

## Equations for Competitive Markets

**Linear Demand:**  $q_d = a - bp$  **Linear Supply:**  $q_s = x + yp$

**Log-linear demand:**  $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$  **Log-Linear Supply:**  $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$

**Total Surplus**=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost

**Total Cost**=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost

**Quantity Tax** (tax per unit):  $p_d = p_s + t$ ; **Value Tax** (tax on percentage spent):  $p_d = (1 + t)p_s$

**Price Elasticity of Demand:**  $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p} \frac{p}{D}$ ; If  $|\varepsilon| > 1$  then curve is elastic

**Tax Incidence Formula:**  $p_s(t) = p^* - \frac{t|D'|}{S' + |D'|}$ ;  $p_d = p^* + \frac{tS'}{S' + |D'|}$ ; If  $\varepsilon_d$  is constant:  $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d| + \varepsilon_s}$

## Equations for Consumer Choice and Demand

**Marginal Utility:**  $MU_i = \frac{\partial u}{\partial x_i}$ ; **Marginal Rate of Substitution:**  $MRS_{ij} = \frac{\frac{\partial u}{\partial x_j}}{\frac{\partial u}{\partial x_i}}$  and at interior optimum =  $\frac{p_i}{p_j}$

**Perfect Substitutes:**  $u(x_1, x_2) = x_1 + cx_2$ ; **Cobb-Douglas:**  $u(x_1, x_2) = \ln(x_1) + c \ln(x_2)$

**CES Utility:**  $u(x_1, x_2) = \frac{1}{\rho} \ln(x_1^\rho + x_2^\rho)$ ;  $\rho \in (-\infty, 1]$ ; **Quasilinear:**  $u(x_0, x_1) = x_0 + g(x_1)$

**Marshallian Demand:**  $x_i^*(\mathbf{p}, m) : \mathcal{L} = u(x_1, x_2) + \lambda(m - \mathbf{p} \cdot \mathbf{x})$

**Dual Problem; Hicksian Demand:**  $h_i^*(\mathbf{p}, u_0) : \min_{\mathbf{x}} \mathbf{p} \cdot \mathbf{x} \text{ s.t. } u(\mathbf{x}) \geq u_0$

**Roy's Identity:**  $x_i^*(\mathbf{p}, m) = -\frac{\partial v}{\partial p_i} / \frac{\partial v}{\partial m}$ ; **Shepard's Lemma:**  $h_i^*(\mathbf{p}, u) = \frac{\partial e(\mathbf{p}, u)}{\partial p_i}$

**Slutsky Equation:**  $\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, m))}{\partial p_i} - \frac{\partial x_i^*(\mathbf{p}, m)}{\partial m} x_i^*(\mathbf{p}, m)$ ; **(Elasticity Form):**  $\varepsilon_i = \varepsilon_i^h - s_i \varepsilon_m$ ;  $s_i = \frac{p_i x_i}{m}$

**Demand Elasticity** for product i, homogeneous of degree 0:  $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j \neq i} \varepsilon_{ij} = 0$

## Equations for Cost and Technology

**Technical Rate of Substitution:**  $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j}$ ; **MC:**  $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$  **MC to VC:**  $\int MC = VC$

**Factor Prices:**  $\mathbf{w} = (w_1, w_2, \dots, w_n)$ ; **Production Function:**  $y = f(x_1, y_2)$

**Cost Function** with two factors:  $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$

$= \min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2 \text{ s.t. } y = f(x_1, x_2)$

**Shepard's Lemma** conditional factor demand:  $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w}, y)}{\partial w_i}$

**Learning Curve:** The typical specification is for  $Y_t = \sum_{s \leq t} y_s$ , AC falls proportionately,  $\ln AC_t = AC_0 - b \ln Y_t$

## Equations for Competitive Firms

**SR Profit Maximization:**  $\max_{y, x_v \geq 0} \pi = \max_{y \geq 0} [\max_{x_v \geq 0} R(y) - w_v x_v - w_f x_f \text{ s.t. } y = f(x_v, \bar{x}_f)] = \max_{y \geq 0} [R(y) - c(y)]$

**Revenue** if firm is competitive:  $R(y) = py = pf(x_v, \bar{x}_f)$  **FOC of unconditional factor demand:**  $p \frac{\partial f(x_v, \bar{x}_f)}{\partial x_v} = w_v$

**Hotelling's Lemma, Supply:**  $y^*(p, \mathbf{w}) = \frac{\partial \pi(p, \mathbf{w})}{\partial p}$ ; unconditional factor demands:  $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p, \mathbf{w})}{\partial w_i}$

**Shutdown Condition** (Competitive Firms):  $-F > py - c_v(y) - F \implies AVC = \frac{c_v(y)}{y} > p$

## Equations for Monopolies

**FOC** for a monopolist:  $p(y) + p'(y)y = c'(y)$  which can be rewritten as  $p = \frac{1}{1 + \frac{1}{\varepsilon}} MC$ ; valid if  $\varepsilon < -1$

**Passing Along Costs:**  $\frac{\partial p}{\partial c} = \frac{1}{2 + yp''(y)/p'(y)}$

## Price Discrimination

Third Degree: Monopolist's Problem:  $\max p_1(x_1)x_1 - cx_1 + p_2(x_2)x_2 - cx_2$

$FOC_{x1} : c = p_1(x_1)[1 - \frac{1}{|\epsilon_1|}]$  Markup factor:  $M_i = \frac{1}{1 - \frac{1}{\epsilon_i}} = \frac{|\epsilon_i|}{|\epsilon_i| - 1}$

Quasilinear utility:  $\max u_i'(x) + y$  s.t.  $px + y = m$ ; FOC (inverse demand curve):  $p = u_i'(x)$

## Decision Theory

Probability Identities:  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ ;  $P(A \cap B) = P(B \cap A)$ ; Given probability sets A,B,C,D:  $P(A) = P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D)$

Bayes Theorem:  $P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$  and given that A is a binary variable  $\frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$

Also:

$\frac{p(s|m)}{p(t|m)} = \left[ \frac{p(m|s)}{p(m|t)} \right] \left[ \frac{p(s)}{p(t)} \right]$ . Can also take logs to get linear expression.

## Cournot

Given  $D(Y) = a - bY$ .  $BR_i(Y_{-i}) = \arg\max_{y_i} \pi_i = P(\sum_{i=1}^n y_i)y_i - c(y_i) \implies P(\sum_{i=1}^n y_i) + P'(\sum_{i=1}^n y_i)y_i - MC_i(y_i) = 0$

To solve for the Nash equilibrium, we want to find where the Best Response functions intersect.  $\Rightarrow NE_{Cournot} : Y^* = \frac{N}{(N+1)b}(a - c) \implies y_i^* = \frac{Y^*}{N} = \frac{a-c}{(N+1)b}$

## Stackelberg

$\arg\max D(Y) = a - bY \rightarrow BR_L = \max_{y_L} \pi_L(y_L, BR_F(y_L)) = D(y_L + BR_F)y_L - cy_L$

## Intertemporal Preference

Given  $U(c_0, c_1)$ , we have  $\frac{\partial_0 U}{\partial_1 U} = \frac{\frac{\partial U}{\partial c_0}}{\frac{\partial U}{\partial c_1}} = MRS_{01} = 1 + MRT_P$

Given Initial Endowment  $E = (e_0, e_1)$  and intertemp prod function  $y = f(x)$ , the PPF is  $\{(q_0, q_1) : q_0 = e_0 - x \leq 0, q_1 = e_1 + f(x) \leq 0\}$

ROI =  $f(x) - x$ ; AROI =  $\frac{f(x)}{x} - 1$ ; MROI =  $f'(x) - 1$

Present Value:  $PV_r(C) = c_0 + \frac{c_1}{1+r}$

Agent Optim:  $\max_x w = PV(Q) = q_0 + \frac{q_1}{1+r} = e_0 - x + \frac{e_1 + f(x)}{1+r}$  FOC:  $1 + r = f'(x) = 1 + MROI \implies r = MROI$

Optimal individual borrowing, consumption and lending:  $\max_{c_0, c_1 \leq 0} U(c_0, c_1)$  s.t.  $PV_r(Q) = PV_r(C) = w$ ,

given  $c_0 = q_0 + b$  and  $c_1 = q_1 - (1+r)b \Rightarrow \max_b U(q_0 + b, q_1 - (1+r)b)$

Fisher's Equation:  $k \approx r + \pi$  (obtained from  $1 + k = (1+r)(1+\pi)$ )

Present Value of discrete cash stream  $X = (x_0, x_1, \dots, x_t)$ :

$$PV_k(X) = \sum_{t=0}^T \frac{X_t}{(1+k)^t}$$

Present Value of continuous cash stream:

$$PV_k(X) = \int_{t=0}^t x_t e^{-kt} dt$$

General formula for interest rates and asset yields:  $k_a = r^* + \pi^e + RP_a \pm T_a$

$T_a$ : Transaction Costs  $RP_a$ : Risk Premium