Formulas for Econ 200, UCSC, Fall 2016

Equations for Competitive Markets

Linear Demand: $q_d = a - bp$ **Linear Supply**: $q_s = x + yp$
Log-linear demand: $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$ **Log-Linear Supply**: $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$
Total Surplus=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost
Total Cost=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost
Quantity Tax (tax per unit): $p_d = p_s + t$; **Value Tax** (tax on percentage spent): $p_d = (1+t)p_s$
Price Elasticity of Demand: $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p}\frac{p}{D}$; If $|\varepsilon| > 1$ then curve is elastic.
Tax Incidence Formula: $p_s(t) = p^* - \frac{t|D'|}{S'+|D'|}$; $p_d = p^* + \frac{tS'}{S'+|D'|}$; If ε_d is constant: $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d|+\varepsilon_s}$

Equations for Consumer Choice and Demand

Marginal Utility: $MU_i = \frac{\partial u}{\partial x_i}$; **Marginal Rate of Substitution**: $MRS_{ji} = \frac{MU_i}{MU_j}$ and at interior optimum $= \frac{p_i}{p_j}$
Perfect Substitutes: $u(x_1, x_2) = x_1 + cx_2$; **Cobb-Douglas**: $u(x_1, x_2) = \ln(x_1) + c\ln(x_2)$
CES Utility: $u(x_1, x_2) = \frac{1}{\rho}\ln(x_1^\rho + x_2^\rho)$; $\rho \in (-\infty, 1]$; **Quasilinear**: $u(x_0, x_1) = x_0 + g(x_1)$
Marshallian Demand: $\mathbf{x}^* = (x_1^*(\mathbf{p}, m), x_2^*(\mathbf{p}, m), ...)$ is the solution to $\max_{\mathbf{x}\geq 0}$ s.t. $m - \mathbf{p}\cdot\mathbf{x}$. The Lagrangian is $\mathcal{L} = u(\mathbf{x}) + \lambda(m - \mathbf{p}\cdot\mathbf{x})$. The FOCs can be written $MU_i = \lambda p_i$ or $MRS_{ji} = \frac{p_i}{p_j}$.
The solutions $x_i^*(\mathbf{p}, m)$ are homogeneous degree 0.
Demand Elasticity identity for product i: $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j\neq i}\varepsilon_{ij} = 0$

Equations for Cost and Technology

Technical Rate of Substitution: $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j} < 0$;
MC: $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$, and $\int MC = VC$.
Factor Prices: $\mathbf{w} = (w_1, w_2, ..., w_n)$; **Production Function**: $y = f(x_1, y_2)$
Cost Function with two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$
$= \min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2$ s.t. $y = f(x_1, x_2)$
Shepard's Lemma conditional factor demand: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w}, y)}{\partial w_i}$
Learning Curve: The typical specification is for $Y_t = \Sigma_{s\leq t} y_s$, AC falls proportionately, $\ln AC_t = AC_0 - b\ln Y_t$

Equations for Competitive Firms

SR Profit Maximization: $\max_{y, x_v \geq 0} \pi = \max_{y\geq 0}[\max_{x_v\geq 0} R(y) - w_v x_v - w_f x_f$ s.t. $y = f(x_v, \bar{x}_f)] = \max_{y\geq 0}[R(y) - c(y)]$
Revenue if firm is competitive: $R(y) = py = pf(x_v, \bar{x}_f)$ **FOC of *un*conditional factor demand**: $p\frac{\partial f(x_v, \bar{x}_f)}{\partial x_v} = w_v$
Hotelling's Lemma, Supply: $y^*(p, \mathbf{w}) = \frac{\partial \pi(p, \mathbf{w})}{\partial p}$; unconditional factor demands: $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p, \mathbf{w})}{\partial w_i}$
Shutdown Condition (Competitive Firms): $-F > py - c_v(y) - F \implies AVC = \frac{c_v(y)}{y} > p$