3. Game settings

   - Players = $\{1, 2\}$
   - Actions $S_1 = \{H, T\}$, $S_2 = \{h, t\}$
   - Preference : Maximized expected payoff.

   (a) Verify that there no pure NE

      i. Best responses:

$$B_1(h) = H \qquad\qquad B_1(t) = T$$
$$B_2(H) = t \qquad\qquad B_2(T) = h$$

We can see that there is no profile S^* such that for each player i and action s this is holds $s_i^* \in B_i(s_{-i}^*)$. So there is no pure strategy Nash Equilibrium in this game.

      ii. Denote by p the probability that player 1's mixed strategy assigns to H and by q the probability that player 2's mixed strategy assigns to h. Then, given player 2's mixing, player 1's expected payoff to the pure strategy H is

$$q \times a + (1 - q) \times 0 = qa$$

And player 1's expected payoff to the pure strategy T is

$$q \times 0 + (1 - q) \times b = (1 - q)b$$

From these two, I solve for q, which is $q = \dfrac{b}{a + b}$

Likewise, given player 1's mixing, player 2's expected payoff to the pure strategy h is

$$p \times 0 + (1 - p) \times c = (1 - p)c$$

And player 1's expected payoff to the pure strategy T is

$$p \times d + (1 - p) \times 0 = pd$$

From these two, I solve for p, $p = \dfrac{c}{d + c}$

So the unique mixed NE is $\big((\dfrac{c}{d+c}, \dfrac{d}{d+c}), (\dfrac{b}{a+b}, \dfrac{a}{a+b})\big)$

   (b) Comparative statics,

$$\frac{\partial p}{\partial d} = \frac{\partial}{\partial d}(\frac{c}{d+c}) = -\frac{c}{(d+c)^2}; \quad \text{and} \quad \frac{\partial p}{\partial d} = \frac{\partial}{\partial c}(\frac{c}{d+c}) = \frac{d}{(d+c)^2}$$
$$\frac{\partial q}{\partial a} = \frac{\partial}{\partial a}(\frac{b}{a+b}) = -\frac{b}{(a+b)^2}; \quad \text{and} \quad \frac{\partial q}{\partial b} = \frac{\partial}{\partial b}(\frac{b}{a+b}) = \frac{a}{(a+b)^2}$$

   - Player 1 will put a higher probability playing p when d increases. If c increases then player 1 will put higher probability by playing $1 - p$.
   - Player 2 will put a higher probability playing q when b increases. If a increases then player 2 will put higher probability by playing $1 - q$.

   (c) As we are in strategic settings and the findings in part b clearly shows that player i mixing is determined solely by what player $-i$'s value of the outcomes. If the valuation from the other player $-i$ change then player i should rationally change his mixing strategy.

4. (a) Game settings

- Players = {1, 2}
- Strategies $S_1 = \{T, M, B\}$, $S_2 = \{L, C, R\}$

Since the strategy C for player 2 is dominated by L and R, then player 2 will remove this strategy. As player 1 knows that player 2 will remove strategy C then player 1 removes strategy B.

	L	R
T	$2, 0$	$4, 2$
M	$3, 4$	$2, 3$

- Best responses in this new set up are as follow:

$$B_1(L) = M \qquad B_1(R) = T$$
$$B_2(T) = R \qquad B_2(L) = M$$

So the pure strategy NE = $((M, L), (T, R))$

- Now check whether there is a NE where player 1 play T vs player 2 mixing L, R: If player 1's strategy is T, then player 2's payoff to her two actions $(0, 2)$ which is different. For NE to exist, the payoff player 2 assigns positive probability must be the same.[1] Using the same logic, I could eliminate the possible pair between pure strategy and mixed.
- Denote by p the probability that player 1's mixed strategy assigns to T and by q the probability that player 2's mixed strategy assigns to L. Then, given player 2's mixing, player 1's expected payoff to the pure strategy T is

$$q \times 2 + (1 - q) \times 4 = 4 - 2q$$

And player 1's expected payoff to the pure strategy M is

$$q \times 3 + (1 - q) \times 2 = 2 + q$$

From these two, I solve for q, which is $q = \frac{2}{3}$

Likewise, given player 1's mixing, player 2's expected payoff to the pure strategy L is

$$p \times 0 + (1 - p) \times 4 = (1 - p)4$$

And player 1's expected payoff to the pure strategy R is

$$p \times 2 + (1 - p) \times 3 = 3 - p$$

From these two, I solve for p, $p = \frac{1}{3}$

So the unique mixed NE is $\big((\frac{1}{3}, \frac{2}{3}), (\frac{2}{3}, \frac{1}{3})\big)$

(b) (Looked at the separated paper).

Using backward induction, I found the Sub Game Perfect Nash Eq, that is (T, R)

[1] I apply preposition 116.2 of Osborne's (2004)

	LL	L	M	R
U	**100, 2**	-100, 1	0, 0	-100, -100
D	-100, -100	100, -49	1,0	**100, 2**

8.D.9 from MWG

a) I would choose M to avoid a large loss.

b) The pure NE are (U,LL) and (D,R), in bold in the matrix. For mixed NE, there are 11 possibilities:

- Mixes (L, M), (L, R), (M, R), and (L, M, R) will cause P1 to play D. This will cause P2 to play R, and (D, R) is already a pure NE

- For a mix of (LL, R) we get:
  2p + (1-p)(-100) = 100p + (1-p)2 ==> p = 1/2
  But this would imply that u(LL) = u(R) = -49, while u(M) = 0. So this cannot be part of a NE.

- This implies that (LL, M, R), (LL, L, R), and (LL, L, M, R) also cannot be part of a NE.

- For a mix of (LL, M), we need p = 50/51. This implies that u(LL) = u(M) = 0. But we know u(L) = 1/51 >0, so this cannot be part of a NE.

- This implies that (LL, L, M) cannot be part of a NE.

- This leaves only (LL, L)
  - For P2 to mix, we need 2p + (1-p)(-100) = p + (1-p)(-49), which gives p = 51/52
    - This gives utilities u(LL) = u(L) = 1/26, u(M) = 0, and u(R) < 0.
  - For P1 to mix, we need 100q + (1-q)(-100) = -100·q + 100·(1-q), which gives q = 1/2
    - This gives utilities u(U) = u(D) = 0
  - Therefore, (1/26, 1/2) is a mixed NE.

c) M is clearly not part of any NE, mixed or pure. But, if P1 plays (1/2, 1/2), then M is a unique best response, and is therefore rationalizable.

d) If we can talk beforehand, we can agree to play one of the pure NE, so I will play either LL or R (depending on the agreement).

8.E.1)

4 hours by myself and using McWG's notes.

		Strong: Attack	Strong: Don't Attack	Weak: Attack	Weak: Don't Attack
Strong	Attack	$-S, -S$ (1)	$M, 0$ (2)	$M-S, -W$ (3)	$M, 0$ (4)
Strong	Don't Attack	$0, M$ (5)	$0, 0$ (6)	$0, M$ (7)	$0, 0$ (8)
Weak	Attack	$-W, M-S$ (9)	$M, 0$ (10)	$-W, -W$ (11)	$M, 0$ (12)
Weak	Don't Attack	$0, M$ (13)	$0, 0$ (14)	$0, M$ (15)	$0, 0$ (16)

Please look at next page

	AA	$A^S O^W$	$O^S A^W$	OD
Attack if strong or weak AA	* $\frac{M}{4}-\frac{S+W}{2}, \frac{M}{4}-\frac{S+W}{2}$	* $\frac{M}{2}-\frac{S+W}{4}, \frac{M}{4}-\frac{S}{2}$	* $\frac{3M}{4}-\frac{S+W}{4}, -\frac{W}{2}$	** $M, 0$
Attack if strong, Don't Attack if weak $A^S D^W$	** $\frac{M}{4}-\frac{S}{2}, \frac{M}{2}-\frac{S+W}{2}$	** $\frac{M-S}{4}, \frac{M-S}{4}$	** $\frac{M}{2}-\frac{S}{4}, \frac{M-W}{4}$	*** $\frac{M}{2}, 0$
Attack if weak, Don't Attack if strong $D^S A^W$	$-\frac{W}{2}, \frac{3M}{4}-\frac{S+W}{4}$	$\frac{M-W}{4}, \frac{M}{2}-\frac{S}{4}$	$\frac{M-W}{4}, \frac{M-W}{4}$	$\frac{M}{2}, 0$
Don't Attack if strong or weak ND	$0, M$	$0, \frac{M}{2}$	$0, \frac{M}{2}$	$0, 0$

* combination of box numbers : 1, 3, 9, 11 $\Rightarrow \frac{M-(2S+2W)}{4} = \frac{M}{4} - \frac{S+W}{2}$

explain more.

: $\frac{M-2S-2W}{4} = \frac{M}{4} - \frac{S+W}{2}$

* combination of box numbers : 1, 4, 9, 12

If $S<W$: NE . (AA, AD), (AD, AA) (if $\frac{M}{4} > \frac{S}{4} + \frac{W}{2}$)

✓

* Combination of 2, 3, 10, 11 $\frac{M+M-S+M-W}{4} = \frac{3M}{4} - \frac{S+W}{4}$, ...

** " " 2, 4, 10, 12 $\frac{M+M+M+M}{4} = 4M/4 = M$, 0

** " " 1, 3, 13, 15 $\frac{M-2S}{4}$, $\frac{2M-S-W}{4}$

** " " 1, 4, 13, 16 $\frac{M-S}{4}$, $\frac{M-S}{4}$

** " " 2, 3, 14, 15 $\frac{M+M-S}{4}$, $\frac{M-W}{4}$, ...

*** " 2, 4, 14, 16 $\frac{2M}{4}$, 0

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Mas-Collel: 8.D.5

a)

The boardwalk stretches from 0 to 1.

Define $\propto \in [0, 1]$ to be distance of a vendor from point 0.

The profits of the vendors (V1 and V2) are:

1) If $2 \quad h \quad h \quad 1$:

$1= \quad 1 + 12(\quad 2- \quad 1)$

$2=(1- \quad 2) + 12(\quad 2- \quad 1)$

2) 2) If $1 \quad h \quad h \quad 2$:

$1=(1- \quad 1) + 12(\quad 2- \quad 1)$

$2= \quad 2 + 12(\quad 2- \quad 1)$

Therefore the best response functions are:

For $\quad h \quad h \quad \lim \rightarrow (\ - \ ) >$

For vendor 2:

$2 \quad 1= \quad 1 +$ If $1 < 12$

$2 \quad 1= \quad 1 -$ If $1 > 12$

$2 \quad 1=(1- \quad 1)$ If $1 = 12$

For Vendor 1:

$1 \quad 2= \quad 2 +$ If $2 < 12$

$1 \quad 2= \quad 2 -$ If $2 > 12$

$1 \quad 2=(1- \quad 2)$ If $2 = 12$

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Nash Equilibrium Exists when a fixed point exists:

$*= \quad *$

This happens when $2 = \quad 1 = 12$

$\Longrightarrow \quad 1 \quad 2= \quad 2 \quad 1$

b)

If $\propto 1 \qquad \propto 2 < 12 \Longrightarrow \quad 3 \propto 1, \propto 2=max\{\propto 1+ \quad , \propto 2+ \quad)$

If $\propto 1 \qquad \propto 2 > 12 \Longrightarrow \quad 3 \propto 1, \propto 2=$min $\{\propto 1- \quad , \propto 2- \quad)$

If $\propto 1 > 12$ and $\propto 2 < 12 \Longrightarrow \quad 3 \propto 1, \propto 2=$

$\propto \quad + \propto \quad - \propto$ If $\propto 2 - \propto 1 \ 2=max\{ \propto 2 - \propto 1 \ 2, \ \propto 1- \quad , 1- \propto 2+ \quad \}$

$\propto 1- \qquad \propto 1- \quad =max\{ \propto 2 - \propto 1 \ 2, \ \propto 1- \quad , 1- \propto 2+ \quad \}$

$\propto 2+ \qquad \propto 2+ \quad =max\{ \propto 2 - \propto 1 \ 2, \ \propto 1- \quad , 1- \propto 2+ \quad \}$

This changes as the position of $\propto 1 \qquad \propto 2$ changes $\Longrightarrow$ This game does not have a Pure Nash Equilibrium.