

### Present Value

$PV = [x_1/(1+i)^1] + [x_2/(1+i)^2] + \dots + [x_n/(1+i)^n]$ ;  $= x_1/i$  if  $x_i$  const and  $n = \infty$ .

### Elasticity

$$E_{Q,y} = \frac{\partial \ln Q}{\partial \ln y} = \frac{\partial Q}{\partial y} \frac{y}{Q} \approx \frac{\% \Delta Q}{\% \Delta y}$$

Own price elasticity: set  $y = P_x$

Cross-price elasticity: set  $y = P_y$

Income elasticity: set  $y = I$

Log-linear Demand function:

$$\ln D_x = a + b \ln P_x + c \ln P_y + d \ln I$$

### Production Process and Costs

Production Function:  $Q = F(K, L)$

Marginal Product of Labor:

$$MP = \Delta Q / \Delta L$$

Average Product of Labor:  $APL = Q/L$

Cost Function:  $C(Q) = VC + FC$

Avg Cost:  $AC = C(Q)/Q = AVC + AFC$

Marginal Cost ( $\approx$  Incremental Cost):

$$MC = \partial C / \partial Q \approx \Delta C / \Delta Q$$

Economies of Scope:

$$C(Q_1, Q_2) < C(Q_1, 0) + C(0, Q_2)$$

Cost Complementarity:  $\partial MC_1 / \partial Q_2 < 0$ .

Learning Curve:  $AC = a - b \ln A$ , where  $A$  = accumulated output

Economies of Scale:  $\partial AC / \partial Q < 0$ .

### Nature of Industry

Four-Firm Concentration Ratio:

$$C4 = FFI = w_1 + w_2 + w_3 + w_4$$

$$HHI = 10,000 \times \sum w_i^2$$

Rothschild Index:  $R = E_I / E_F$

Lerner Index:  $L = (P - MC) / P$

Markup Factor:  $P = (1/(1-L)) MC$

### Managing Markets

Perfect competitive firms max profits when

$$MC = MR = P$$

Long run equilibrium:  $P = \min AC$

Multi-plant monopoly:

$$Q = Q_1 + Q_2$$

$$MR(Q_1 + Q_2) = MC_1(Q_1)$$

$$MR(Q_1 + Q_2) = MC_2(Q_2)$$

### Optimal Ad budget

$$A/R = -E_A / E_P$$

### Oligopoly

Given linear (inverse) demand:

$$P = a - b(Q_1 + Q_2), \text{ and}$$

Constant MC w/zero FC:

$$C_1(Q_1) = c_1 Q_1$$

$$C_2(Q_2) = c_2 Q_2, \text{ the}$$

Reaction function (Cournot) is:

$$Q_1 = r(Q_2) = (a - c_1) / 2b - Q_2 / 2; \text{ and the}$$

Stackelberg Leader's output is

$$Q_1 = a + c_2 - 2c_1 / 2b.$$

In Bertrand Oligopoly with homogeneous goods, the price is driven down to the second lowest MC.

In Contestable markets, the price is driven down to the second lowest AC.

### Simple Markup Rule

$$P = [E / (1 + E)] \times MC$$

For **Cournot Oligopoly** with  $N$  firms, elasticity is  $E = N \times E_M$ , where  $E_M$  is the market elasticity.

For a **3<sup>rd</sup> degree price discriminator**, use different  $E$  for each segment, e.g.,

$$P_1 = [E_1 / (1 + E_1)] \times MC$$

$$P_2 = [E_2 / (1 + E_2)] \times MC$$

For basic **2-part pricing** set price = MC, compute consumer surplus, and set the fixed-fee equal to consumer surplus.

### Transfer Pricing

$$NMR_d = MR_d - MC_d = MC_u$$

$$\text{Mean} = E[x] = q_1 x_1 + q_2 x_2 + \dots + q_n x_n,$$

$$\text{Variance} = \sigma^2 = \text{Var}[x] = q_1 (x_1 - E[x])^2 + q_2 (x_2 - E[x])^2 + \dots + q_n (x_n - E[x])^2$$

$$\text{Std dev} = \sigma = \text{sqrt of Var}[x]$$

$$CE = E[x] - RP, \text{ where } RP = (1/2) r \text{Var}[x]$$

### Profit max in uncertainty

$$E[MR] = MC$$

**Auctions**- 1<sup>st</sup> price auction (or Dutch) with  $n$  bidders and uniformly distributed valuations above  $L$ : Optimal bid is  $B = v - ((v-L)/n)$ .

Revenue comparisons: for IPV, risk-neutral

English = 1<sup>st</sup> price = 2<sup>nd</sup> Price = Dutch.

For Affiliated Values, English > 2<sup>nd</sup> Price > 1<sup>st</sup> price  $\geq$  Dutch.

### Limit Pricing is profitable if

$$\frac{(\Pi^L - \Pi^D)}{i} > \Pi^M - \Pi^L$$