7. Oligopoly

I. Overview

A. So far we have only looked at two extreme types of markets.

1. Competitive markets have only price-taking firms (presumably lots of them).
2. Monopolist markets have one firm with unilateral pricing power.

B. We now look at markets with firms that have some pricing power, but not unilateral.
Oligopoly: from Greek, more than one but less than "many."

C. We use game theory to study behavior in oligopolies.

1. Firm decisions affect one another $\iff$ strategic interaction.
   a. Need an equilibrium concept that describes multiple agents trying to optimize.
2. Given the way we construct equilibria in game theory, the strategic variable chosen will matter.
   a. We solved the monopolist's problem by describing its choice of quantities.
   b. We could have just as easily (and with the same result) had the monopolist choose a price.
   c. This symmetry disappears in our study of oligopoly.

II. (Normal Form) Game theory

A. A normal form (simultaneous play) game (NFG) is defined by three elements

1. A list of N players
2. A set of strategies for each player, $s_i \in S_i$. E.g.,$S_i = \{s_{i1}, s_{i2}, ..., s_{ik}\}$.
3. A payoff function for each player, $\pi_i(\mathbf{s})$, where the profile of all players' strategies is $\mathbf{s} = (s_i, \mathbf{s_{-i}})$.

B. Let $\mathbf{s_{-i}}$ be a vector of strategies of all players other than i. The **best response function** (or correspondence) is $BR_i(\mathbf{s_{-i}}) = argmax_{s_i \in S_i} \pi_i(s_i, \mathbf{s_{-i}})$. In words, for a given profile $\mathbf{s_{-i}}$ of other players' strategies, player i's best response is the strategy (or subset of strategies) that maximizes his payoff.

C. A **Nash equilibrium** is a strategy profile $\mathbf{s}^*$ in which every player is making a best response to the other players' strategies, i.e.,

$$s_i^* \in BR_i(\mathbf{s}^*_{\mathbf{-i}}), \quad i = 1, ..., n. \tag{1}$$

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D. These definitions are quite general and apply in politics, biology, business, traffic engineering, etc. etc. Here we will apply them to oligopoly.

III. Quantity Setting: Cournot Markets

A. The Duopoly NFG

1. $N = 2$ players, called firms.
2. Strategy is the output quantity $y_i \in [0, \infty) = S_i$.
3. The choices y_1, y_2 are made simultaneously (logically speaking).
4. We'll keep things simple in computing the payoff functions (profit functions). Set $Y = \sum_{i=1}^{N} y_i$ to be total output, and assume a linear inverse demand curve (for perfect substitutes)

   $p = a - bY$

5. We'll also assume a linear cost curve, i.e., identical marginal cost c for all firms and zero fixed cost.
6. Then the profit to any firm i is:

   $\pi_i = y_i(a - bY) - y_i c = (a - c - bY)y_i$

7. The most important part of this is that the aggregate output quantity choices of other firms $Y_{-i} = Y - y_i$ affects firm $i's$ profits and therefore his optimal choice.

B. Best Response Function

1. A best response function BR_i describes firm i's best choice of quantity y_i as a function of the quantity choices of everyone else.
   a. Note this doesn't imply that firms actually *know* the quantity choice of others.
   b. Action is simultaneous.
2. We will see that in this quantity setting game, $\frac{\partial BR_i}{\partial y_j} < 0$
   a. If you know your rival's output is high, you want your output to be low.
   b. Quantity is a *strategic substitute.*

   Ex: Getting the BR from the FOCs with 2 firms. FOC is

$$0 = \frac{\partial \pi_i}{\partial y_i} = a - c - 2by_i - bY_{-i} \tag{2}$$

   so BR is

$$BR_i(Y_{-i}) = [\frac{a-c}{2b} - \frac{1}{2}Y_{-i}]_+ \tag{3}$$

   where $[z]_+ = \max\{0, z\}$. [Draw BR's in duopoly case]

C. Nash Equilibrium

1. A Nash equilibrium is a profile of strategies at which no player has an incentive to change their behavior given what others are doing.
2. Or, all firms are simultaneously playing their best response.
3. Adding the FOCs (2) for $i = 1, ..., m$, we get
   $0 = N(a-c) - 2bY - b(N-1)Y$,
   so total output in Cournot Nash equilibrium is
   $Y^* = \frac{N}{(N+1)b}(a-c)$, and by symmetry,
   $y_i^* = \frac{Y^*}{N} = \frac{a-c}{(N+1)b}$.
   For duopoly, $y_i^* = \frac{a-c}{3b}$, and NE payoff is
   $\pi_i^* = (a - c - bY^*)y_i^* = (by_i^*)y_i^* = \frac{(a-c)^2}{9b}$.

D. Asymptotics

1. Can we describe the equilibrium behavior of Cournot firms in terms of the number of firms (N)?
   a. By doing this we can look at the relationship between oligopolies and both monopolies and competition.
2. Denoting s_i as $\frac{y_i}{Y}$, profit maximization gives us Marginal Revenue = Marginal Cost. Using familiar tricks on Marginal Revenue, we get

   $p(Y)(1 + \frac{s_i}{\epsilon}) = c_i'(y_i)$

3. If all firms have the same constant marginal cost c and fixed costs that they can cover in Nash equilibrium, then

   $p(Y)[1 + \frac{1}{N\epsilon}] = c$

4. The main result is a price somewhere between competition and monopoly:
   a. If $N = 1$ this price is just the monopoly price.
   b. As N approaches infinity, price converges to the competitive level.
   c. With N in between price remains above marginal cost, but below the monopoly level.

E. Problems with Cournot Analysis

1. We usually think of firms setting price – not quantity.
2. Where do prices come from in Cournot markets?
   a. We know prices come from the inverse demand curve.
   b. The Cournot model implicitly assumes that firms just dump their output on the market and accept the market clearing price.

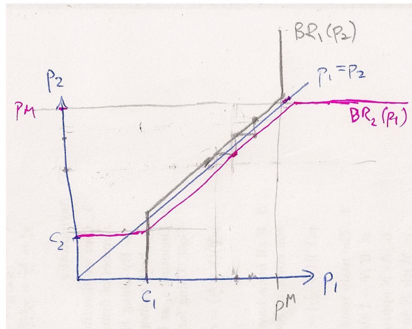

Figure 1: NE in Bertrand Model. Monopoly price is p_M, firms have marginal costs c_1, c_2.

3. The next model assumes direct price setting, and comes to a very different predicted (equilibrium) outcome.

IV. Price Setting: Bertrand Markets

A. Describing a Duopoly

1. Two firms simultaneous choose a price to sell unit at with a given demand curve $D(p)$.
2. We'll assume firms face constant marginal costs c_i.
3. Consumers will obviously buy from the lower priced firm, since they produce perfect substitutes.
4. Firm i's demand is given by
   a. $D(p_i)$ if $p_i < p_j$
   b. $D(p_i)/2$ if $p_i = p_j$
   c. 0 if $p_i > p_j$

B. Best Response

1. Note now that the strategic variables – p_i – are *strategic complements*.
2. The lower a rival's price, the lower you'd like your price to be.

C. Nash Equilibrium

1. A Nash Equilibrium in the Bertrand game is a set of prices at which no firm has an incentive to change his or her price.
2. Assume, without loss of generality, that $c_j > c_i$.
3. Then in a Nash equilibrium:
   a. $p_i = c_j$
   b. $p_j \geq c_j$
4. The price in the market is competitive – it is equal to marginal cost.
   a. This is especially striking if we assume (as we often do) that firms share a common marginal cost.
   b. If $c_i = c_j = c$ then we have $p_i = c$.

5. In the price setting game, then, the prediction is that the competitive outcome obtains *even with only 2 firms*.

D. The Bertrand Paradox

1. Some call the result the "Bertrand paradox."
2. Intuition tells us that, say, five firms should compete much more fiercely than two firms.
   a. Indeed there is experimental evidence to this effect.
3. One way of reclaiming supercompetitive behavior in pricing games with few firms is to feature repeated games.
4. Another is to have firms selling slightly different products, as in one of your homework problems.

V. (Extensive Form) Game Theory: Basic ideas, assuming perfect info.

VI. Quantity Setting With A Leader: Stackelberg Markets.

- Draw "tree" for 2 player sequential game. Solve by backward induction.

- Take the duopoly example with linear demand, const MC, say $N = 2, a = 30, b = 1, c = 6$.
- For comparison, Cournot-Nash equilibrium is $y_1^* = y_2^* = \frac{30-6}{3} = 8$, price is $p^* = a - bY = 30 - 16 = 14$, and profits are $\pi_1^* = \pi_2^* = (p^* - c)y_i^* = 8 * 8 = 64$.
- The Stackelberg leader, firm 1, chooses output y_1 to maximize her profit, knowing how the follower will react. That is, she assumes that $y_2 = BR_2(y_1)$.
- Using equation (3), we see that $BR_2(y_1) = \frac{a-c}{2b} - \frac{1}{2}y_1 = 12 - \frac{1}{2}y_1$.
- Hence she solves

$$\begin{aligned}\max_{y_1 \geq 0} \pi_1(y_1, BR_2(y_1)) &= (a - c - b(y_1 + BR_2(y_1)))y_1 = (30 - 6 - (y_1 + 12 - \frac{1}{2}y_1))y_1 \\ &= (12 - \frac{1}{2}y_1)y_1, \qquad (4)\end{aligned}$$

- which is easily seen to have solution $y_1^{SB} = 12$.
- Hence $y_2^{SB} = BR_2(y_1^{SB}) = 12 - \frac{1}{2}y_1^{SB} = 6$, and $p = 30 - (12 + 6) = 12$, so $\pi_1^{SB} = (12 - c)12 = 72$ and $\pi_1^{SB} = (12 - c)6 = 36$.
- Compared to Cournot, price is lower, profit for leader is higher, but follower profit and total profit are lower in Stackelberg-Nash equilibrium.

VII. Price Setting With Differentiated Products: Hotelling Markets

- Diagrams only, on line segment and on circle.
- Horizontal differentiation (aka versioning) vs vertical (price or quality).
- Sunk cost to entry implies an equilibrium number of firms.