Midterm Practice Problems - ECON 200 - Fall 2016
ANSWER KEY

October 23, 2016

1 Grants

You manage a department whose mission involves two quantities, x_1 (say expenditures on safety) and x_2 (say expenditures on education). Preferences are strictly monotone and convex, and current consumption (expenditure levels in \$million) is $(x_1, x_2) = (12, 36)$.

1.1 a

The department is eligible for a $g = 10$ grant for x_1 from the Department of Homeland Security. Compare your optimal consumption here to that for an unresticted lump sum increase to your budget of 10. Does your answer depend on whether safety is a normal or inferior good? How would your answer change if the grant were 15 instead of 10? Graphs may help you make your point.

Answer: **Note**: This problem goes beyond the usual budget constraint, and so will be covered in more detail in lectures after the midterm.

Let $m > 0$ denote the original budget, so $p_1x_1 + p_2x_2 = m$. If we normalize the prices to 1 (we are purchasing 'expenditures'), then the pre-grant budget constraint becomes $x_1 + x_2 = 48$, graphed in Figure 1 as a black diagonal line.

Because the grant can only be spent on x_1, the budget shifts out by 10 but only at or below $x_2 = 48$, as shown in the blue line segments. (A lump sum would generate the orange budget line.) There is some grant size (around 16 or 17 for homothetic utility) for which this "kink" in the budget would constrain the choice. $g = 15$ is graphed in red.

Assuming homothetic preferences, we can identify the optimal choice, since any interior optimum would satisfy:

$$MRS(x_1, x_2) = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} = 1$$
$$\text{implies } x_2 = 3x_1 \text{ based on observed consumption}$$

Such optima lie on the ray thru $(x_1, x_2) = (12, 36)$, on which $x_2 = 3x_1$, shown as a black dashed ray in Figure 1.

If preferences were not homothetic, and safety were an inferior good, then this relationship does not continue to hold as your income effectively rises with the grant. Instead, you would shift away from x_1 and toward even more x_2, implying that the kink in the budget line might constrain you to $(g, 48)$.

1.2 b

The grant changes to a 1:1 matching grant for x_1 only. What is your optimal consumption now? You may assume preferences are homothetic for simplicity.

Answer: Instead of a "kinked" budget, the budget line now acts as if the price for x_1 is half of what it was before, as shown in blue in Figure 2. An interior solution satisfies $\frac{MU_1}{MU_2} = \frac{p_1}{p_2} = \frac{1}{2}$. With homothetic preferences, each such point lies on a ray such as that graphed in a dashed line in Figure 2. Thus the solution is the intersection of that ray with the new budget line.

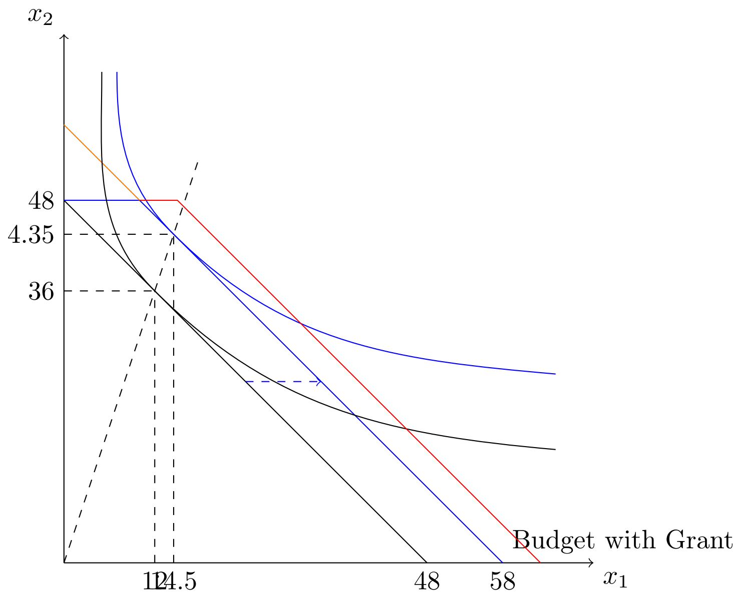

Figure 1: Grant v Lump Sum, Q1a

If preferences are Cobb-Douglas (a stronger assumption than homothetic), then we can say that expenditure shares of both goods remain constant as prices and income vary. Hence $\hat{x}_2$ remains at 36, while $\hat{x}_1 = \frac{12}{1/2} = 24$.

2 Estimating Demand

Your boss is considering a price increase for your firm's main product. He is not sure whether demand is best approximated as linear, or log linear. Your coworker tells him that either semilog or LES actually might be a better approximation. Before investing time and resources in determining the best approximation, your boss asks you to tell him the percentage change in demand he should expect for a 2-10% price increase in each case. He also wants to know which of these specifications have a choke price (above which demand is 0) or a saturation level (a maximum quantity demanded even at price 0). For each of the 4 specifications, write down the answer to his questions in terms of the relevant coefficient (which will be estimated from the data).

1. Linear:

$$D(P) = a - b \cdot P$$

   (a) There is a choke price: demand is zero when $P \geq \frac{a}{b}$
   (b) There is a maximum quantity demanded: a
   (c) The elasticity is $-b \cdot \frac{P}{Q}$, so the answer depends on current price and quantity. A 10% increase would lead to a decrease in quantity demanded of $10b\frac{P}{Q}$ percent

2. Log linear:

$$\log D(P) = a - b \cdot \log P$$

   (a) There is no choke price nor maximum quantity demanded.
   (b) The elasticity is $-b$, so a 10% increase in price would lead to a decrease in quantity demanded of $10b$ percent

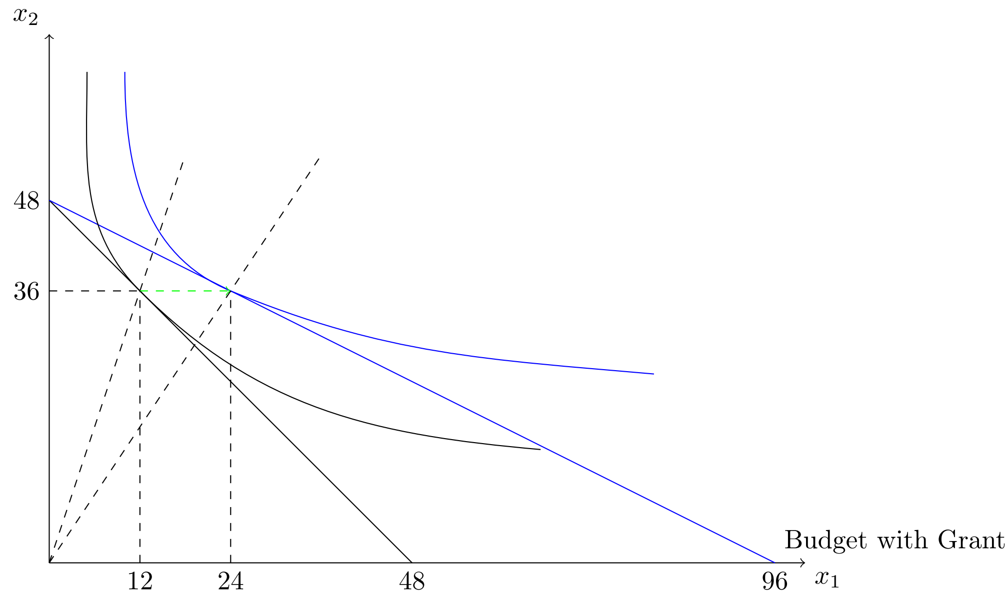

Figure 2: Matching Grant, Q1b

3. Semilog:

$$D(P) = a - b \cdot \log P$$

Note: Semi-log and LES have not been covered in class yet this quarter, so these parts should be considered very optional.

(a) There is a choke price at $\log P \geq \frac{a}{b}$

(b) There is no maximum quantity demanded

(c) The elasticity is $-b/Q$, so the answer depends on current quantity. A 10% increase would lead to a decrease in quantity demanded of $10b/Q$ percent.

4. LES [Linear Expenditure System]

$$D(P) = b + \frac{a \cdot I}{P} - \sum_j \frac{P_j}{P} b_j$$

(a) There is a choke price $P \geq \frac{1}{b} \cdot \left[\sum p_j b_j - a \cdot I\right]$ that depends on the prices of other goods, the marginal rates of substitution, and income

(b) There is no maximum quantity demanded

(c) The elasticity is $\frac{-aI + \sum p_j b_j}{PQ} < 0$, so the answer depends on current revenue, income, and the prices of other goods. A 10% increase in price would lead to a decrease in quantity demanded of $\frac{10 \sum p_j b_j - 10aI}{PQ}$ percent

3 Income Elasticity

Your firm sells scentroids to middle-income teenagers. Experience with these customers, who spend approximately 10% of their income on your product, indicates that a 1% increase in their income tends to increase your sales by 3%, while a 1% price increase tends to decrease the quantity they buy by about 0.8%. The opportunity soon will arise to sell scentroids to high-income teenagers. Assuming that they respond to changes in income and price the same way as current customers (except that they currently spend less than 1% of income on products like yours) estimate how they would respond to a 1% price decrease.

Answer: **Note:** Once again, we did not yet cover the Slutsky equation in class yet; that will be done in the first lecture after the midterm.

The Slutsky equation in elasticities (and expenditure share s_i) is:

$$\begin{aligned} \varepsilon &= \varepsilon^{Hicks} - s_i \cdot \varepsilon_m \\ -.8 &= \varepsilon^{Hicks} - .1 \cdot 3, \end{aligned}$$

So the pure substitution elasticity is $\varepsilon^{Hicks} = (-.8 + .3) = -0.5$. For the High Income (HI) kids we then have

$$\begin{aligned} \varepsilon_{HI} &= \varepsilon^{Hicks} - .01 \cdot 3 \\ \varepsilon_{HI} &= -0.5 - .01 \cdot 3 \\ &= -0.53 \end{aligned}$$

Thus the response would be an increase in quantity demanded of about 0.53 percent.

4 Preferences

4.1 Blutarsky

Blutarsky is planning a fraternity party. He cares only about alcohol content. Write down a utility function for him for x_1=bottles of beer and x_2= bottles of vodka, given that each bottle of vodka has the same amount of alcohol as three six packs of beer.

Answer: Alcohol content of a bottle of vodka is $3 * 6 = 18$ times that of a bottle of beer, so

$$u(x_1, x_2) = 18 \cdot x_1 + x_2.$$

4.2 Justine

Justine's preferences can be represented by $u(x_1, x_2) = \ln x_1 + \ln x_2$. Which of the following utility functions (if any) also represent her preferences? $v(x_1, x_2) = x_1 + x_2$, $w(x_1, x_2) = x_1^{0.4} x_2^{0.4}$, $U(x_1, x_2) = x_1 + g(x_2)$. Explain very briefly.

Answer:

Only $w(x_1, x_2)$, since it can be transformed as follows:

$$\begin{aligned} w(x_1, x_2) &= x_1^{0.4} x_2^{0.4} \\ \ln w &= 0.4 \cdot \ln x_1 + 0.4 \cdot \ln x_2 \\ 2.5 \ln w &= \ln x_1 + \ln x_2 = u(x_1, x_2) \end{aligned}$$

Thus, they represent the same preferences, since u is a positive monotonic transformation of w.

v and U have constant marginal utility of x_1, which is not a feature of u.

5 Tricounty Organic Strawberries

Your client, the Tricounty Organic Strawberry Association, provides data on their sales revenue, from which you estimate the demand function for their product.

5.1 Data

What other data will you need to estimate income elasticity η, own price elasticity ε, and cross price (for inorganic strawberries) elasticity ε_c?

Answer:

Prices for organic (P) and inorganic (P_C) strawberries, and mean income of consumers.

5.2 Estimation

Write out a convenient equation to estimate these elasticities from that data.

Answer:

We can impose a log-linear form:

$$\log D(P) = \alpha + \varepsilon \log P + \eta \log Y + \varepsilon_c \log P_C$$

5.3 Interpretation

Suppose that you estimate $\eta = 1.7$, $\varepsilon = -1.2$, and $\varepsilon_c = 0.4$. A former classmate comments that your estimates can't be right because the elasticities should sum to 0. How should you respond?

Answer:

It is unlikely that we have included all of the relevant other products' prices, and perhaps we missed some important substitutes (e.g., other fruit) and (more importantly, since the sum of elasticities is too high) complements (e.g., shortcake). However, if we believe that there aren't that many omitted complements, then we should be concerned. It likely implies that we have imposed a poor functional form, or we may be estimating from non-exogenous changes in prices and income. Or perhaps we made a simple mistake in entering the data, or something like that. We should see what happens when, after checking the data and estimation procedure carefully, we impose the constraint that the estimated elasticities sum to 0.

6 North Ifstan

North Ifstan (NI) has domestic suppliers of internet services whose monthly supply curve is well approximated by $S(p) = 10p$, while monthly demand is well approximated by $D(p) = 1000 - 10p$. International suppliers can provide any amount of access at $p = 25$.

6.1 a

Compute the current competitive equilibrium (CE) price, domestic producers surplus (PS), and consumer surplus (CS).

Answer:

$p_{CE} = 25$

$PS = \frac{1}{2}25 \cdot 250 = 3,125$

$CS = \frac{1}{2}75 \cdot 750 = 28,125$

6.2 b

The NI government is considering a rule that would eliminate foreign supply. How much (if at all) would domestic suppliers benefit? How much would consumers lose?

Answer:

Benefit to domestic suppliers:

$25 \cdot 250 + \frac{1}{2} \cdot 25 \cdot 250 = 9,325$

Loss to consumers:

$25 \cdot 500 + \frac{1}{2} \cdot 25 \cdot 250 = 15,625$

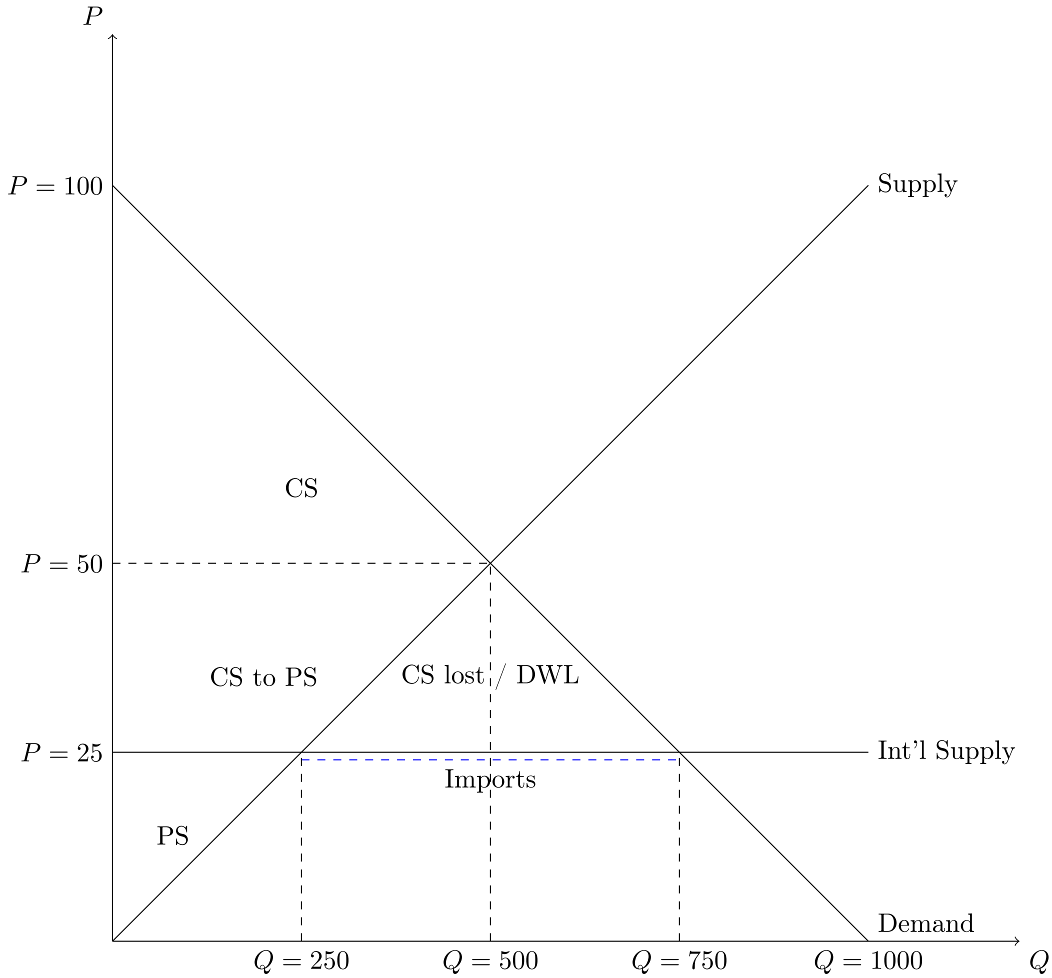

Figure 3: North Ifstan, Q6