2. Preferences and Demand

Microeconomic Analysis, Chapters 7-9

I. Preference Orderings

A. Bundles

1. Our main goal with this section is to model how people choose between consumption options.
2. We call these options **bundles**.
3. A bundle is represented as a vector $\mathbf{x}$ with a dimension equal to the number of goods.
4. If consumers only make choices between two goods (1 and 2), we could represent this as a vector $\mathbf{x} = (x_1, x_2)$
   a. $\mathbf{x} = (7, 4)$ represents a bundle consisting of 7 units of good 1 and 4 units of good 2.
   b. For simplicity we'll assume there are only two goods through most of this section
   c. These results, however, extend to an arbitrarily large number of goods.

Ex: Bundles in a 2-dimensional consumption space.

B. Preferences Over Bundles

1. We assume that people can compare pairs of bundles.
2. We call this comparison a **preference**.
3. In other words, a preference is really a *relation(ship)* between between two bundles.
4. Taking any two bundles $\mathbf{x}$ and $\mathbf{y}$ we write
   - $\mathbf{x} \sim \mathbf{y}$ if the consumer is indifferent between $\mathbf{x}$ and $\mathbf{y}$, i.e., she is just as happy with $\mathbf{x}$ as she is with $\mathbf{y}$.
     OR
   - $\mathbf{x} \succ \mathbf{y}$ or $\mathbf{x} \prec \mathbf{y}$ if the consumer either prefers $\mathbf{x}$ to $\mathbf{y}$ or prefers $\mathbf{y}$ to $\mathbf{x}$, i.e., she is happier with $\mathbf{x}$ than $\mathbf{y}$ or is happier with $\mathbf{y}$ than $\mathbf{x}$.
     This is called **strict preference.** Given the choice, the consumer would always choose one to the other.
   - $\mathbf{x} \succeq \mathbf{y}$ or $\mathbf{x} \preceq \mathbf{y}$ if the consumer is either indifferent between $\mathbf{x}$ and $\mathbf{y}$ or prefers $\mathbf{x}$ to $\mathbf{y}$ (alternatively the consumer is either indifferent between y and $\mathbf{x}$ or prefers $\mathbf{y}$ to $\mathbf{x}$). This is called **weak preference**.

Ex: Indifference curves and better sets in a 2-dimensional consumption space.

C. Preferences are *always* assumed to satisfy three properties:

1. Complete: Either $\mathbf{x} \succeq \mathbf{y}$, $\mathbf{y} \succeq \mathbf{x}$ or both (in which case $\mathbf{x} \sim \mathbf{y}$).
2. Reflexive: $\mathbf{x} \succeq x$
3. Transitive: If $\mathbf{x} \succeq \mathbf{y}$ and $\mathbf{y} \succeq \mathbf{z}$ then $\mathbf{x} \succeq \mathbf{z}$.

Ex: Preference no-nos [Indifference curves that cross, ...]

D. Preferences are often assumed to have two extra properties (when someone talks about **well behaved preferences** this is what they mean).

1. (positive) Monotone: More is better: $x_i \geq y_i \forall i \implies \mathbf{x} \succeq \mathbf{y}$.
2. Convex: If $\mathbf{x} \sim \mathbf{y}$ then for any $0 \leq \alpha \leq 1$, $(\alpha x_1 + (1 - \alpha y_1), \alpha x_2 + (1 - \alpha y_2)) \succeq (x_1, x_2)$

Ex: Not well behaved preferences

II. Utility Functions

A. It seems pretty hard to model decision making using the preference theory we just talked about because we'd have to specify the preference relations between every pair of bundles.

Luckily if we assume that preferences (automatically complete, reflexive and transitive) are also continuous (a property awkward to define formally, but intuitively clear) and monotone, then it is known that we can represent those preferences with a continuous function. [Varian, p.97 sketches a proof.]

B. A **utility function** u assigns a real number $u(\mathbf{x})$ to each bundle $\mathbf{x}$ such that

1. $\mathbf{x} \succ \mathbf{y}$ if and only if $u(\mathbf{x}) > u(\mathbf{y})$, and
2. $\mathbf{x} \sim \mathbf{y}$ if and only if $u(\mathbf{x}) = u(\mathbf{y})$.

C. Utility functions aren't unique – they're just shorthand for underlying preferences.

1. Any monotonic transformation of a utility function can be used to represent the same set of preferences – i.e. does the same job.
2. No harm in choosing a smooth (continuously differentiable) utility function.

D. The partial derivative of a utility function is its **marginal utility**: $mu_i \equiv \frac{\partial u}{\partial x_i}$. It depends on the choice of a utility function.

E. The **marginal rate of substitution** between two goods i and j is $MRS_{ij} \equiv -mu_j/mu_i \equiv -\frac{\partial u}{\partial x_j} / \frac{\partial u}{\partial x_i}$.

1. It is invariant to the choice of utility function to represent given preferences!
2. It is just the slope of the indifference curve between the two goods.
3. In higher dimensions, there is an indifference hypersurface, and there MRS_{ij} is the slope of that surface in the $i - j$ plane.

4. You can think of MRS_{ij} as the number of micro-units of j that the consumer is just willing accept to give up a micro-unit of i — the consumer's trade-off rate between the two goods.

Ex: Perfect substitutes: $u(x_1, x_2) = x_1 + cx_2$. Then $MRS_{12}(x_1, x_2) = c > 0$; the tradeoff rate is constant.

Ex: Cobb-Douglas utility: $u(x_1, x_2) = \ln x_1 + c \ln x_2$. Then $MRS_{12}(x_1, x_2) = \frac{cx_1}{x_2} > 0$. Convex, Inada. What can we say about $v(x_1, x_2) = \exp(u(x_1, x_2))$?

Ex: CES utility: $u(x_1, x_2) = \frac{1}{\rho} \ln(x_1^\rho + cx_2^\rho)$, where $\rho \in (-\infty, 1]$. Can show that this nests the previous two cases, which correspond respectively to $\rho = 1, 0$. Generalizes directly to more than 2 goods. Very useful in applied work.

Ex: Quasilinear utility: $U(x_0, x_1) = x_0 + g(x_1)$. Think of good 0 as money, or purchasing power. $MRS_{ij}(x_0, x_1) = g'(x_1)$. Very very useful in applied work.

III. The Direct Consumer Problem and (Marshallian) Demand

A. Putting this machinery to use – we get to model choice and figure out where demand comes from.

B. Let me follow textbooks for the next hour. Later I will show you a streamlined approach I developed with Jozsef Sakovics (with roots in Marshall).

C. Suppose that the consumer is constrained by available income and by the prices of the goods. We call this the budget constraint.

   1. We'll denote the consumer's money as m and p_1 the price of good 1, p_2 the price of good 2 etc.
   2. Then $m \geq \mathbf{p} \cdot \mathbf{x} = p_1 x_1 + p_2 x_2 + ...$ (i.e. you can't spend more than you have)
   3. If she has strictly monotone preferences, a consumer will spend all of her money and so
   4. $m = \mathbf{p} \cdot \mathbf{x} = p_1 x_1 + p_2 x_2 + ...$

   Ex: Budget constraints and budget sets.

D. Then we have the following constrained optimization problem for the two good case (we're assuming strong monotonicity, smooth indifference curves and an interior optimum):

   max $u(x_1, x_2, ...)$
   s.t. $m = p_1 x_1 + p_2 x_2 + ...$

   1. We form the Lagrangian

   $$\mathcal{L} = u(x_1, x_2) + \lambda(m - \mathbf{p} \cdot \mathbf{x})$$

2. Differentiating, we get two first order conditions:

   a. $\frac{\partial u}{\partial x_1} - \lambda p_1 = 0$

   b. $\frac{\partial u}{\partial x_2} - \lambda p_2 = 0$

3. Simultaneously solving this FOC we can get the exact consumption decisions, at least if the SOC holds.

4. But even in this abstract form we get a property of optimal choice that may look familiar:

   $\frac{\partial u}{\partial x_i} / \frac{\partial u}{\partial x_j} = p_i/p_j$

5. The indifference curve will be tangent to the budget line – their slopes (the MRS and the absolute price ratio) will be equal.

   Ex: An example with Cobb-Douglas preferences.

   Ex: Direct consumer problem with indifference curves.

E. Solving this problem (which we will call the *direct consumer problem*) for x_i gives us the quantity demanded for good i as a function of prices and income. (The solution will be unique if preferences are *strictly* convex.)

F. This is the individual's **Marshallian demand** curve, $x_i(\mathbf{x}, m)$.

   1. If we hold the price of other goods and income constant, we get the old demand curve we studied in the last section: $x_i(p_i)$

Ex: Marshallian demand from Cobb-Douglas preferences.

Ex: Indirect utility.

Ex: Expenditure function

Ex: Hicksian demand