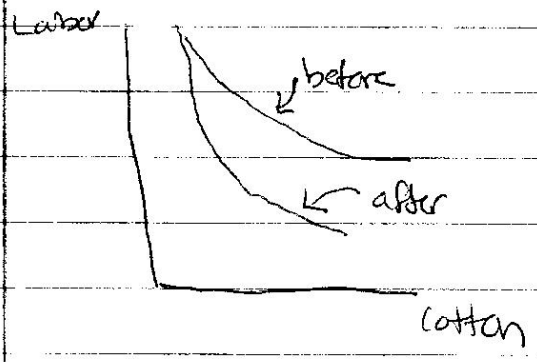
Problem Solution Set 3

Chapter 16

14) Technical progress is not neutral. Less labor was required to spin the same amount of cotton.



21) a) $MP_R^W = (2.5)(0.64)A^{0.36}R^{-0.36} = 1.6(A/R)^{0.36}$

$MP_R^P = (2.5)(0.75)A^{0.25}R^{-0.25} = 1.875(A/R)^{0.25}$

b) $MRTS^W = -MP_A/MP_R = -\left(\frac{9}{16}\right)\left(\frac{R}{A}\right)$) assuming A on x-axis

$MRTS^P = -MP_A/MP_R = -\frac{1}{3}(R/A)$

c) Yes. Consider:

$G_W = 3A^{0.25}R^{0.75}$ and $G_P = 2.5A^{0.25}R^{0.75}$

These have same MRTS but different marginal products.

26) a) constant returns to scale

b) if $\alpha+\beta=1$, constant

if $\alpha+\beta>1$, increasing

if $\alpha+\beta<1$, decreasing

c) same as above

d) if $d=1$, constant $d<1$, decreasing

$d>1$, increasing

extra credit

30) If $f(xL, xK) = x^y(L, K)$ then $\frac{\partial f}{\partial L} = xf_1'(xL, xK) = x^y f_1'(L, K)$
and $f_1'(xL, xK) = x^{y-1} f_1'(L, K)$ then $\frac{\partial f}{\partial K} = f_2'(xL, xK) = yx^{y-1} f_2'(L, K)$

31) from above with $y=1$, $f_1'(xL, xK) = f_1'(L, K)$ and
$f_2'(xL, xK) = f_2'(L, K)$ which implies:

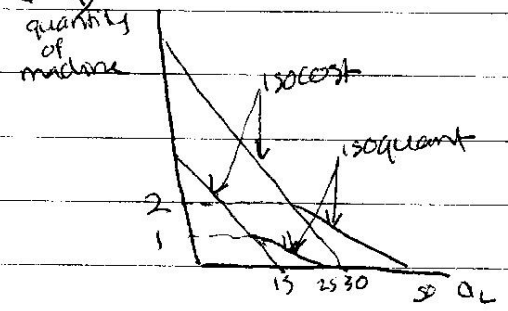
$$\frac{f_1'(xL, xK)}{f_2'(xL, xK)} = \frac{f_1'(L, K)}{f_2'(L, K)}$$

and the MRTS is independent of x.

32) If $f(xL, xK) = x^y f(L, K)$ then $\frac{\partial f}{\partial x} = Lf_1'(xL, xK) + Kf_2'(xL, xK) = yx^{y-1} f(L, K)$.
Set $x=1$: $Lf_1'(L, K) + Kf_2'(L, K) = yf(L, K)$

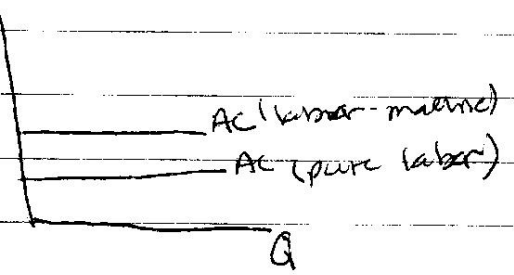
Chapter 7

16 a)



b) The firm chooses labor/machine technology

c)



23) $AC(q) = 0.55q^{0.67} + 800q^{-2}$

$\frac{\partial AC(q)}{\partial q} = 0.3685q^{-0.33} + 1600q^{-3} = 0$

$q^{2.67} = 4341.93$

$q \approx 23$

with the tax

$AC(q) = 0.55q^{0.67} + 800q^{-2} + 400q^{-1}$

$\frac{\partial AC}{\partial q} = 0.3685q^{-33} + -1600q^{-3} - 400q^{-2} = 0$

$q \approx 68$

27) a) You must set $\frac{\partial AC}{\partial q} = 0$ for each firm. The minimum point of AC_1 is at $q = 2$. At plant 2, the min is 1

b) The firm should produce 3 units in plant 1, and 1 unit in plant 2.

31) Suppose capital is fixed at $\bar{K}$ $F = r\bar{K} = 20\bar{K}$

$VC = wL = 10L$

Total Cost: $C = F + VC = 20\bar{K} + 10L$

$q = 10L^{0.32}\bar{K}^{0.32} \Rightarrow$ ~~$L = (0.1q)^{3.125}$~~ $L = 0.1q\bar{K}^{-0.32})^{0.32} =$

$AVC = VC/q =$ ~~$0.0075q^{3.125}$~~ $\frac{10L}{q}$

Suppose $\bar{K} = 1$ $L = (0.1q)^{3.125}$; $VC = 10L = 0.0075q^{3.125}$

$AVC = VC/q = 0.0075q^{2.125}$

$MC = dV/dq = d(0.0075q^{3.125})/dq = 0.023q^{2.125}$

36) $MP_L / MP_K = [0.7q/L] / [0.3q/K] = \frac{w}{r}$

In the US, $w/r = 7/3 \rightarrow L = K$; $q = L^{0.7} K^{0.3} = K$

when $q = 100$, $K = L = 100$ $C = 1000$

In Asia, $w/r = 7/9 \rightarrow L = 3K$ $q = L^{0.7} K^{0.3} = 2.16K$

when $q = 100$, $K = 46.3$, $L = 138.9$, $C = 694.5$

If it had to use same factor quantities as US ($K = L = 100$) $C = 350 + 450 = 800$

extra credit

37)