Midterm Exam

Econ 200

Instructions. When insufficient information is provided, write down an explicit and reasonable assumption, and proceed. Show calculations where relevant. Keep answers brief. Closed book, except for attached formula sheet. 50 points total, as marked.

1. North Ifstan (NI) has domestic suppliers of internet services whose monthly supply curve is well approximated by $S(p) = 10p$, while monthly demand is well approximated by $D(p) = 100010p$. International suppliers can provide any amount of access at $p = 25$.

a. Compute the current competitive equilibrium (CE) price, domestic producers surplus (PS), and consumer surplus (CS). (6pts)

b. The NI government is considering a rule that would eliminate foreign supply. How much (if at all) would domestic suppliers benefit? How much would consumers lose? (4pts)

c. Extra credit. What is the maximum amount that domestic suppliers would pay the NI government decision makers to impose that rule? [max 2pts]

Solution: Given $D(p) = 1000 - 10p$ and $S(p) = 10p$, we can derive competitive price, p^* from $D(p^*) = S(p^*)$, or

$$1000 - 10p^* = 10p^*. \tag{1}$$

Thus, $\underline{p^* = 50}$, and substituting this equilibrium price into demand or supply function yields competitive equilibrium quantity, $\underline{q^* = 500}$.

When international suppliers are willing to supply any amount at $p = 25$, supply function changes and the new supply function, $S_w(p)$, is given by,

$$S_w(p) = \begin{cases} 10p & \text{if } p < 25 \\ \infty & \text{if } p \geq 25. \end{cases} \tag{2}$$

Given the demand function, $D(p)$, and the new supply function, $S_w(p)$, we can derive new competitive equilibrium. Since demand equals supply at $p = 25$, the competitive equilibrium price is, $\underline{p_w^* = 25}$ and by substituting the equilibrium price into demand function yields, competitive equilibrium quantity, $\underline{q_w^* = 750}$.

By substituting the equilibrium price into supply function, the amount domestic firms supply at this equilibrium price is, $\underline{q^*_{w,d} = S(p^*_w) = 250}$.

a) As calculated above, competitive equilibrium price with international suppliers is,

$$p^*_w = 25. \tag{3}$$

Domestic producer's surplus is the area between equilibrium price and domestic supply function,

$$PS_w = \frac{1}{2}(25 - 0)(250 - 0) = 3125. \tag{4}$$

Consumer surplus is the area between demand function and equilibrium price,

$$CS_w = \frac{1}{2}(100 - 25)(750 - 0) = 28125. \tag{5}$$

b) In the same way, producer surplus and consumer surplus without international suppliers are

$$PS = \frac{1}{2}(50 - 0)(500 - 0) = 12500, \tag{6}$$

$$CS = \frac{1}{2}(100 - 50)(500 - 0) = 12500. \tag{7}$$

Thus, the change in PS and CS respectively are,

$$\Delta PS \equiv PS - PS_w = 9375, \tag{8}$$

$$\Delta CS \equiv CS - CS_w = -15625. \tag{9}$$

c) Therefore, the maximum willingness to pay for domestic suppliers to maintain the governmental import policy is 9375 per month. The present value of that payment stream is the max WTP as a lump sum. E.g., if the policy is expected to remain for decades, and the interest rate is 1% per month, then PV $\approx \frac{1}{.01}9375 = 937500$.

2. Your client, the Tricounty Organic Strawberry Association, provides data on their sales revenue, from which you estimate the demand function for their product.

a. What other data will you need to estimate income elasticity , own price elasticity and cross price (for inorganic strawberries) elasticity c? (2pt)

b. Write out a convenient equation to estimate these elasticities from that data. (3pts)

c. Suppose that you estimate $\eta = 1.7$, $\varepsilon = -1.2$, and $\varepsilon_c = 0.4$. A former classmate comments that your estimates cant be right because the elasticities should sum to 0. How should you respond? (4pts)

Solution: We only have to estimate Marshallian demand functions. Marshallian demand functions are a function of own price, cross prices, and income.

a) We need data on the price of the product, the price of the rival product, and average income to estimate own price elasticity, cross price elasticity, and income elasticity.

b) Then we can run a regression such as,

$$\ln q_1 = \alpha_0 + \alpha_1 \ln p_1 + \alpha_2 \ln p_2 + \alpha_3 \ln m + error. \tag{10}$$

The coefficients α_1, α_2, α_3 are own price elasticity, cross price elasticity, and income elasticity respectively.

c) The classmate is right. We must have $\varepsilon_{11} + \sum_{i=2}^{n} \varepsilon_{1i}+$ all other cross price elasticities $+\eta_i = 0$, because everyone's budget constraint is homogeneous of degree 1. One possible reason for the discrepancy is that we did not include prices of products other than strawberry. If there are complements, cross price elasticity of those goods are positive and the discrepancies diminishes by including those prices in the regression. Another possible reason is that we mis-specified the regression, e.g., the elasticities are not constant. Finally, the data might be inadequate, especially the income data; using mean income assumes that all households' income move proportionately. Still, these effects should not be large, so it probably would be more accurate to impose the coefficient constraint on the regression, that the elasticities sum to 0.

3. Briefly explain the differences between (a) increasing returns to scale, (b) learning curve, (c) decreasing (average) cost, and (d) economies of scope. For (a, b, c), mention the implications for competitive equilibrium price in the long run. (6pts)

Solution: Several concepts on cost functions.

(i) Only learning curve has the relationship between cost and the amount produced in the past. The more you produced in the past cumulatively, average cost today is lower.

(ii) Only economies of scope talks about the cost relationship across different goods. It is cheaper to produce jointly than produce two goods separately.

(iii) IRS implies decreasing AC and vice versa. The more you produce, average cost is lower.

Learning curve implies that competitive equilibrium price decreases over time. IRS and decreasing AC implies that competitive equilibrium can only exist when IRS and decreasing AC disappear at the scale firms produce to maximize profits.

4a. Blutarsky is planning a fraternity party. He cares only about alcohol content. Write down a utility function for him for x_1=bottles of beer and x_2= bottles of vodka, given that each bottle of vodka has the same amount of alcohol as three six packs of beer. (3pts)

b.Justines preferences can be represented by $u(x_1, x_2) = \ln x_1 + \ln x_2$. Which of the following utility functions (if any) also represent her preferences? $v(x_1, x_2) = x_1 + x_2$, $w(x_1, x_2) = x_1^{0.4} x_2^{0.4}$, $U(x_1, x_2) = x_1 + g(x_2)$. Explain very briefly. (3pts)

Solution: Utility functions.

a) Blutarsky's utility comes only from alcohol content. One bottle of vodka, good 2, contains the same amount of alcohol as 18 bottles of beer, good 1, so these are perfect substitutes at that ratio. Therefore, (one of the) utility functions is,

$$U_B(x_1, x_2) = x_1 + 18x_2. \tag{11}$$

b) Justine's utility function is $u(x_1, x_2) = \ln x_1 + \ln x_2$. Any monotonic transformation of utility function preserves preference relationship, but you can also check whether two utility functions represent the same preference or not by comparing MRS because Marshallian demand requires MRS equals the slope of budget constraint and the slope is fixed here.

MRS of Justin's utility function is,

$$MRS_J = -\frac{u_1}{u_2} = -\frac{x_2}{x_1}. \tag{12}$$

MRS of three candidates are,

$$MRS_v = -\frac{1}{1} = -1, \tag{13}$$

$$MRS_w = -\frac{0.4x_1^{-0.6}x_2^{0.4}}{0.4x_1^{0.4}x_2^{-0.6}} = -\frac{x_2}{x_1}, \tag{14}$$

$$MRS_U = -\frac{1}{g'(x_2)}. \tag{15}$$

Therefore, only utility function $w(x_1, w_2)$ represents the same preference as $u(x_1, x_2)$.

5. Suppose that each firm in an industry has long run total cost function $c(y_i) = y_i^3 - 9y_i^2 + 36y_i$, and they face industry demand curve $y_T = 200 - 10p$.

a. What is each firms marginal cost function? Average cost function? Fixed cost? Supply curve? (4pts)

b. What is long run competitive equilibrium price?Output per firm? Number of firms? (3pts)

c. Compute the producer surplus (PS), consumer surplus (CS) and total surplus (TS) at long run competitive equilibrium. (4 pts)

Solution: Individual cost function is given as $c(y_i) = y_i^3 - 9y_i^2 + 36y_i$

a) Since cost function is given, it is immediate to derive marginal cost function, average cost function, and fixed cost. Since there is no term which does not depend on y_i, fixed cost is 0 and this means that average variable cost equals average cost. Therefore,

$$FC = 0, \tag{16}$$

$$MC \equiv c'(y_i) = 3y_i^2 - 18y_i + 36, \tag{17}$$

$$AC \equiv \frac{c(y_i)}{y_i} = y_i^2 - 9y_i + 36 = AVC. \tag{18}$$

Supply function is a portion of the MC curve, but firms produce positive amount only if the profit from producing is higher than not producing. Firms earn higher profit from producing when price is higher than the lowest value of AVC. Since we can rewrite AVC as

$$AVC = y_i^2 - 9y_i + 36 = \left(y_i - \frac{9}{2}\right)^2 + \frac{63}{4}, \tag{19}$$

the lowest value of AVC is $\frac{63}{4}$ when $y_i = \frac{9}{2}$. You can also derive this using $MC = AVC$. Therefore, the inverse supply function is,

$$p = S_i^{-1}(y_i) = 3y_i^2 - 18y_i + 36 \quad \text{for } y_i \geq \tfrac{9}{2}. \tag{20}$$

To obtain the supply function, solve that equation for y_i in terms of p in the relevant range. Using the quadratic formula and simplifying, you get

$$y_i = S_i(p) = \begin{cases} 3 + (\frac{p}{3} - 3)^{\frac{1}{2}} & \text{if } p \geq \frac{63}{4}, \\ 0 & \text{if } p < \frac{63}{4}. \end{cases} \tag{21}$$

b) In the long run, as long as firms can earn positive profit, potential entrants enter the market. Therefore, in the long run, price equals the lowest value of AC. In this simple example, since $FC = 0$, the lowest value of AC equals to that of AVC, which we derived already. Therefore, equilibrium price, p^*, is

$$p^* = \frac{63}{4}, \tag{22}$$

and given this equilibrium price, each firm determines the amount of supply by the optimality condition $P = MC$. However, since MC goes through the minimum of AC and p^* is the minimum value of AC, we already know that the optimal amount to supply is,

$$y_i^* = \frac{9}{2}. \tag{23}$$

You can also find this value by $MC = AC$.

Aggregate supply function, $S(p)$, is the sum of individual supply functions. As we derive, if $p < \frac{63}{4}$, no firms enter the market and if $p = \frac{63}{4}$, every firm is willing to supply $y_i^* = \frac{9}{2}$. Therefore, aggregate supply function is perfectly elastic at $p = \frac{63}{4}$,

$$S(p) = \begin{cases} \infty & \text{if } p \geq \frac{63}{4}, \\ 0 & \text{if } p < \frac{63}{4}. \end{cases} \tag{24}$$

Given aggregate supply function and aggregate demand function, we can derive competitive equilibrium. Since demand equals supply at $p = \frac{63}{4}$, the competitive equilibrium price, p^*, is

$$p^* = \frac{63}{4}. \tag{25}$$

By substituting equilibrium price into demand function yields equilibrium quantity,

$$q^* = 200 - 10\left(\frac{63}{4}\right) = \frac{85}{2}. \tag{26}$$

Since each firm produces $y_i^* = \frac{9}{2}$ and aggregate output is $q^* = \frac{85}{2}$, the number of firms are

$$N = \frac{\frac{85}{2}}{\frac{9}{2}} = \frac{85}{9} < 10. \tag{27}$$

So, N=9 in LRCE, because a 10th firm would earn a negative profit by entering this market.

Consumer surplus is the area between demand curve and equilibrium price, producer surplus is the area between equilibrium price and supply curve,

$$CS = \frac{1}{2}(20 - \frac{63}{4}) * \frac{85}{2} = \frac{1445}{16}, \tag{28}$$

$$PS = \frac{1}{2}(\frac{63}{4} - \frac{63}{4})\frac{85}{2} = 0, \tag{29}$$

$$TS = CS + PS = \frac{1445}{16}. \tag{30}$$

6. Your client wants to plan how to respond to changes in key input prices (electricity, ore, labor) to achieve given levels of output. What sort of data should you gather, and how might you analyze it? (8pts)

Solution: There are mainly two steps to derive conditional factor demand functions.

First, we have to estimate cost function. We need data on cost c, input prices w_i, and output y. Then we can run a regression such as

$$\ln\frac{c}{y} = \alpha_0 + \alpha_1 \ln w_1 + \alpha_2 \ln w_2 + error. \tag{31}$$

Then solve the fitted equation for c.

Second, we can derive conditional factor demand functions from cost function using Shephard's lemma. Now we can analyze how to respond to changes in input prices. So to answer your client's question, you just differentiate the c function you obtained wrt the two input prices.