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To whom it may concern,

Please note that the following working paper is based on initial pilot results.

However in early October we have nine production sessions involving over fifty subjects scheduled. If accepted, the presenter will discuss these results.

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Hotelling in Continuous Time

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We study Hotelling's classic location model in continuous time with flow payoffs accumulated over time and the price dimension made explicit. We find that the principle of minimum differentiation generally holds, although in the majority of cases positions — especially on the price dimension — do not stabilize. Our results also support literature that the ability to respond quickly increases cooperation.

Harold Hotelling's seminal paper Stability in Competition is well-known by any economist or economics student, and continues to gather citations even 85 years after its publication. In the classic model, two firms compete on geographic location only, which Hotelling suggested but did not prove would lead to price equilibrium between firms. This has since been proven to be incorrect, as firms can always improve profits by moving to a new position after a competitor's move, be it a new location or new price point. The disincentives to settle contrast starkly with Hotelling's result of firms located adjacent to one another to maximize profits, which has been applied extensively to phenomena ranging from industrial organization to politics.

We adhere closely to the original model and question whether the ability to rapidly adjust position can induce firms to cooperate to achieve higher profits. In a continuous time setting in the laboratory, we examine variants of the Hotelling model in which pairs of anonymously matched subjects can adjust their price

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and location during four-minute periods. Treatments vary in how often they are allowed to adjust their position, ranging from free adjustment on either dimension to being limited to adjustment on only one dimension during set blocks of time. Subjects accrue flow payoffs throughout the period that depend simply on their position relative to their counterparts in that moment in time. Subjects are randomly rematched after each period, with sessions running between 11 and 13 periods.

Our results indicate that free and unlimited adjustment leads to higher payoffs for subjects, while limiting adjustment lowers prices and payoffs and decreases cooperation. We provide circumstantial evidence that cooperation arises from signaling from one of the subject pairs, either through momentary jumps to desired positions or willful loss in payoffs while waiting for a counterpart to fall in line. However, with cooperation rates at thirty percent at most, it is clear that the Hotelling model is not one that leads easily to a settled state, and it should only be used very cautiously when explaining real world events. Section 1 provides background on Hotelling's seminal model and its evolution, and Section 2 recalls his original notation. Section 3 details the experimental design, Section 4 gives the results of the experiment, and Section 5 concludes.

I. Background

Hotelling's original model characterized the stylized fact that individuals buy commodities from different sellers despite modest differences in price. The model is a simple location model in which firms decide how to position their product in a linear product space. This space is taken to be location for the purposes of this discussion, although the model has been adapted extensively to fit the political spectrum after Hotelling's suggestion in his paper. Firms face a constant price and a uniformly distributed mass of potential consumers that will buy at most one unit from one of the firms as determined by a specified utility function. Consumer utility is decreasing in distance, and as such consumers will purchase the homogenous good from the closest vendor. With two firms, this will lead firms located back-to-back in the middle; this is to say, the firms produce identical products. This is known as the *principle of minimum differentiation*.

It is appropriate to distinguish between horizontal and vertical differentiation. With horizontal differentiation, not all consumers will agree on which firm or product is preferred. With fixed prices, Hotelling's paper only discusses horizontal differentiation in the form of a location choice by the firm. With vertical differentiation, all consumers will agree which product is preferred, all else held constant. The dimension is taken to be be price for the purposes of this discussion, although it could also be product quality with prices held constant.

A. *Evolution of Hotelling's Original Model*

The original model has been appropriated to attempt to explain a wide range of phenomena, concentrated most densely in the political science and industrial organization literatures. These applications range from voting habits (Downs, 1957) to entry deterrence (Schmalensee, 1978) to competition in specific industries (Baum and Mezias (1992) and Calem and Rizzo (1995), for example).

Unfortunately, however, many of these applications do not take into account how sensitive the Hotelling location model is to small changes to the setup and set of assumptions. Eaton and Lipsey (1975) detail equilibria for more than two players, and show that minimum differentiation does not generalize easily even if local clustering tends to emerge. With four players, for example, the equilibrium has two players on each of the first and third quartiles. With three players, however, there is no way to satisfy the pure equilibrium conditions. Salop (1979) changed the linear city to a circle — among other alterations — effectively making the product space a line of indefinite distance. This results in maximum differentiation in product space such that firms are evenly distributed around the circle.

Perhaps most significantly, D'Aspremont et al. (1979) show that the principle of minimum differentiation does not hold due to the non-existence of a price equilibrium when firms are not sufficiently far from each other. This is because demand is discontinuous when firms are located close together. They propose a simple modification to the consumer utility function — quadratic instead of linear transport costs — that restores the continuity of demand and allows for a price equilibrium anywhere. However, this changes the sign of the derivative of the firms' profit functions with respect to location from positive to negative; this is to say, firms then locate as far from each other as possible.

This prompted a number of authors to implement alterations to the model to remedy this equilibrium non-existence problem. Graitson (1980) assumes "maximin" behavior, in which a firm that is too close to an opponent sets a price that maximizes its profit function taking the other firm's price to be zero. This leads to a "maximin equilibrium" which gives an equilibrium with firms located at the first and third quartiles charging Nash-Cournot prices. Neven (1985) integrated the D'Aspremont et al. suggestion of quadratic transport costs and gave the model two stages, where firms rst engage in horizontal differentiation before choosing a price.[1] He shows that a pure strategy price equilibrium exists for every pair of products, and confirms that firms maximize horizontal differentiation in equilibrium. Generally, subsequent work — Cremer et al. (1991), for example — followed the quadratic transport costs approach, although Economides (1986) examined a range of utility costs of transportation and showed that not all specifications resulted in maximal differentiation. However, the problem of equilibrium existence within Hotelling's original setup lingered, and increasingly nuanced approaches attempted to tackle the problem.[2]

Despite these result-altering breakthroughs, Hotelling's conclusions are still widely cited, if only casually. This paper explores the applicability of his original

[1]Hotelling himself hinted at a two stage approach when prices were decision variables, for the reason that prices are easier to adjust than locations or product variety. Other papers — such as Graitson (1980) and Economides (1984) — tacitly followed this approach.

[2]See Gabszewicz and Thisse (1986) for an early overview and Caplin and Nalebuff (1991) for an approach that gives conditions for a pure strategy price equilibrium.

result as well as the robustness of the more nuanced conclusions of subsequent models in an experimental setting.

B. Previous Experimental Work

There have been a number of attempts to test the Hotelling model in an experimental setting. Brown-Kruse and Schenk (2000) — along with its predecessor, Brown-Kruse et al. (1993) investigate a two-player uncertain endpoint model, but focus on the effect of communication on collusion. They find that communication led participants to locate near the quartiles to maximize joint profit, but the principle of minimum differentiation did seem to hold in their results when communication was limited. Huck et al. (2002) were the first to test a four-person Hotelling game, but found little support for the equilibrium hypothesis. Other work from our lab showed that the four-player Nash equilibrium emerged more quickly with the ability to adjust location instantly.

Very few authors have included vertical differentiation in an experimental setting. This could be due to the equilibrium existence problems discussed above, or due to a lack of technological capability. To our knowledge, only three works have tested a game with price as a choice variable. The first attempt was by Mangani and Patelli (2002), who specified their model with quadratic transport costs such that theory would suggest subjects should follow a "minimax" strategy first outlined by Graitson (1980). The authors tested this with three treatments: a two-stage location then price game mirroring Neven's setup, a treatment with only periodic location adjustments, and a treatment in which price and location were chosen in the same period. Subjects, however, tended to minimize differentiation, although prices were generally higher than competitive levels. The authors suggested risk aversion as an explanation, but their conclusions are weakened by using successive research stages. Kusztelak (2011) allowed limited communication between subjects as well as also including quadratic transport costs, and subjects behaved as expected with maximal differentiation in a location only treatment.

But when a price decision was added, the differentiation decreased significantly. Finally, Barreda et al. (2011) attempted to test the hypothesis that firms use product differentiation to relax price competition by focusing on a limited, discrete location decision. Specifically, subjects could only choose among either seven or eight location slots, depending on the treatment. In their most relevant treatments, the authors found less product differentiation than theory would predict, and relatively few high prices.

This work contributes to the literature in several important ways. First, this experiment is the cleanest test to date of a simple two-player Hotelling model with horizontal and vertical decision variables and linear transport costs, thus giving a better view into how the ability to adjust quickly affects firms behavior. Second, this experiment takes a novel approach by blurring the sharp distinction between the Hotelling continuous choice model and sequential models, providing insight into how Hotelling's seminal result should be applied. Finally, superior lab software and programming gives participants a more intuitive interface and faster learning experience to allay concerns of participant apprehension skewing results in the competitive setting.

II. Model and Predictions

First, we begin by recalling Hotelling's assumptions and notation. There are customers evenly distributed on a line of length l, with firms A and B selling a homogenous product with zero production cost.[3] For simplicity, we normalize l to be 1. Each customer consumes one unit of the good, and will buy from the seller who gives the least delivered price. Firms locate at points a and b ($a+b=l$, $a>0$,$b>0$) and set prices p_A and p_B. Transport costs are linear and are denoted by c.

First, consider the case that price and location are chosen simultaneously. Payoff functions for A and B are given by:

[3]This experiment only examines the two-person game, but can be generalized to n sellers. See Brenner (2005) for a derivation of the model with more than two players.

$$\pi_A(p_A, p_B, a, b) = \begin{cases} ap_A + \frac{1}{2}(l-a-b)p_A + \frac{1}{2c}p_Ap_B - \frac{1}{2}p_A^2 & \text{if } |p_A - p_B| \leq c(l-a-b) \\ lp_A & \text{if } p_A < p_B - c(l-a-b) \\ 0 & \text{if } p_A > p_B + c(l-a-b) \end{cases}$$

$$\pi_B(p_A, p_B, a, b) = \begin{cases} bp_B + \frac{1}{2}(l-a-b)p_B + \frac{1}{2c}p_Ap_B - \frac{1}{2}p_B^2 & \text{if } |p_A - p_B| \leq c(l-a-b) \\ lp_B & \text{if } p_B < p_A - c(l-a-b) \\ 0 & \text{if } p_B > p_A + c(l-a-b) \end{cases}$$

These profit functions are clearly discontinuous at the points where the delivered price of one firm is equal to the price of a rival at the rival's location. At these points, a whole group of consumers will be indifferent between the two firms. D'Aspremont et al. showed that there is a Nash-Cournot equilibrium point only if sellers are sufficiently far from each other, or such that:

$$\left(l + \frac{a-b}{3}\right)^2 \geq \frac{4}{3}(a+2b)l \tag{1}$$

$$\left(l + \frac{b-a}{3}\right)^2 \geq \frac{4}{3}(b+2a)l \tag{2}$$

When sellers locate close to one another, it is optimal for them to undercut each other and capture the entire market. But if (1) and (2) hold, then both $\partial\pi_A/\partial_a$ and $\partial\pi_B/\partial_b$ are strictly positive, implying each firm should move closer to her rival. Once the firms are relatively close to one another, (1) and (2) are violated, implying a Nash equilibrium does not exist.[4] Therefore, subjects in the experiment have an incentive to push toward the center, then try to undercut each other in the price dimension to grab the entire market. This prediction would see subjects follow each other closely in the action space, with frequent adjustment

[4]It is worth noting that there is a trivial Nash equilibrium at $p_A^* = p_A^* = 0$ if $a = b$. This follows from Bertrand competition in which there always exists an equilibrium uniquely determined by zero prices.

to price and location and large volatility in profits.

A possible evasion of this problem is to assume the maximin strategy introduced by Graitson (1980). Here, the seller charges the profit maximizing price if she is likely to be undercut by her competitor when charging the Nash-Cournot price, and the Nash equilibrium price if not. Graitson proves that a socially optimal equilibrium — i.e. one that minimizes transport costs — exists with this strategy in which firms charge Nash-Cournot prices and locate at the first and third quartiles.[5]

In the two-stage game, firms first simultaneously choose location, then simultaneously choose a price with full information from the first stage. Apart from this feature, the setup is the same as above. Dasgupta and Maskin (1986) prove that each price-setting stage has an equilibrium in mixed strategies and Osborne and Pitchik (1987) examine the equilibrium that results from firms using mixed strategies in this stage. They characterize a unique perfect equilibrium in the first stage in which each firm locates 0.27 from the endpoints of the unit interval, which is clearly quite close to the equilibrium that arises from minimax behavior. In the price-setting stage, for a symmetric location pair, the equilibrium price strategy is a union of two intervals — such that the CDF will be kinked. Prices then fall between .9 and 1, with most of the probability weight falling on price of 1. This prediction would see participants in the experiment at or near the profit maximizing positions.

In both versions, players can gain higher profits from collusion, but have incentive to cheat. This mirrors the classic Prisoner's Dilemma, albeit with far more intermediate outcomes. Friedman and Oprea (2012) showed that continuous time treatments greatly increase cooperation; as such, we would predict successful collusion to be much more prevalent in continuous time treatments.

[5]Similar to Graitson, Tabuchi (1994) purposes that firms will maximize on one dimension and minimize on the other in the two-stage game.

III. Experimental Design

The experiment was performed in sessions differing only in the timing of the game. We study three treatments: Discrete, Continuous Instant, and Continuous Slow. Sessions consisted of both either continuous or both discrete games, so that subjects in a session played both variants of a given timing set up. In all treatments, participants choose their location and price by clicking in the x-y action space. Subjects are randomly matched into two-person pairs, and rematched with a new counterpart each period. Sessions contained eight students, and subjects were put into four-person silos — subgroups of players that are only matched with each other in a session. Note that for all treatments, there was no difference between counterparts in any of the game's parameters.

In the Discrete treatment, subjects play an n-stage game in which location is selected first, followed by price with full information about location decisions. Subjects are given 10 seconds to choose their location, indicated by a progress bar on the top of their computer screen. The screen then adjusts to reflect the location the subject and her counterpart have chosen, and subjects are given 10 seconds to choose price, again indicated by a progress bar. Some subject pairs alternated between choosing location and price (Two-Stage), while others had four subperiods of price decisions before they were allowed to readjust location (N-Stage). Figure 1a gives a screenshot of the user interface for this treatment. "Flow" payoffs are shown as bars in the graph on the right, and are updated after every subperiod. The blue dot indicates the subject's position in the last subperiod, while the green dot indicates her counterpart's position. The black line shows the subject's current choice for that subperiod, while the grey line simply follows the mouse.

In the Continuous Instant treatment, subjects choose both location and price freely and instantaneously.[6] A screenshot of this treatment is shown in Figure 1b.

[6]The latency between a subject's click and seeing the action on the computer screen is around 50 milliseconds, or far faster than human reaction time. This latency did increase slightly during periods of

Flow payoffs are shown in the graph on the right, and are updated continuously. The blue dot indicates the subject's current position, and the pink dot shows her counterpart's current position. The grey crosshairs simply follow the mouse.

The Continuous Slow treatment is identical to the previous treatment except for a "speed limit" on subject movement in action space. When a subject chooses a new location and price coordinate, a grey dot appears at that location while her actual position adjusts slowly to that point. Subjects experienced both periods in which their position adjusted on both axes equally, and periods in which position could be adjusted four times quicker on the price dimension than on the location dimension.

Some periods gave more information to subjects about their current payoffs. In these periods, subjects see the linear transport costs running away from their position, the cutoff that determined the edge of the area they control, and a shaded region showing the area they control. Figure 3 shows the user interface with the additional payoff information. Figure 1c shows the user interface with the additional payoff information.

Subjects in all sessions were randomly selected using online recruiting software at the University of California, Santa Cruz from our pool of volunteers, who are primarily undergraduates from all major disciplines. Instructions given for each treatment are included in the appendix.

IV. Results

To provide an overview of the results, Figure 2 gives heat maps of all players' price and location decisions by treatment, respectively. In these figures, "hotter" colors mean players spent more time in these positions. The black lines on each figure represent the mixed strategy equilibrium derived by Osborne and Pitchik (1987). The most striking feature of these figures is the heat distribution between continuous and discrete time treatments. Subject positions were clearly more

very frequent position adjustment by subjects, but not above tolerable levels that would disrupt subject behavior.

concentrated in continuous time treatments, with discrete time positions more evenly distributed in the action space. In continuous time, players tended to be centrally located on the x-dimension, while price positions varied more by treatment. Prices in continuous instant treatments tended to be the highest of any treatment, and putting a speed limit on adjustment seemed to lower prices somewhat. When players could adjust prices much faster than location, subjects tended to be less centrally located on both dimensions.

Table 1 gives basic summary statistics for continuous time treatments. In the

Table 1—: Summary Statistics, Continuous Time

	Continuous Instant		With Info		No Info	
	Mean	SE	Mean	SE	Mean	SE
Location	0.5527	(0.0052)	0.4528	(0.0137)	0.5862	(0.0067)
Price	0.6480	(0.0058)	0.5470	(0.0187)	0.6819	(0.0088)
Profit	0.1484	(0.0037)	0.1296	(0.0078)	0.1547	(0.0042)
	Continuous Slow Equal		With Info		No Info	
	Mean	SE	Mean	SE	Mean	SE
Location	0.4877	(0.0059)	0.4555	(0.0090)	0.4984	(0.0075)
Price	0.5127	(0.0078)	0.4958	(0.0163)	0.5183	(0.0083)
Profit	0.1108	(0.0031)	0.1116	(0.0067)	0.1105	(0.0036)
	Continuous Slow Fast Y		With Info		No Info	
	Mean	SE	Mean	SE	Mean	SE
Location	0.4098	(0.0067)	0.4682	(0.0104)	0.3962	(0.0077)
Price	0.5106	(0.0101)	0.5416	(0.0225)	0.5034	(0.0115)
Profit	0.1103	(0.0037)	0.1171	(0.0080)	0.1087	(0.0040)

Notes: Means and block bootstrapped standard errors of location, price, and profits by treatment.

Continuous Instant treatment, subjects have higher prices and profits when they have less information about the action space that they control, i.e. their current payoffs. This treatment also has the highest average prices and profits of any treatment. In the Continuous Slow treatment with equal adjustment, subjects react to the speed limit by decreasing both price and location positions. This is to say that locations are further to the left, which is expected given the slow adjustment rate and starting point at the origin. When subjects can adjust price quickly but not location, prices were nearly the same as with equal adjustment, but having additional payoff information increased prices and subject profits. Subjects were even more hesitant to venture from the starting point, as this treatment had the furthest left average location position of any treatment. Both speed limit treatments reduced prices and profits in comparison to periods with instant adjustment.

Table 2 provides basic summary statistics for discrete time treatments. Locations, prices, and profits were very similar both when subjects alternated between choosing on each dimension and when subjects could only adjust location every fifth subperiod. Note that additional payoff information was not given to subjects

Table 2—: Summary Statistics, Discrete Time

	Discrete Equal	
	Mean	SE
Location	0.5472	(0.0049)
Price	0.4947	(0.0061)
Profit	0.1004	(0.0029)
	Discrete Four Y	
	Mean	SE
Location	0.5303	(0.0048)
Price	0.4546	(0.0050)
Profit	0.0997	(0.0026)

Notes: Means and block bootstrapped standard errors of location, price, and profits by treatment.

in discrete time sessions except during practice periods.

Tables 3 and 4 gives summary statistics on mean and median distance from a

Table 3—: Comparison to Counterpart Statistics, Continuous Time

	Cont.	Cont. Instant	Cont. Slow Equal	Cont. Slow Fast Y
Location Distance (distance on x-axis from counterpart)				
Mean	0.1508	0.1759829	0.1669	0.1103
	(0.0049)	(0.0009)	(0.005)	(0.0108)
Median	0.0843	0.0950	0.0973	0.0730
	(0.0044)	(0.0098)	(0.0045)	(0.0117)
Price Distance (distance on y-axis from counterpart)				
Mean	0.1452	0.1667	0.1753	0.0952
	(0.005)	(0.0093)	(0.0046)	(0.0010)
Median	0.0818	0.0840	0.0950	0.0722
	(0.0038)	(0.0084)	(0.0037)	(0.0089)
Euclidean Distance (from counterpart)				
Mean	0.2284	0.2640	0.2616	0.1617
	(0.007)	(0.0127)	(0.0066)	(0.0014)
Median	0.1515	0.1731	0.1793	0.1271
	(0.0059)	(0.015)	(0.0052)	(0.0124)

Notes: Mean and median distances on specified dimension by treatment. Block bootstrapped standard errors in parentheses. The first column aggregates all continuous time treatments.

subject's counterpart, with "distance" specified as purely location, purely price, and euclidean distances. Note that both axes are scaled to one, so that a distance of .1 is very close while a distance of .5 is quite far from a counterpart. In the continuous time treatments, subjects are closest together when they can only adjust price quickly, and not being able to adjust location nor price quickly drives subjects further apart relative to instant adjustment. Subjects did not seem to distinguish between horizontal and vertical differentiation, as the distances in each dimension are nearly identical. In the discrete time treatments, it seemed to

Table 4—: Comparison to Counterpart Statistics, Discrete Time

	Discrete Time	Discrete Equal	Discrete More Y
Location Distance (distance on x-axis from counterpart)			
Mean	0.3142	0.3161	0.3123
	(0.017)	(0.02432)	(0.0195)
Median	0.2740	0.2790	0.2570
	(0.0227)	(0.0332)	(0.0302)
Price Distance (distance on y-axis from counterpart)			
Mean	0.2771	0.2900	0.2641
	(0.0144)	(0.0292)	(0.0128)
Median	0.2020	0.215	0.190
	(0.0134)	(0.0257)	(0.0191)
Euclidean Distance (from counterpart)			
Mean	0.4672	0.4753	0.4591
	(0.0208)	(0.0369)	(0.022)
Median	0.4435	0.4501	0.4365
	(0.0242)	(0.0242)	(0.0299)

Notes: Mean and median distances on specified dimension by treatment. Block bootstrapped standard errors in parentheses. The first column aggregates all discrete time treatments.

matter very little how many periods subjects were able to adjust price. However, discrete time subjects were generally much further away from their counterparts than continuous time subjects, and seemed to differentiate slightly more on location than on price.

As detailed previously, subjects have an ever-present incentive to undercut on either dimension. We compile collusion rates ik as the fraction of time player pair i and her counterpart j successfully collude in session k. We will define collusion as a situation where two players are able to settle into relatively stable positions. Specifically, players have payoffs that are within ten percent of each other, although this is a conservative threshold and our results are robust to a range of changes to this threshold.

Table 3 shows mean and median collusion rates by treatment. Perhaps surprisingly, the Continuous Slow treatment with fast price adjustment was the most conducive to successful collusion. We speculate that this is because the harsh location speed limit forced subjects to focus on price competition almost exclusively once they were close to being centrally located. When the speed limit was on both dimensions, collusion rates were considerably lower. Note that median values were generally well below mean values. This is because some subject pairs were able to quickly come to an agreeable state — thus spending large portions of periods in cooperation — while others could only manage to collude for short periods, or not at all.

Table 5—: Collusion Rates by Treatment

	10%		*20%*	
	Mean	Median	Mean	Median
Continuous Instant	.1795	.0738	.2740	.1792
	(.0517)	(.0508)	(.0579)	(.0976)
Continuous Slow Equal Adjustment	.0759	.0751	.1601	.1678
	(.0113)	(.0111)	(.0226)	(.0268)
Continuous Slow Fast Y Adjustment	.1915	.0817	.3061	.1614
	(.0775)	(.0292)	(.0758)	(.0961)
Discrete Equal Adjustment	.0918	.0801	-	-
	(.0106)	(.0239)		
Discrete More Y Adjustment	.1503	.0999	-	-
	(.0369)	(.0222)		

Notes: Mean and median cooperation rates with bootstrapped standard errors. The percentage refers to the threshold defining when subjects are cooperating.

Figure 3a gives circumstantial evidence of how players were able to coordinate. It shows a subject pair in the middle of the session playing a Continuous Instant treatment without additional payoff information. Shaded regions indicate collusion between subjects. In the bottom panel on each figure, the thick lines are smoothed flow payoffs for each subject. In the very beginning of the period, player 8 adjusts her price to the maximum allowed (normalized to one) several times until player 7 follows. The subject pair cooperated for almost the entirety of the period, indicated by the blue bars in the payoff figure. The subject pair obtained much higher than average payoffs in this period as a result. Figure 3b shows a typical case of players following each other in the action space throughout the period.

V. Discussion

If we take these pilot results seriously, we can come to two main conclusions. First, our results are consistent with previous laboratory experiments that showed the ability to respond quickly can increase cooperation, though in this case it is not as dramatic as in a simpler game such as a Prisoner's Dilemma. We found mean collusion rates between 17-28 percent, compared to about 9 percent on discrete time treatments. It is worth noting that median collusion rates were meaningfully lower than mean rates, and subjectively it seemed only a portion of subjects were willing to attempt to collaborate to achieve higher profits.

Second, we verified several theoretical predictions. Subjects tended to locate in the center, and many subjects were unable to settle on a price equilibrium. While this was perhaps expected, it does reject several alternative equilibrium concepts that have been floated as fixes to the original model. In light of this, the many applications of Hotelling's model — from voting theory to gas station placement — should be viewed with extreme caution. For instance, according to the median voter theorem in a two-party system, political parties should track to the center to capture the maximum amount of votes. But it is clear that this works only if party differences can be characterized only by horizontal differentiation, as introducing vertical differentiation — as we have done with a price decision — will obscure the clean result and require cooperation to achieve some degree of stability.

Going forward, we will seriously consider giving subjects more payoff information on all periods. Our data weakly show that it will increase payoffs for subjects, possibly intimating at confusion on how to earn payoffs when the information is not given. We may also wish to run a simple location game as a benchmark test. Because that model has a clear Nash equilibrium, an expected result would ensure that the game interface is well understood by subjects, and allay concerns that the interface is simply too complicated. Finally, there seems to be little to no effect in changing the amount of times subjects can adjust price between location adjustments in discrete time. While it draws a rough parallel to the continuous time treatment with a harsh price speed limit, it may be the treatment that serves the least functional purpose of those tried thus far.

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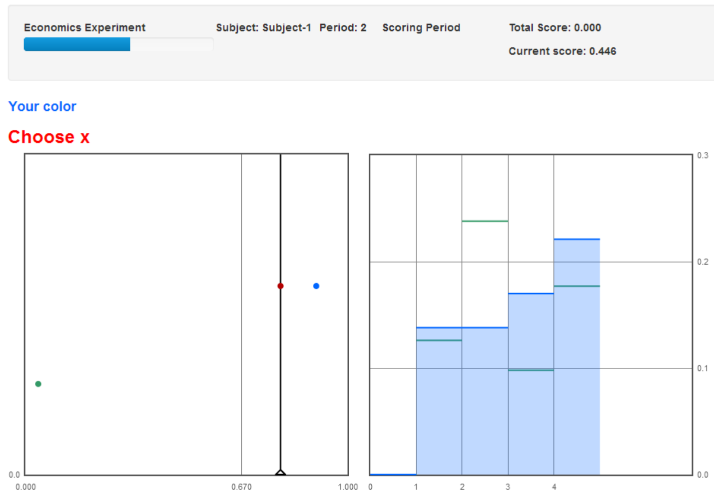
Figure 1. : User Interfaces

(a) Discrete Time



(b) Continuous Time

(c) Continuous Time with Additional Information about Payoffs

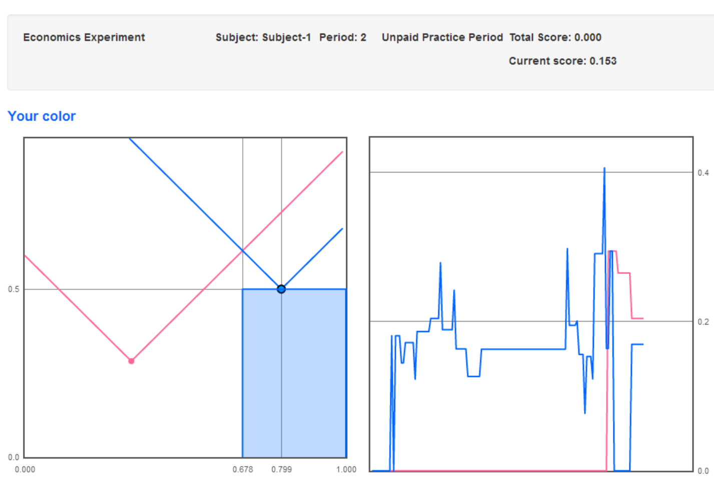
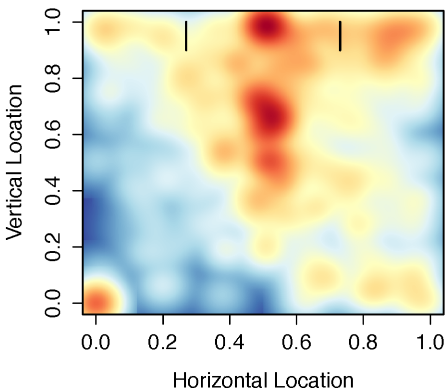
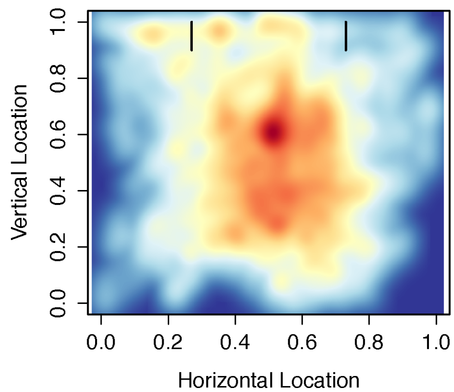


Figure 2. : Heat Maps of Subject Price and Location Decisions by Treatment

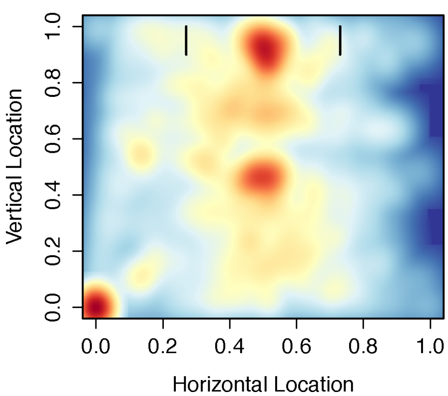
(a) Continuous Instant Adjustment



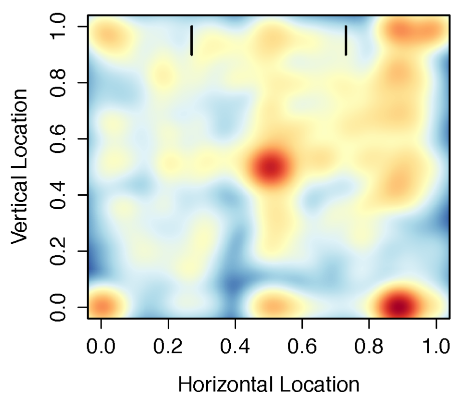
(b) Continuous Slow Equal Adjustment



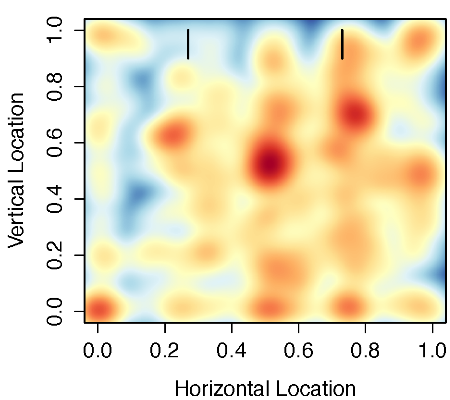
(c) Continuous Slow Fast Y Adjustment



(d) Discrete Equal Adjustment



(e) Discrete More Y Adjustment

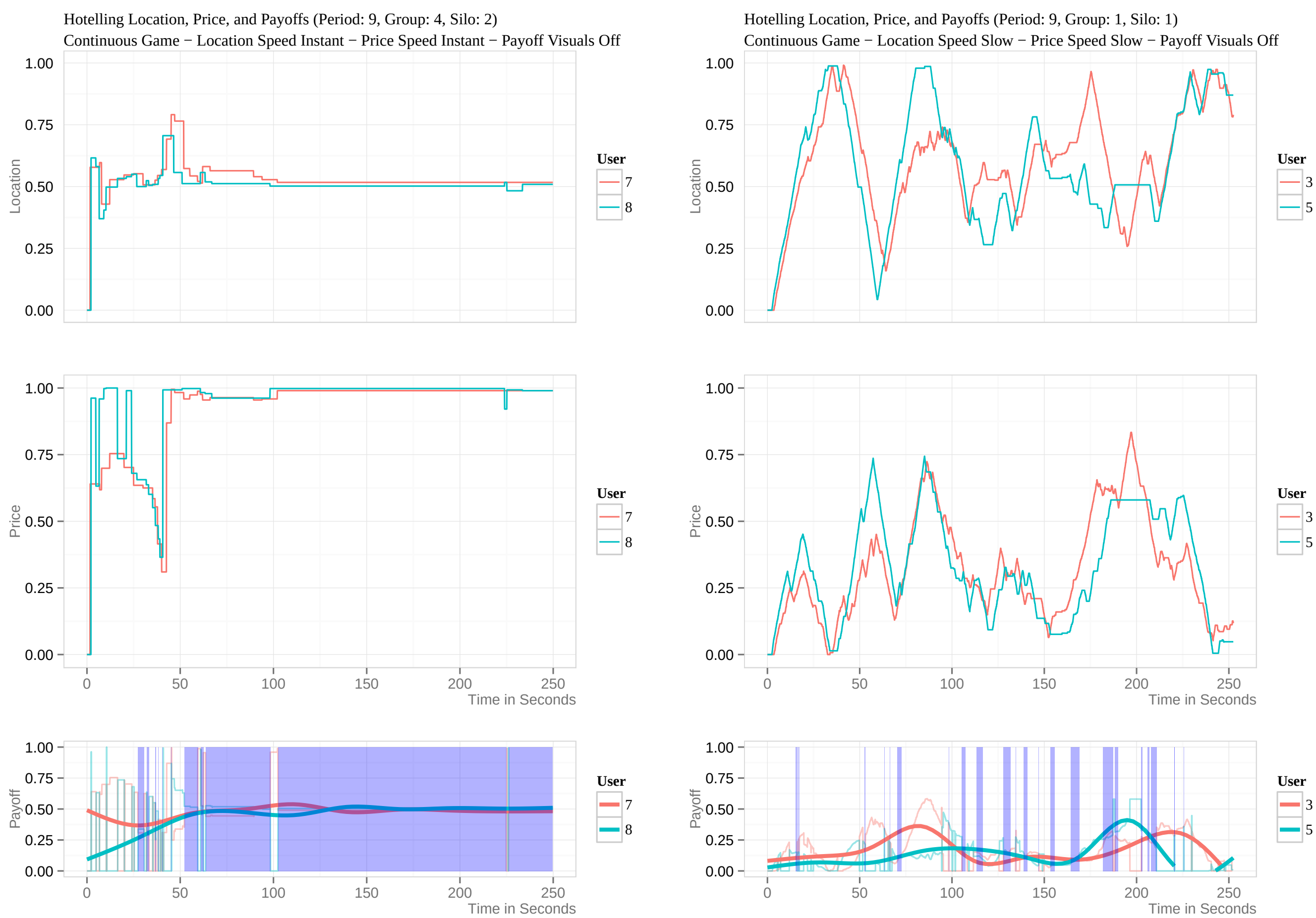


These figures show all players' price and location decisions by treatment. The heat maps run from cool to hot colors, with "hotter" colors indicating that players spent more time in those positions. The black bars indicate the mixed strategy equilibrium outlined in Osborne and Pitchik (1987).

Figure 3. : Collusion Evidence

(a) Successful Collusion

(b) Unable to Collude



Instructions Given to Continuous Time Subjects

Welcome! You are about to participate in an experiment in the economics of decision making. If you listen carefully and make good decisions, you could earn a considerable amount of money that will be paid to you in cash at the end of the experiment.

Please remain silent and do not look at other participants' screens. If you have any questions, or need assistance of any kind, please raise your hand and we will come to you. Do not attempt to use the computer for any other purpose than what is explicitly required by the experiment. This means you are not allowed to browse the internet, check emails, etc. If you disrupt the experiment by using your smart phone, talking, laughing, etc., you may be asked to leave and may not be paid. We expect and appreciate your cooperation today.

The Basic Idea

The experiment will be divided into a number of periods, and each period you will be matched anonymously with a counterpart, who is another participant in today's experiment.

Throughout each period you will choose a position within a range of possible positions. The number of points you earn each period will depend on your choice and the choice(s) of your counterpart.

At the end of the session, two periods will be randomly chosen to be the paid periods. The number of points you earned in those periods will be converted to US dollars at a rate shown on the whiteboard, and paid to you in cash.

Choosing Your Position

Figures A1 and A2 show the user interface. Crosshairs will follow your mouse as it moves over the screen, but your location will not be adjusted until you click the mouse. You will be able to select your position by clicking on your desired position with your mouse on the screen. However, in some periods your position will *not* adjust instantly. Instead, your position will slowly adjust to the location you selected, which will be indicated by a grey dot on your screen. Note that your position may be able adjust horizontally and vertically equally quickly, or your position may be able to adjust more quickly along one dimension. Also note that your counterpart will always have the same ability to adjust her position as you.

How You Earn Points

You earn points at a rate proportional to the size of the action area you control. At every moment during the experiment you will see your position and earnings, and the position and earnings of your counterpart. In Figure A1, your position is indicated by the blue dot in the box on the left side. Note that your color will always be indicated by the color of the words "Your color" above the box on the left. Your flow of earnings is shown in the box on the right by a blue line. Similarly, the position of your counterpart is indicated by the pink dot on the left, and the current earnings of your counterpart is shown by the pink line on the right. Note that your flow of earnings is also shown as a number in the box at the top of the screen under "Current score."

Note that the figure also includes lines running away from your position and from your counterpart's position, as well as a shaded area indicating the action area you control. These lines and shaded areas will only be visible to you during two unpaid practice periods. The intersection of your line with your counterpart's line determines the size of the action area you control. Figure A2 shows the user interface you will see during the remaining practice periods as well as during scoring periods.

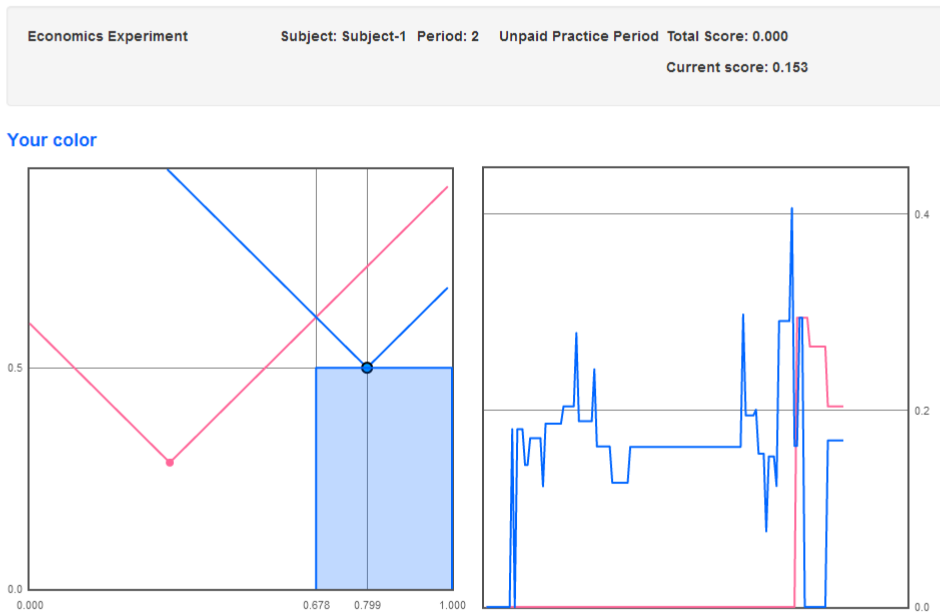
Periods

Periods will last several minutes. After a brief intermission, a new period will begin with new counterparts who may be different participants than in the previous period.

Frequently asked questions

1) Is this some kind of psychological experiment with an agenda you haven't told us?

Figure A1. : User Interface During Practice Periods

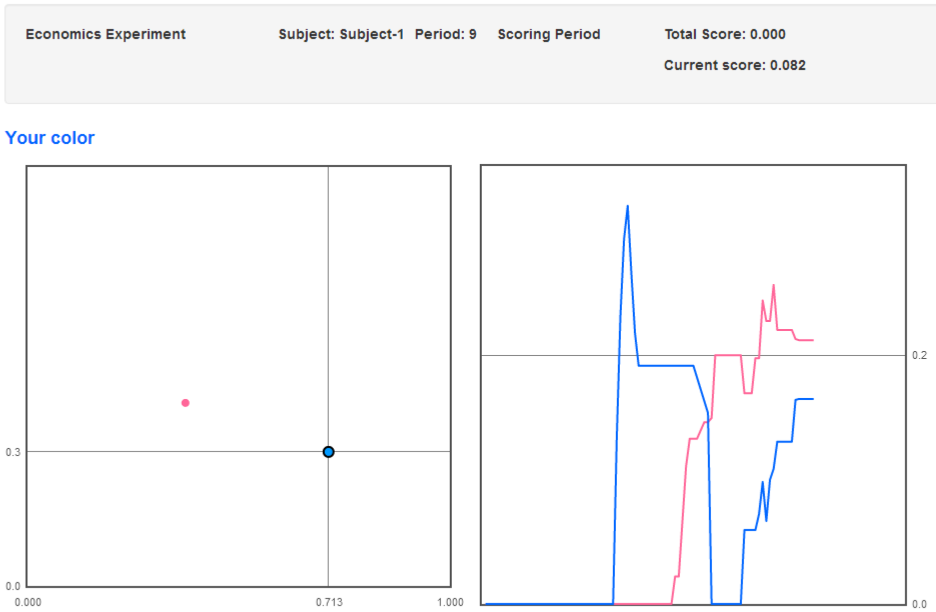


a) Answer: No. It is an economics experiment. If we do anything deceptive or don't pay you cash as described then you can complain to the campus Human Subjects Committee and we will be in serious trouble. These instructions are meant to clarify the game and show you how you earn money; our interest is simply in seeing how people make decisions.

2) Will there be practice periods?

a) Answer: Yes. You will have three unpaid 30-second practice periods. Use this time to familiarize yourself the controls of the software, and how your choice and the choices of your counterparts affect your flow payoffs.

Figure A2. : User Interface Scoring Periods



Instructions Given to Discrete Time Subjects

Welcome! You are about to participate in an experiment in the economics of decision making. If you listen carefully and make good decisions, you could earn a considerable amount of money that will be paid to you in cash at the end of the experiment.

Please remain silent and do not look at other participants' screens. If you have any questions, or need assistance of any kind, please raise your hand and we will come to you. Do not attempt to use the computer for any other purpose than what is explicitly required by the experiment. This means you are not allowed to browse the internet, check emails, etc. If you disrupt the experiment by using your smart phone, talking, laughing, etc., you may be asked to leave and may not be paid. We expect and appreciate your cooperation today.

The Basic Idea

The experiment will be divided into a number of periods, and each period will be broken up into subperiods. For each period, you will be matched anonymously with a counterpart, who is another participant in today's experiment.

Throughout each period you will choose a position within a range of possible positions. The number of points you earn each period will depend on your choice and the choice(s) of your counterpart.

At the end of the session, two periods will be randomly chosen to be the paid periods. The number of points you earned in those periods will be converted to US dollars at a rate shown on the whiteboard, and paid to you in cash.

Choosing Your Position

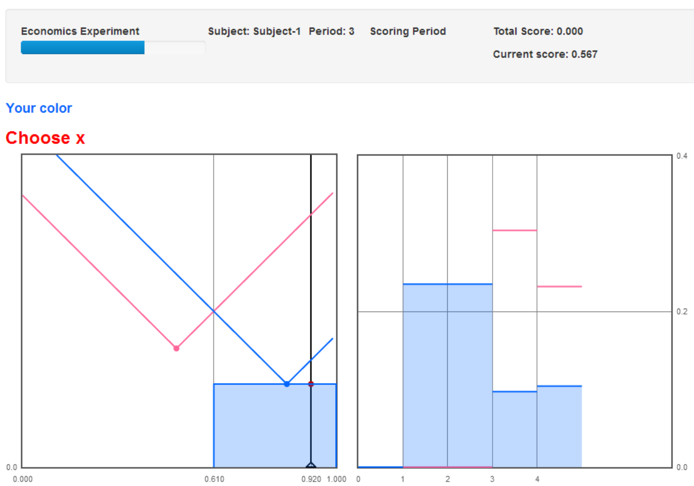
Figures 1 and 2 show the user interface. A gray line will follow your mouse as it moves over the screen, but your location will not adjust until you click the mouse. You will be able to select your position by clicking on your desired position with your mouse on the screen. This will proceed in stages. At the beginning of the period, you are in the first subperiod. In this subperiod, you will only be able to adjust your position along the x-axis (horizontally). In the next subperiod, you will be able to adjust your position only along the y-axis (vertically). There will be between one and four subperiods in which you can only adjust your position vertically. You will be able to adjust your position within a subperiod, but your final position is the position you have selected when the subperiod ends.

How You Earn Points

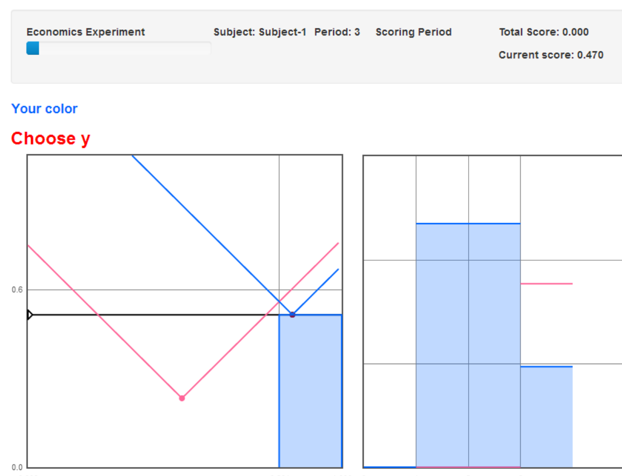
You earn points at a rate proportional to the size of the action area you control. At every subperiod during the experiment you will see your position and earnings from the pervious subperiod, and the position and earnings of your counterpart from the previous subperiod. In Figure B2a, your position from the previous subperiod is indicated by the blue dot in the box on the left side. Note that your color will always be indicated by the color of the words "Your color" above the box on the left. Your flow of earnings is shown in the box on the right by a blue area. Similarly, the position of your counterpart is indicated by the pink dot on the left, and the current earnings of your counterpart is shown by the pink line on the right. Note that your flow of earnings is also shown as a number in the box at the top of the screen under "Current score." Also note that you do not receive earnings after making only a horizontal adjustment of your position, as your earnings depend on both your horizontal *and* vertical positions. The black line and red dot indicate the position you currently have selected for your horizontal position, which you are free to adjust until the end of the subperiod. Figure B2b is identical to Figure B2a, except that the black line and red dot indicate the position you currently have selected for your vertical position.

Figure B1. : User Interfaces During Practice Periods

(a) Adjusting Position Horizontally



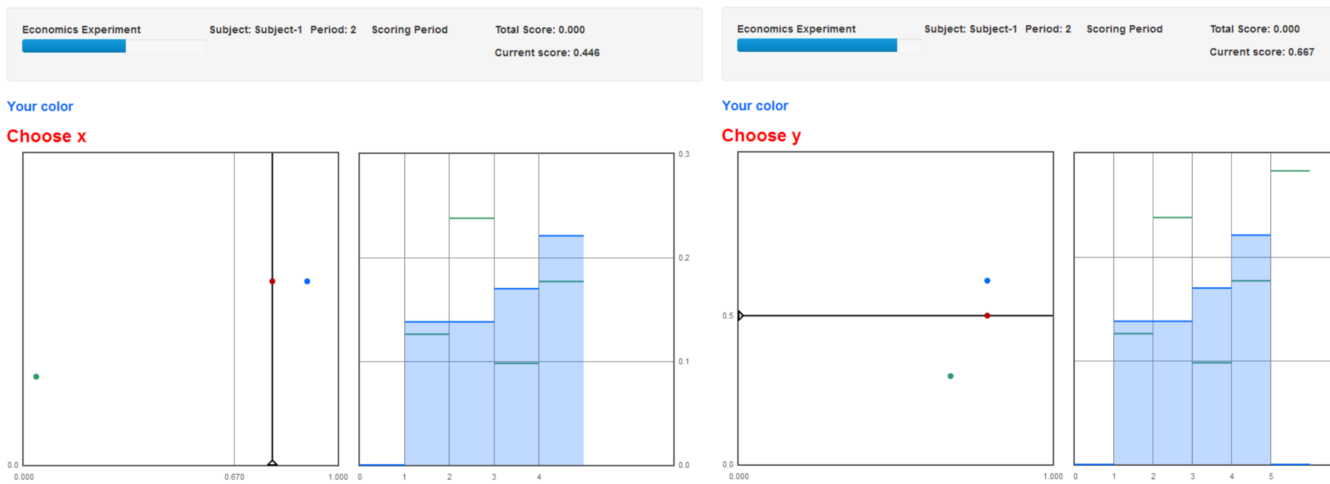
(b) Adjusting Position Vertically



Note that the figure also includes lines running away from your position and from your counterpart's position, as well as a shaded area indicating the action area you control. These lines and shaded areas will only be visible to you during two unpaid practice periods. The intersection of your line with your counterpart's line determines the size of the action area you control. Figure B2 shows the user interface you will see during the third and final practice period as well as during scoring periods.

Figure B2. : User Interfaces During Scoring Periods

(a) Adjusting Position Horizontally (b) Adjusting Position Vertically



Periods

The time remaining in the subperiod is shown by the progress bar at the top of the screen. After a brief intermission, a new subperiod will begin. At the end of each *period*, you will be matched with new counterparts who may be different participants than in the previous period.

Frequently asked questions

1) Is this some kind of psychological experiment with an agenda you haven't told us?

    a) Answer: No. It is an economics experiment. If we do anything deceptive or don't pay you cash as described then you can complain to the campus Human Subjects Committee and we will be in serious trouble. These instructions are meant to clarify the game and show you how you earn money; our interest is simply in seeing how people make decisions.

2) Will there be practice periods?

a) Answer: Yes. You will have three unpaid 100-second practice periods. Use this time to familiarize yourself the controls of the software, and how your choice and the choices of your counterparts affect your flow payoffs.