Expected Utility Thm. With "reasonable" preferences, $\exists$ a bernoulli fx u s.t. $L \succeq L' \Leftrightarrow \sum p_i u_i \geq \sum p'_i u_i$. (choose lottery with higher expected utility)

SD - $G(x) \leq F(x) \forall x$ (1st - function always "under") $\int_{-\infty}^{s} G(x)dx \leq \int_{-\infty}^{s} F(x)d$ (2nd - function mostly under)

Risk Aversion $\frac{-u''(s)}{u'(s)}$ARA $\frac{-u''(s)\cdot s}{u'(s)}$ RRA

Treasure chest type questions - 1. sort in order of highest $\frac{pr_i}{c_i}$ to lowest. 2. try in order until success or $\frac{pr_i}{c_i} < \frac{1}{value}$.

CE - lottery gives x or y with prob p and $1-p$. Then $u[CE] = pu(x) + (1-p)(u(y))$.

Bayes $p(s|z) = \frac{p(z|s)p(s)}{\sum_t p(z|t)p(t)}$ states(s,t) signals(z)

$\frac{p(s|z)}{p(t|z)} = \frac{p(z|s}{p(z|t)}\frac{p(s)}{p(t)}$ posterior = likelihood ratio·priors

s	$p(s)$	$p(z_1\|s)$	z_2	$p(z_1 s)$	$p(z_2 s)$	$p(s\|z_1)$
s_1	.4	.9	.1	.36	.04	$\frac{.36}{.54}$
s_2	.6	.3	.7	.18	.42	$\frac{.18}{.54}$
	1			.54+	$x = 1$	1

Cookbook - 1. Draw decision tree, fill in payoffs and nature probs 2. Solve by BI, take EU at N-moves and max at decision nodes 3. Write induced utilities at each now terminal node, repeat until at start 4. write out complete contingency plan.

V(I) =informed payoff-uninformed payoff-cost of getting info

Game Solutions IDDS $\subset$ SPNE $\subset$ NE $\subset$ CE/RE SPNE rules out NE not obtainable by BI

Generalized BI - 1. Find all NE at any irreducible terminal subgame 2. Write out reduced game with a NE payoff vector replacing subgame 3. Iterate until start, always at least one SPNE

Incomplete Info (Harsanyi) - 1. Specify types and connect with N-move, drawing relevant info sets 2. Assume common prior for N-move 3. Solve for NE(BNE) and SPNE(PBE) by normal methods.

Given beliefs μ and strategy profile $\sigma \to$ 1) μ is the Bayesian posterior given common prior and σ. 2) each component of σ is a BR to μ. 1+2 is a BNE, and is a PBE if 1+2 hold in every subgame.

Repeated Games - For known finite number of repetitions of the Prisoner's dilemma stage game, always-defect is the unique NE. For infinite number of repetitions, grim can sustain cooperation in NE if $\delta > \delta_0$.

Folk Thm - any feasible payoff vector that dominates the NE is achievable as a SPNE if players are sufficiently patient.

Coop Games 2+ players, transferable utility. Start with characteristic fx V (typically list of outcomes of all possible K). Convex ftn - If S$\subset$ T and i $\in$ N-T, ν (S$\bigcup$i)-$\nu(s) \leq \nu(T \bigcup i) - \nu(T)$

Core - Coalition K blocks allocation u if $\sum_{i \in K} u_i < v(K)$ i.e. if they can do better by themselves. Core is all allocations unblocked by any K. **Shapley Value** - $\phi(v)$ exists(possibly empty), unique, and pareto optimal.

	1	2	3
123	0	1	1
132	0	0	2
213	1	0	1
231	2	0	0
312	2	0	0
321	2	0	0
$\phi(v)$	7/6	1/6	4/6

Example:$V(i) = 0$
$V(12) = 1, V(13) = 2$
$V(23) = 0, V(123) = 2$. Then the core is half of side of simplex ($100 \leq x_1 \leq 200, x_2 = 0, 0 \leq x_3 \leq 100$). Note that $\phi(v)$ is outside the core.

NBS - Given threat point, NBS is pareto optimal (on NE frontier) max $g(u,v) = (u - \bar{u})(v - \bar{v})$. Example: Feasible utilities given by $20 - u^2 = v$ and $(\bar{u}, \bar{v}) = (2,2)$. max $g = (u-2)(v-2)$. Plug in $v \to (u-2)(18-u^2) = u^2+18u-36$. Then $\frac{dg}{du} = -3u^2+4u+18 = 0$. This gives u^*, plug in for v^*.

Evo Games Describes ongoing strategic interaction.

2-pop example: $U(I,r) = 2r - 1$
$U(D,r) = 2 - 6r \to r = \frac{3}{8}$
$w(A,s) = 2 - 10s$
$w(B,s) = s - 1 \to s = \frac{1}{9}$ Then these are the breaks in state space (1x1 unit square).

	B	G	
I	1,-8	-1,0	(s)
D	-4,2	2,1	(1-s)
	(r)	(1-r)	

Monopolistic normal problem: max $qp(q) - c(q)$. Take derivative w.r.t q and solve. parametric example: $p = a - bq_T$ const. cost $= cq_j$ $q_{-j} = \frac{q_T - q_j}{J-1}$ avg. of other firms (n=J). In stack. leader gets twice as much, but total profits < Cournot. Better to be 2nd in Bertrand. $q^m < q^c < q^s < q^b = q^{comp}$.

	monop	cournot	stack
q_1	$\frac{1}{2}\frac{a-c}{b}$	$\frac{1}{3}\frac{a-c}{b}$	$\frac{1}{2}\frac{a-c}{b}$
q_2		$\frac{1}{3}\frac{a-c}{b}$	$\frac{1}{4}\frac{a-c}{b}$
q_T	$\frac{1}{2}\frac{a-c}{b}$	$\frac{2}{3}\frac{a-c}{b}$	$\frac{3}{4}\frac{a-c}{b}$
π_1	$\frac{1}{4}\frac{(a-c)^2}{b}$	$\frac{1}{9}\frac{(a-c)^2}{b}$	$\frac{1}{8}\frac{(a-c)^2}{b}$
π_2		$\frac{1}{9}\frac{(a-c)^2}{b}$	$\frac{1}{16}\frac{(a-c)^2}{b}$
π_T	$\frac{1}{4}\frac{(a-c)^2}{b}$	$\frac{2}{9}\frac{(a-c)^2}{b}$	$\frac{3}{16}\frac{(a-c)^2}{b}$

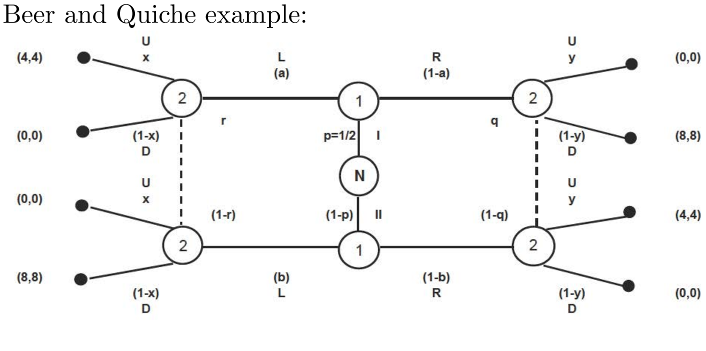
Entry Games - Stage 1:[in with cost K,out]. Stage 2:K is sunk, J entrants. Cournot: Stage 1:[1,0]. Stage 2:get $\pi_j^{NE} - K$ if in, 0 if out. From parametric case:$\pi_j^{NE} - K = \frac{(a-c)^2}{b(J+1)^2} - K$. J must be s.t. $\frac{(a-c)^2}{b(J+1)^2} \approx K$, then can solve for J so that if anyone else enters, it is unprofitable.

Adverse Selection - Asymmetric Info. Ex: Seller knows quality θ=value to buyer. Seller values at $r(\theta)$.$\Theta(p) = \{\theta : r(\theta) \leq p\}$ is the subset of sellers willing to sell at price p. Then a competitive eqm. in a market with asymmetric info is $(p^*, \Theta\ (p^*)$ s.t. $p^* = E(\theta | \theta \in \Theta^*)$ and $\Theta(p^*) = \{\theta : r(\theta) \leq p^*\}$ (i.e. expected quality among those that are selling is the price). Used car ex - $\theta = [2,3]$. $r(\theta) = \theta - .1$ and $\Theta(p) = \{\theta : \theta - .1 \leq p\}$. Then $p^* = \frac{2+(p+.1)}{2}$ solving for p gives $p^* = 2.1$, and $\Theta^* = [2, 2.2]$. 80% market failure.

Signalling - N-move θ, I sends message $m(\theta)$ and U picks action $a(m)$ after forming beliefs $\mu(\theta|m)$. PBE is $[m^*, a^*, \mu]$ s.t. 1. $m^* \in$argmax $u_s(m, a^*; \theta) \forall \theta$ (for every possible state, send m that max u given U's BR to m). 2. $a^* \in$argmax $\sum_\theta \mu \cdot u_r(a, m; \theta)$ (pick a that max EV) 3. μ is consistent with Bayes given N-move and m^*

Kinds of PBE - 1. Separating (each state θ a different m^*) 2. pooling (m^* constant) 3. partial pooling (not 1:1) 4. hybrid (mixed). For separating, a false message must be too costly if not true (i.e. sending H when state is L).

Beer and Quiche example:



	1's Plan	2's assesments	2's b.r.	Optimal for 1?
Separating Equilibria				
$A)$	$I \longrightarrow L; II \longrightarrow R$	$r = 1; q = 0$	$L \longrightarrow U; R \longrightarrow U$	YES
$B)$	$I \longrightarrow R; II \longrightarrow L$	$r = 0; q = 1$	$L \longrightarrow D; R \longrightarrow D$	YES
Pooling Equilibria				
$C)$	$I \longrightarrow L; II \longrightarrow L$	$r = \frac{1}{2}; q = ?$		
		Case $q < \frac{1}{3}$	$L \longrightarrow D; R \longrightarrow U$	YES
		Case $q > \frac{1}{3}$	$L \longrightarrow D; R \longrightarrow D$	$NO\ (I \longrightarrow R)$
$D)$	$I \longrightarrow R; II \longrightarrow R$	$r = ?; q = \frac{1}{2}$		
		Case $r < \frac{2}{3}$	$L \longrightarrow D; R \longrightarrow D$	$NO\ (II \longrightarrow L)$
		Case $r > \frac{2}{3}$	$L \longrightarrow U; R \longrightarrow D$	YES

Then there are 2 separating eqm: 1. Type I plays L, II plays R, and player 2 always plays U. (r=1, q=0) 2. Type I plays R, II plays L, and player 2 always plays D. (r=0, q=1).

There are 2 pooling eqm: 1. Both types play L, player 2 plays D when observes L and U when observes R (r=$\frac{1}{2}$, q< $\frac{1}{3}$)

Screening - U moves first and provides "menu" of choices to induce I to reveal info

P/A Model - $max_e\ EU_P$ s.t. $IC[e] : EU_A(e) \geq EU_A(\tilde{e}) \forall e \in \{e_L, e_H)$ and $PC : EU_A(e) \geq \bar{u}_A\}$. $u_A = v(w) - g(e)$. $u_P = E(\pi|e) - E(w|e)$. $f(\pi|e_H)$FOSD$f(\pi|e_L)$. Reduces to P minimize wage schedule that induces e_H:

$L = -\int\ w(\pi)f(\pi|e_H)d\pi + \gamma[\int v(w(\pi))f(\pi|e_H)d\pi - \bar{u}_A] + \mu[\int v(w(\pi))[(f(\pi|e_H) - f(\pi|e_L)]d\pi - g(e_H) + g(e_L)]$

FOC w.r.t $w(\pi)$ gives: $\frac{1}{v'(w(\pi))} = \gamma + \mu(1 - \frac{f(\pi|e_L)}{f(\pi|e_H)})$. Case 0: e is observable - only one level of effort, wage = $1/\mu$. Case 1: e unobservable, but A is risk neutral. LHS is 1, and $\gamma = 1, \mu = 0$. Case 2: If not, γ is the base pay and μ is extent of effort-based bonus.

Assorted Info, just in case PDF of normal

$P(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$ CDF $C(x) = \frac{1}{\sqrt{2\pi}}\int e^{t^2/2}dt$

General monopoly vs. Cournot FOC: $p'(q^m)q^m + p(q^m) = c$ vs. $p'(q_T)\frac{q_T}{J} + p(q_T) = c$ (non-zero profit $\pi_i = pq - qmc$)