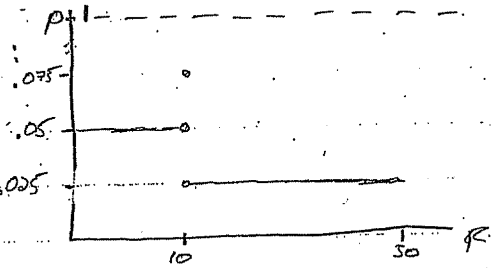
Economics 204 B
Dan Friedman

Answer Key for
Final Exam

Winter 2010
UCSC

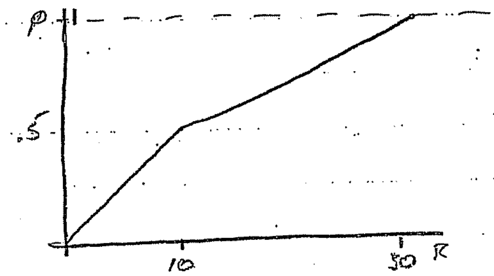
[1] $e_L = \frac{1}{2}[0,10] + \frac{1}{2}[10,30]$

a) pdf:



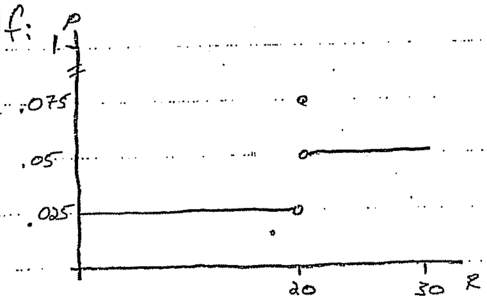
$$f(R_L) = \begin{cases} .05 & \text{if } 0 \le R < 10 \\ \cancel{.075} & \cancel{\text{if } R = 10} \\ .025 & \text{if } 10 < R \le 30 \end{cases}$$

cdf:



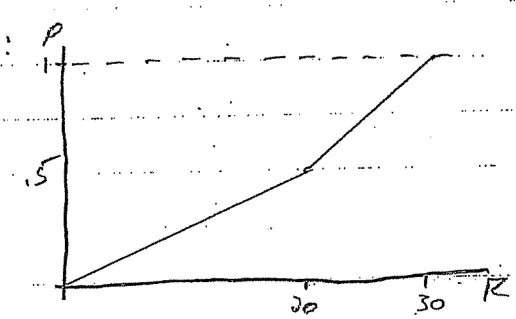
b) $e_H = \frac{1}{2}[0,20] + \frac{1}{2}[20,30]$

pdf:



$$f(R_H) = \begin{cases} .025 & \text{if } 0 \le R < 20 \\ .075 & \text{if } R = 20 \\ .05 & \text{if } 20 < R \le 30 \end{cases}$$

cdf:



8/8

c) e_H 1° and 2° SD e_L, $F(e_H) \le F(e_L)$ $\forall e_i$ and 6/6
$\int_0^e F(e_H) \le \int_0^e F(e_L)$ (One can also say 2°SD doesn't apply since means are not the same.)

The expected utility

#1 (d)

$$EU(w) = \int_0^{30} 2w^{0.5} f(w)\,dw$$

$$w = \frac{x}{4}$$

$$EU(x) = \int_0^{30} 2\times\left(\frac{x}{4}\right)^{0.5} f(x)\,dx$$

$$= \int_0^{30} x^{0.5} f(x)\,dx$$

For "Take it Easy"

$$EU(x) = \int_0^{10} x^{0.5}\cdot\frac{1}{20}\,dx + \int_{10}^{30} x^{0.5}\cdot\frac{1}{40}\,dx$$

$$= \left.\frac{x^{1.5}}{1.5\times 20}\right|_0^{10} + \left.\frac{x^{1.5}}{1.5\times 40}\right|_{10}^{30}$$

$$= \frac{10^{1.5}}{30} + \frac{30^{1.5}-10^{1.5}}{60}$$

$$= \frac{30^{1.5}}{60} + \frac{10^{1.5}}{60} = \frac{195.939}{60} = 3.27$$

For "work hard"

$$EU(x) = \int_0^{20} x^{0.5}\frac{1}{40}\,dx + \int_{20}^{30} x^{0.5}\cdot\frac{1}{20}\,dx$$

$$= \left.\frac{x^{1.5}}{1.5\times 40}\right|_0^{20} + \left.\frac{x^{1.5}}{1.5\times 20}\right|_{20}^{30}$$

$$= \frac{20^{1.5}}{1.5\cdot 40} + \frac{30^{1.5}-20^{1.5}}{1.5\cdot 20}$$

$$= \frac{30^{1.5}}{30} - \frac{20^{1.5}}{60} = \frac{239.191}{60} = 3.99$$

Since $u(w_{CE}) = EU(w)$, we have

$$u(w_{CE}^{Easy}) = 2\sqrt{w_{CE}^{easy}} = 3.2$$

$$\Rightarrow w_{CE}^{easy} \approx \left(\frac{3.2}{2}\right)^2 = (1.6)^2 = \underline{\underline{2.56}}$$

$$w_{CE}^{Hard} \approx \left(\frac{3.99}{2}\right)^2 \simeq \underline{4.}$$

#3

3a

a. For a consumer at $z = x$, he faces

$$\begin{cases} P_A + tx & \text{if he buy from } A \\ P_B + t(1-x) & \text{if he buy from } B. \end{cases}$$

he chooses A iff

$$P_A + tx \le P_B + t(1-x).$$

A's consumer group:

$$P_A + tx = P_B + t(1-x)$$

$$\Rightarrow t(2x-1) = P_B - P_A$$

$$\boxed{\hat{x} = \frac{P_B - P_A}{2t} + \frac{1}{2}}$$ territorial devide

A's ~~revenue~~ profit (payoff function)

$$= (P_A - 2)\cdot\left(\frac{P_B - P_A}{2t} + \frac{1}{2}\right)$$

FOC: $\frac{\partial \text{Revenue}}{\partial P_A} = 0$

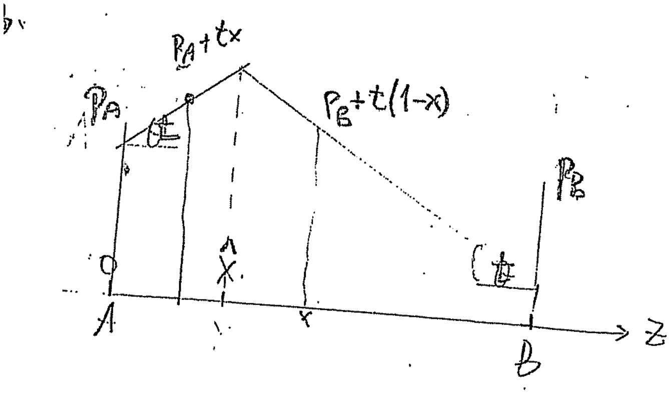
$$\Rightarrow \left(\frac{P_B - P_A}{2t} + \frac{1}{2}\right) - \frac{P_A - 2}{2t} = 0$$

$$\Rightarrow P_B - P_A + t - P_A + 2 = 0$$

$$\Rightarrow P_A = \frac{P_B + t + 2}{2} \checkmark$$

This is A's best reply function given P_B.

b.



By Symmetry, B's best reply

$$BR_B(P_A) = \min\left\{2, \frac{P_A+t+2}{2}\right\}$$

Nash Equilibrium:

$$\text{if } t>0 \begin{cases} P_A = \frac{P_B+t+2}{2} \\ P_B = \frac{P_A+t+2}{2} \end{cases} \Rightarrow P_A^* = P_B^* = t+2$$

a. If $t=0$, the game is the

$\Rightarrow$ standard Bertrand game. ✓

$$BR_A(P_B) = \begin{cases} P_B - \varepsilon & \text{if } P_B > 2 \\ 2 & \text{if } P_B \le 2. \end{cases}$$

$$BR_B(P_A) = \begin{cases} P_A - \varepsilon & \text{if } P_A > 2 \\ 2 & \text{if } P_A \le 2. \end{cases}$$

Nash Equilibrium:

$$P_A^* = P_B^* = 2.$$

8/8

b. As derived above (see previous page), at $t=3$, eqm:

$$P_A^* = P_B^* = 5.$$

The market segment for A:

$x \in [0, \frac{1}{2}]$

$Q_A = \frac{1}{2}$

$[0, \hat{x}]$
$\hat{x} = \frac{1}{2}$

The market segment for B:

$x \in [\frac{1}{2}, 1] = [\hat{x}, 1]$

$Q_B = \frac{1}{2}$

Equilibrium payoff:

$$\pi_A = (P_A - C) \times Q_A = (5-2) \times \frac{1}{2} = \frac{3}{2}$$

$$\pi_B = (P_B - C) \times Q_B = (5-2) \times \frac{1}{2} = \frac{3}{2}$$

10/10

c. · $t=0$ case:

In equilibrium no firm will enter. ✓

Reason:

The industry is like a Bertrand game, if new firm enter, equilibrium payoff is zero minus fixed cost.

· $t>0$ case:

In equilibrium there can be firms enter, depends on t. if t is larger, more firms will enter.

For $t=3$, it seems that 1 more firm can enter, most profitably at $z = \frac{1}{2}$.

2. a. Need to know that boss's payoff is $[R-w]$ (risk neutral) and ~~whether~~ e is ~~observable~~ (assume not).

b. Cost-min FOC to motivate $e = e_H$ is

$$(*) \quad \underbrace{\frac{1}{v'(w(R))}}_{} = \gamma + \mu\left(1 - \frac{f(R|e_L)}{f(R|e_H)}\right).$$

Since $v(w) = 2\sqrt{w}$ by problem #1,

The RHS is ~~Example problem~~ $\dfrac{1}{\frac{d}{dw}\left(2\sqrt{w(R)}\right)} = \dfrac{1}{\frac{2}{2\sqrt{w(R)}}} = \sqrt{w(R)}$.

The likelihood ratio from (1ab) e.g. $\dfrac{\frac{1}{20}}{\frac{1}{40}} = 2$.

$$\frac{f(R|e_L)}{f(R|e_H)} = \begin{cases} 2 & \text{if } R \in [0,10) \\ 1 & \in [10,20) \\ \frac{1}{2} & [20,30) \end{cases}$$

So LHS of $(*)$ is $\gamma - \mu$, γ, or $\gamma + \frac{1}{2}\mu$ in these 3 cases

~~Hence $w(R) = (\gamma +$~~

Conclusion:

Pay salesman
$$w(R) = \begin{cases} (\gamma - \mu)^2 & \text{if } R \in [0,10) \\ \gamma^2 & \text{if } [10,20) \\ (\gamma + \frac{1}{2}\mu)^2 & \text{if } [20,30) \end{cases}$$

c. Caveats: – Best to phrase as "base pay" and as bonuses.

– Think about whether salesman reacts badly to changes in commission schedule.

– Most important: is it more profitable to incentivize H or L effort?

d. For extra credit, use [IC] to find that $\gamma = 4$, and use [PC] to obtain μ.

4. a. NBS

$$\begin{cases} \max\ (u_A - \underline{u}_A)(u_B - \underline{u}_B) \\ \text{s.t.}\quad u_A + u_B = 100 \\ \qquad \underline{u}_A = \underline{u}_B = 0 \end{cases}$$

FOC $\Rightarrow u_A^* = u_B^* = 50$ ✓ 6/6

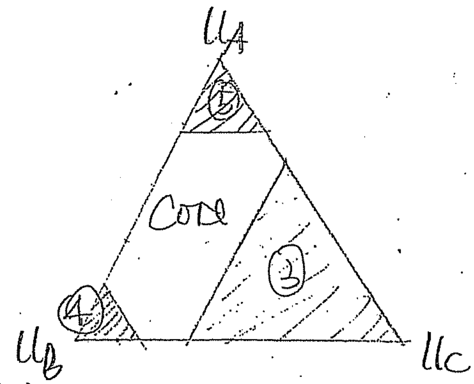
b. Characteristic function

$$\begin{cases} v(\phi) = 0 & ① \\ v(A) = v(B) = v(C) = 0 & ② \\ v(A,B) = 100/150 = \frac{2}{3} & ③ \\ v(A,C) = 30/150 = \frac{1}{5} & ④ \\ v(B,C) = 40/150 = \frac{4}{15} & ⑤ \\ v(A,B,C) = 150/150 = 1 & ⑥ \end{cases}$$

where $v(A,B)$ reads as CF for coalition $\{A,B\}$, etc. 4/4

c. Core: Core is the set of all feasible utility outcomes unblocked by any coalition.

$$\begin{cases} v(A,B) = 100/150 \Rightarrow u_A + u_B \geq 100 \\ v(A,C) = 30/150 \Rightarrow u_A + u_C \geq 30 \\ v(B,C) = 40/150 \Rightarrow u_B + u_C \geq 40 \\ v(A,B,C) = 150/150 \Rightarrow u_A + u_B + u_C \geq 150 \end{cases}$$



So the core is any allocation satisfies

$$\begin{cases} u_A + u_B \geq 100 \\ u_A + u_C \geq 30 \\ u_B + u_C \geq 40 \\ u_A + u_B + u_C = 150 \end{cases}$$

As indicated in the graph, the triangle is $u_A + u_B + u_C = 150$.

$$\begin{cases} ③ \text{ is blocked by } v(A,B) \\ ④ \text{ ------------ } v(A,C) \\ ⑤ \text{ ------------ } v(B,C) \end{cases}$$

The rest area is not blocked by any coalition, which is the core.

8/8

4d. Sharpley Value.

permutation	MC_A	MC_B	MC_C
{A, B, C}	0	100	50
{A, C, B}	0	120	30
{B, A, C}	100	0	50
{B, C, A}	110	0	40
{C, A, B}	30	120	0
{C, B, A}	110	40	0
Total.	350	380	170
Sharpley Value	$\frac{350}{900}$	$\frac{380}{900}$	$\frac{170}{900}$
Simplify:	0.39	0.42	0.19

5a. Standard definitions

b. Thiefs would like to pool w/ other suspects. Detectives look for a separating equilibrium.

The dog will bark at strangers, but not at people it knows. Sherlock infers that the thief is someone the dog knows.

another answer.

b) The case of the barking dog is an example of a signaling mechanism. The dog can signal after a nature move (N-presence of theif or not). The question Sherlock needs to ask, however, is what type of signal it is. If the dog will bark no matter what (whether ∃ a theif or not) it is not useful and if she will never bark, it is equally unuseful. This is known as a pooling signal. If the dog barks in the presence of a theif and does not otherwise, this is useful (similarly if she doesn't bark if a thief and does without). This type of signal, called a separating signal, will be useful in cracking the case. Sherlock must now see if the dog is (NB_{NT}, NB_T), (NB_{NT}, B_T), or (B_{NT}, NB_T)