Problem (1)**.** The payoff is $u(a,s) = 1-(a-s)^2$, where the true state s is uniformly distributed on $[0,1]$. Act before observing s, and feasible actions $a \in [-10, 20]$.

Answer. (a) The expected payoff conditional on taking action a is

$$E\left[1-(a-s)^2 \,|a\right] = E\left(1-a^2-s^2+2as|a\right) = 1-a^2-E\left(s^2\right)+2aE(s).$$

Therefore the optimal action is

$$a^* = \arg\max_{a\in[-10,20]} 1-E\left(s^2\right)-a^2+2aE(s),$$

and the FOC implies

$$a^* = \arg\min_{a\in[-10,20]} |a-E(s)|. \tag{1}$$

When $s \sim U[0,1]$, then the optimal action $a^* = E(s) = 1/2$, and the maximal expected payoff is $E(u^*) = 11/12$.

(b) If the distribution for s had an arbitrary cdf F, let's assume the corresponding pdf is f. Then Equation (1) implies that the optimal action is

$$a^* = \begin{cases} -10, & if\ E(s) < -10 \\ E(s), & if\ -10 \le E(s) \le 20 \\ 20, & if\ E(s) > 20 \end{cases},$$

where $E(s) = \int_{-\infty}^{\infty} sf(s)\,ds$, and the corresponding maximal expected payoff is

$$\begin{aligned} E(u^*) &= 1-E\left(s^2\right)-(a^*)^2+2a^*E(s) \\ &= 1-\int_{-\infty}^{\infty} s^2 f(s)\,ds-(a^*)^2+2a^*\int_{-\infty}^{\infty} sf(s)\,ds. \end{aligned}$$

□

#2

A) Compute the joint p(d, t1, t2)

*p(+,+,A)=p(+ test1|A)*p(+ test2|A)*p(A)=p(d=A,t1=+,t2=+)*

pr(A)	0.6
pr(B)	0.4

Test1

p(+|A)	0.7
p(-|A)	0.3
p(+|B)	0.4
p(-|B)	0.6

Test2

p(+|A)	0.2
p(-|A)	0.8
p(+|B)	0.5
p(-|B)	0.5

	A++	A+-	A-+	A--
p(d,t1,t2)	0.084	0.336	0.036	0.144
	B++	B+-	B-+	B--
p(d,t1,t2)	0.08	0.08	0.12	0.12

Marginal(t1,t2) **aka signal pulse*

	++	+-	-+	--
p(t1,t2)	0.164	0.416	0.156	0.264

B) Prior Probabilties

p(t1=+)=Σp(d,t1=+,t2)

	+	-
p(t1)	0.58	0.42
p(t2)	0.32	0.68

pr(A)	0.6
pr(B)	0.4

C) Posterior Probabilities with 1 test

*pr(A|+ test1)=pr(+|A)*pr(A)/(p(+|A)p(A)+p(+|B)p(B))*

	A|+	A|-	B|+	B|-
p(d|t1)	0.724138	0.428571	0.275862	0.571429
	A|+	A|-	B|+	B|-
p(d|t2)	0.375	0.705882	0.625	0.294118

D) Posterior Probabilities with 2 tests

pr(d|t1,t2)=pr(d,t1,t2)/pr(t1,t2)

	A|++	A|+-	A|-+	A|--
p(d|t1,t2)	0.512195	0.807692	0.23769	0.545455
	B|++	B|+-	B|-+	B|--
p(d|t1,t2)	0.487805	0.192308	0.769231	0.454545

Problem (3)**.** Pfissle firm has adequate or substandard quality control, the prior is 0.5 for adequate. Testing pfissles costs 1 per unit, and the test outcomes are iid normal with variance 9 and mean 1 if adequate (-1 if substandard). Loss is 1000 when err and 0 when correct.

Answer. (a) Let c denotes the control quality, $c = A$ (or S) means the quality is adequate (or substandard), then the likelihood pdf of the outcomes of testing n units is

$$f\left(t_1, t_2, \ldots, t_n | c = A\right) = \frac{1}{3^n} \prod_{i=1}^{n} \phi\left(\frac{t_i - 1}{3}\right), \ and\ f\left(t_1, t_2, \ldots, t_n | c = S\right) = \frac{1}{3^n} \prod_{i=}^{n} \phi\left(\frac{t_i + 1}{3}\right),$$

where t_i, $i = 1, 2, \ldots n$, are the test outcomes of the n units and $\phi(\cdot)$ is the pdf of standard normal distribution. Therefore, the signal pdf of the n test outcomes is

$$\begin{aligned} f\left(t_1, t_2, \ldots, t_n\right) &= f\left(t_1, t_2, \ldots, t_n | c = A\right) \Pr\left(c = A\right) + f\left(t_1, t_2, \ldots, t_n | c = S\right) \Pr\left(c = S\right) \\ &= \frac{1}{2} \frac{1}{3^n} \left[\prod_{i=1}^{n} \phi\left(\frac{t_i - 1}{3}\right) + \prod_{i=}^{n} \phi\left(\frac{t_i + 1}{3}\right)\right]. \end{aligned}$$

So the posterior pdf are

$$\begin{aligned} f\left(c = A | t_1, t_2, \ldots, t_n\right) &= \frac{f\left(t_1, t_2, \ldots, t_n | c = A\right) \Pr\left(c = A\right)}{f\left(t_1, t_2, \ldots, t_n\right)} \\ &= \frac{\frac{1}{2} \frac{1}{3^n} \prod_{i=1}^{n} \phi\left(\frac{t_i - 1}{3}\right)}{\frac{1}{2} \frac{1}{3^n} \left[\prod_{i=1}^{n} \phi\left(\frac{t_i - 1}{3}\right) + \prod_{i=}^{n} \phi\left(\frac{t_i + 1}{3}\right)\right]} \\ &= \frac{1}{1 + \exp\left[-\frac{2}{9} \sum_{i=1}^{n} t_i\right]}, \end{aligned}$$

and

$$f\left(c = S | t_1, t_2, \ldots, t_n\right) = 1 - f\left(c = A | t_1, t_2, \ldots, t_n\right) = \frac{1}{1 + \exp\left[\frac{2}{9} \sum_{i=1}^{n} t_i\right]}.$$

Therefore, after observing the n test outcomes, the expected loss is $n + 1000 * f\left(c = S | t_1, t_2, \ldots, t_n\right)$ if deciding adequate, and $n + 1000 * f\left(c = A | t_1, t_2, \ldots, t_n\right)$ if deciding substandard. Hence the threshold is $\sum_{i=1}^{n} t_i = 0$.

If the decision of how many units to test is made in advance, then the firm minimizes its expected loss

$$\min_n E\left[L\left(n\right)\right] = -n - 1000 \left\{\Pr\left(\sum_{i=1}^{n} t_i < 0 | c = A\right) \Pr\left(c = A\right) + \Pr\left(\sum_{i=1}^{n} t_i > 0 | c = S\right) \Pr\left(c = S\right)\right\}. \tag{1}$$

If $c = A$, $\sum_{i=1}^{n} t_i \sim N\left(n, 9n\right)$; otherwise, if $c = S$, $\sum_{i=1}^{n} t_i \sim N\left(-n, 9n\right)$. That is, we have

$$\Pr\left(\sum_{i=1}^{n} t_i < 0 | c = A\right) = \Phi\left(-\frac{1}{3}\sqrt{n}\right), \tag{2}$$

$$\Pr\left(\sum_{i=1}^{n} t_i > 0 | c = S\right) = 1 - \Phi\left(\frac{1}{3}\sqrt{n}\right) = \Phi\left(-\frac{1}{3}\sqrt{n}\right), \tag{3}$$

where Φ is the cdf of standard normal distribution. Substituting Eq. (2) and (3) into Eq. (1) gives us the firm's objective function

$$\min_n E\left[L\left(n\right)\right] = n + 1000\Phi\left(-\frac{1}{3}\sqrt{n}\right),$$

and the FOC is

$$1-\frac{500}{3}\frac{1}{\sqrt{n}}\phi\left(-\frac{1}{3}\sqrt{n}\right)=0,$$

which implies that the optimal decision should be $n^*=42$, as n should be an integer, and the minimized expected loss is $E\left[L\left(n^*\right)\right]=57.3768$.

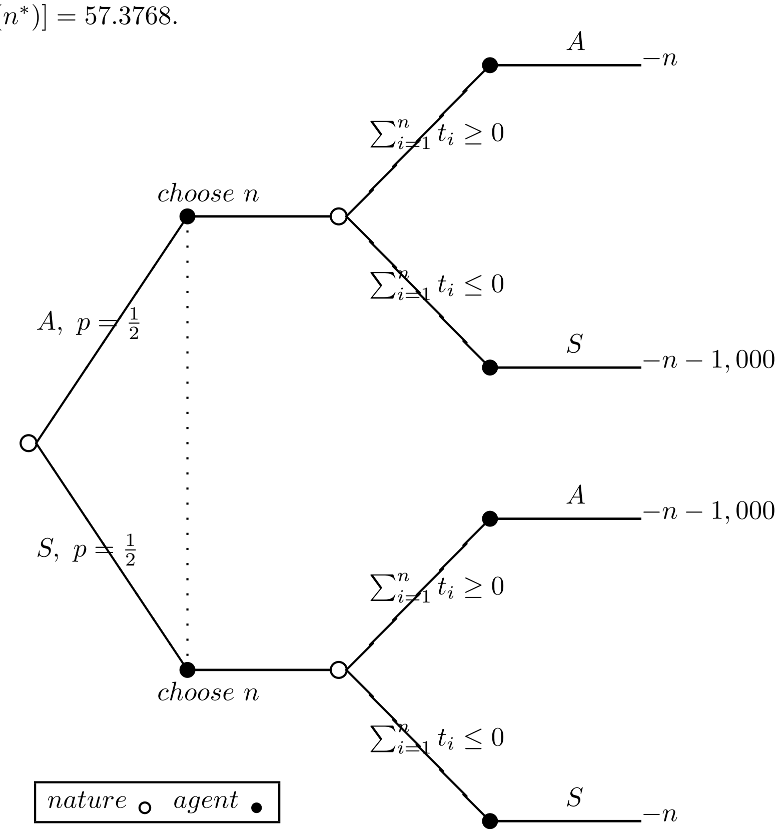


Figure 1: Decision Tree

(b) If $\Pr(c=S)=0.2$, and the loss is 400 if decide control is inadequate when it actually is adequate, then we have the signal pdf as

$$\begin{aligned}
f\left(t_1,t_2,\ldots,t_n\right) &= f\left(t_1,t_2,\ldots,t_n|c=A\right)\Pr\left(c=A\right)+f\left(t_1,t_2,\ldots,t_n|c=S\right)\Pr\left(c=S\right)\\
&= \frac{1}{3^n}\left[\frac{4}{5}\prod_{i=1}^n\phi\left(\frac{t_i-1}{3}\right)+\frac{1}{5}\prod_{i=}^n\phi\left(\frac{t_i+1}{3}\right)\right]
\end{aligned}$$

and posterior pdf are

$$\begin{aligned}
f\left(c=A|t_1,t_2,\ldots,t_n\right) &= \frac{f\left(t_1,t_2,\ldots,t_n|c=A\right)\Pr\left(c=A\right)}{f\left(t_1,t_2,\ldots,t_n\right)}\\
&= \frac{\frac{4}{5}\frac{1}{3^n}\prod_{i=1}^n\phi\left(\frac{t_i-1}{3}\right)}{\frac{1}{3^n}\left[\frac{4}{5}\prod_{i=1}^n\phi\left(\frac{t_i-1}{3}\right)+\frac{1}{5}\prod_{i=}^n\phi\left(\frac{t_i+1}{3}\right)\right]}\\
&= \frac{4}{4+\exp\left[-\frac{2}{9}\sum_{i=1}^n t_i\right]},
\end{aligned}$$

$$f\left(c=S|t_1,t_2,\ldots,t_n\right)=1-f\left(c=A|t_1,t_2,\ldots,t_n\right)=\frac{1}{1+4\exp\left[\frac{2}{9}\sum_{i=1}^n t_i\right]}.$$

Therefore, after observing the n test outcomes, the expected loss is $n + 1000 * f(c = S|t_1, t_2, \ldots, t_n)$ if deciding adequate, and $n + 400 * f(c = A|t_1, t_2, \ldots, t_n)$ if deciding substandard. And we can compute the threshold $\sum_{i=1}^{n} t_i = T$.

So the firm is to minimize its expected loss

$$\begin{aligned}\min_n E[L(n)] &= n + 400 \Pr\left(\sum_{i=1}^{n} t_i < T | c = A\right) \Pr(c = A) \\ &\quad + 1000 \Pr\left(\sum_{i=1}^{n} t_i > T | c = S\right) \Pr(c = S) \\ &= n + 320\Phi\left(\frac{T-n}{3\sqrt{n}}\right) + 200\left[1 - \Phi\left(\frac{T+n}{3\sqrt{n}}\right)\right],\end{aligned}$$

and the FOC is

$$1 - 160\phi\left(\frac{T-n}{3\sqrt{n}}\right)\frac{n+T}{3n^{3/2}} - 100\phi\left(\frac{T+n}{3\sqrt{n}}\right)\frac{n-T}{3n^{3/2}} = 0.$$

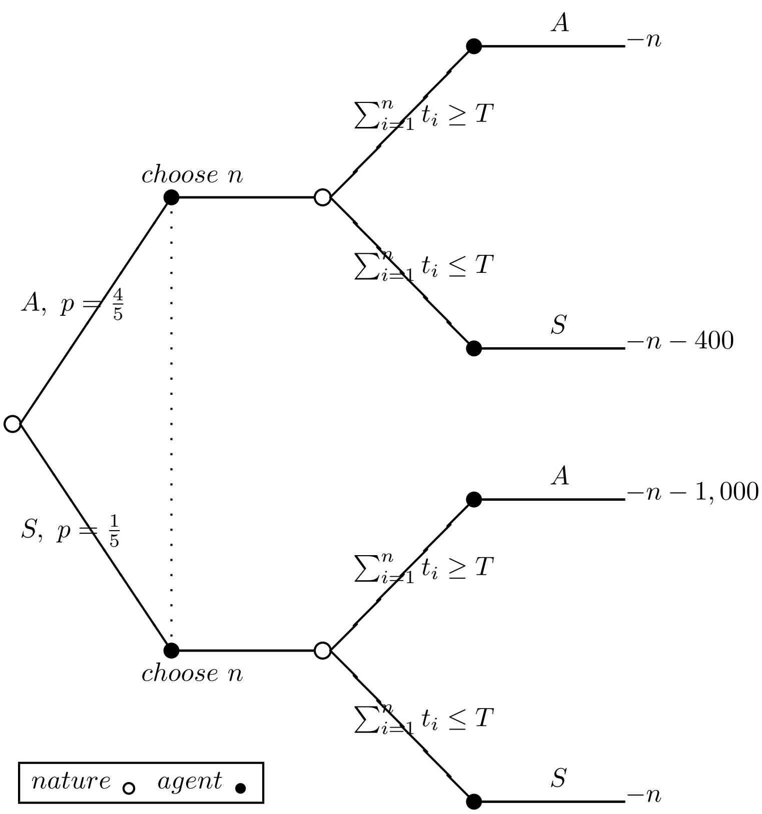


Figure 2: Decision Tree

(c) The optimal solution takes the form upper and lower thresholds for the test result. If the upper (lower) threshold is crossed, then say A (S), and if not crossed, then take another test sample. See DeGroot for an explanation of why this is true and how to compute the thresholds. □

CAN KADIRGAN

$\boxed{\text{Question 4:}}$

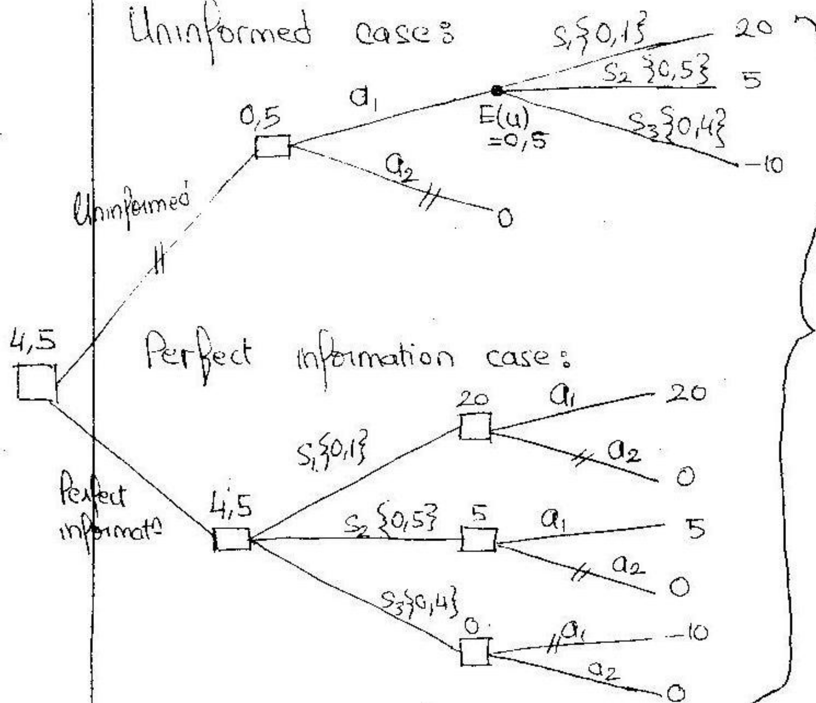
Decision to hire : $\{a_1, a_2\}$

a_1 → hire, a_2 → don't hire

a) States: $\{s_1, s_2, s_3\}$ where $s_1 \rightarrow$ great salesman

$s_2 \rightarrow$ good "

$s_3 \rightarrow$ poor "



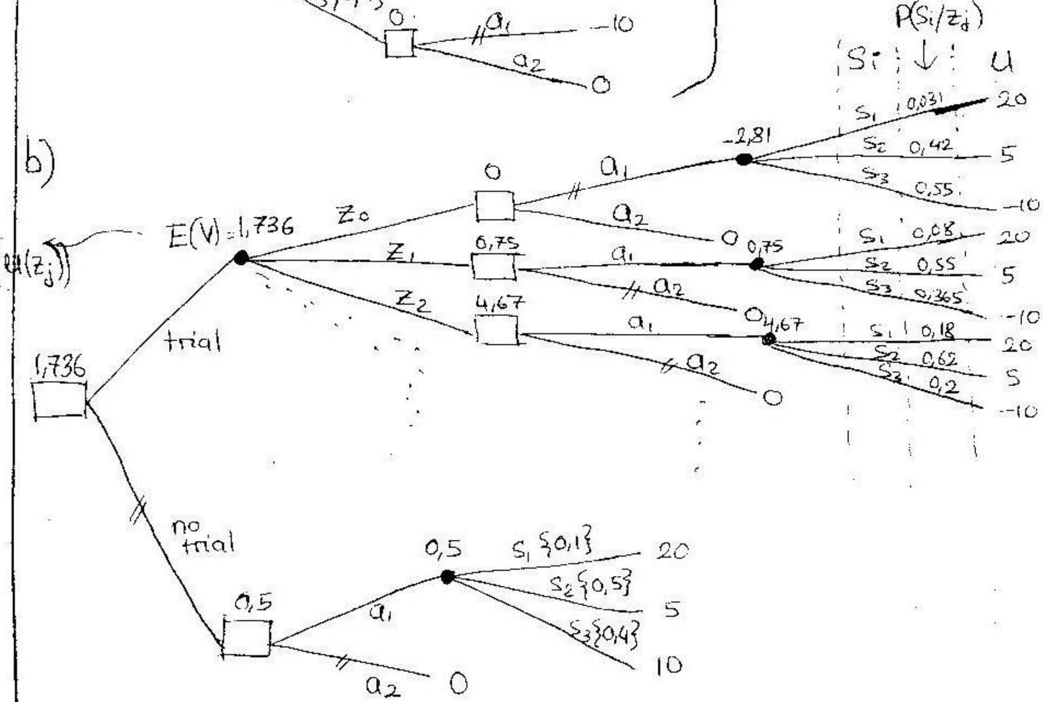
Gross Value of Perfect Inform.

= V(Perfect inf.) − V(Uninformed)

$\Rightarrow \boxed{V(I) = 4,5 - 0,5 = 4}$

b)

$$E(V) = \sum_j P(z_j)\, E[u(z_j)] = 1,736$$



Results: Gross $E(V(I)) = 1{,}736 - 0{,}5 = 1{,}236$

Net $E(V(I))$ = "Gross $E(V(I))$" − cost $= 1{,}236 - 0{,}04 = 1{,}196$

Explanations for excel sheet attached to 4(b):

$$P(s_i / z_j) = \frac{P(s_i, z_j)}{P(z_j)} = \frac{P(z_j | s_i).\,P(s_i)}{\sum_{s_i \in S} P(z, s_i)}$$

→ We obtain those from Poisson Probabilities

→ We calculate this from $P(z_j, s_i) = \frac{P(z_j|s_i)}{P(s_i)}$

c) Same methodology as in (b) except;

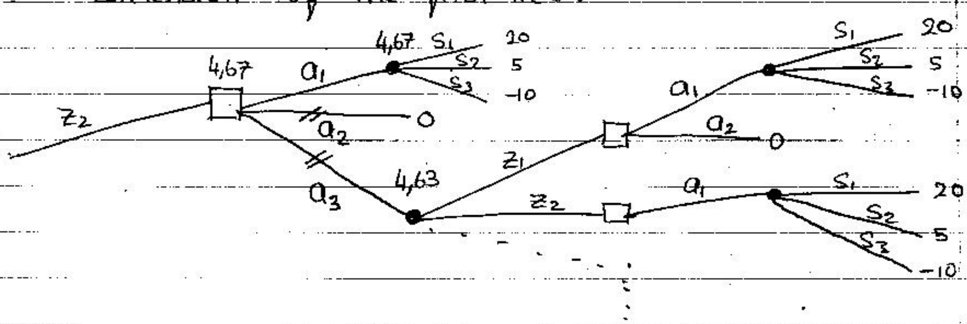
old posterior = new prior

⇒ We replace $P(s_i)$ by $P(s_i/z_2)$ where

$P(s_i/z_2) = \{0{,}181; 0{,}616; 0{,}203\}$

a_3 → hire for another week for second trial

Extension of the first tree:



Conclusion: If the owner tries the salesman an extra week after him selling 2 cars the 1^{st} week; the expected utility (payoff) will be less than hiring him after the 1^{st} week ($4{,}63 < 4{,}67$).

⇒ The owner should decide "a_2" (hire the salesman directly after first week of trial).

t=4		λ	0.5			λ	0.25			λ	0.125		
z	P(z)	z	$P(z/s_1)$	$P(s_1)$	$P(z,s_1)$	z	$P(z/s_2)$	$P(s_2)$	$P(z,s_2)$	z	$P(z/s_3)$	$P(s_3)$	$P(z,s_3)$
0	0.3742	0	0.1353	0.181	0.0245	0	0.36788	0.616	0.2266	0	0.60653	0.203	0.12313
1	0.3372	1	0.2707	0.181	0.04899	1	0.36788	0.616	0.2266	1	0.30327	0.203	0.06156
2	0.1777	2	0.2707	0.181	0.04899	2	0.18394	0.616	0.1133	2	0.07582	0.203	0.01539
3	0.073	3	0.1804	0.181	0.03266	3	0.06131	0.616	0.0378	3	0.01264	0.203	0.00257
4	0.0261	4	0.0902	0.181	0.01633	4	0.01533	0.616	0.0094	4	0.00158	0.203	0.00032
5	0.0085	5	0.0361	0.181	0.00653	5	0.00307	0.616	0.0019	5	0.00016	0.203	3.2E-05
6	0.0025	6	0.012	0.181	0.00218	6	0.00051	0.616	0.0003	6	1.3E-05	0.203	2.7E-06
7	0.0007	7	0.0034	0.181	0.00062	7	7.3E-05	0.616	4E-05	7	9.4E-07	0.203	1.9E-07
8	0.0002	8	0.0009	0.181	0.00016	8	9.1E-06	0.616	6E-06	8	5.9E-08	0.203	1.2E-08
9	4E-05	9	0.0002	0.181	3.5E-05	9	1E-06	0.616	6E-07	9	3.3E-09	0.203	6.6E-10
10	7E-06	10	4E-05	0.181	6.9E-06	10	1E-07	0.616	6E-08	10	1.6E-10	0.203	3.3E-11
Sum	1												

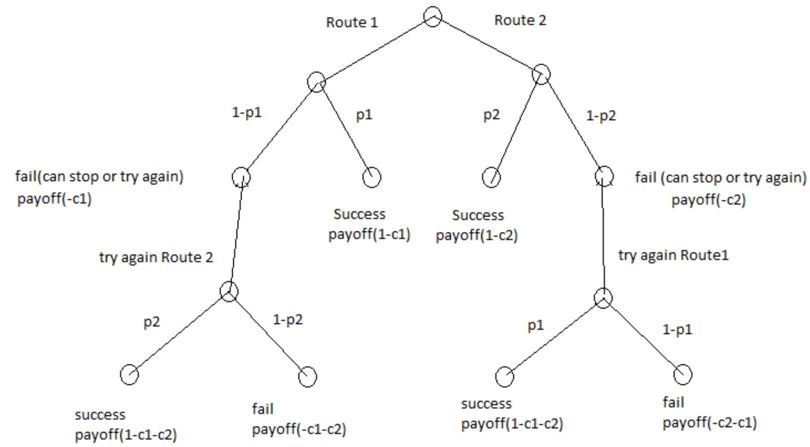
$u(s_1)$	$u(s_2)$	$u(s_3)$
20	5	-10

z	$P(s_1/z)$	$P(s_2/z)$	$P(s_3/z)$	$E(u(z,a_1))$	$E(u(z,a_2))$	E(u(z))	P(z)E(u(z))	ΣP(z)E(u(z))
0	0.0655	0.6055	0.329	1.046735373	0	1.046735373	0.391725165	4.67
1	0.1453	0.6721	0.1826	4.440716918	0	4.440716918	1.49726751	
2	0.2757	0.6377	0.0866	7.836472605	0	7.836472605	1.392454635	
3	0.4474	0.5174	0.0351	11.18449205	0	11.18449205	0.816411888	
4	0.6258	0.3619	0.0123	14.20339432	0	14.20339432	0.370613946	
5	0.7728	0.2234	0.0038	16.53499418	0	16.53499418	0.139765259	
6	0.8728	0.1262	0.0011	18.07549111	0	18.07549111	0.045094873	
7	0.9323	0.0674	0.0003	18.98066022	0	18.98066022	0.01266516	
8	0.9651	0.0349	7E-05	19.47466299	0	19.47466299	0.003138546	
9	0.9822	0.0177	2E-05	19.73321991	0	19.73321991	0.000694352	
10	0.991	0.009	5E-06	19.86555628	0	19.86555628	0.000138559	

ΣP(z)E(u(z))	Cost
4.67	0.04

E(V) with second trial

4.63



#5

<u>Road 1 payoff:</u>

to decide whether to stop or continue:

EV(continue)=p2(1-c1-c2)+(1-p2)(-c1-c2)=p2-c1-c2

EV(stop)=-c1

if EV(continue) > EV(stop) continue else stop:

if p2-c1-c2 > -c1 implies continue only if p2>c2

if c2>p2 then stop

if (p2>c2)

EV(Road1) = p1(1-c1) +(1-p1)(p2-c1-c2)

=p1+p2-c1 c2-p1*p2 +c2*p1

if (c2>p2)

EV(Road1)=p1(1-c1) + (1-p1)(-c1)

<u>Road 2 payoff:</u>

to decide whether to stop or continue:

EV(cotinue)=p1(1-c2-c1)+(1-p1)(-c2-c1)=p1-c1-c2

EV (stop)=-c2

if EV(continue) > EV(stop) continue else stop:

if p1-c1-c2>-c2 implies continue only if p1>c1

if c1>p1 then stop

if (p2>c2)

EV(Road2) = p2(1-c2)+(1-p2) (p1-c1-c2)

=p1 + p2 -c1 -c2-p1*p2+c1*p2

if (c2>p2)

EV(Road2)=p2(1-c2) + (1-p2)(-c2)

<u>4 cases:</u>

if: p1>c1 & p2>c2

EV(Road1)=p1+p2-c1-c2-p1*p2+c2*p1

Ev(Road2)=p1+p2-c1-c2-p1*p2+c2*p1

if EV(Road1) > EV(Road2) take Road 1 else Road 2

EV(Road1)> EV(Road2) if **p1/c1 > p2/c2** then Road 1 else Road 2

if c1>p1 & c2>p2

EV(Road1)=p1(1-c1)+(1-p1)(-c1)=p1-c1

Ev(Road2)=p2-c2

Both EV are negative so **STAY HOME**

if p1>c1 & c2>p2

EV(Road1)=p1-c1

EV(Road2)=p1+p2-c1-c2-p1*p2+c1*p2

EV(Road1) > EV(Road2) ***(p2-c2-p1*p2+c1*p2 <0)

<u>So Road 1 only</u>

if c1>p1 & p2>c2

EV(Road1)=p1+p2-c1 c2-p1*p2 +c2*p1

Ev(Road2)=p2-c2

EV(Road2) > EV(Road1) ***(p1-c1-p2*p2+p1 <0)

<u>So Road 2 only</u>

For K=2: if cost(road)>prob(road) no road; else sort prob(road)/cost(road) and try highest to lowest ratio. As long as prob/cost >1

To prove for K>2: Do it for K=3, K=4 show its same as K=2. Then by induction implies same strategy as K=2. Show if P(roadX)/Cost(roadX) > P(roadY)/Cost(roadY), go with road X

Problem (6)**.** The prior probability that the rival has new tech is 0.1, their reported productivity is normally distributed with variance 1.0 and mean 10 (12) with the old (new) tech.

Answer. So we have the prior $\Pr(tech = new) = 0.1$, and the likelihood pdf are

$$f(prod = x | tech = new) = \phi(x - 12) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x-12)^2}{2}\right], \text{ and}$$

$$f(prod = x | tech = old) = \phi(x - 10) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x-10)^2}{2}\right],$$

where $prod = x$ is the reported productivity. Therefore the posterior pdf are

$$f(tech = new | prod = x) = \frac{\frac{1}{10}\phi(x-12)}{\frac{1}{10}\phi(x-12) + \frac{9}{10}\phi(x-10)} = \frac{1}{1 + 9\exp[-2x + 22]}$$

$$f(tech = old | prod = x) = \frac{\frac{9}{10}\phi(x-10)}{\frac{1}{10}\phi(x-12) + \frac{9}{10}\phi(x-10)} = \frac{9}{9 + \exp[2x - 22]}.$$

So $f(tech = new | prod = x) > f(tech = old | prod = x)$ if and only if the reported productivity $x > \ln 3 + 11 \approx 12.0986$. $\square$

$\boxed{6.B.4}$

a) set $u_A = 1$ and $u_D = 0$. Then $u_B = p \cdot 1 + (1-p) \cdot 0 = p$

$u_C = q \cdot 1 + (1-q) \cdot 0 = q$

b)

Criterion 1	2
$P_A = .99 \cdot .9 = .891$	$P_A = .8415$
$P_B = .99 \cdot .01 = .099$	$P_B = .1485$
$P_C = .9 \cdot .01 = .009$	$P_C = .0095$
$P_D = .1 \cdot .01 = .001$	$P_D = .0005$

$EU_1 = .891 + .099 \cdot p + .009 \cdot q + .001 \cdot 0$

$EU_2 = .8415 + .1485p + .0095q + .0005 \cdot 0$

Prefer (1) to (2) if:

$$.099p + .009q + .0495 > .1485p + .0095q$$

$$\boxed{99 > 99p + q}$$

$\boxed{6.C.1}$

We have $q > \pi$. Then:

$$\max\ (1-\pi)u(w-\alpha q) + \pi u(w - \alpha q - D + \alpha)$$

$$\text{FOC: } -q(1-\pi)u'(w-\alpha q) + \pi u'(w - \alpha(1-q) - D) \leq 0$$

If $\alpha = D \Rightarrow -q(1-\pi)u'(w-Dq) + \pi u'(w-Dq) \leq 0$

$$= u'(w-Dq)(\pi - q) \leq 0$$

$\Rightarrow$ This cannot be true if $q > \pi$.

$\boxed{6.C.2}$ $u(x) = \beta x^2 + \gamma x$. Say the distribution is $F(x)$

then:

a) $$\int u(x)\, dF(x) = \int [\beta^2 x^2 + \gamma x]\, dF(x) = \beta \underbrace{\int x^2 dF(x)}_{= \text{mean}^2 \text{ of } F(x) + \text{var } F(x)} + \gamma \underbrace{\int x F(x)}_{\text{mean of } F(x)}$$

b) $u(F) = \text{mean of } F - r \cdot \text{var. of } F \quad r > 0$

Proof

As before, $u(F) = \int u(x)\, dF(x)$

Say $u(x) = x$

then $u(F) = \int x\, dF(x)$. This means that $r \cdot \text{var of } F = 0$

This implies risk neutrality, which is contradictory to the inclusion of the variance in $U(F)$.

Therefore, $U(F)$ is not compatible with any Bernoulli utility function

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18. Individual bernoulli utility function $u(x) = \sqrt{x}$.

    a.) Arrow-Pratt coefficients of absolute r_A and relative risk aversion r_R at the level of wealth $w = 5$. Applying definiton 6.C.3 and 6.C.5

    $$r_A(x) = \frac{0.25x^{-3/2}}{0.5x^{-1/2}}$$

    and

    $$r_R(x) = \frac{5 \times 0.25x^{-3/2}}{0.5x^{-1/2}}$$

    evaluated at $x = 5$,

    $$r_A(5) = 0.1$$
    $$r_R(5) = 0.5$$

    b.) Certainty equivalent $c(F, u)$ and probability premium $\pi(x, \epsilon, u)$ for gamle $(16, 4; \frac{1}{2}, \frac{1}{2})$. Applying the formula 6.C.3 and 6.C.4

    $$u(c(F, u)) = \Sigma u(x)p(x)$$
    $$\sqrt{c(F, u)} = \frac{1}{2}\sqrt{4} + \frac{1}{2}\sqrt{16}$$

    Solve for $c(F, u)$, we have $c(F, u) = 9$

    $$u(x) = (\frac{1}{2} + \pi(x, \epsilon, u))u(x + \epsilon) + (\frac{1}{2} - \pi(x, \epsilon, u))u(x - \epsilon)$$
    $$\sqrt{10} = (\frac{1}{2} + \pi(x, \epsilon, u))\sqrt{16} + (\frac{1}{2} - \pi(x, \epsilon, u))\sqrt{4}$$

    Solve for $\pi(x, \epsilon, u)$, we get $\pi(x, \epsilon, u) = 0.0811$

    c.) Certainty equivalent $c(F, u)$ and probability premium $\pi(x, \epsilon, u)$ for gamle $(36, 16; \frac{1}{2}, \frac{1}{2})$. Applying the formula 6.C.3 and 6.C.4

    $$\sqrt{c(F, u)} = \frac{1}{2}\sqrt{16} + \frac{1}{2}\sqrt{36}$$

    Solve for $c(F, u)$, we have $c(F, u) = 25$

    $$\sqrt{26} = (\frac{1}{2} + \pi(x, \epsilon, u))\sqrt{36} + (\frac{1}{2} - \pi(x, \epsilon, u))\sqrt{16}$$

    Solve for $\pi(x, \epsilon, u)$, we get $\pi(x, \epsilon, u) = 0.045$

19. A lottery L over monetary outcomes that pays

    $$L = \begin{cases} x + \epsilon & \text{w.p } \frac{1}{2} \\ x - \epsilon & \text{w.p } \frac{1}{2} \end{cases}$$

    Compute

    $$\frac{\partial^2 c(F, u)}{\partial \epsilon^2}$$

    show that

    $$\lim_{\epsilon \to 0} \frac{\partial^2 c(F, u)}{\partial \epsilon^2} = -r_A(x) = \frac{u''}{u(x)}$$

$$u(c(F,u)) = \frac{1}{2}u(x+\epsilon) + \frac{1}{2}u(x-\epsilon) \qquad \text{by defn} \tag{1}$$

$$u'(c(F,u)) = \frac{1}{2}u'(x+\epsilon) - \frac{1}{2}u'(x-\epsilon) \qquad \text{by Implicit funct theorem} \tag{2}$$

$$\frac{\partial c(F,u)}{\partial \epsilon} = \frac{1}{2}\frac{u'(x+\epsilon)}{u'(c(F,u))} - \frac{1}{2}\frac{u'(x-\epsilon)}{u'(c(F,u))} \tag{3}$$

$$\frac{\partial^2 c(F,u)}{\partial \epsilon^2} = \frac{1}{2}\frac{u''(x+\epsilon)}{u'(c(F,u))} + \frac{1}{2}\frac{u''(x-\epsilon)}{u'(c(F,u))} \tag{4}$$

$$\lim_{\epsilon \to 0} \frac{\partial^2 c(F,u)}{\partial \epsilon^2} = -r_A(x) = \frac{u''}{u(x)} \tag{5}$$