LITIGATION WITH SYMMETRIC BARGAINING
AND TWO-SIDED INCOMPLETE INFORMATION

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Abstract.

We construct game theoretic foundations for bargaining in the shadow of a trial. Plaintiff and defendant both have noisy signals of a common-value trial judgment and make simultaneous offers to settle. If the offers cross, they settle on the average offer; otherwise, both litigants incur an additional cost and the judgment is imposed at trial. We obtain an essentially unique NE and characterize its conditional trial probabilities and judgments. Some of the results are intuitive, e.g., an increase in trial cost (or a decrease in the range of possible outcomes) reduces the probability of a trial. Other results reverse findings from previous literature. For example, trials are possible even when the defendant's signal indicates a higher potential judgment than the plaintiff's signal, and when trial costs are low, the middling cases (rather than the extreme cases) are more likely to settle.

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1. Introduction.

Seminal work on bargaining in the shadow of a trial by Landes (1971), Gould (1973), Shavell (1982), and others assumed that the participants had differential expectations. Later writers criticized this literature for not having the proper game theoretic underpinnings. The main points of this criticism were that these early authors (1) did not explicitly model how each side would take into account that the other side had a different but equally valid estimate of the trial outcome; and (2) often assumed that there would always be a settlement if the plaintiff expectations were below the defendant's expectations. These criticisms are valid, but the intuition behind the early work – that the litigants have differential expectations – is compelling. In this article, we provide a a game-theoretic foundation for this intuition. In the process, we derive new results, the most startling being that when trial costs are low, the probability of a trial is highest when the probability of the plaintiff winning is either very high or very low.

Plaintiffs and defendants have access to different information and therefore have different expectations about the outcome of a trial.[1] While pre-trial discovery may reduce some of this differential in information, much information cannot or will not be credibly conveyed to the other side. Examples include the nature of the argument that will be made to the jury, the questions to be presented to the opposing side's witnesses, and private information regarding the biases of the judge. Indeed, if the plaintiff communicated to the defendant that the judge scheduled to hear the

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[1] Our discussion is in terms of plaintiffs and defendants in the context of a civil trial. Clearly, the same type of logic holds for prosecutors and defendants in a criminal trial.

case was biased in favor of the plaintiff, then the defendant would ask for a different judge. In this paper, we consider two-sided incomplete information where both players take into account that their own information is partial.

In a recent review article, Daughety (2000) lists over 50 articles on litigation and settlement. A natural question is to ask is how the present paper distinguishes itself from this intellectual thicket.

A large majority of these articles deal with one-sided incomplete (asymmetric) information, e.g., the plaintiff, but not the defendant, knows the actual damages. Restricting our attention to those three articles that deal with two-sided asymmetric information, our paper differs from the previous work on a number of dimensions. First, the nature of the two-sided incomplete information is different. In previous articles, each side is privy to a special kind of information. For example, one side knows the extent of damages and the other side knows the probability of winning (Daughety and Reinganum, 1994) or the plaintiff knows the strength of the plaintiff's case while the defendant knows the strength of the defendant's case (Kennan and Wilson, 1993).[2] While there are some mathematical similarities, here we take the issue of common values head on – both parties are getting a signal of the expected outcome and both must take this into account, just as players in an auction must account for the other players' draws in order to avoid the winner's curse.

Another way that this paper differentiates itself from the previous literature is the bargaining protocol. In virtually all of the literature, one side makes an offer and then the other side observes the offer and either accepts it or rejects the offer, in which case there is a trial.[3] This

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[2] Schweizer (1989) is the third paper that deals with two-sided incomplete information, but he allows only two discrete information types, good news and bad news. Here we deal with a continuum of information types.

[3] There are extended versions, where there may be multiple offers by one party as in Spier (1992). The infinite-horizon offer-and-counter-offer model of Rubinstein (1982) does not fit the litigation game very well as there is usually a fixed court date, and in many situations there is no cost of delay (or required costly bargaining) to one or both sides. Furthermore, the Rubinstein model gives greater bargaining power to the side that makes the first offer. Here, we focus on games with

creates vastly different outcomes, depending on which side makes the (first) offer. The problem is so serious that Daughety and Reinganum (1994) considered two models -- one where the plaintiff makes the offer and the other where the defendant makes the offer. Here, we present the following symmetric bargaining protocol. The plaintiff submits his demand and the defendant submits her offer to a third party (perhaps a computer). If the offer is greater than or equal to the demand, then there is a settlement half way between the two; otherwise, the case proceeds to trial. [4]

Our bargaining protocol is an extension of Chaterjee and Samuelson (1983). They consider a buyer and a seller, each having a private value for the good drawn from a uniform distribution. If the demand by the seller is less than the offer by the buyer, then there is a trade at the half-way point; otherwise, the seller keeps the item and the buyer keeps her money. Our paper differs in two fundamental ways. As noted earlier, we are concerned with common values: the true value depends on both signals. Second, the truth can be revealed through a costly trial.

Our symmetric game has symmetric Nash equilibrium strategies that turn out to be piecewise linear and essentially unique. The simple equilibrium structure yields several useful results that were either not accessible or less intuitive in earlier work. For example, we derive exact expressions for the distribution of trial settlements as a function of trial costs, and characterize the cases that go to trial. Priest and Klein (1984) argued that those cases where the plaintiff has close to a 50 percent chance of winning are more likely to go to trial than cases where the plaintiff's probability of winning is closer to 0 or 1. We show that the results depend on the cost of the trial. When there are high costs of trial, the results confirm the Priest-Klein conjecture; however, when the costs of trial are low, the results are contrary.

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symmetric bargaining power. See Ausubel, Crampton and Deneckere (2002) for a recent review of Rubinstein type models.

[4] One might ask why the computer does not just announce a settlement at the average of the demands and offers even if there is not an overlap or undertake a more sophisticated analysis and figure out the true observations from the litigants' demands and offers and then announce a settlement at the average of their observations. Unfortunately that protocol provides perverse incentives, e.g., the plaintiff should demand infinity.

The next section presents a general symmetric model, and defines a tractable simple case. Section 4 derives precise quantitative results for the special case. Section 5 shows that the ordinal properties extend to the general model (and somewhat beyond). A discussion appears in Section 6, Appendix A gives formal proofs of the propositions, and Appendix B presents derivations supporting the main arguments in Section 5.

2. The Model

The litigation game has two players, referred to as the plaintiff and the defendant, and three stages. At the preliminary stage 0, the players have common knowledge about the structure of the game, including the payoff function and the distributions of signals. In particular, both players know that the lowest possible judgment is $L \geq 0$ and the largest is $U > L$. At stage 1, the plaintiff privately observes a signal θ_p drawn according to the cumulative distribution function F^P, and chooses a demand p. Simultaneously, the defendant privately observes an independent signal θ_d drawn from her cdf F^D, and chooses a demand d. At the final stage 2, the payoffs are determined as follows. (a) If $d \geq p$, the case is settled at the average offer $[p + d]/2$. (b) If $d < p$, the offers are inconsistent and the case goes to trial. Each player then incurs cost $c \geq 0$, and the judgment is $[\theta_p + \theta_d]/2$.

We consider pure strategies contingent on the realized signal. Thus a plaintiff strategy is a measurable function P defined on the support of $F^P \subseteq [L, U]$ that assigns the demand $p = P(\theta_p) \in [0, \infty)$ when he observes signal θ_p. Similarly, a defendant strategy is a measurable function D with support $F^D \subseteq [L, U]$ that assigns the offer $d = D(\theta_d) \in (-\infty, \infty)$ when she observes signal θ_d.

The objective of the plaintiff is to maximize the expected payments (net of any court costs that might be incurred), conditioned on his realized signal θ_p and the defendant's strategy D. The payoff function for the plaintiff is

(1) $\quad \Pi^P(p, \theta_p, D, F^D) = .5 \int\limits_{[d \geq p]} [p + D(x)] dF^D(x) + .5 \int\limits_{[d < p]} [\theta_p + x - 2c] dF^D(x).$

The notation $[d < p]$ indicates the set of defendant signals $\{x: d=D(x) < p\}$ that the given D function maps to numbers less that the plaintiff's chosen demand p, and $[d \geq p]$ indicates the complementary signal set. Hence the first term represents the expected payment due to a settlement and the second term represents the expected payment due to a trial judgment. A plaintiff strategy P is a *best response* to defendant strategy D and we write $P \in PBR(D)$ if, for each possible signal realization θ_p, the value $p = P(\theta_p)$ solves the problem $\max\limits_{y} \Pi^P(y, \theta_p, D, F^D)$.

Similarly, the defendant's strategy D is a best response to plaintiff's strategy P, or $D \in DBR(P)$, if it minimizes the expected payment (including possible court costs) at each signal realization θ_p. That is, $d = D(\theta_d)$ solves the problem $\min\limits_{x} \Pi^D(x, \theta_d, P, F^P)$, where

(2) $\Pi^D(d, \theta_d, P, F^P) = .5 \int\limits_{[d \geq p]} [d + P(y)] dF^P(y) + .5 \int\limits_{[d < p]} [\theta_d + y + 2c] dF^P(y).$

Of course, the notation $[d < p]$ now indicates the set of plaintiff signals $\{y: d < P(y)\}$, etc.

Definition. A Nash equilibrium (*NE*) of the litigation game is a strategy pair (D, P) such that $P \in PBR(D)$ and $D \in DBR(P)$.

Remarks.

The judgment at stage 2b is a version of common values. In another popular version of common values, each signal is equal to the judgment plus an independent mean zero error. That version appears to give the same qualitative results as the simpler version we use here.

At first glance, the game might appear to be zero-sum because the plaintiff maximizes an objective function that resembles the objective function minimized by the defendant. However, the objective functions are different in two respects. First, the trial cost c offsets the plaintiff's receipts but increases the defendant's payment, so there is an overall gain when the case is settled. Second, with two-sided incomplete information, the expectations in the two objective functions are taken over different signal spaces, so the objective functions actually are not comparable.

Several normalizations and simplifications will be useful. First, in equilibrium analysis, we can restrict attention to non-decreasing functions. As shown in Lemma 1 in Appendix A, if a strategy is a best response (to *any* opponent's strategy, whether or not an equilibrium strategy or even monotone) then it can be represented as a non-decreasing left-continuous function. Such functions have inverses,[5] so the integrals in (1) and (2) are from L to z and from z to U, where $z = D^{-1}(p)$ in (1) and $z = P^{-1}(d)$ in (2).

Next, without loss of generality we can normalize $L = 0$ and $U = 1$. This normalization can be obtained using a positive linear transformation of all variables (including cost, signals and bids) that has no effect on the optimization problems. However, the transformation has implications, discussed in section 4 below, for the assumptions that the trial cost c is positive and that the plaintiff will proceed to trial if there is no settlement.

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[5] A technical clarification: the inverse image of a point z in the range of a bounded non-decreasing left-continuous function D is either a single point $\{b\}$ or an interval $[a, b]$ (or $(a, b]$). We always set $b = D^{-1}(z)$; i.e., the function D^{-1} is defined as the supremum of the inverse image correspondence.

A *basic litigation game* is a litigation game with the $L = 0$, $U = 1$ normalization, the restriction to non-decreasing strategies, and (more substantively) with $c \geq 0$ and uniform signal distributions. That is, $F^D(\theta) = F^P(\theta) = \theta$ and $dF^D(\theta) = dF^P(\theta) = d\theta$ for all θ in [0, 1]. The uniform distribution assumption does involve loss of generality but, as discussed in section 4, perhaps less than one might think. It is very convenient because the objective functions in the basic litigation game take the form:

$$(3) \quad \Pi^P(p, \theta_p, D, F^D) = 0.5 \int_0^{D^{-1}(p)} [p + D(x)]dx + 0.5 \int_{D^{-1}(p)}^1 [\theta_p + x - 2c]dx$$

$$(4) \quad \Pi^D(d, \theta_d, P, F^P) = 0.5 \int_0^{P^{-1}(d)} [d + P(y)]dy + 0.5 \int_{P^{-1}(d)}^1 [\theta_d + y + 2c]dy.$$

3. Basic Results

The first question one might ask is whether a Nash equilibrium exists. An affirmative answer is not automatic because we have restricted strategy sets that exclude mixed strategies. In turns out, however, that the basic litigation game has an equilibrium of a very simple form.

Proposition 1. The basic litigation game has a NE in the piecewise linear, continuous bid functions graphed in Figure 1. The functions are:

(5) $P(\theta_p) = 2\theta_p/3 - 2c + 1/2$, truncated above at $min\{1, 2c + 1/2\}$ and below at $max\{0, 2c - 1/6\}$, and

(6) $D(\theta_d) = 2\theta_d/3 + 2c - 1/6$, truncated above at $min\{1, 7/6 - 2c\}$ and below at $max\{0, - 2c + 1/2\}$.

The minimum and maximum values of $d = D(\theta_d)$ are $2c - 1/6$ and $2c + 1/2$ when $\theta_d = 0$ and 1, respectively. So $p = P(\theta_p)$ is never strictly greater than the largest value of D nor strictly below the smallest value. Similarly, d is bounded by the natural range of P.

FIGURE 1A — D above P; equivalently c > 1/6

FIGURE 1B — P above D; equivalently, c < 1/6
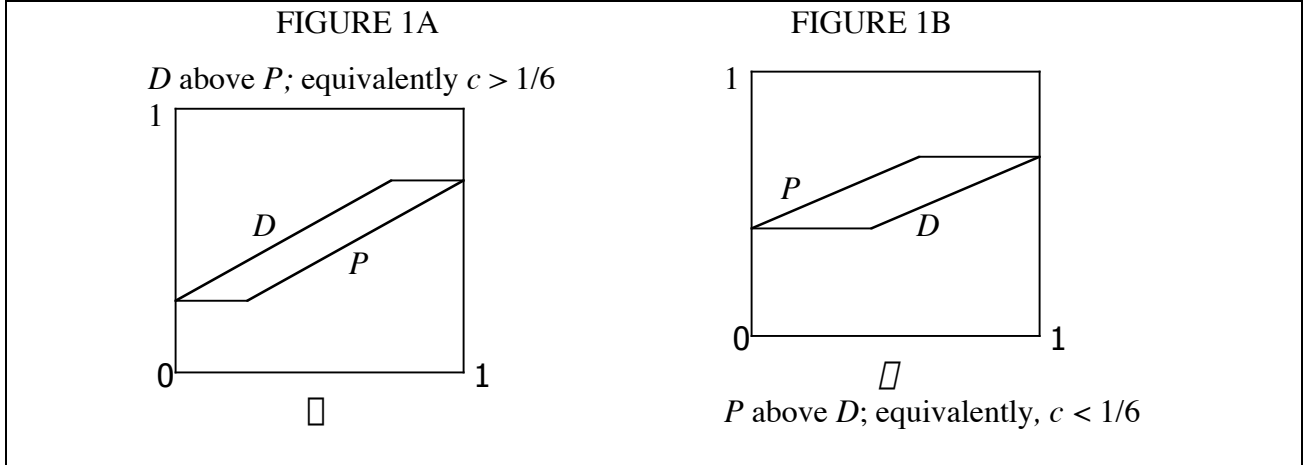
All proofs appear in the Appendix. The intuitive reason for the lower truncation in Figure 1A is that the plaintiff has no incentive to demand less than the lowest possible offer from the defendant. Demanding less would not increase the likelihood of a settlement (it is already 100%), but would reduce what the plaintiff collects in a settlement. Likewise, the upper truncation reflects the fact that the defendant has no incentive to offer more than the plaintiff's highest demand.

Figure 1B shows the perhaps more intuitive case where, at equal signals, the plaintiff demands more than the defendant offers. This case arises when $P(x) = 2x/3 - 2c + 1/2 > D(x) = 2x/3 + 2c - 1/6$. That is, when $c < 1/6$, the plaintiff's demand curve is above the defendant's offer curve. When $c > 1/6$, the plaintiff's demand curve is below the defendant's offer curve as in Figure 1A.

Next, one might ask whether there are other NE. Of course, there are *trivial* NE, where all cases end in trial because both the plaintiff and the defendant make offers certain to be rejected, e.g., $D(\theta) = 1$ and $P(\theta) = 0$ for all signals θ.[6] There are also variations on strategies (5) and (6) that are *inessential* in that they induce the same outcomes (i.e., the same mapping from signals to payoffs). To illustrate, suppose that $c = 1/12$. Then by (6) the maximum defendant offer is 2/3, and by (5) the plaintiff demand curve is higher and the case will go to trial with probability 1 whenever $\theta_p > .5$. Of course, the trial outcome depends only on the signal, not the demand. Hence the outcome will always be the same if we replace (5) by any plaintiff demand function that coincides with (5) for $\theta_p \le .5$ and has arbitrary values in $[2/3, \infty)$ for $\theta_p > .5$. Thus some of the truncations in Figure 1 are inessential. However, they keep the graphs of the functions within the unit square, which simplifies later calculations.

Our uniqueness result covers equilibria that are symmetric in the sense that corresponds to the symmetry of the basic litigation game. For example, suppose that when the defendant observes the signal 1/4 she offers 1/3. Symmetry would imply that when the plaintiff observes 3/4 he demands 2/3. With this in mind, we make the following

Definition. The strategies P and D of the simple litigation game are symmetric if for all θ in [0,1] we have $P(\theta) = 1 - D(1 - \theta)$ or, equivalently, $D(\theta) = 1 - P(1 - \theta)$.

It is easy to see that the trivial NE strategies mentioned above are symmetric. So are the piecewise linear strategies (5-6). Our next result is that the equilibrium is essentially unique in its class.

Proposition 2. All non-trivial piecewise-linear symmetric Nash equilibria of the basic litigation game induce the same outcome as strategies (5-6).

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[6] We will drop the subscripts from θ_p and θ_d when the meaning is clear.

In the rest of this section, "equilibrium" refers to the outcome generated by strategies (5-6). We focus on the probability of trial and the distribution of cases that go to trial, because for empirical work trial data are much easier to collect than data on settlements. The first comparative statics result shows that the probability of a trial decreases nonlinearly in the trial cost c.

Proposition 3. The equilibrium probability of trial is $1 - 18c^2$ for $0 \leq c \leq 1/6$; $2(1 - 3c)^2$ for $1/6 \leq c \leq 1/3$; and 0 for $c \geq 1/3$.

Inspection of (5-6) provides much of the intuition. The probability of a trial is 1 if the highest defendant offer $(2/3 + 2c - 1/6)$ is below the lowest plaintiff demand $(-2c + 1/2)$, or equivalently, if $c \leq 0$. The probability of a trial is 0 if the lowest defendant offer $(2c - 1/6)$ is weakly above the highest plaintiff demand $(7/6 - 2c)$; that is, if $c \geq 1/3$. For intermediate costs, the case goes to trial when $p = (2/3)\theta_p - 2c + 1/2 > (2/3)\theta_d + 2c - 1/6 = d$, i.e., when $\theta_p - \theta_d > 6c - 1$. The last inequality is more likely to be satisfied, and thus the case is more likely to go to trial, when c is smaller. The Appendix derives the exact expressions.

Now consider the probability of a trial conditional on the value of the potential judgment $J = (\theta_p + \theta_d)/2$. Intuition (and the Priest-Klein result) suggests that cases with extreme judgments are more likely to be settled, and cases with judgments near the average of 0.5 are more likely to go to trial. Our next result confirms this intuition for high trial costs, but reverses it for low costs.

Proposition 4. The equilibrium probability of a trial increases in $|J - 0.5|$ when $c < 1/6$ and decreases in $|J - 0.5|$ when $c > 1/6$.

The intuition can be extracted from Figure 2. Recall from the discussion of Proposition 3 that a trial will take place if and only if $\theta_p - \theta_d > 6c - 1$, i.e. if and only if the signal combination (θ_p, θ_d) lies Northwest of the lines of slope +1 labeled $6c - 1 = K$. Consider first the case $K > 0$, i.e., $c > 1/6$; say $K = 0.5$ as in Figure 2. The trial region now is the triangle to the northwest of the dotted line. Lines of given value $J = (\theta_p + \theta_d)/2$ have slope -1. For J near zero (and for J near 1)

the negatively sloped J line does not intersect the trial triangle. Hence the trial probability conditioned on such J is zero. For a value of J closer to 0.5, such as that shown in Figure 2, the J line does intersect the trial triangle. Since signals are independent and uniform, their joint distribution is uniform over the unit square. Hence the desired conditional trial probability is the length the J line segment inside the triangle as a fraction of its length inside the unit square. This is clearly maximized when the J line meets the corner of the triangle, i.e., when $J = 0.5$.

FIGURE 2: The probability of a trial
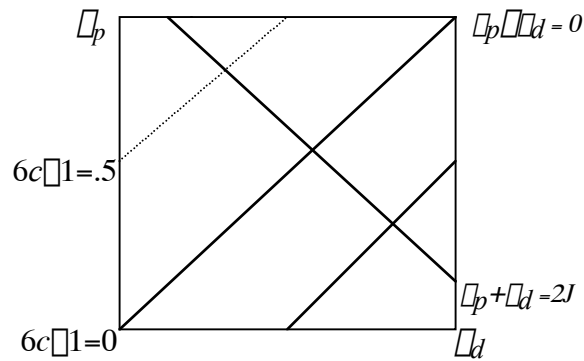
To see the less intuitive case, suppose $c < 1/6$ so $K < 0$ as in the unlabeled upward sloping line below the diagonal in Figure 2. The settlement region now is the triangle Southeast of this line, while the trial region is the rest of the unit square, Northwest of the line. In this case, lines with extreme values of J do not intersect the settlement region, and the trial probability is 1. As J moves towards 0.5, a larger fraction of the J segment lies in the settlement region and the trial probability decreases.

The geometry comes from the fact that when c is small, the plaintiff's demand curve in Figure 1b is above the defendant's offer curve. When J is very high (or very low), the litigants' signals must be fairly similar because J is the average of the signals. When the litigants' signals are similar and c is small, p will tend to be above d even if θ_p is slightly smaller than θ_d. As a result, the case goes to trial. However, when J is close to .5, it is possible that the defendant observed a signal close to 1 and the plaintiff observed a signal close to 0. In such situations, the

plaintiff's demand will be below the defendant's offer even though the plaintiff's demand curve is above the defendant's offer curve. If we interpret θ as probability (and assume that the award is fixed at, say, one million dollars), then this result says that those cases where the plaintiff has a 50 percent chance of winning are the least likely to go to trial. Thus, the Priest and Klein 50% rule does not hold when the cost of a trial is low.

Of course, when the cost of trial is high, then the plaintiff's demand curve is below the defendant's offer curve (as in Figure 1a). As a consequence, small differences that arise when the trial outcome is either close to 0 or 1 will not be sufficient to make the plaintiff's demand above the defendant's offer. So a trial will not take place. On the other hand, when J is near .5, it is quite possible for θ_p to be sufficiently larger than θ_d so that p is greater than d, resulting in a trial.

Our last result for the basic litigation game requires the following definition.

Definition. A random variable x has a *tent distribution* on $[L, U]$ if it has a piecewise linear continuous density with four pieces: a piece with slope $s > 0$ from L to an intermediate point $M = aU + (1 - a)L$ for some $a \in (0, .5)$, a second piece with slope $m \in [0, s]$ from M to $(L + U)/2$, a third piece with slope $-m$ from $(L + U)/2$ to $N = aL + (1 - a)U$, and a final piece from N to U with slope $-s$.

The better-known *triangle distribution* is the degenerate case $m=s$; see Appendix A for further discussion. The uniform distribution is the degenerate case at the other extreme, where $s \to \infty$ and $m = 0$. Hence tent distributions are unimodal, piecewise-linear distributions between the uniform and the triangular.

Proposition 5. The distribution of *potential* trial judgments is a triangle on $[0, 1]$. The equilibrium distribution of *actual* trial judgments is a tent on $[0, 1]$ if $0 < c < 1/6$, and is a triangle on $[K/2, (1-K)/2]$ if $1/6 \le c < 1/3$, where $K = 6c - 1$.

It is well known that the sum of two independent, uniformly distributed random variables has the triangle distribution, and potential judgments are simply half the sum of the signals. Hence the first part of the proposition is routine. The equilibrium distribution is less obvious, but the logic from Proposition 4 extends as follows.

FIGURE 3: Tent distribution of trial outcomes.
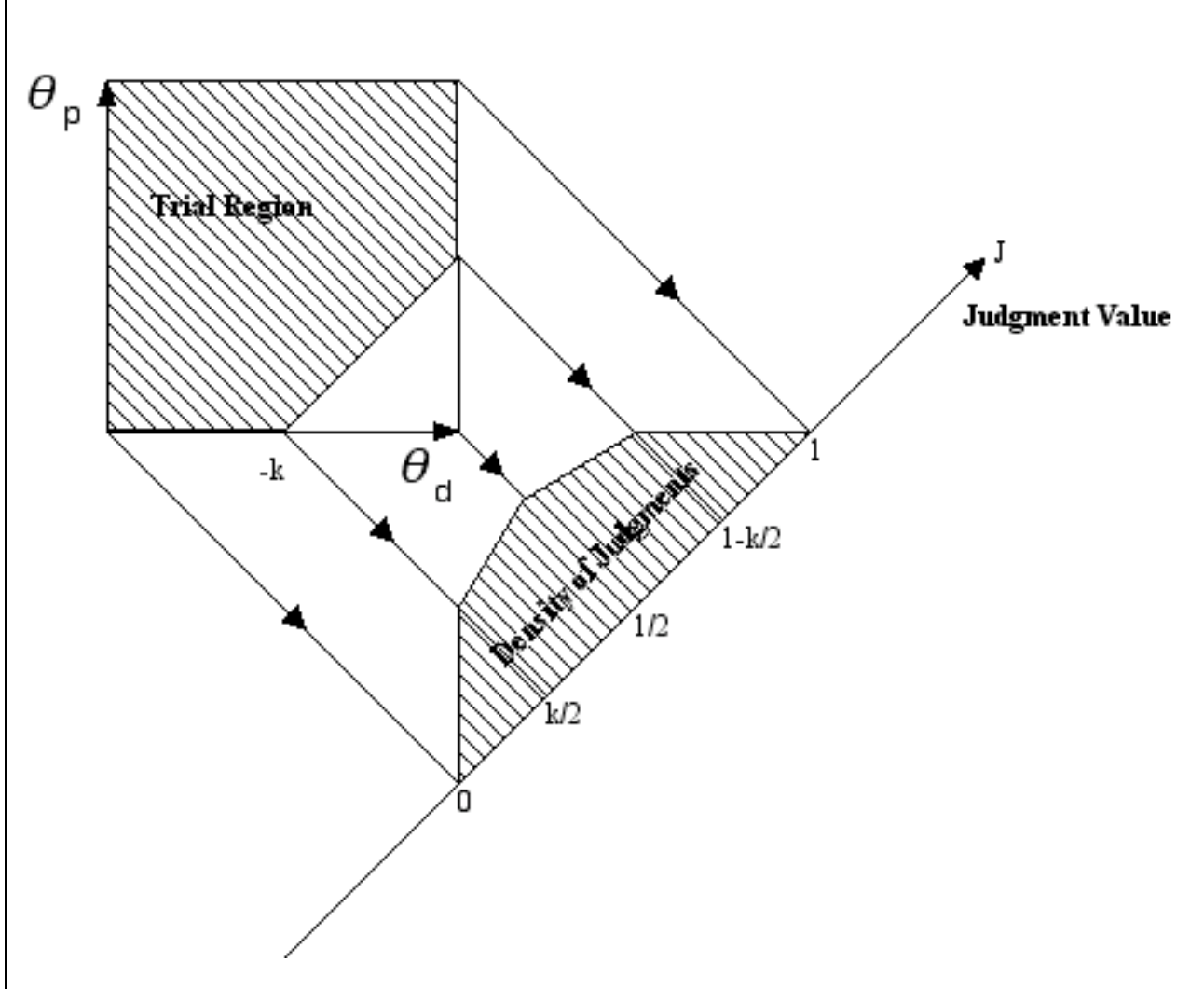
When c is relatively large, the cases that go to trial come from the Northwest triangle, and all the extreme cases are settled. Hence the distribution of observed trial judgments is triangular, and concentrated at the center of potential judgments.

When *c* is relatively small, cases from the Southeast triangle are settled, so the observed trial judgments come from the shaded region of Figure 3. The density height is just a renormalization of the length of the J-lines in the trial region. Hence the distribution in this case is a tent, and is more dispersed (closer to uniform) than the overall distribution of potential judgments.

4. Extensions.

Additional insights can be gleaned from relaxing the assumptions of the basic litigation game. Consider first shifting the distribution of trial outcomes by adding or subtracting a constant *A* to the upper and lower endpoints *U* and *L* of the range of possible outcomes. In the litigation game, the probability of trial depends on the differential in expectations, not the level of these expectations. Hence such shifts in the outcome range have no effect, other things equal. However, thinking carefully about such shifts leads to a more subtle issue concerning the way the litigation game is specified. We shall return to this issue shortly.

Next, consider increasing the width *U - L* of the trial outcome range, holding constant the cost $c \geq 0$, and maintaining the assumption that signals are independent and uniform on [*L, U*]. We also assume for simplicity (a weaker assumption will do) that *L* does not increase, and refer to such a stretching of possibilities as an *outcome spread*. We have a sharp comparative statics result.

Proposition 6. An outcome spread increases the equilibrium probability of a trial.

Here equilibrium still refers to outcomes generated by the Nash equilibrium strategies (5-6), except that the variables are linearly transformed to apply to [*L, U*] ≠ [0, 1]. The normalized trial cost *c* has *U - L* in the denominator, hence the relative cost of a trial falls in an outcome spread. In Figure 2, this is represented by a downward (or Southeasterly) shift in the *K* line, hence an increase in the area of the trial region (as a fraction of the area of the entire square). Appendix A shows that the desired result turns out to be a corollary of Proposition 3, which tells us that a (relative) cost decrease indeed increases the trial probability.

What happens if we relax the assumption of independent uniformly distributed signals? Signal dependence *per se* does not appear to introduce any interesting new issues. The signal can be decomposed into a common component that shifts the outcome midrange, and idiosyncratic components that are independent. As long as the litigants are risk neutral, it seems that only the idiosyncratic components matter.

The more important generalizations are to independent signals from a distribution that may not be uniform, to trial costs that may not be symmetric, and to bargaining protocols that might not split the difference equally. The ordinal properties of our main propositions survive quite well, and extend the intuition of the basic litigation game. There are three parts to the argument.

First, an increase in own trial cost induces more moderate offers and demands. This result is intuitive, but the generality of the setting requires some extra work. Appendix B derives the direct effect d_c, the rate at which the defendant's best response offer increases as her cost increases, holding constant the plaintiff's demand function. It also derives the indirect effect d_s, the rate at which her offer increases as the plaintiff's demand shifts up. The plaintiff has an analogous direct effect p_c for an increase in his cost, and indirect effect p_s for a shift in the defendant's offer function. Appendix B shows that that the direct effects are always towards moderation: $d_c > 0 > p_c$ in the relevant region. The direct effects cause shifts in the best response functions that reverberate via the indirect effects. The total effect of a change in own trial cost is the sum of a geometric series, $d_c/(1 - d_s p_s)$ for the defendant and $p_c/(1 - d_s p_s)$ for the plaintiff. Thus in the usual case $d_s p_s < 1$, the total effect has the same sign as the direct effect.

In the basic litigation game the effects turn out to be $d_c = -p_c = 4/3$ and $d_s = p_s = -1/3$. Hence a unit increase in defendant's cost only will increase her offer by $(4/3)/(8/9) = 3/2$ and reduce the plaintiff's demand by $(-1/3)(3/2) = -1/2$. Similarly, a unit increase in plaintiff's cost only will reduce his demand by $-3/2$ and increase plaintiff's offer by $1/2$. Thus we confirm the equilibrium cost coefficients (for equal shifts in both litigants' costs) in equations (5-6) of $3/2 +$

$1/2 = 2$ and $-1/2 - 3/2 = -2$, and now see that 75% of the impact is due to own cost and 25% to the other player's cost.

The second part of the argument notes the consequences for extreme signal values. Figure 4 illustrates two key cases. Suppose that in equilibrium D is above P, as in panel A. For reasons given earlier, P will never be below the minimum value of D; hence $P(\theta_p) = D(L)$ for $\theta_p \leq \theta_p^{CL}$.

Similarly, D will never be above the maximum value of P; hence $D(\theta_d) = P(H)$ for $\theta_d \geq \theta_d^{CH}$.

Panel B shows the opposite case where P is above D, and again each litigant's function has a flat segment over a range of extreme signal values. The same thing happens if the functions were to intersect in the interior. Indeed, there is always a flat segment at both extremes unless (a) both untruncated functions are strictly increasing and (b) $D(L) = P(L)$ and/or $D(H) = P(H)$. But the first part of the argument shows that ties as in (b) are exceptional and will be broken by slightly increasing or decreasing either litigant's trial cost. Hence we can safely assume a flat segment in one of the litigant's functions at both extremes.

FIGURE 4A

D above P

FIGURE 4B

P above D
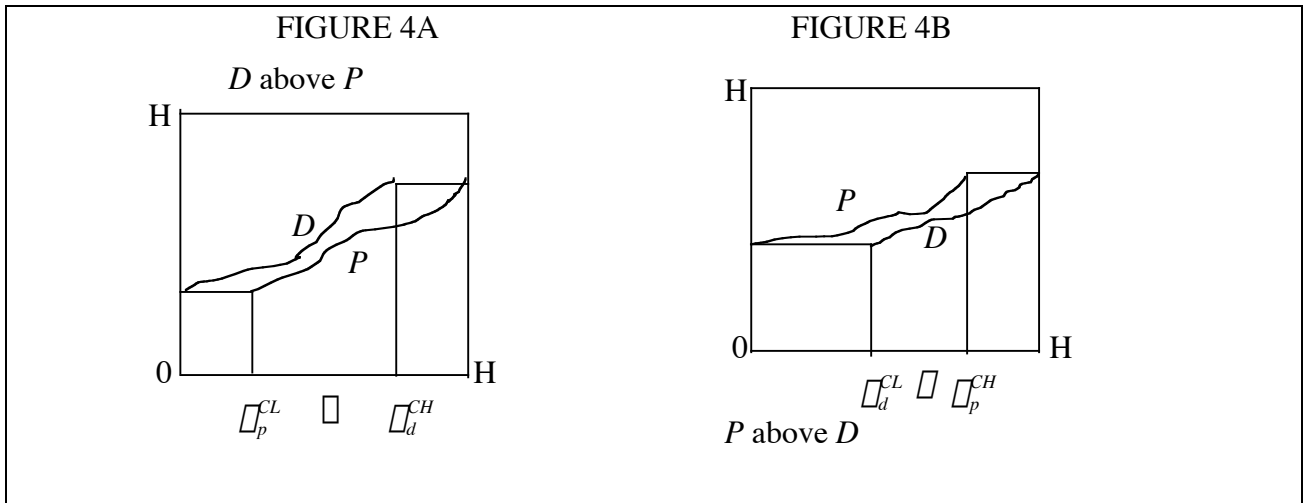
The third and last part of the argument is the same as for Proposition 4, and so is the intuition. When J is either very large or very small, the observations by the litigants must be similar. If the demand curve is below the offer curve, this means that the case will be settled; if the demand curve is above the offer curve, this means that the case will be tried. More specifically, in

Case A (where D is above P), a settlement is certain for all judgments in the more extreme half of the flat segments ($J < (L + \theta_d^{CL})/2$ and $J > (H + \theta_p^{CH})/2$ in Panel A), and in general the cases that go to trial oversample the middling judgments. By the first part of the argument, case A is to be expected when trial costs are high. By the same token, when trial costs are low we expect to see case B (where P is above D). Now trial cases oversample the more extreme judgments, and the most extreme judgments ($J < (L + \theta_p^{CL})/2$ and $J > (H + \theta_d^{CH})/2$ in Panel B) are certain to go to trial. Hence our main qualitative results appear to be quite robust.

To counter a bias we perceive in much of the literature, we have modeled litigation as entirely symmetric. We see no reason to suppose informational or bargaining asymmetries based on who makes the offer, but there is a natural strategic asymmetry: the plaintiff can always drop the case, or not bring it to begin with, while the defendant has no such options (unless there is a potential for a countersuit). The general litigation model presented in section 2 does not allow for this asymmetry, but it may now be worth a brief discussion.

The key question is when the plaintiff would prefer to drop the case after his demand has been rejected. A numerical example will help fix ideas. Suppose that $\theta_p = 0$, then $p = 1/2 - 2c$ according to equation (5). If the plaintiff's demand is rejected, he can infer $(2/3)\theta_d + 2c - 1/6 < 1/2 - 2c$, i.e., $\theta_d < -6c + 1$. If the case goes to trial, he expects to get $.5[0] + .5(.5)[1 - 6c] - c = .25 - 2.5c$. Hence $c < .1$ implies that the threat of a trial is always credible in the basic litigation game. For $c > .1$ in more general models, the threat of a trial is still credible if $c < L$. Assuming $c < L$ is the standard approach used in the literature to insure that the plaintiff's threat of going to trial is always credible.

5. CONCLUDING REMARKS

We obtained a nontrivial NE for the basic litigation game and showed it was essentially unique in its class (Propositions 1-2). In this equilibrium, a trial takes place if and only if $\theta_p - \theta_d > 6c - 1$. It is instructive to compare this result to the traditional divergent expectations story, where a trial

takes place if and only if the differential in the litigants' draws are greater than the total cost of going to trial, i.e., a trial takes place if and only if $\theta_p - \theta_d > 2c$. Our model shows that the traditional condition is neither necessary nor sufficient. If $c < 1/4$, then $6c - 1 < 2c$ and a trial is possible in equilibrium even though the condition $\theta_p - \theta_d > 2c$ is violated. For example, if $c = 1/6$, then, inconsistent with the traditional divergent expectations story, there will be a trial whenever θ_p is between θ_d and $\theta_d + 1/3$. Similarly, if $c > 1/4$ then $6c - 1 > 2c$ and there will be equilibrium settlements inconsistent with the traditional story. For example, if $c = 1/3$, there will always be a settlement, while the divergent expectations story would predict a trial whenever $\theta_p - \theta_d > 2/3$.

We derived several comparative static results regarding the probability of a trial. We showed that an increase in c (or a decrease in the range of possible outcomes) reduces the probability of a trial (Propositions 3, 6).

We also showed that when the trial cost is relatively high, trials are more apt to come from the cases with potential judgments near the median (Propositions 4, 5). To compare this result with that of Priest and Klein, it is again useful to characterize the payoff as being one million dollars if the plaintiff wins, and to interpret θ_p and θ_d as the plaintiff's and defendant's estimates of the plaintiff's probability of winning. With this interpretation, our result is that cases are less likely to go to trial when the plaintiff's probability of winning is either very low or very high. Hence our result here is consistent with the argument put forth by Priest and Klein that trials tend to select from cases with a 50% chance of winning.

However, when the trial cost is relatively low, trials are more apt to come from the cases with potential judgments at the extremes (Propositions 4, 5), contrary to Priest and Klein. Of course, exact comparisons are difficult because their analysis applies to a negligence rule and differs in other respects from our model. We suspect that the primary reason for the contrary results is their use of the traditional divergent expectations approach. Another possible reason is that they implicitly assume an additional error component that increases as the expected outcome approaches the mean outcome.

The work presented here provides several avenues for further research. One suggested by recent events is diplomacy in the shadow of war. Stage 1 of our litigation game can be reinterpreted as diplomatic negotiations between two sovereign nations with conflicting interests. A settlement is possible, but if the demands at stage 1 are incompatible, then there is a costly armed conflict. It seems reasonable to represent the expected outcome of the war by the common value expression used in the litigation model, $[\theta_p + \theta_d]/2$. To pursue this application, it would make sense to relax the condition $L \geq 0$.[7]

Theorists may enjoy exploring a second general avenue for further research. We suspect that stronger versions of Proposition 2 can be obtained, demonstrating essential uniqueness within broader classes of strategies. One perhaps could characterize precisely the class of distributions for and payoff asymmetries which the ordinal comparative statics results hold. Appendix B and the discussion in Section 5 is a step in this direction. There also are many variants of the litigation game (e.g., with plaintiff options to not file and to drop after negotiation) or the diplomacy game that could be formalized and explored.

Most importantly, there is now a better opportunity for empirical tests. The ordinal comparative statics results can be tested indirectly on existing field data, or tested directly on lab data. It will be particularly interesting to see whether the low-trial-cost result can account for observed data, since it contradicts previous intuition and conventional wisdom.

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[7] Of course, one might prefer a less static model of bargaining where war and diplomacy are available every period. But the static model is still useful as it is an upper bound on bargaining efficiency.

APPENDIX A

Lemma 1. Any best response in the litigation game can be represented by a non-decreasing left-continuous function.

Proof. Let D be an arbitrary defendant strategy, and suppose that plaintiff strategy P does not have the desired monotonicity property. We will show that there is a better response, Q, that is closer to monotonic. If P isn't non-decreasing, then there are points $a < b$ s.t. $A = P(a) > P(b) = B$. Let $Q(a) = B$ and $Q(b) = A$ and let $Q(x) = P(x)$ for $x \neq a, b$. The payoff sum at points a and b for Q is half of:

$$\Pi_Q = \int_{[d \geq B]} [B + D(y)]dF(y) + \int_{[d < B]} [a + y - 2c]dF(y) + \int_{[d \geq A]} [A + D(y)]dF(y) + \int_{[d < A]} [b + y - 2c]dF(y)$$

The corresponding sum for P is

$$\Pi_P = \int_{[d \geq A]} [A + D(y)]dF(y) + \int_{[d < A]} [a + y - 2c]dF(y) + \int_{[d \geq B]} [B + D(y)]dF(y) + \int_{[d < B} [b + y - 2c]dF(y)$$

The payoff difference is

$$\Pi_Q - \Pi_P = \int_{[d \in [B,A]]} [(b + y - 2c) - (a + y - 2c)]dF(y) = (b - a)\Pr_F[D(y) \in [B, A]].$$

Since $b > a$ by hypothesis, the last expression is non-negative, and it is strictly positive if the defendant's offers fall between B and A with positive probability. Hence Q is a better response, and strictly better except in inessential cases. This establishes the desired non-decreasing property for plaintiff's best response. The argument for the defendant is similar.

Non-decreasing bounded functions on a compact set are well-known to have at most a countable set of discontinuity points; hence left-continuity is without loss of generality (see Loeve, 175, for the proof).///

The key intuition is that for given demand levels A vs B, the tradeoff between trial and settlement favors trial more at the higher signal b, because a settlement is directly unaffected by the signal, but a trial outcome increases in the signal. The main idea in the proof is analogous to the "bubble sort" algorithm: whenever one finds a lower demand at a higher signal, one switches

the demands, thereby creating a better response. The ultimate effect is that the demand is everywhere (weakly) increasing in the signal, as desired.

Proposition 1. The basic litigation game has a NE in the piecewise linear, continuous bid functions graphed in Figure 1. The functions are:

(A1) $P(\theta_p) = 2\theta_p/3 - 2c + 1/2$, truncated above at $min\{1, 2c + 1/2\}$ and below at $max\{0, 2c - 1/6\}$, and

(A2) $D(\theta_d) = 2\theta_d/3 + 2c - 1/6$, truncated above at $min\{1, 7/6 - 2c\}$ and below at $max\{0, -2c + 1/2\}$.

Proof.

Note first that it is impossible for both the demand and offer to equal 1. $P(\theta_p) = 2\theta_p/3 - 2c + 1/2 \geq 1$ only if $c \leq 1/12$. But if $c \leq 1/12$, then $D(\theta_d) = 2\theta_d/3 + 2c - 1/6 \leq 2/3$. A similar exercise shows that both the demand and offer cannot equal 0. Thus the 0, 1 truncations are inessential because when the truncations are operative, the case would go to trial whether there was a truncation or not, and the demands and offers do not affect the trial outcome. The truncation of $P(\theta_p)$ above at $2c + 1/2$ and the truncation of $D(\theta_d)$ below at $-2c + 1/2$ are also inessential for similar reasons.

The defendant minimizes his expected payment

(A3) $\Pi^D(d, \theta_d, P, F^P), = 0.5 \int_0^{P^{-1}(d)} [d + P(y)]dy + 0.5 \int_{P^{-1}(d)}^1 [\theta_d + y + 2c]dy$.

Differentiating this expression with respect to d, one obtains the first order condition:

(A4) $d/P'(P^{-1}(d)) + .5P^{-1}(d) - [.5P^{-1}(d) + .5\theta_d + c]/P'(P^{-1}(d)) = 0$.

Multiplying (A4) through by $P'(P^{-1}(d))$, we get the more convenient expression:

(A5) $d + .5P^{-1}(d)P'(P^{-1}(d)) - .5\theta_d - c - .5P^{-1}(d) = 0$

We first look at the case where $c < 1/6$. In this situation the plaintiff's demand curve is above the defendant's offer curve and the truncations are inessential; so we can ignore them. We are checking the defendants' best response to $P(\theta_p) = 2\theta_p/3 - 2c + 1/2$, so we substitute $P^{-1}(d) = 3d/2 + 3c - 3/4$ and $P' = 2/3$ into (A5) to obtain

(A6) $0 = d - 3d/12 - 3c/6 + 3/24 - .5\theta_d - c = .75d - 1.5c + 1/8 - .5\theta_d$

The unique solution is $d = 2\theta_d/3 + 2c - 1/6$, as desired. Moreover, the second derivative of the objective function is

(A7) $\Pi^D_{dd} = 1.5/P'(P^{-1}(d)) - .5/[P'(P^{-1}(d))]^2 = 1.5/(2/3) - .5/(4/9) = 9/4 - 9/8 = 9/8 > 0,$

so we are indeed at a minimum. Hence we have verified the best response when $c < 1/6$.

Next suppose that $c \geq 1/6$ so that the defendant offer curve is on or above the plaintiff demand curve and the demand and offer functions involve essential truncations as in Figure 1A. As explained in the text, the truncation does not reduce the probability of a settlement, but does make the settlement more favorable for the litigant. The computations above for $c < 1/6$ still hold, but we still need to check the truncations. It suffices to show that even when $\theta_d = 0$, the defendant will still want to settle. Here $d = 2c - 1/6$ and there will be a settlement for that amount. If the defendant were to raise his offer, he would clearly be worse off. If the defendant were to reduce his offer, then there would be a trial. Recall that the plaintiff will demand $2c - 1/6$ for all values of $\theta_p \leq 6c - 1$. Hence, the expected cost of the trial to the defendant would be $.5(0) + .5[.5(6c - 1)] + c = 2.5c - .25$. The term in brackets is the expected value of θ_p given that $\theta_p \leq 6c - 1$. When is the expected cost of a trial, $2.5c - .25$, greater than the cost of a settlement, $2c - 1/6$? When $.5c > 1/12$; equivalently, when $c > 1/6$ – the condition for the plaintiff's demand curve to be below the defendant's offer curve.

The game being symmetric, a similar argument verifies that the given piecewise-linear plaintiff strategy is a best response to the defendant's given strategy. ///

Proposition 2. All non-trivial piecewise-linear symmetric Nash equilibria of the basic litigation game induce the same outcome as strategies (5-6).

Proof. We will now make use of the symmetry conditions to rewrite (A5) as a function of D rather than P. We first explicitly consider the following symmetry relationships.

Let $y = P(z) = 1 - D(1 - z)$. Then $z = P^{-1}(y)$ and $D^{-1}(1 - y) = 1 - z$. Equivalently, $z = 1 - D^{-1}(1 - y)$. So $P^{-1}(y) = 1 - D^{-1}(1 - y)$ and $P'(P^{-1}(d)) = D'(1 - P^{-1}(d)) = D'(1 - 1 + D^{-1}(1 - d)) = D'(D^{-1}(1 - d))$.

Substituting these relationships into (A5) we get:

(A8) $d + .5[1 - D^{-1}(1 - d)]D'(D^{-1}(1 - d)) - .5\theta_d - c - .5[1 - D^{-1}(1 - d)]$

$= d + .5[1 - D^{-1}(1 - d)][D'(D^{-1}(1 - d)) - 1] - .5\theta_d - c = 0$

D is assumed to be piecewise-linear. Let $d = a_d + b_d c + e_d \theta_d$, then $\theta_d = (d - a_d - b_d c)/e_d$ and $D' = e_d$. Equation (A8) can be rewritten as follows:

(A9) $d + .5(e_d - 1) - .5(e_d - 1)(1 - d - a_d - b_d c)/e_d - .5\theta_d - c = 0$

Equivalently,

(A10) $d(1.5e_d - .5)/e_d = -.5(e_d - 1)(1 - 1/e_d) - .5(e_d - 1)a_d/e_d - .5(e_d - 1)(b_d c)/e_d + c + .5\theta_d$

or

(A11) $d = -.5(e_d - 1)(1 - 1/e_d)e_d/(1.5e_d - .5) - .5(e_d - 1)a_d/(1.5e_d - .5)$

$- .5(e_d - 1)(b_d c)/(1.5e_d - .5) + ce_d/(1.5e_d - .5) + .5\theta_d e_d/(1.5e_d - .5)$

Now the coefficient of θ_d is e_d. So from (A11), we have the following relationship:

(A12) $e_d = .5e_d/(1.5e_d - .5) = e_d/(3e_d - 1)$

The solutions are $e_d = 2/3, 0$. Clearly, the pair (A1-A2) satisfies these conditions. The inessential truncations need not hold because, as shown in Proposition 1, the probability of a trial is 1 regardless. So the outcome is not changed as long as the slope is greater than or equal to zero once the point of truncation is reached.

The only question remaining is whether there is another piecewise-liner function with this set of slopes but in a different combination. We first show that $P = 1/2 + 0\theta_p$, $D = 1/2 + 0\theta_d$ is not a NE for all $0 < c < 1/3$. Suppose that $\theta_p = 1$, then the expected outcome if the case goes to trial is 3/4. So the plaintiff will raise his demand above .5 if $c < .25$.

Every other horizontal line where D is below P always results in trial and therefore is trivial. The reverse can never be an equilibrium as all cases would be settled and the plaintiff would want to increase her demand and the defendant would want to decrease his offer.

There are two other families of possibilities:

(i) The 2/3-slope line is broken up by one or more 0 slope lines.
(ii) The 2/3-slope line is still in the middle, but the 0 slope line(s) start or stop at a different place.

We will focus on (i). The argument for (ii) is a blend of the previous arguments.

Let us consider a horizontal portion between two line segments with 2/3 slope. Moving along the horizontal portion, the defendant's loss from going to trial strictly increases continuously as θ_d increases. Furthermore, we know that that the probability of trial is less than 1 as the defendant increases his offer for still larger values of θ_d to reduce the probability of trial. Thus, the defendant should continuously strictly increase his offer to continuously reduce the probability of a trial. But a 0 slope says otherwise. Hence we are led to a contradiction. ///

Proposition 3. The equilibrium probability of trial is $1 - 18c^2$ for $0 \leq c \leq 1/6$; $2(1 - 3c)^2$ for $1/6 \leq c \leq 1/3$; and 0 for $c \geq 1/3$.

Proof. A case goes to trial when $p = (2/3)\theta_p - 2c + 1/2 > (2/3)\theta_d + 2c - 1/6 = d$, i.e., when $\theta_p - \theta_d > 6c - 1$. Clearly, as c increases the probability of a trial decreases. More specifically, the probability of a trial is 0 if the lowest defendant offer ($2c - 1/6$ in equilibrium) is weakly above the highest plaintiff demand ($7/6 - 2c$ in equilibrium); so in equilibrium the condition reduces to $c \geq 1/3$. In this situation, the plaintiff will demand the minimum defendant offer and the defendant will offer the maximum plaintiff demand. That is, $P(\theta) = D(\theta) = .5$ (remember that we are restricting our analysis to non-trivial symmetric equilibria).

Since signals are independent and uniformly distributed, the equilibrium trial probability is the area of the trial region, i.e., the subset of signal combinations in the unit square where $\theta_p - \theta_d > 6c - 1$. For $1/6 \leq c \leq 1/3$, the trial region is the isosceles triangle Northwest of the line of slope $+1$ with θ_p-intercept $6c - 1$. Hence the triangle has height $h = 1 - (6c - 1) = 2 - 6c$ and area $0.5h^2 = 2(1 - 3c)^2$. For $0 \leq c \leq 1/6$, the trial region excludes only the isosceles right triangle Southeast of the line of slope $+1$ with θ_d-intercept $1 - 6c$. That triangle has height $h = 6c$ and area $0.5h^2 = 18c^2$, and so the trial probability is the remaining area of the unit square, or $1 - 18c^2$. ///

Proposition 4. The equilibrium probability of a trial increases in $|J - 0.5|$ when $c < 1/6$ and decreases in $|J - 0.5|$ when $c > 1/6$.

Proof. For fixed $J \in [.5, 1)$, consider the set of signals on the line $(\theta_p + \theta_d)/2 = J$. Since $\theta_p = 2J - \theta_d$ and $J \in [.5, 1)$, the maximum value of θ_d is 1, in which case $\theta_p = 2J - 1$. Similarly, the maximum value of θ_p is 1, in which case $\theta_d = 2J - 1$. The J line has slope $\Delta\theta_p / \Delta\theta_d$ equal to -1. Hence the difference $K = \theta_p - \theta_d$ is uniformly distributed over the interval $(2J - 2, 2 - 2J)$.

Recall that a trial only takes place when θ_p - $\theta_d > 6c - 1$. Hence the probability $T(J)$ of a trial is the length of the subinterval where $K > 6c - 1$ divided by the length of the interval. The interval is empty and $T(J) = 0$ if $6c - 1 > 2 - 2J$, i.e., if $c > 0.5 - J/3$. Otherwise,

(A13) $T(J) = [(2 - 2J) - (6c - 1)]/[(2 - 2J) - (2J - 2)] = .25 [3 - 2J - 6c]/[1 - J]$.

To find the effect of an increase in J (the expected trial outcome) on the probability of a trial (T), we take the derivative of T with respect to J over the relevant interval, and simplify to obtain

(A14) $T'(J) = -2(.25)/[1 - J] + .25 [3 - 2J - 6c]/[1 - J]^2 = .25 [2J - 2 + 3 - 2J - 6c]/(1 - J)^2$

$$= .25[1 - 6c]/[1 - J]^2.$$

The sign of T' therefore is the sign of $1 - 6c$. When $c > 1/6$, the probability of a trial decreases, but when c < 1/6 the probability of a trial increases as the expected outcome goes from .5 to 1. So, the proposition is proven for $J \geq .5$. The analysis is similar when $J < .5$. ///

Proposition 5. The distribution of potential trial judgments is a triangle on [0, 1]. The equilibrium distribution of actual trial judgments is a tent on [0, 1] if $0 < c < 1/6$, and is a triangle on $[K/2, (1-K)/2]$ if $1/6 \leq c < 1/3$, where $K = 6c$ - 1.

Proof. For potential trial judgments, the density is the (integral = 1) normalization of the length $m(J)$ of the constant J line inside the unit square. Direct computation yields $m(J) = \sqrt{2} \min\{J, 1\text{-} J\}$, whose integral from $J = 0$ to 1 is $1/\sqrt{2}$. Hence the density is $2 \min\{J, 1\text{-} J\}$, which defines a triangular distribution.

For actual trial judgments, recall from proofs of the last two propositions the construction of trial regions in the unit square and the lines of constant signal difference K. First consider $K \geq 0$, i.e., $1/6 \leq c < 1/3$, and recall that in this case the trial region is an isosceles triangle Northwest of the line with slope +1 and y-intercept K. The constant-J line through that corner of the triangle has

value $J = K/2$. Hence the trial probability is 0 for $J \leq K/2$. For $J > K/2$, the length of the constant-J segment in the trial region increases linearly in J at rate $\sqrt{2}$ until the line intersects the apex of the trial triangle at $J=0.5$, as depicted in Figure 3. With further increases in J, the length decreases linearly at the same rate until the line meets the other corner of the trial triangle at $J = (1 - K)/2$. After the integral 1 normalization, we obtain a triangular distribution whose density peaks at $2/(1 - K)$ when $J = 0.5$.

The argument for the case $K < 0$, or $0 < c < 1/6$, is only slightly more complicated and is illustrated in Figure 3. The trial settlement region again is Northwest of the line of slope $+1$ whose x-intercept is $-K$. The entire J line lies inside the trial region for positive values of $J \leq |K/2|$, and its length (by simple geometry) is $2\sqrt{2}J$. When $J \in (|K/2|, 0.5)$, a portion of the J-line falls outside the trial region so its length increases only half as fast, at rate $\sqrt{2}$. Again, for $J > 0.5$ we have a symmetric decrease in segment length inside the trial region. The desired result follows from the proportionality of the density to the segment lengths and the definition of the tent distribution. ///

Proposition 6. An outcome spread increases the equilibrium probability of a trial.

Proof. Transform a litigation game on a nontrivial interval $[L, U]$ to a basic litigation game on $[0, 1]$ using the mapping $x \rightarrow (x + L)/(U - L)$. It is easy to check that independent uniform signal distributions retain those properties under the transformation. Note that the transformed trial cost is $c = (C + L)/(U - L)$ if the non-normalized cost is C. A spread is defined as an increase in $U - L = W$, so we consider the partial derivative of $c = (C + L)/W$ with respect W. It is easily calculated to be $(WL' - C - L)/W^2 \leq - (C + L)/W^2 < 0$, where the weak inequality comes from the proviso that L does not increase in W. Since the normalized cost decreases, we apply Proposition 3 to conclude that the equilibrium trial probability increases.///

APPENDIX B

We analyze the general effect of cost shifts when defendant has an offer function $d = D(\theta_d)$ and plaintiff has a demand function $p = P(\theta_p)$. (The notation d distinguishes the offer from the symbol d for a derivative.) We assume that litigants' signals are independent, and the distribution functions F^P and F^D have strictly positive densities f^P and f^D that are differentiable almost everywhere on $[L, H]$. By Lemma 1, we can assume that P', $D' > 0$ and have inverses with positive derivatives in the regions of interest.

Let $Z = P^{-1}(d)$. Then $P(Z) = d$ and $Z' = P^{-1'} > 0$. The defendant seeks to minimize

(B1) $\quad 2\Pi^D(d, \theta_d, c, P, F^P) = \int_L^Z [d + P(y)]dF^P(y) \; + \; \int_Z^H [\theta_d + y + 2c]dF^P(y).$

First order conditions for an interior solution:

(B2) $\; 2\,\Pi_d^D(d, \theta_d, c, P, F^P) \; = F^P(Z) + [d + P(Z)]f^P(Z)Z' - [\theta_d + Z + 2c]f^P(Z)Z'$

$= F^P(Z) + 2d\,f^P(Z)Z' - [\theta_d + Z + 2c]f^P(Z)Z'$

$= F^P(Z) - [\theta_d + Z + 2c - 2d]f^P(Z)Z' = 0.$

Since $F^P(Z), f^P(Z)Z' > 0$, equation (B2) implies that

(B3) $\; [\theta_d + Z + 2c - 2d] = \dfrac{F^P(Z)}{f^P(Z)Z'} > 0.$

Second order conditions for a minimum:

(B4) $2\,\Pi_{dd}^D(d, \theta_d, P, F^P) = \; 3f^P(Z)Z' - [\theta_d + Z + 2c - 2d][f^{P'}(Z)\,Z'^2 + f^P(Z)\,Z''] - f^P(Z)\,Z'^2$

$$= 3f^P(Z)Z' - \frac{F^P(Z)}{f^P(Z)Z'} [f^{P'}(Z) Z'^2 + f^P(Z) Z''] - f^P(Z) Z'^2 > 0$$

The inequality might not be strict at a set of isolated points, which we can safely ignore.

Equation (B4) is equivalent to

(B5) $\frac{F^P(Z)}{[f^P(Z)Z']^2} [f^{P'}(Z)Z'^2 + f^P(Z) Z''] < 3 - Z'.$

Let $\psi = \frac{1}{(\ln(F^P(Z))'} = \frac{F^P(Z)}{f^P(Z)Z'}$. Then by (B3) we have

(B6) $\psi = \theta_d + Z + 2c - 2d.$

Using the definition of ψ and (B5), we obtain the following relationship:

(B7) $\psi' = 1 - \frac{F^P(Z)[f^{P'}(Z)Z'Z' + f^P(Z)Z'']}{[f^P(Z)Z']^2} > Z' - 2.$

In the basic litigation game (BLG), $Z'' = 0 = f^{P'}$ and thus $\psi' = 1$.

We now compute the direct effect of an increase in the defendant's cost on the demand, d_c, holding constant the plaintiff's demand function, $P(\theta_p)$. Implicitly differentiating (B6), we get:

(B8) $\psi' d_c = Z' d_c + 2 - 2 d_c$

Thus the direct effect is

(B9) $d_c = 2/(\psi' + 2 - Z').$

Hence $d_c > 0$ if the denominator is greater than 0. But this follows from (B7). Thus the direct effect of an increase in c on D's offer is to increase d.

We next compute the effect p_s of a shift in D's offer on P's demand.

Let $W = D^{-1}(p)$, then $D(W) = p$. $W' > 0$ because $D' > 0$. The plaintiff seeks to maximize

$$(B10) \quad 2\Pi^P(p, \theta_p, c, D, F^D) = \int_W^H [p + D(x)]dF^D(x) + \int_L^W [\theta_p + x - 2c]dF^D(x).$$

The first order conditions for an interior solution are then:

$$(B11) \quad 2\Pi_p^P(p, \theta_p, c, D, F^D) = 1 - F^D(W) + [\theta_p + W - 2c - 2p]f^D(W)W' = 0.$$

Since $1 - F^D(W), f^D(W)W' > 0$, equation (B11) implies that

$$(B12) \quad [\theta_P + W - 2c - 2p] = \frac{F^D(W) - 1}{f^D(W)W'} < 0.$$

Second order conditions for a maximum:

$$(B13) \quad 2\Pi_{pp}^P(p, \theta_P, c, D, F) = -3f^D(W)W' + [\theta_p + W - 2c - 2p][f^{D'}(W)\ W'^2 + f^D(W)\ W''] + f^D(W)W'^2$$

$$= -3f^D(W)W' + \frac{F^D(W) - 1}{f^D(W)W'}[f^{D'}(W)W'^2 + f^D(W)W''] + f^D(W)W'^2 < 0$$

In the BLG this reduces to $-9/2 + 0 + 9/4 < 0$.

Equation (B13) is equivalent to

(B14) $\dfrac{F^D(W)-1}{[f^D(W)W']^2} [f^{D'}(W)W'^2 + f^D(W)\,W''] < 3 - W'$

Let $\xi = \; = \dfrac{F^D(W)-1}{f^D(W)W'}$. Then

(B15) $\xi \; = \; \theta_p + W - 2c - 2p$ by (B13).

W is the cognate of Z and ξ is the cognate of ψ.

(B16) $\xi' = 1 - \dfrac{[F^D(W)-1][f^{D'}(W)W'W' + f^D(W)W'']}{[f^D(W)W']^2} > W' - 2$ by (14).

Differentiating (B15) with respect to c, we obtain

(B17) $p_c = \dfrac{-2}{\xi' - W' + 2}$

The denominator is positive by (B16); so the expression is negative.

Now consider a shift in the defendant's offer function, D, by a small amount s (in either direction) and derive the sensitivity, p_s. At the margin $p = D(\theta_p) + s$, so now set $W = D^{-1}(p - s)$, and the first order condition is:

(B18) $\xi(p - s) \; = \; \theta_p + W(p - s) - 2c - 2p$.

Holding c constant, we take the derivative of this expression with respect to s and obtain an implicit equation for the desired effect p_s.

(B19) $\xi'(p - s)[p_s − 1] + 2p_s = W'(p − s) [p_s − 1]$. In turn this implies

(B20) $p_s = \dfrac{\xi'−W'}{\xi'−W'+2}$.   The denominator is positive by (B16).

We next compute the indirect effect, d_s, of a shift upwards in the plaintiff's demand on the defendant's offer. At the margin $d = P(\theta_d) + s$, so now set $Z = P^{-1}(d - s)$, and the first order condition is:

(B21) $\psi(d - s) = \theta_d + Z(d − s) + 2c − 2d$.

Holding c constant, we take the derivative of this expression with respect to s and obtain

(B22) $\psi'(d - s)[d_s − 1] + 2d_s = Z'(d − s) [d_s − 1]$. In turn this implies

(B23) $d_s = \dfrac{\psi'−Z'}{\psi'−Z'+2}$.

The total effect of the defendant's cost increase on the defendant's offer includes

(i) the direct effect of an increase in cost on the defendant's offer (d_c); plus

(ii) the first round indirect effect on the defendant's offer: the effect of the change in the defendant's offer on the plaintiff's demand which in turn has an effect on the defendant's offer ($d_c p_s d_s$); plus

(iii) the second round indirect effect on the defendant's offer due to the shift at the end of the first round ($d_c[p_s\, d_s]^2$); etc.

Hence the total effect is

$$(B24) \quad d_c + d_c p_s d_s + d_c [p_s \, d_s]^2 \ldots = d_c[1 + p_s d_s + [p_s d_s]^2 \ldots] = \frac{d_c}{1 - d_s p_s}$$

This expression is positive if $p_s \, d_s < 1$. Inspection of (B20) and (B23) shows that $0 < p_s < 1$ when $\xi' - W' > 0$ and that $0 < d_s < 1$ when $\psi' - Z' > 0$. In this case the indirect effect magnifies the direct effect of an increase in c. When either $\xi' - W' < 0$ or $\psi' - Z' < 0$, but not both, then $p_s \, d_s < 0$ and the indirect mitigates, but does not change the sign of the direct effect. And if both p_s and d_s are negative but the product is less than 1, then the indirect effect will again magnify the direct effect.

The total effect (B24) has a different sign from the direct effect only in the unusual case (a) $\xi' - W' < 0$, (b) $\psi' - Z' < 0$, and (c) the geometric mean of the absolute indirect effects exceeds 1, so $p_s \, d_s > 1$. We call the case unusual because we have not yet been able to find an example.

A similar analysis shows that the total effect of a change in the plaintiff's trial costs is $\frac{p_c}{1 - d_s p_s}$. Since the denominator is the same as for defendant's total effect, the same cases determine when the direct effect is magnified, mitigated or reversed.

In the basic litigation game, $\psi' = \xi' = 1$ and $Z' = W' = 3/2$. So $d_c = 2/(\psi' + 2 - Z') = 4/3$ and $d_s = \frac{\psi' - Z'}{\psi' - Z' + 2} = -1/3 = p_s$. Hence, $\frac{p_c}{1 - d_s p_s} = (4/3)(9/8) = 3/2$. That is, following a unit increase in his own cost, the defendant raises his offer (over the relevant range) by 3/2. The impact on the plaintiff is the indirect effect times this total effect, or $(-1/3)(3/2) = -1/2$. By symmetry, if the plaintiff also experiences a unit cost increase, then he reduces his demand by 3/2, inducing an in increase in the defendant's offer by 1/2. Hence, if both experience a unit cost increase, then the total effect is 2, as seen in the equilibrium bid functions.

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