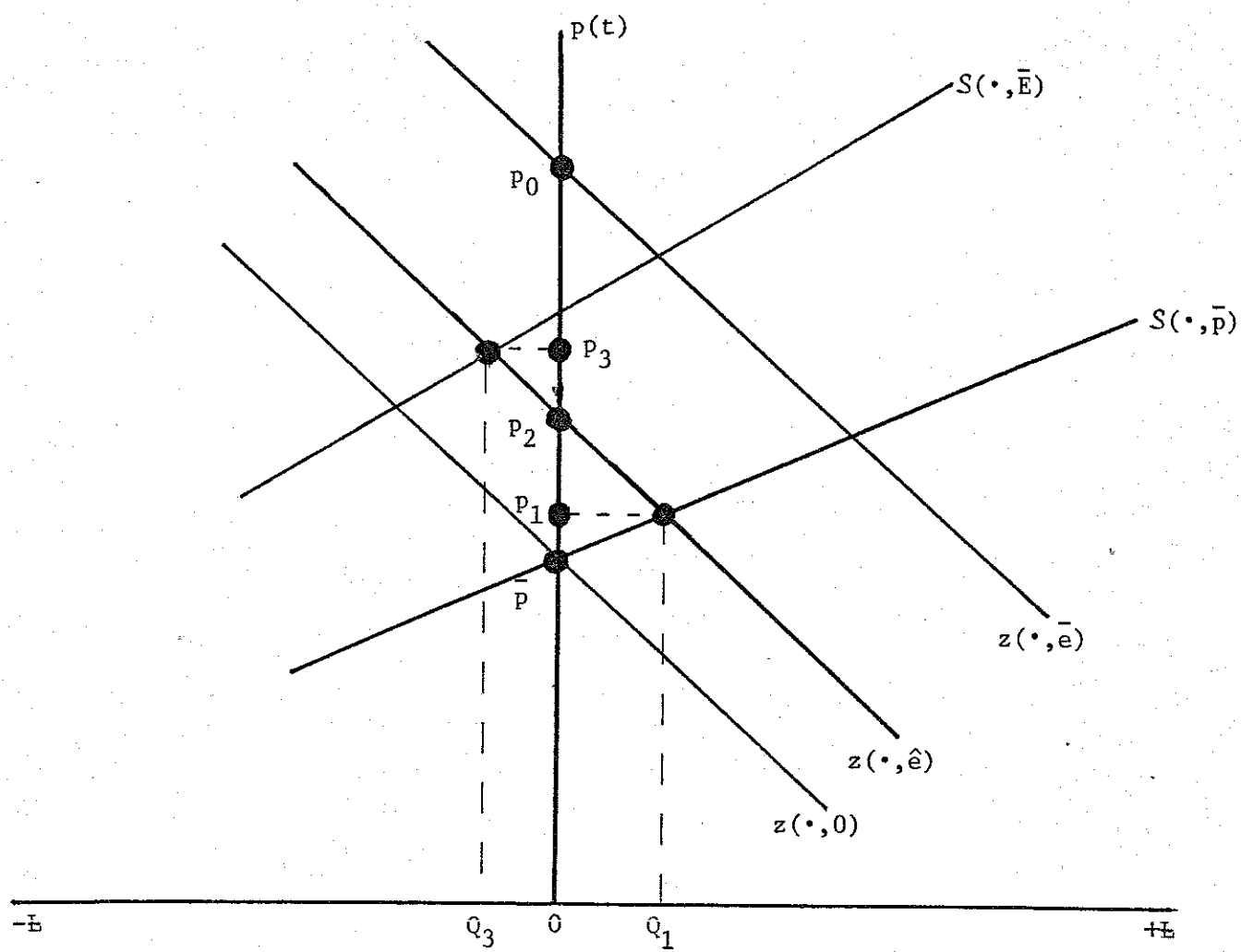
A technical point should be mentioned. In defining  $W_t$  and elsewhere, it was implicitly assumed that certain sums over all forward dates  $s$  converged. This assumption is justified if one either assumes that forward markets will not exist beyond some arbitrary horizon  $h$  (so  $s-t \leq h$ ), or else makes assumptions (e.g., bounded risk tolerance and lognormal processes) which ensure asymptotic convergence.

Much of the argument is summarized diagrammatically in Figure 2. On the vertical axis is the spot rate  $p(t)$ ; given the interest rate term structures  $r_f$  and  $r_E$ , all forward rates can be inferred from  $p(t)$  and (A1), so they need not be depicted. The horizontal axis measures net quantities of  $E$  "supplied" and "demanded." The  $z$  curves are related to excess trading demand for  $E$ ;  $z(\cdot, 0)$  is the graph for the "unshocked" case  $e = 0$ ,  $z(\cdot, \bar{e})$  is for a given shock  $\bar{e} > 0$ , and  $z(\cdot, \hat{e})$  is the graph of  $z(p, \bar{e}) - x(t) - g(t-1, t)$ , in which the shock is partially offset by hedging and previous speculation. The  $S$  curves represent the spot supply of  $E$  due to current "pure" speculation in forward FX, transmitted to the spot market by arbitrage, or equivalent "spot" speculation.

Specifically,  $S$  and  $\bar{E}$  are defined by  $S(p, \bar{E}) = -W_t(\bar{E}_t - p(t)) = -\sum_{i \in S} \sum_{s > t} \hat{a}_{its} (e^{-(s-t)r_E} E_{it} p(s) - p(t))$ . Thus  $z$  incorporates the first two terms of equation (3\*) and  $S$  the last term so that their intersection  $(p, Q)$  represents the equilibrium spot rate  $p$  and net amount of speculation  $Q$ .

For a given shock  $\bar{e}$ , the equilibrium spot rate would be  $p_0$  if there

Figure 2: Determination of FX Rates



were no hedging or speculation, but would be  $p_2$  had there been speculation (including hedging) at previous dates. If one also takes into account current speculation (and hedging) oriented towards future dates, the outcome depends on the "market average expectation"  $\bar{E}$  (as well as aggregate willingness to speculate, which determines the slope of  $S$ ). If  $\bar{E} = \bar{p}$ , then in the case illustrated, the equilibrium spot rate is reduced to  $p_1$ , but if speculators expect some higher average  $\bar{E}$ , then the spot rate is raised to  $p_3$ . The corresponding speculative supplies are  $Q_1 > 0$  (i.e., speculators are in some average sense, "short"  $\epsilon$ ) and  $Q_3 < 0$  ("long"  $\epsilon$ , implying a negative supply of  $\epsilon$ ).

I conclude this section with two numerical illustrations. First, suppose that there are only two market dates, today ( $t = 0$ ) and tomorrow ( $s = 1$ ). Traders obey the linear  $z$ -rule, with  $e(0) = 0$  for certain, and  $e(1) = 0$  or  $1$  with equal probability. There is only one speculator, who forms his expectations "rationally," knows the distribution of the shock  $e$ , and follows (1) with  $a_{ts} = R/\text{Var}_t p(s)$ , with constant risk acceptance parameter  $R$ . An arbitrageur stands ready to accommodate the speculator, and interest rate differentials are zero, so  $p(0) = p(0,1) \equiv p_0$ . There are no inherited positions or scheduled (hedgable) cash flows.

Under these assumptions, (7') implies  $p_0 = \bar{p} + G/d$ ,  $p_{10} = \bar{p} - G/d$ , and  $p_{11} = \bar{p} + (1-G)/d$ , where  $G = G(0, 1)$  is the speculator's contract, and  $p_{10}$  and  $p_{11}$  denote tomorrow's spot prices for the realizations  $e(1) = 0$  and  $e(1) = 1$ , respectively. Clearly  $E_0 p_1 = \bar{p} + (\frac{1}{2} - G)/d$  and  $\text{Var}_0 p_1 = (\frac{1}{2}/d)^2 = 1/4d^2$ . Hence  $G \equiv (E_0 p_1 - p_0) \cdot R/\text{Var}_0 p_1 = ((\frac{1}{2} - 2G)/d) \cdot R \cdot 4d^2$ , so  $G = 2dR/(1+8dR)$ . Consequently,

$$p_0 = \bar{p} + 2R/(1+8dR),$$

$$\text{and } E_0 p_1 = \bar{p} + (\frac{1}{2d} + 2R)/(1 + 8dR).$$

Note that as risk acceptance  $R \rightarrow \infty$ , we have  $G \rightarrow \frac{1}{2}$ , and tomorrow's expected spot rate  $E_0 p_1 \rightarrow p_0 = p(0,1)$ , today's forward rate. For finite values of  $R$ , however, speculation only partially compensates for the expected "shock," and the forward rate is a biased predictor of tomorrow's spot rate.

It is more difficult to compute such rational-expectations equilibrium prices when there are several forward dates. In principle, one can solve for speculators' positions and spot rates for further-term dates contingent upon nearer-term realizations of  $e$ ,  $p$  and  $G$ , and recursively work backward towards an explicit solution of the current spot rate. One finds that the computational burden increases fairly rapidly (e.g., the solution to a simple 3-date example requires finding the roots of a 6th degree polynomial).

Computation is much simpler when one takes the marginal probability distributions for future spot prices as given. For example, suppose that there are now two forward dates ( $s = 1, 2$ ) and one speculator, who again has no hedgables or inherited positions and chooses his  $G$ 's as above. Assume now that current his information leads him to believe that the possible realizations of  $p(1)$  are distributed normally  $(\mu_1, \sigma_1)$  and those of  $p(2)$  are independently distributed normally  $(\mu_2, \sigma_2)$ . If all interest rates are zero and there is no current period shock, (3\*) and (4\*) yield

$$p(0) = p(0,1) = p(0,2) = \left(\frac{d}{d+w}\right)\bar{p} + \left(\frac{w}{d+w}\right)E p, \text{ where}$$

$$w = a_{01} + a_{02} = R(\sigma_1^2 + \sigma_2^2)/\sigma_1^2\sigma_2^2, \text{ and } E p = \left(\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\right)\mu_1 + \left(\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\right)\mu_2.$$

Thus, today's spot rate and forward rates are all weighted averages of expected future spot rates ( $\mu_1$  and  $\mu_2$ ) and today's "non-speculative" spot rate ( $\bar{p}$ ), the relative weights being  $\sigma_1^{-2}$ ,  $\sigma_2^{-2}$  and  $\frac{d}{R}$ , respectively. Of course,  $\mu_1$  and  $\mu_2$  depend on our speculator's plans, but as long as  $\frac{d}{R}$  is not insignificant, it seems clear that there must be distributions of  $e(s)$  which "rationalize" given  $(\mu, \sigma)$ 's.

#### §4. Discussion

What insights does the partial equilibrium model suggest for exchange rate determination? Suppose that the UK is emerging from recession and her trade balance will tend to deteriorate, so  $e(s)$  will tend to decrease, say for dates  $s$  six to twelve months hence. To the extent that this supposition is valid and common knowledge, speculators' desired  $\mathbb{E}$  positions will decrease for this time horizon, and arbitrageurs' accommodating transactions will lower the current term structure  $p(t,s)$ , i.e., sterling will depreciate. Unless aggregate willingness to speculate is infinite, however,  $p(t,s)$  will tend to overestimate the market expectation  $\mathbb{E}_t p(s)$  over this horizon and underestimate it for  $s$  under 6 months and beyond 12 months. Correspondingly, net speculative positions would be short  $\mathbb{E}$  from 6 to 12 months out, and long at other maturities.

Of course, the model presented in the previous section is very incomplete, but the "spreading effect" it highlights survives considerable generalization. Although the notational problems would be severe, it seems conceptually straightforward to let speculators deal simultaneously in several currencies that are not perfectly correlated, and/or employ more sophisticated decision rules <sup>14</sup> than (1). In these circumstances, individual or market expectations would be more difficult to

derive<sup>15</sup> but clearly discrepancies between (adjusted) expectations and corresponding forward rates are present a fortiori. One could also specify how expectations are formed as part of the model; as long as aggregate willingness to speculate is finite and future foreign exchange rates are not known with certainty, the results of the previous section are not affected, as illustrated by the numerical examples.

Perhaps the most serious omission from the present model is a specification of interest rate determination. Clearly, \$ and £ interest rate term structures are determined simultaneously with the exchange rate term structures, and some information (e.g., inflation expectations) is relevant to both. If, for instance, £ is the currency of a small open economy and £ interest rates are quite malleable<sup>16</sup> (i.e.,  $r_{£}(t, \cdot)$  could be easily reshaped by arbitrage borrowing and lending, induced by net speculation), the interest rate differentials could align themselves to market expectations, so  $\bar{E}_t p(s) = p(t)e^{(s-t)r}$  and blurring effects vanish. On the other hand, even if strong domestic influences on interest rates made the term structures quite rigid with respect to (covered interest) arbitrage borrowing and lending, then market expectations still could align themselves to interest rate differentials if speculators induced sufficient deposit flows. That is, in terms of the partial equilibrium model, we could have  $\hat{e}(s) - \hat{g}(s)$  reshaped sufficiently by speculation to rationalize  $p(t)e^{(s-t)r}$  as the market expectation. Either way, however, speculation must be a "strong force" (in a domestic money market and/or the forward foreign exchange market) for such an alignment to occur.

In its omission of interest rate determination and classification of agents, the present model appears similar to Tsiang's (1959) Modern Theory (MT). I have already explained that my classification is really of roles



rather than agents, with arbitrary choices disappearing upon aggregation so some of the problems in the MT associated with this classification do not arise in the present approach (see also footnote 5). Perhaps the most serious defect of the MT, according to Adler and Dumas (1983, page 958), arises from the assumption that the interest rate parity (IRP) forward rate is independent of expectations. In my approach, expectations regarding all future dates affect the equilibrium forward rate, which is equal<sup>17</sup> to the IRP forward rate (see equations 3\* and 4\*). Consequently, my omission of interest rate determination does not lead to the "demonstrably false" result that the equilibrium forward rates lies between the IRP forward and the expected future spot rates.

In my opinion, it would be premature to commit to any existing theory of interest rate determination. Given its assumptions of homogeneous beliefs and no market segmentation, the standard Asset Market approach hardly provides an adequate basis for modeling money markets. Moreover, the AM approach itself is quite incomplete in postulating an exogenous (and often stationary and even Normal) random process generating asset returns.

Dornbusch (1976) pioneered a now-popular alternative to the AM approach, in which interest rates are determined from an LM-curve in a small open-economy macro model. In fact, Wilson (1979) employed such a model to obtain a result in some way, analogous to spreading effects:<sup>18</sup> if expectations shift regarding a future exogenous variable (such as forthcoming monetary policy) then the entire future path of spot rates will respond. Presumably the term structure of forward rates would also respond if incorporated in such a model. However, models of this sort rely heavily uncovered interest parity, justified by perfect foresight at almost every moment or the equivalent, so speculation is a "strong force." One could consider relaxing this assumption, but coherence requires that uncertainty

and limited aggregate willingness to speculate must appear in domestic money markets if present in exchange markets, in which case the LM-approach is inadequate.

Certain empirical implications follow if one accepts my argument for spreading effects. First, the quest for the elusive risk premium (referenced in the introduction and footnotes 2 and 3) can be abandoned on the grounds that spreading effects will probably dominate systematic risk adjustments to market expectations.<sup>19</sup> Second, the range of legitimate forecasting models for foreign exchange rates is broadened: anything that affects participants' beliefs regarding future supply and demand for foreign exchange (that is, regarding  $e(s)$  for any  $s > t$ , in terms of the partial equilibrium model) is a potential explanatory variable. More constructively, the partial equilibrium model suggests that the following sort of variables could provide useful information: (1) the determinants of near-term changes in the current account, which affect current expectations of  $e(s)$  and therefore the spot rate; (2) aggregate Commercial Bank foreign currency positions by maturity date, as a proxy for arbitrage and short-term speculative position, indicating the sign of the spreading effect at a particular maturity; and (3) estimates of recent exchange rate variances (and covariances) as a proxy for changes expected variance and thence willingness to speculate and the magnitude of the spreading effect.

To summarize, I have argued that the relationship between forward and expected future spot rates is not as simple as it might seem. If my analysis provokes new empirical work or more complete theoretical models, it will have served its purpose.

### FOOTNOTES

1. See for instance the Swiss Bank Corporation (1978) handbook, or McKinnon (1980, pp. 216-217).
2. Adler and Dumas (1983) are perhaps the most ecumenical, specifying the risk premium as the sum of two components: the first (the "inflation premium") being a weighted average of covariances of spot rate changes with inflation rates for the various currencies, and the second (the "real risk premium") being an average of asset/currency covariances weighted by shares of the world real portfolio (real outside assets); in both components the weights also reflect (absolute) risk aversion indices for investors whose wealth is denominated in the various currencies. Previously, Stulz (1980) identified covariances of exchange rates with world real consumption, and Grauer, Litzenberger and Stehle (1976) identified covariances with world nominal GNP, as determinants of the "risk premium."
3. See Adler and Dumas (1983, pp. 956-957) for a brief recent survey and Kohlhaugen (1978) for a broader survey of the relevant literature. As Dooley and Isard (1981) and Bilson (1980) point out, the variance of unadjusted forecast errors generally swamps the variance of any reasonable specification of the risk premium, so tests comparing different specifications of the risk premium are usually inconclusive. Moreover, there is evidence that current spot rates (e.g., Meese and Rogoff, 1983; Hsieh, 1981) or commercial forecasts (e.g., Levich, 1978) can outperform the forward rate, as predictors of subsequent spot rates. Pre-

sumably the modest risk adjustments suggested by theories cited in the previous footnote would not reverse this conclusion.

4. As an empirical matter, covered interest parity virtually always holds up to bid-ask spreads, with respect to Eurocurrency interest rates, due to the pricing practices of forward dealers mentioned in the introduction. See Aliber (1973) for the classic discussion of interest parity. Note that, given covered interest parity, foreign currency can be regarded as a storable good, the (opportunity) cost of storage being the interest rate differential.
5. Hence the determination of foreign exchange rates in the model is unaffected if some speculators chose indirect "spot" speculation (i.e., borrow one currency in the Euromarkets, exchange it spot and deposit) rather than direct forward speculation as postulated in Section 3, whereas this choice undermines the Modern Theory because interest rate parity does not typically hold there -- see Phaup (1981).
6. For example, a foreign dockworkers' strike that will affect next month's imports, or an impending replacement of monetary policymakers.
- 6A. If arbitrage is like picking up a \$1 bill lying in clear view on the sidewalk, then speculation is like putting your own \$1 bill into a slow and faulty change machine that you believe gives (on average) more change than it should. Especially if a long and tedious wait is required on each attempt to use the machine, it presumably will attract less immediate and overwhelming attention than the \$1 on the sidewalk. The analogy is crude but indicates why arbitrage should normally exert a stronger force than speculation.

7. One must also assume unanimity of beliefs across speculators in this case, since otherwise one has "irresistible" speculative forces pulling in opposite directions.
8. See McKinnon [1980, p. 151] for the contrary position that may be negative at least in the very short run.
9. Here and elsewhere, an omitted agent subscript indicates summation. Thus  $g(t-1, t) = \sum_{i \in S} g_i(t-1, t)$ .
10. Actually, a shock  $e(t)$  could be reinforced by speculators if it were believed to be permanent. In discussions I will for simplicity write as if  $e(t)$  were "temporary" and had the same sign as  $g(t-1, t)$ .
11. See Danthine (1978, pp. 82-83) for an analogous result.
12. The weights in (4') may differ very slightly from those in (4) because risk aversion  $R_i$  is evaluated at a wealth that takes the hedgeables  $x$  into account. A complication not discussed here is that the scheduling of cash flows could be endogenous, in which case (4') would still be correct but less informative.
13. As with exchange rates, I ignore bid-ask spreads for simplicity. I also ignore default risk.
14. For instance, rules that take into account the covariance of future spot rates across different dates, or rules for "forward-forward" trading.

15. In the multi-currency case, for example, one would have to insert terms involving spot rate covariances.
16. See Beenstock and Longbottom (1981) for an analysis of this case.
17. If domestic interest rates, rather than Eurocurrency interest rates, are used in my approach, the IRP and equilibrium forward rates can differ, but in the absence of actual or anticipated capital controls or other regulatory barriers, the differential should be rather small and predictable.
18. My thanks to an anonymous referee for pointing out this analogy.
19. If interested in risk premia for their own sake, rather than as a means to improve the predictive power of forward rates, one could proceed as follows. Find appropriate weights for a set of forward maturities, leased on average open interest in exchange rate futures, for example. Then try to explain weighted average error in the implied forward forecasts by the usual explanatory variables for a risk premium.

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