ECON 200 - Final Exam - Fall 2016
Solutions

December 6, 2016

1 Risky Choice

Suppose that Anna makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = m^{2/3}$. She is faced with a situation in which she will receive either 0 (with probability 0.7) or 27 (with probability 0.3)

a) What is the expected (mean) outcome? Variance of outcome? (4 pts)
The mean is as follows:

$$\begin{aligned}\mu &= \sum_i p_i v_i \\ &= 0.7 \cdot 0 + 0.3 \cdot 27 = 8.1\end{aligned}$$

The variance is as follows:

$$\begin{aligned}\text{Var} &= \sum_i p_i (v_i - \mu)^2 \\ &= 0.7 \cdot (0 - 8.1)^2 + 0.3 \cdot (27 - 8.1)^2 \\ &= 45.927 + 107.163 = 153.090\end{aligned}$$

b) What is Anna's certainty equivalent for this situation? What is the maximum amount she would be willing to pay an insurer to get the mean outcome for sure? (6 pts)
We calculate this by setting equal the utility of the certainty equivalent c and the expected utility of the situation:

$$\begin{aligned}u(c) &= 0.7 \cdot u(0) + 0.3 \cdot u(27) \\ c^{2/3} &= 0 + 0.3 \cdot 27^{2/3} = 2.7 \\ c &= 2.7^{3/2} \approx 4.437\end{aligned}$$

Since she is receiving 8.1 with certainty, we could say the WTP for insurance (also known as the risk premium) is as follows:

$$\begin{aligned}u(8.1 - wtp) &= u(c) \\ 8.1 - wtp &= 4.437 \\ wtp &\approx 3.66\end{aligned}$$

c) What is Anna's coefficient of absolute risk aversion at the mean outcome? Her coefficient of relative risk aversion? (4 pts)

This is formulaic:

$$\begin{aligned}
\text{ARA}(x) &= -\frac{u''(x)}{u'(x)} \\
&= \frac{\frac{2}{9}x^{-4/3}}{\frac{2}{3}x^{-1/3}} = \frac{1}{3x} \\
\text{ARA}(8.1) &= \frac{1}{3 \cdot 8.1} = \frac{1}{24.3} = 0.0412 \\
\text{RRA}(x) &= x \cdot \text{ARA}(x) = x\frac{1}{3x} = \frac{1}{3}.
\end{aligned}$$

2 Bayesian Updating

Exams are either tough or easy; overall 60% are tough. Looking at the first two problems on the exam gives a clue: these problems seem tough 80% of the time when the exam is tough, and the seem tough only 10% of the time when the exam is easy.

a) What is the updated probability that the exam is tough, after seeing that the first two problems are easy? (6 pts)
This is a form of Bayesian updating. We are looking for the posterior probability that the exam is tough given that the first two problems are easy:

$$\begin{aligned}
p(\text{Exam} = \text{tough}|\text{first two} = \text{easy}) &= \frac{p(\text{first two} = \text{easy}|\text{Exam} = \text{tough})\, p(\text{Exam} = \text{tough})}{p(\text{first two} = \text{easy})} \\
p(E = t|f2 = e) &= \frac{p(f2 = e|E = t)\, p(E = t)}{p(f2 = e|E = t)\, p(E = t) + p(f2 = e|E = e)\, p(E = e)} \\
p(E = t|f2 = e) &= \frac{0.2 \cdot 0.6}{0.2 \cdot 0.6 + 0.9 \cdot 0.4} \\
&= \frac{0.12}{0.12 + 0.36} = \frac{1}{4}
\end{aligned}$$

The posterior probability is 0.25

b) Would you find it psychologically comforting to know the day before whether the first two problems seem easy? (1 pt) When would that information have economic value? (3 pts)
Any coherent answer gets the 1pt for psych comfort. You get the 3pts if you point out that the information is economically valuable to the extent that it would change your optimal plan. Otherwise put, the information would have economic value if and only if it enabled you to allocate your time more efficiently away from studying and toward something more productive, such as studying for another exam that would be relatively more difficult or towards a paying job. The key here is that the change in how likely the exam is to be difficult (in terms of your perception) must enable you to do something more productive; otherwise, there is no economic value. (That's the neoclassical answer. You could argue as a behavioral economics matter, there may be some willingness to pay to reduce anxiety).

3 Yambits

Agil Corp recently launched a distinctive line of yambits. It finds that inverse demand for this product is $p = 80 - 2y$, while it can produce y units per month at cost $c(y) = 75 + 20y$.

a) What is the maximized profit, and corresponding price (p) and output (y)? (6 pts)
Take from the question that Agil has a distinctive product and therefore a monopoly pricing environment. We begin with profit maximization:

$$\max_y py - c(y)$$

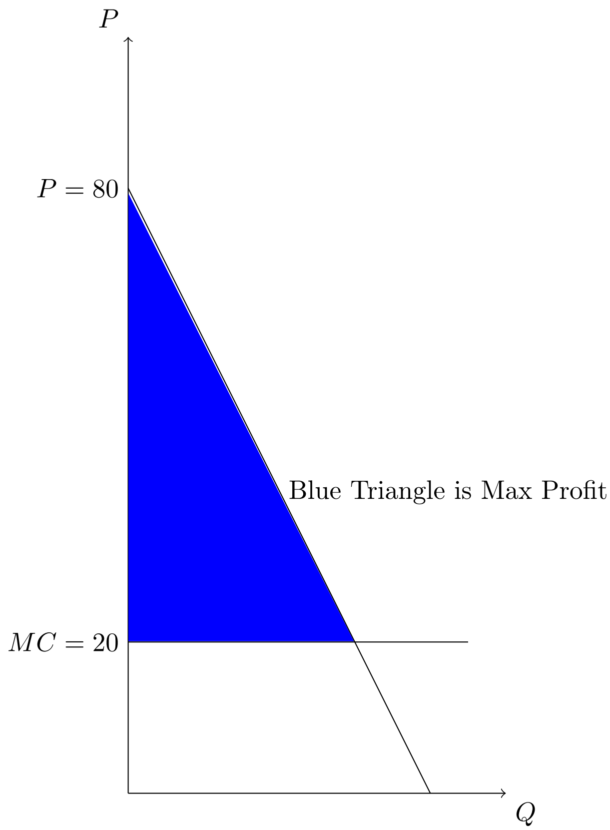

Figure 1: Perfect Price Discrimination

$$\max_y (80 - 2y)\, y - 75 - 20y = 60y - 2y^2 - 75$$

The first order condition is:

$$60 - 4y = 0$$
$$y^M = 15$$

We can check the second order condition:

$$-4 < 0$$

So this is a maximum. What is price?

$$p^M = 80 - 2y^M = 80 - 2 \cdot 15 = 50.$$

The corresponding profit is

$$\pi^M = p^M y^M - c\left(y^M\right) = 50 \cdot 15 - 75 - 20 \cdot 15 = 375.$$

b) Suppose Agil can charge different prices on different units sold. What is the largest profit that (absent obstacles) could then be obtained, and what is the corresponding output level? (3 pts)
It is instructive to draw a graph, but the point is essentially the following. If Agil Corp can charge every customer its willingness to pay, it the demand curve is the marginal revenue curve. As a result, the intersection of demand and marginal cost (20) is the optimal output:

$$p = 80 - 2y = 20 = MC$$
$$y = 30$$

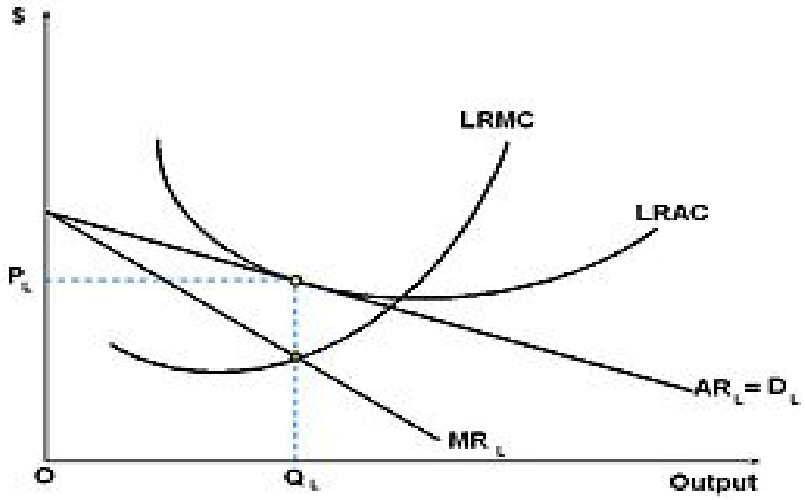

Figure 2: LR equilibrium for a Monopolistic Competitive firm. From Wikpedia article.

The price of each unit sold is different, so profit is the integral of the area under the marginal revenue (also demand) curve and above the marginal cost line:

$$\begin{aligned}
\pi^{max} &= \int_0^{30} (p(y) - mc)\, dy \\
&= \int_0^{30} (80 - 2y - 20)\, dy \\
&= \int_0^{30} (60 - 2y)\, dy \\
&= 60y - y^2|_0^{30} \\
&= 60(30) - (30)^2 - 0 \\
&= (60 - 30) \cdot 30 = 30^2 = 900
\end{aligned}$$

c) What practical considerations limit Agil's ability to profitably charge different prices on different units? What are some possible ways to deal with these considerations? (4 pts)
Other than legal and moral (or customer attitude) considerations, (a) Agil likely has little information on each customer's willingness to pay. Furthermore, it is may be difficult to prevent arbitrage, which would allow all customers to buy at the lowest price. Some methods discussed in class to mitigate these problems include offering price/quantity menus, segmenting the market by observable characteristics correlated with WTP (or price elasticity), loyalty programs that collect data useful in determining willingness to pay, and in generated non-transferrable discounts.

d) Now assume that Agil must offer a unified price on all units sold each month. Suppose that there are no barriers to entry in the yambit business. What does standard (e.g., good undergrad level) economics predict regarding Agil's long run profit? Explain how the prediction works (e.g., in terms of shifts in the cost function or demand functions). A diagram may help you make your points. (6 pts)
The underlying intuition here is that economic profit is positive in the short run, which leads to entry. More firms will enter, especially those with relatively close substitutes (not exact substitutes, Agil's product remains distinctive). This will lower and flatten the demand curve until average cost is tangent to the demand curve, at which point further entry is unprofitable, as illustrated in Figure 2. Agil may earn economic profit during this transition, but long-run profit will be zero afterwards.

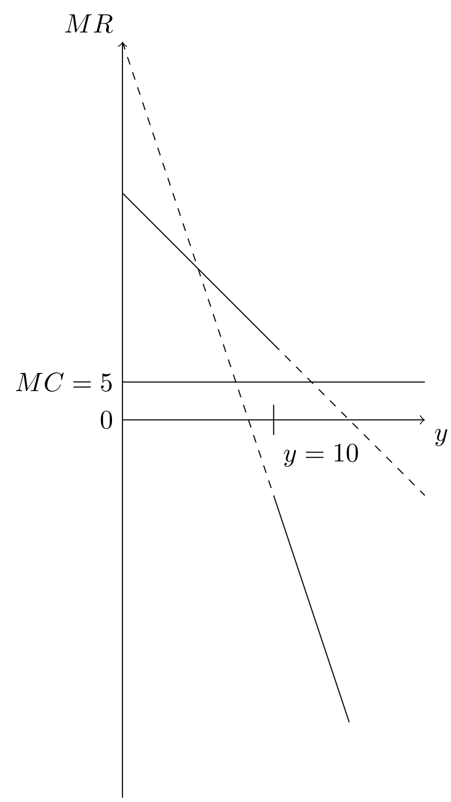

Figure 3: Discontinuous Marginal Revenue

4 Price Match Guarantee

Betamin Inc. currently sells $y = 10$ units per month of its main product at the prevailing price $p = 20$. When all firms in the industry shift their prices together, Betamin's inverse demand curve is well approximated by $p = 50 - 3y$. When rivals stay with the prevailing price, Betamin's inverse demand is approximately $p = 30 - y$. Its cost is $c(y) = 10 + 5y$.

a) What are Betamin's own price elasticities at the prevailing price when only it shifts price? When all firms shift in parallel? (2 pts)
When it is the only one, the own price elasticity is:

$$\varepsilon = \frac{\Delta y}{\Delta p} \cdot \frac{p}{y} = -1 \cdot \frac{20}{10} = -2$$

When all firms shift together, the own price elasticity is:

$$\varepsilon = \frac{\Delta y}{\Delta p} \cdot \frac{p}{y} = -\frac{1}{3} \cdot \frac{20}{10} = -\frac{2}{3}$$

b) Rivals offer a "not-undersold" policy in which they match the lowest price offered in the market if it is below $p = 20$, and otherwise stay at $p = 20$. Is it profit-maximizing for Betamin to continue to produce $y = 10$ units per month? Show your calculations. (5 pts)
Marginal cost is always 5. Marginal revenue is discontinuous at $y = 10$. For $y < 10$, $p > 20$, Betamin is the only one changing its price. Here, marginal revenue is given by:

$$\frac{\partial}{\partial y}(30 - y)\, y = 30 - 2y,$$

which always exceeds MC $= 5$. So Betamin would profitably increase output whenever it is below $y = 10$, i.e., would decrease price if above $p = 20$. On the other hand, for $y > 10$, $p < 20$, marginal revenue is given by:

$$\frac{\partial}{\partial y}(50 - 3y)\, y = 50 - 6y < 0,$$

which also needlessly lowers profit. Clearly, the best situation is for Betamin to remain at the prevailing price with $y = 10$.

c) Betamin's COO is worried that its cost function might shift, and asks you what sort of economic factors could change it. Please list the standard items. (3 pts)
Standard things include shocks to input prices, changes in transportation costs (gasoline prices, etc), or changes in technology. Also economies of scope (if there are related product) and a learning curve (which tends to lower MC as experience accumulates).

d) For what range of marginal costs and fixed costs would Betamin find it profit-maximizing for Betamin to continue to charge the prevailing price? (6 pts)
From the diagram you can see the main condition is MC ≤ 10. If MC goes higher than that, Betamin should charge above 20 and reduce output below 10. The other condition is that Betamin covers its average cost (or in the SR, its avoidable cost). If the fixed cost is sunk (as the formula suggests) it is irrelevant in the SR. In the LR all costs are avoidable, and the revenue $10 \cdot 20 = 200$ must cover total cost $= y\cdot$ MC + FC. E.g., at MC=5, the firm will exit eventually if FC $> 200 - 10 \cdot 5 = 150$.

5 Ingots and Externalities

Demand in the ingot industry is $Y = 300 - 2p$. Supply is via 100 identical firms with production cost functions $c(y_i) = 50y_i + 0.5y_i^2$. People who live in towns that produce ingots suffer health and other costs increasing in total output approximated by $e(Y) = 0.5Y^2$.

a) What is the industry supply curve (also known as the private MC function)? (2 pts)
First we need to find individual private marginal cost:

$$mc(y_i) = \frac{\partial c(y_i)}{\partial y_i} = 50 + y_i$$

Each of 100 firms will supply according to this condition (noting min avc=0, supply at $p = mc$), so:

$$Y^S = 100y_i = 100 \cdot (p - 50) = 100p - 5000 \quad \forall p \geq 50$$

b) Find the competitive equilibrium output Y^c, and the corresponding total surplus $TS = PS + CS$. Assume that the costs $e(Y)$ are not included in producer surplus (PS) but are subtracted from consumer surplus.
Find competitive output by setting demand equal to supply:

$$\begin{aligned}
300 - 2p &= 100p - 5000 \\
102p &= 5300 \\
p &= 51.96 \\
Y &= 300 - 2 \cdot \frac{5300}{102} \\
&= 196.08
\end{aligned}$$

Note that the producer surplus is above the supply curve and below the competitive price, from zero to the competitive output. This is

$$PS = \frac{1}{2}(51.96 - 50) \cdot 196.08 = 192.16$$

The consumer surplus before accounting for the externality is above the price and below the demand curve:

$$CS_{pre} = \frac{1}{2}(150 - 51.96) \cdot 196.08 = 9611.84$$

Calculate the externality

$$e(Y) = \frac{1}{2}(196.08)^2 = 19223.68$$

Total consumer surplus:

$$CS = 9611.84 - 19223.68 = -9611.84$$

Total surplus:

$$TS = CS + PS = 192.16 - 9611.84 = -9419.68$$

b2) What is the social MC function (which includes e as well as c costs)? (3 pts)
We can add the marginal external damages, $e'(Y) = Y$ directly to the aggregate industry supply function:

$$\begin{aligned}
P\left(Y^S\right) &= 50 + \frac{1}{100}Y^S + e'\left(Y^S\right) \\
&= 50 + 1.01Y^S \\
Y^S &= \frac{100}{101}P - \frac{5000}{101} \quad \forall p \geq 50
\end{aligned}$$

c) Compute the output level Y^o that maximizes total surplus ($TS = PS + CS$). How much higher is TS here than at Y^c? (4 pts)
We know that the intersection of Y^S_{SMC} and demand maximizes TS:

$$\begin{aligned}
\frac{100}{101}P - \frac{5000}{101} &= 300 - 2p \\
\frac{302}{101}p &= \frac{30300 - 5000}{101} \\
p &= \frac{25300}{302} \\
p &= 83.77 \\
Y^o &= 300 - 2 \cdot \frac{25300}{302} \\
&= 132.45
\end{aligned}$$

Note that producer surplus is still calculated as under the price line and above the supply curve. However, this is the triangle of surplus above social marginal cost and below the price line plus the external damages:

$$\begin{aligned}
PS &= \frac{1}{2}(83.77 - 50) \cdot 132.45 + e(132.45) \\
&= 2236.42 + \frac{1}{2}(132.45)^2 \\
&= 2236.42 + 8771.50 = 11007.92
\end{aligned}$$

The pre-externality consumer surplus is still under the demand curve and above the price line

$$CS_{pre} = \frac{1}{2}(150 - 83.77) \cdot 132.45 = 4392.70$$

The externality is

$$e(Y) = \frac{1}{2}(192.31)^2 = 8771.50$$

Consequently,

$$CS = -4378.80$$

Note that while consumer surplus is negative, total surplus is positive and equal to:

$$\begin{aligned} TS &= \frac{1}{2}(83.77 - 50) \cdot 132.45 + \frac{1}{2}(150 - 83.77) \cdot 132.45 \\ &= 2236.42 + 4392.70 = 6692.12 \end{aligned}$$

d) What are the main possible approaches to restore efficiency? Name a specific policy that seems most effective in this example. (3 pts)
The most effective method is to tax sales or production at Y, but because this needs to change with scale, it may need to be an approximation. Assuming perfect information, a tax of 31.81 would result in maximal total surplus.

6 Imperfect Substitutes

Anyway Inc. and Belton Co. produce imperfectly substitutable products, with inverse demand $p_A = 8 - q_A - 0.5q_B$ for Anyway and $p_B = 8 - q_B - 0.5q_A$ for Belton, where q_A and q_B denote their respective output quantities. They have cost functions $c(q_i) = 2 + 3q_i$, for $i = A, B$. Both firms choose quantity simultaneously and independently.

a) Find the best response functions for both firms. (4 pts)
This is a symmetric prolem,

$$\max_{q_i} \left(8 - q_i - \frac{1}{2}q_j\right) q_i - 2 - 3q_i$$

FOC

$$5 - 2q_i - \frac{1}{2}q_j = 0$$

Rearranging, for both A and B:

$$q_i = \frac{5}{2} - \frac{1}{4}q_j$$

b) Find Nash equilibrium outputs, prices, and payoffs (profits). (4 pts)
Note:

$$\begin{aligned} q_i &= \frac{5}{2} - \frac{1}{4}q_j \\ q_i^\star &= \frac{5}{2} - \frac{1}{4}\left(\frac{5}{2} - \frac{1}{4}q_i^\star\right) \\ &= \frac{15}{8} + \frac{1}{16}q_i^\star \\ q_i^\star &= 2 = q_A^\star = q_B^\star \end{aligned}$$

Note that

$$\begin{aligned} p_i &= 8 - q_i - \frac{1}{2}q_j \\ p_i^\star &= 8 - 2 - \frac{1}{2} \cdot 2 = 5 \end{aligned}$$

for both A and B. The profits are therefore:

$$\pi_i = 5 \cdot 2 - 2 - 3 \cdot 2 = 2$$

for both A and B.

c) What is each firm's conjectural variation? What actual variation do the BR functions imply (e.g., $d\text{BR}_B/dq_A$)? (4 pts)
The conjectural variation is the assumption that firm i cannot affect firm j's level of output. In other words, i assumes that conjectural variation is zero. The actual variation is $-1/4$.

d) Now assume that both firms choose price (not quantity) simultaneously and independently. What now are their best response functions? NE prices, outputs and payoffs? Conjectural and implied actual variations? (6 pts)
Choosing price,

$$\begin{aligned} p_A &= 8 - q_A - .5q_B \\ p_B &= 8 - q_B - .5q_A \end{aligned}$$

$$\begin{aligned} q_i &= 8 - p_i - .5q_j \\ &= 8 - p_i - .5\left(8 - p_j - .5q_i\right) \\ &= 4 - p_i + .5p_j + .25q_i \\ q_i &= \frac{16}{3} - \frac{4}{3}p_i + \frac{2}{3}p_j \end{aligned}$$

$$\max_{p_i} p_i \cdot \left(\frac{16}{3} - \frac{4}{3}p_i + \frac{2}{3}p_j\right) - 2 - 3\left(\frac{16}{3} - \frac{4}{3}p_i + \frac{2}{3}p_j\right)$$

$$\max_{p_i} \frac{28}{3}p_i - \frac{4}{3}p_i^2 + \frac{2}{3}p_j p_i + constant$$

FOC

$$\begin{aligned} \frac{28}{3} - \frac{8}{3}p_i + \frac{2}{3}p_j &= 0 \\ p_i &= \frac{7}{2} + \frac{1}{4}p_j \\ p_i &= \frac{7}{2} + \frac{1}{4}\left(\frac{7}{2} + \frac{1}{4}p_i\right) \\ p_i^\star &= \frac{5}{4} \cdot \frac{7}{2} \cdot \frac{16}{15} = \frac{14}{3} \end{aligned}$$

for both A and B. Output is

$$q_i^\star = \frac{16}{3} - \frac{4}{3} \cdot \frac{14}{3} + \frac{2}{3} \cdot \frac{14}{3} = \frac{20}{9}$$

for both A and B. Conjectural variation is again zero, because firm i takes firm j's price as given. Yet, actual variation is $1/4$.

e) Now suppose that they are able to form a cartel. What is the maximized total profit for the two firms? What are the corresponding prices and outputs? What are the corresponding conjectural variations?
Maximize jointly

$$\max_{q_A, q_B} \left(8 - q_A - \frac{1}{2}q_B\right) q_A - 2 - 3q_A + \left(8 - q_B - \frac{1}{2}q_A\right) q_B - 2 - 3q_B$$

$$\max_{q_A, q_B} 5q_A + 5q_B - q_A^2 - q_B^2 - q_B q_A - 4$$

FOC

$$\begin{aligned} 5 - 2q_A - q_B &= 0 \\ 5 - 2q_B - q_A &= 0 \end{aligned}$$

Solving

$$\begin{aligned} q_A &= 5 - 2q_B \\ &= 5 - 2\left(5 - 2q_A\right) \\ &= -5 + 4q_A \\ q_A &= \frac{5}{3} = q_B \end{aligned}$$

Prices are as follows:

$$p = 8 - \frac{5}{3} - \frac{1}{2}\frac{5}{3} = 5.5$$

Profits are as follows:

$$\pi = 5.5 \cdot \frac{5}{3} - 2 - 3 \cdot \frac{5}{3} = \frac{13}{6} > 2$$

Collusion conjectural variation is that the other firm will maintain its share of output. Written properly, actual variation here is $-1/2$ for quantities.