Answers for Final practice problems

UCSC, Econ 200, Fall 2016

1. Business is good at Acme Products. George, the owner, estimates that he can earn an additional 12 per year indefinitely starting next year if he invests in expansion now, George is risk-neutral: he is not averse to risk, but neither does he seek it. The investment will be sunk once George makes the decision.

(a) (4 points) If George can finance at rate $k = 0.10$, what is his maximum willingness to pay for that investment?

Solution: Maximum willingness to pay for this investment is the present value of the expansion,

$$\begin{aligned} \frac{12}{(1+0.1)} + \frac{12}{(1+0.1)^2} + \cdots &= \left(\frac{12}{1-\frac{1}{1.1}}\right)\left(\frac{1}{1+0.1}\right) \\ &= 120. \end{aligned} \quad (1)$$

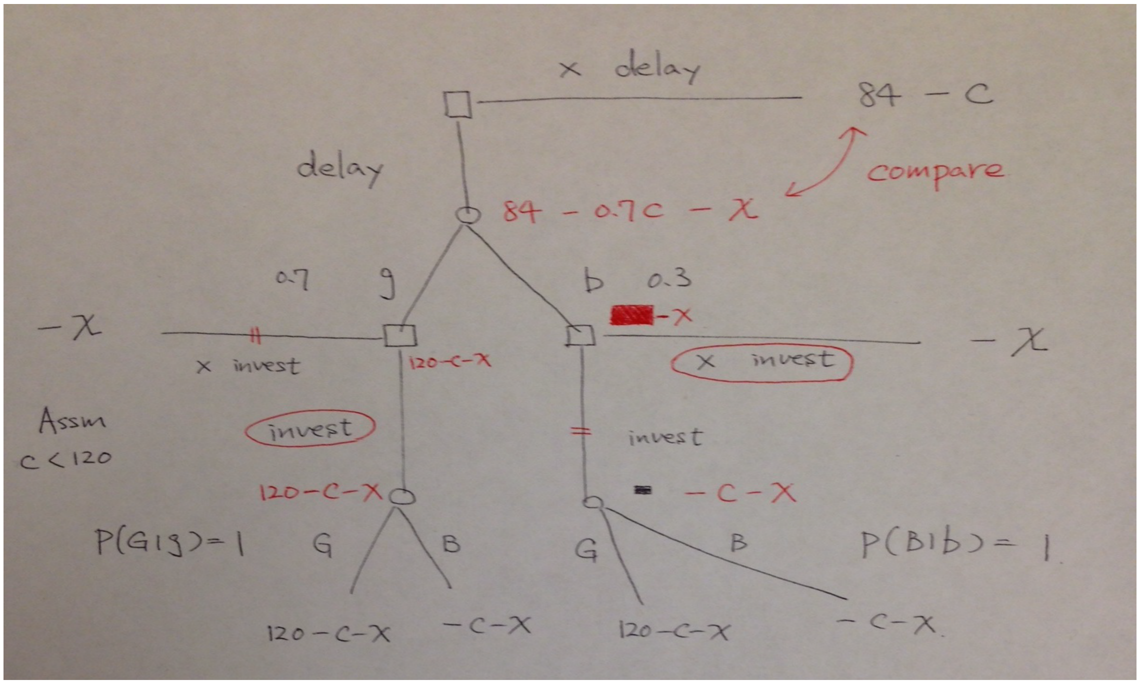
(b) (6 points) Suddenly George realizes that things might go wrong during the next year, and if so the additional earnings will be zero. He estimate the probability if 0.3 that things will go wrong. Now how much is he willing to pay for that investment.

Solution: Expected present value of the investment with possible failure is,

$$0.7(120) + 0.3(0) = 84. \quad (2)$$

(c) (8 points) George then realizes that, at some cost, he can delay making the investment until he knows whether or not things will go wrong. How much cost of delay is he willing to incur?

Solution: Let c be the investment cost and x be the cost to delay. The decision tree is as follows. ("Not delay" is marked "x delay" and similarly for Not invest.) Therefore, willingness to pay for delaying is $(84 - 0.7c) - (84 - c) = 0.3c$.



3. Suppose that Abe makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = \ln(m+1)$. He is faced with a situation in which he will receive either 0 or 24; the outcomes are equally likely.

(a) (4 points) What is the expected outcome? Variance of outcome?

Solution: Expected value and the variance of the situation is,

$$E[m] = \frac{1}{2}(0) + \frac{1}{2}(24) = 12,$$
$$V[m] = E[(m - E(m))^2] = \frac{1}{2}(0-12)^2 + \frac{1}{2}(24-12)^2 = 144. \tag{3}$$

(b) (6 points) What is Abe's certainty equivalent of the risky outcome? What is the maximum amount he would be willing to pay an insurer to get the mean outcome for sure?

Solution: Certainty equivalence, m^{CE}, satisfies,

$$\begin{aligned} \ln(m^{CE}+1) &= \frac{1}{2}\ln(0+1) + \frac{1}{2}\ln(24+1), \\ &= \ln(24+1)^{\frac{1}{2}} = \ln 5 = \ln(4+1). \end{aligned} \tag{4}$$

Thus, $m^{CE} = 4$. Max insurance payment = RP = $E[m] - m^{CE} = 12 - 4 = 8$.

(c) (4 points) What is Abe's coefficient of relative risk aversion at the mean outcome?

> **Solution:** Since $u'(m) = (m+1)^{-1}$ and $u''(m) = -(m+1)^{-2}$, the coefficient of relative risk aversion is,
>
> $$r(m) = -\frac{-(m+1)^{-2}}{(m+1)^{-1}}(m) = \frac{m}{m+1}. \tag{5}$$
>
> Thus, $r(E[m]) = \frac{12}{13}$.

4. Ajax Inc and Bestco produce imperfectly substitutable products, each at per unit marginal costs of 1. Inverse demand for Ajax is $p_A = 6 - q_A - 0.5q_B$ and symmetrically Bestco's inverse demand is $p_B = 6 - q_B - 0.5q_A$, where q_A and q_B denote the quantities of the two firms. Find the Nash equilibria of the following games, where payoffs are profits.

(a) (6 points) Both firms choose quantity simultaneously and independently.

> **Solution:** (Cournot competition) Firm A maximizes profit with respect to q_A,
>
> $$\max_{q_A}(6 - q_A - \frac{1}{2}q_B)q_A - q_A. \tag{6}$$
>
> First-order condition is,
>
> $$5 - \frac{1}{2}q_B - 2q_A^* = 0. \tag{7}$$
>
> Since firms are symmetric,
>
> $$5 - \frac{1}{2}q_A - 2q_B^* = 0. \tag{8}$$
>
> Thus, in Nash equilibrium, $q_A^{NE} = q_B^{NE} = 2$.

(b) (4 points) Ajax chooses quantity first, then Bestco observes it and then chooses its own quantity.

> **Solution:** (Stackelberg) A moves first and B moves second. We solve this problem by backward induction. The best response function of firm B is the same as Cournot competition case,
>
> $$\begin{aligned} 5 - \frac{1}{2}q_A - 2q_B^* &= 0, \\ \Leftrightarrow \quad BR_B(q_A) &= \frac{5}{2} - \frac{1}{4}q_A. \end{aligned} \tag{9}$$
>
> Firm A maximizes profit taking this best response function of firm B into consider-

ation,

$$\max_{q_A} \quad \left(6 - q_A - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{4}q_A\right)\right) q_A - q_A. \tag{10}$$

First-order condition is,

$$\frac{15}{4} - \frac{7}{4}q_A^* = 0. \tag{11}$$

Thus, $q_A^{NE} = \frac{15}{7}$ and $q_B^{NE} = \frac{55}{28}$.

(c) (6 points) Both firms choose price simultaneously and independently.

Solution: (Bertrand) By rewriting the demand system, we obtain,

$$q_A = 4 - \frac{4}{3}p_A + \frac{2}{3}p_B, \tag{12}$$

$$q_B = 4 - \frac{4}{3}p_B + \frac{2}{3}p_B. \tag{13}$$

Firm A maximizes its profit with respect to p_A,

$$\max_{p_A} \quad p_A(4 - \frac{4}{3}p_A + \frac{2}{3}p_B) - (4 - \frac{4}{3}p_A + \frac{2}{3}p_B). \tag{14}$$

First-order condition is,

$$\frac{16}{3} - \frac{8}{3}p_A^* + \frac{2}{3}p_B = 0. \tag{15}$$

Since firms are symmetric,

$$\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0. \tag{16}$$

Thus, $p_A^{NE} = p_B^{NE} = \frac{8}{3}$.

(d) (4 points) Ajax chooses price first, then Bestco observes it and then chooses its own price.

Solution: Again, we solve this problem by backward induction. The best response function of firm B is the same as Bertrand case,

$$\begin{aligned} &\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0 \\ \Leftrightarrow\ \ & BR_B(p_A) = 2 + \frac{1}{4}p_A. \end{aligned} \tag{17}$$

Firm A maximizes profit taking this best response function of firm B into consideration,

$$\max_{p_A} \ (p_A - 1)\left(4 - \frac{4}{3}p_A + \frac{2}{3}\left(2 + \frac{1}{4}p_A\right)\right). \tag{18}$$

First-order condition is,

$$-\frac{7}{3}p_A^* + \frac{39}{6} = 0. \tag{19}$$

Thus, $p_A^{NE} = \frac{39}{14}$ and $p_B^{NE} = \frac{151}{56}$.

5. Name at least three techniques of price discrimination commonly used by airlines. How does each technique overcome the main obstacles to price discrimination? Comment very briefly on the efficiency implications of each technique. (18pts)

Solution: Laws against one customer reselling an airline ticket to another customer virtually eliminate the arbitrage obstacle for any form of price discrimination. Self-selection, or incentive compatibility, helps cope with the unobservability of WTP obstacle, as noted below.

- Timing of purchase: Charge higher prices for those who buy tickets on closer dates to departure dates. Incentive compatible because last minute customers tend to have less price-elastic demand. Can be regarded as Third-degree PD.
- First-class vs Economy-class: Price way higher prices for consumers with high willingness to pay. Incentive compatible because of the difference in service and quality. Can be regarded as Second-degree PD.
- Mileage: Give bonuses to customers with low willingness to pay. Incentive compatible because customers with high willingness to pay does not have incentive to earn mileage because of opportunity cost. Similar to Second-degree/ quantity discount, and a little like a 2-part tariff.
- Payment history: This is similar to the example from one of the clip in class. Charges different prices to different customers based on histories of purchase, which reflect willingness to pay. Time will tell us whether this new way of price discrimination will work or not. Might approximate First-degree.

6. The price of crude oil decreased about 60% over the last 1.5 years. Suppose that a sector of the economy can be approximated reasonably well by a production function with crude oil as one input and labor and capital as the other inputs. If that production function is Leontieff, what impact would you expect to see on the sector's input demands, cost and output? How would you answer change if the production function were CES with substitution elasticity between 0 and 1? Which production function seems a more reasonable description in the short run of a month or two? (14pts)

Solution:

- When the relative price of input changes, ratios of input quantities do not change if production function is Leontieff. One way to see this is to draw isoquant. Isoquant has a kink and regardless of the input price ratio, cost is minimized at the kink. Still, the overall marginal cost is lower, and thus total cost. With lower marginal cost, the profit-maximizing output should increase.
- When the elasticity of substitution is not zero, there is a substitution toward the lowered priced goods. This additional effect even more lowers the total cost.
- In the short-run, it is hard to switch to different input ratios. As a result, production function with low elasticity of substitution is suitable.

1 Question 7

You have been hired as a consultant by a firm that produces a new, irreplaceable office gizmo (with no close substitutes) exclusively under a patent. The firm has two factories, one in Santa Cruz, the other in San Jose. The managers of the two factories have an ongoing dispute over what the company should do. The Santa Cruz manager argues that the San Jose plant should be closed because leasing and other fixed costs are much higher in San Jose and it makes no sense to produce at such an expensive facility. The SJ factory manager counters that the Santa Cruz plant should be closed because even though the factory lease is cheap, Santa Cruz workers are less productive. When the surf is big, they all call in sick and this reduces the plant's productivity. Your research shows that inverse demand is $P = 100 - Q$, where $Q = Q_{SC} + Q_{SJ}$. Costs at the San Jose factory are $C_{SJ} = 75 + 2 \cdot Q_{SJ}^2$, and costs are $C_{SC} = 25 + 3 \cdot Q_{SC}^2$ at Santa Cruz. Assume that you can't alter these cost functions.

a) With both plants operating, how should production be allocated between the two plants? What are the profits for the firm at this level of joint production?

The main point here is that you want to make sure you are producing at a point where the marginal costs of both plants are equalized. We can do that in a number of ways. Note from the profit maximization problem

$$\max_{Q_{SJ},Q_{SC}} P\left(Q_{SJ}+Q_{SC}\right)\cdot\left(Q_{SJ}+Q_{SC}\right)-C_{SJ}\left(Q_{SJ}\right)-C_{SC}\left(Q_{SC}\right)$$

we find

$$\frac{\partial P}{\partial Q_{SJ}} \cdot (Q_{SJ} + Q_{SC}) + P(Q_{SJ} + Q_{SC}) - C'_{SJ} = 0$$
$$\frac{\partial P}{\partial Q_{SC}} \cdot (Q_{SJ} + Q_{SC}) + P(Q_{SJ} + Q_{SC}) - C'_{SC} = 0$$

Plugging in the numbers,

$$\begin{aligned}100 - 2Q_{SJ} - 2Q_{SC} &= 4Q_{SJ} \\ 100 - 2Q_{SJ} - 2Q_{SC} &= 6Q_{SC}\end{aligned}$$

Solving this,

$$\begin{aligned}
Q_{SJ} &= \frac{1}{6}(100 - 2Q_{SC}) \\
Q_{SC} &= \frac{1}{8}(100 - 2Q_{SJ}) \\
Q_{SJ} &= \frac{50}{3} - \frac{1}{3}\left(\frac{25}{2} - \frac{1}{4}Q_{SJ}\right) \\
&= \frac{75}{6} + \frac{1}{12}Q_{SJ} \\
\frac{11}{12}Q_{SJ} &= \frac{75}{6} \\
Q_{SJ} &= \frac{150}{11} \\
Q_{SC} &= \frac{25}{2} - \frac{1}{4} \cdot \frac{150}{11} = \frac{22 \cdot 25 - 150}{44} = \frac{400}{44} = \frac{100}{11}
\end{aligned}$$

Given this, profit is as follows:

$$\begin{aligned}
\pi = &\left(100 - 2 \cdot \left(\frac{150}{11} + \frac{100}{11}\right)\right) \cdot \left(\frac{150}{11} + \frac{100}{11}\right) \\
&- 75 - 2 \cdot \left(\frac{150}{11}\right)^2 - 25 - 3 \cdot \left(\frac{100}{11}\right)^2 \\
\pi \approx &519.83
\end{aligned}$$

Note that we should look for corner solutions. Suppose $Q_{SJ} = 0$, but we keep the San Jose plant.

$$\max_{Q_{SC}} P(Q_{SC})\, Q_{SC} - 100 - 3Q_{SC}^2$$

$$
\begin{aligned}
P'Q_{SC} + P &= 6Q_{SC} \\
-Q_{SC} + 100 - Q_{SC} &= 6Q_{SC} \\
8Q_{SC} &= 100 \\
Q_{SC} &= 12.5
\end{aligned}
$$

Profit in this scenario is

$$\pi_{SC\ only} = (100 - 12.5) \cdot 12.5 - 100 - 3\,(12.5)^2 = 525$$

Suppose instead $Q_{SC} = 0$

$$\max_{Q_{SJ}} P\,(Q_{SJ})\,Q_{SJ} - 100 - 2Q_{SJ}^2$$

$$
\begin{aligned}
P'Q_{SJ} + P &= 4Q_{SJ} \\
-Q_{SJ} + 100 - Q_{SJ} &= 4Q_{SJ} \\
6Q_{SJ} &= 100 \\
Q_{SJ} &= \frac{50}{3}
\end{aligned}
$$

Profit in this scenario is

$$\pi_{SJ\ only} = \left(100 - \frac{50}{3}\right) \cdot \frac{50}{3} - 100 - 2 \cdot \left(\frac{50}{3}\right)^2 = 733.33$$

b) Given the arguments of the two managers, what is your recommendation? Explain. What are the profits of the firm if they follow your recommendation?
As noted above, if the firm acts as a monopolist with only one or the other plant open, the profits are 600 closing the San Jose plant (add 75 to the SC only scenario) or 758.33 closing the Santa Cruz plant (add 25 to the SJ only scenario). Closing the Santa Cruz plant is optimal, essentially due to high marginal cost there. Of course, the answer could change if demand were to increase or decrease sufficiently.

2 Question 8

The incumbent firm and all potential entrants have marginal cost 4 in an industry with inverse demand $p = 48 - Q$ for a homogeneous product. All customers buy from the lowest price firm.

a. Before other firms can enter, what is the maximum profit for the incumbent?

$$\max_{p}\,(48 - q)\,q - 4q = 44q - q^2$$

$$
\begin{aligned}
44 - 2q &= 0 \\
q^M &= 22 \\
\pi^M &= (48 - 22) \cdot 22 - 4 \cdot 22 = 484
\end{aligned}
$$

b. After entry is possible, what is the maximum profit for the incumbent?

Entry enables unfettered competition, driving economic profit down to zero for all firms, including the incumbent, since marginal cost is flat and common across firms. With no barriers to entry and constant marginal cost, any number of firms $n > 2$ could split demand. At $p = MC = 4$, $Q = 48 - 4 = 44$, the firm's output is $q = 44/n$ and its profit is $\pi = (p - c)\,q = 0 \cdot q = 0$.

c. The incumbent has a unique option to lower marginal cost to 2 before entry is possible. How much would he be willing to pay to exercise this option?

If the incumbent has a lower marginal cost, he can charge $4 - \epsilon$ and take the whole market. Profit would then be

$$\pi = (4 - \epsilon)\,q - 2 \cdot q = (2 - \epsilon)\,q \approx 2 \cdot 44 = 88.$$

The incumbent is willing to pay up to approximately 88 per period to exercise the option. If the discount rate (adjusted for the risk of market disruption) is r and the cost advantage is permanent, then she is WTP $\approx 88/r$, e.g., about 900 if r is a bit under 10% per period.