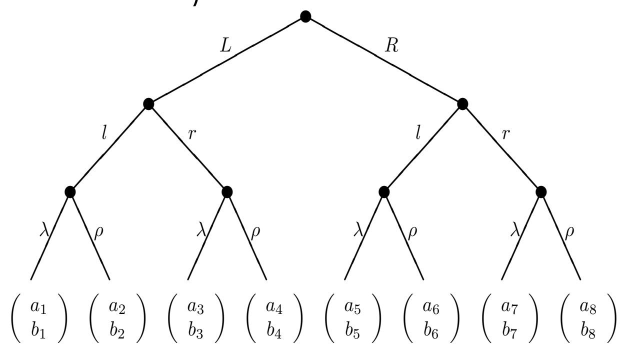
Economics 204 B — Winter 2014
Dan Friedman — UCSC

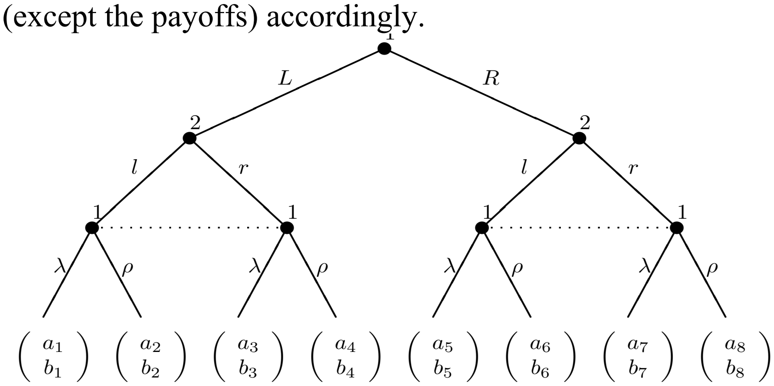
Answer Key to Problem Set #2

Part I. Problems.

1. a. Draw a game form (“tree trunk”) for a three move situation in which the first move is a choice between *R* and *L*, the second move is a choice between *r* and *l*, and the third move is a choice between ρ and λ.



b. Suppose that the first move is owned by player 1, the second by player 2 and the third by player 1 (again). Player 2 observes player 1’s first move. At his last move, player 1 does not observe player 2’s move, though he remembers his own initial move. Complete the game tree (except the payoffs) accordingly.



c. For the tree in part b., write out the complete set of pure strategies for each player. (Hint: remember that each strategy is a *complete* contingency plan!)

$S_1 = \{L\lambda\lambda, L\lambda\rho, L\rho\lambda, L\rho\rho, R\lambda\lambda, R\lambda\rho, R\rho\lambda, R\rho\rho\}$ $\quad S_2 = \{ll, lr, rl, rr\}$

Then write out the full normal form

	ll	lr	rl	rr
$L\lambda\lambda$	a_1, b_1	a_1, b_1	a_3, b_3	a_3, b_3
$L\lambda\rho$	a_1, b_1	a_1, b_1	a_3, b_3	a_3, b_3
$L\rho\lambda$	a_2, b_2	a_2, b_2	a_4, b_4	a_4, b_4
$L\rho\rho$	a_2, b_2	a_2, b_2	a_4, b_4	a_4, b_4
$R\lambda\lambda$	a_5, b_5	a_7, b_7	a_5, b_5	a_7, b_7
$R\lambda\rho$	a_6, b_6	a_8, b_8	a_6, b_6	a_8, b_8
$R\rho\lambda$	a_5, b_5	a_7, b_7	a_5, b_5	a_7, b_7
$R\rho\rho$	a_6, b_6	a_8, b_8	a_6, b_6	a_8, b_8

,

and the reduced normal form for the game (payoffs still unspecified).

	l	r
$L\lambda\cdot$	a_1, b_1	a_3, b_3
$L\rho\cdot$	a_2, b_2	a_4, b_4
$R\cdot\lambda$	a_5, b_5	a_7, b_7
$R\cdot\rho$	a_6, b_6	a_8, b_8

.

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d. Alter the game in part b., as follows. At player 1's last move, he only knows whether or not the earlier moves matched. (Hint: (R, r) and (L, l) match but (R, l) and (L, r) do not.) Given a mixed strategy σ_1 for player 1 and σ_2 for player 2, what is the probability that player 1 chooses ρ on his final move, given that he is in the information set for matched earlier moves?

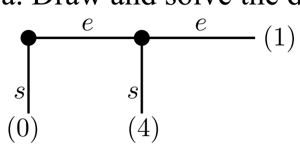
$$p = \frac{\sigma_1(L\rho\cdot)\,\sigma_2(l\cdot) + \sigma_1(R\cdot\rho)\,\sigma_2(\cdot r)}{[\sigma_1(L\lambda\cdot) + \sigma_1(L\rho\cdot)]\,\sigma_2(l\cdot) + [\sigma_1(R\cdot\lambda) + \sigma_1(R\cdot\rho)]\,\sigma_2(\cdot r)}.$$

e. [extra credit, but not especially hard.] Does an arbitrary mixed strategy for player 1 for the game in part b. induce a unique behavior strategy? Does it for the game in part d.? (Hint: think about imperfect recall, and look for a discussion of Kuhn's theorem in MCWG or elsewhere.)

(e) According to Kuhn's theorem, an arbitrary mixed strategy for player for the game in (b) induce a unique behavior strategy. However, it does not for the game in (d). □

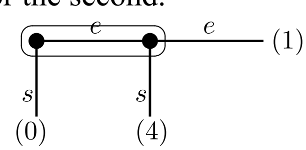
2. [Based on a paper by A. Rubinstein]. An absent-minded driver can either turn south or continue east through two junctions. His payoffs are 0 if he turns at the first junction, 4 at the second, and 1 if he doesn't turn.

a. Draw and solve the decision tree given perfect information.



solution: turn south at the second junction.

b. Draw the tree if the driver can recognize a junction but has no clue whether it is the first or the second.



c. What is the driver's optimal behavior and corresponding expected payoff in b? You should consider mixed strategies that are consistent with imperfect recall.

(c) Denote the probability of turn south as q at either junction, and the probability of continuing east as $1-q$. Then with imperfect recall, the driver seeks to maximize his expected payoff

$$\max_q E(q) = q * 0 + (1-q)\,q * 4 + (1-q)^2 * 1,$$

and the FOC, $-6q+2=0$, implies $q^* = 1/3$. Therefore, the optimal behavior is to turn south whenever a junction appears with probability of 1/3, and the corresponding expected payoff is $E(q^*) = 4/3$.

d. Do you see anything paradoxical (or "time-inconsistent") about your solution in c? If so, resolve the paradox.

(d) The optimal behavior in (c) is calculated before the driver gets to some junction. If the driver sticks to this plan (i.e. with full commitment), then it is reasonable for him to believe when coming to one junction that with probability

$$\frac{1}{1+\frac{2}{3}} = \frac{3}{5} = 0.6$$

he is facing the first junction and with probability 0.4 he is facing the second one. So if he decides to turn south at this junction (i.e. $q=1$), then his expected payoff is $0.4 \times 4 = 1.6 > 4/3$. Hence here comes the paradox.

Resolution: TBA…I want to see how you handled it.

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3. Consider the matching pennies game, where the payoff parameters a,b,c,d are all positive:

	h	t
H	a, 0	0, d
T	0, c	b, 0

a. Verify that there are no pure NE, and compute the unique mixed NE s*.

Denote by p the probability that player 1's mixed strategy assigns to H and by q the probability that player 2's mixed strategy assigns to h. Then, given player 2's mixing, player 1's expected payoff to the pure strategy H is

$$q \times a + (1-q) \times 0 = qa$$

And player 1's expected payoff to the pure strategy T is

$$q \times 0 + (1-q) \times b = (1-q)b$$

From these two, I solve for q, which is $q = \dfrac{b}{a+b}$

Likewise, given player 1's mixing, player 2's expected payoff to the pure strategy h is

$$p \times 0 + (1-p) \times c = (1-p)c$$

And player 1's expected payoff to the pure strategy T is

$$p \times d + (1-p) \times 0 = pd$$

From these two, I solve for p, $p = \dfrac{c}{d+c}$

the unique mixed NE is $((\frac{c}{d+c}, \frac{d}{d+c}), (\frac{b}{a+b}, \frac{a}{a+b}))$

b. Derive the comparative statics—precisely how does each player's NE mix change when each of the parameters change?

$$\frac{\partial p}{\partial d} = \frac{\partial}{\partial d}(\frac{c}{d+c}) = -\frac{c}{(d+c)^2}; \quad \text{and} \quad \frac{\partial p}{\partial d} = \frac{\partial}{\partial c}(\frac{c}{d+c}) = \frac{d}{(d+c)^2}$$

$$\frac{\partial q}{\partial a} = \frac{\partial}{\partial a}(\frac{b}{a+b}) = -\frac{b}{(a+b)^2}; \quad \text{and} \quad \frac{\partial q}{\partial b} = \frac{\partial}{\partial b}(\frac{b}{a+b}) = \frac{a}{(a+b)^2}$$

but own partial derivatives -- p wrt a, b and also q wrt c, d -- are 0!

c. Most people have the intuition that changes in player i's payoff will affect the equilibrium mix mainly for player i, and have little or no effect on the mix for other players j. Try to reconcile this intuition with your findings in part b.

Remember that player i's mix is chosen to keep player(s) –i indifferent. Hence it responds to –i's payoffs, but not to own payoffs.

4. a. Compute all NE of the following 2 player normal form game.

	L	C	R
T	2, 0	1, 1	4, 2
M	3, 4	1, 2	2, 3
B	1, 3	0, 2	3, 0

Since the strategy C for player 2 is dominated by L and R, then player 2 will remove this strategy. As player 1 knows that player 2 will remove strategy C then player 1 removes strategy B.

	L	R
T	$2, 0$	$4, 2$
M	$3, 4$	$2, 3$

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- Best responses in this new set up are as follow:

$$B_1(L) = M \qquad\qquad B_1(R) = T$$
$$B_2(T) = R \qquad\qquad B_2(L) = M$$

 So the pure strategy NE $= ((M, L), (T, R))$

- Now check whether there is a NE where player 1 play T vs player 2 mixing L, R: If player 1's strategy is T, then player 2's payoff to her two actions $(0, 2)$ which is different. For NE to exist, the payoff player 2 assigns positive probability must be the same.[1] Using the same logic, I could eliminate the possible pair between pure strategy and mixed.
- Denote by p the probability that player 1's mixed strategy assigns to T and by q the probability that player 2's mixed strategy assigns to L. Then, given player 2's mixing, player 1's expected payoff to the pure strategy T is

$$q \times 2 + (1 - q) \times 4 = 4 - 2q$$

 And player 1's expected payoff to the pure strategy M is

$$q \times 3 + (1 - q) \times 2 = 2 + q$$

 From these two, I solve for q, which is $q = \dfrac{2}{3}$

 Likewise, given player 1's mixing, player 2's expected payoff to the pure strategy L is

$$p \times 0 + (1 - p) \times 4 = (1 - p)4$$

 And player 1's expected payoff to the pure strategy R is

$$p \times 2 + (1 - p) \times 3 = 3 - p$$

 From these two, I solve for p, $p = \dfrac{1}{3}$

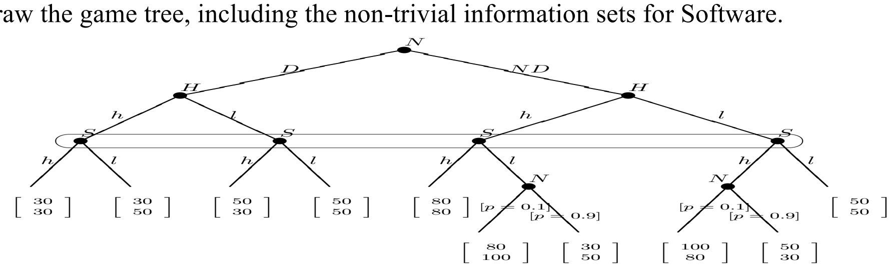
So the unique mixed NE is $\left((\frac{1}{3}, \frac{2}{3}), (\frac{2}{3}, \frac{1}{3})\right)$

b. Write out an extensive form game of perfect information with the same payoffs as above but in which Row moves first, and solve for all NE.

Omitted to conserve space. SGPNE is unique by Zermelo's theorem, and is easily seen to be (T,R)

5. Software and Hardware form a joint venture in which they agree to split revenue evenly. Each can exert either high effort at cost 20 or low effort at cost 0. Hardware moves first but Software does not observe her effort. Revenue is 100 if both exert low effort, or if parts are defective. If parts are not defective, revenue is definitely 200 if both exert high effort, and is 200 with probability 0.1 (and 100 with probability 0.9) if only one partner exerts high effort. Both partners initially believe the probability of defective parts is 0.3. Hardware discovers the truth before she chooses effort level, but Software does not.

a. Draw the game tree, including the non-trivial information sets for Software.



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b. Write out the bimatrix for the corresponding normal form.

Nature=Defective

		Software h	Software l
Hardware	h	30,30	30,50
	l	50,30	50,50

Nature=Not Defective

		Software h	Software l
Hardware	h	80,80	35,55
	l	55,35	50,50

c. Find all NE for the game in b.

(c) Apparently, the pure BNE are (ll, l) and (lh, h).

First, notice that the fact that Hardware knows the realization of nature (whether parts are defective r not) before making decision on effort is a common knowledge. And we know (so does Software) ıat when parts are defective, giving low effort is the dominant strategy that Hardware would play. herefore, denote p as the probability that Hardware gives high effort when parts are not defective, ıd q as the probability that Software gives high effort regardless of the realization of the nature (since oftware does not know the truth). Then in order to have a mix NE, we need to have

$$u_H(h, \sigma_S | N = D) = 80q + 35(1-q) = u_H(l, \sigma_S | N = D) = 55q + 50(1-q) \Rightarrow q = 0.375,$$

If we have covered the relevant topics before the due date, then

d. Compute all Bayesian Nash equilibria (BNE).

$$u_S(h, \sigma_H) = 0.3 * 30 + 0.7 * (80p + 35(1-p)) = u_S(l, \sigma_H) = 0.3 * 50 + 0.7 * (55p + 50(1-p))$$

$$\Rightarrow p = 0.5893.$$

Thus, the mix BNE is such that Software gives high effort with probability 0.375, and Hardware always gives low effort when parts are defective and gives high effort with probability 0.5893 when parts are not defective, i.e. $\sigma_H^*(lh) = 0.5893$, $\sigma_H^*(ll) = 0.4107$, and $\sigma_H^*(hh) = \sigma_H^*(hl) = 0$, while $\sigma_S^*(h) = 0.375$ and $\sigma_S^*(l) = 0.625$.

e. In BNE (if several, pick one), what is Software's belief about Hardware's choice?

(d) In the mixed strategy BNE, the Software's belief about Hardware's choice is that Hardware always gives low effort when parts are defective and gives high effort with probability 0.5893 when parts are not defective.

f. In a branch where Software chose high effort and sees revenue of 100, what is her belief that the parts were defective?

(e) In a branch where Software chose high effort and sees revenue 100, then her belief that the parts were defective is

$$\begin{aligned}
\Pr(Defective | R = 100, S = h) &= \frac{\Pr(Defective, R = 100, S = h)}{\Pr(R = 100, S = h)} \\
&= \frac{0.3q}{0.3q + 0.7(1-p)q * 0.9} = 0.5369.
\end{aligned}$$

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