Formulas for Econ 200, Fall 2015, UCSC

Equations for Competitive Markets

Linear Demand: $q_d = a - bp$ **Linear Supply**: $q_s = x + yp$
Log-linear demand: $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$ **Log-Linear Supply**: $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$
Total Surplus=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost
Total Cost=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost
Quantity Tax (tax per unit): $p_d = p_s + t$; **Value Tax** (tax on percentage spent): $p_d = (1+t)p_s$
Price Elasticity of Demand: $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p}\frac{p}{D}$; If $|\varepsilon| > 1$ then curve is elastic
Tax Incidence Formula: $p_s(t) = p^* - \frac{t|D'|}{S'+|D'|}$; $p_d = p^* + \frac{tS'}{S'+|D'|}$; If ε_d is constant: $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d|+\varepsilon_s}$

Equations for Consumer Choice and Demand

Marginal Utility: $MU_i = \frac{\partial u}{\partial x_i}$; **Marginal Rate of Substitution**: $MRS_{ij} = \frac{MU_j}{MU_i}$ and at interior optimum $= \frac{p_i}{p_j}$
Perfect Substitutes: $u(x_1, x_2) = x_1 + cx_2$; **Cobb-Douglas**: $u(x_1, x_2) = \ln(x_1) + c\ln(x_2)$
CES Utility: $u(x_1, x_2) = \frac{1}{\rho}\ln(x_1^\rho + x_2^\rho)$; $\rho \in (-\infty, 1]$; **Quasilinear**: $u(x_0, x_1) = x_0 + g(x_1)$
Marshallian Demand: $x_i^*(\mathbf{p}, m) : \mathcal{L} = u(x_1, x_2) + \lambda(m - \mathbf{p}\cdot \mathbf{x})$
Dual Problem; Hicksian Demand: $h_i^*(\mathbf{p}, u_0) : \min_x \mathbf{p}\cdot \mathbf{x}$ s.t. $u(\mathbf{x}) \geq u_0$
Roy's Identity: $x_i^*(\mathbf{p}, m) = -\frac{\partial v}{\partial p_i} / \frac{\partial v}{\partial m}$; **Shepard's Lemma**: $h_i^*(\mathbf{p}, u) = \frac{\partial e(\mathbf{p},u)}{\partial p_i}$
Slutsky Equation: $\frac{\partial x_i(\mathbf{p},m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p},v(\mathbf{p},m))}{\partial p_i} - \frac{\partial x_i^*}{\partial m}x_i^*(\mathbf{p}, m)$; (**Elasticity Form**): $\varepsilon_i = \varepsilon_i^h - s_i\varepsilon_m$; $s_i = \frac{p_i x_i}{m}$
Demand Elasticity for product i, homogeneous of degree 0: $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j\neq i}\varepsilon_{ij} = 0$

Equations for Cost and Technology

Technical Rate of Substitution: $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j}$; **MC**: $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$ **MC to VC**: $\int MC = VC$
Factor Prices: $\mathbf{w} = (w_1, w_2, ..., w_n)$; **Production Function**: $y = f(x_1, y_2)$
Cost Function with two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$
$= \min_{x_1, x_2 \geq 0} w_1x_1 + w_2x_2$ s.t. $y = f(x_1, x_2)$
Shepard's Lemma conditional factor demand: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w},y)}{\partial w_i}$
Learning Curve: The typical specification is for $Y_t = \Sigma_{s\leq t}y_s$, AC falls proportionately, $\ln AC_t = AC_0 - b\ln Y_t$

Equations for Competitive Firms

SR Profit Maximization: $\max_{y, x_v \geq 0} \pi = \max_{y\geq 0}[\max_{x_v\geq 0} R(y) - w_v x_v - w_f x_f$ s.t. $y = f(x_v, \bar{x}_f)] = \max_{y\geq 0}[R(y) - c(y)]$
Revenue if firm is competitive: $R(y) = py = pf(x_v, \bar{x}_f)$ **FOC of *un*conditional factor demand**: $p\frac{\partial f(x_v, \bar{x}_f)}{\partial x_v} = w_v$
Hotelling's Lemma, Supply: $y^*(p, \mathbf{w}) = \frac{\partial \pi(p,\mathbf{w})}{\partial p}$; unconditional factor demands: $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p,\mathbf{w})}{\partial w_i}$
Shutdown Condition (Competitive Firms): $-F > py - c_v(y) - F \implies AVC = \frac{c_v(y)}{y} > p$

Equations for Monopolies

FOC for a monopolist: $p(y) + p'(y)y = c'(y)$ which can be rewritten as $p = \frac{1}{1+\frac{1}{\varepsilon}}MC$; valid if $\varepsilon < -1$

Passing Along Costs: $\frac{\partial p}{\partial c} = \frac{1}{2+yp''(y)/p'(y)}$
Price Discrimination. Third Degree: Monopolist's Problem: max $p_1(x_1)x_1 - cx_1 + p_2(x_2)x_2 - cx_2$
$FOC_{x1} : c = p_1(x_1)[1 - \frac{1}{|\epsilon_1}]$ Markup factor:$M_i = \frac{1}{1-\frac{1}{\epsilon_i}} = \frac{|\epsilon_i|}{|\epsilon_i|-1}$

Quasilinear utility: max $u_i(x) + y$ s.t. $px + y = m$; FOC (inverse demand curve): $p = u_i'(x)$
Decision Theory. Probability Identities: $P(A|B) = \frac{P(A \cap B)}{P(B)}$; $P(A \cap B) = P(B \cap A)$; Given probability sets A,B,C,D:
$P(A) = P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D)$
Bayes Theorem: $P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$ and given that A is a binary variable $\frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\neg A)P(\neg A)}$
Also: $\frac{p(s|m)}{p(t|m)} = \left[\frac{p(m|s)}{p(m|t)}\right]\left[\frac{p(s)}{p(t)}\right]$. Can also take logs to get linear expression.

Cournot. Given $D(Y) = a - bY$. $BR_i(Y_{-i}) = \underset{y_i}{\text{argmax}}\ \pi_i = P(\sum_{i=1}^{n} y_i)y_i - c(y_i) \implies P(\sum_{i=1}^{n} y_i) + P'(\sum_{i=1}^{n} y_i)y_i - MC_i(y_i) = 0$
To solve for the Nash equilbrium, we want to find where the Best Response functions intersect. $\Rightarrow NE_{Cournot} : Y^* = \frac{N}{(N+1)b}(a-c) \implies y_i^* = \frac{Y*}{N} = \frac{a-c}{(N+1)b}$
Stackelberg. argmax $D(Y) = a - bY \to BR_L = \max_{y_L} \pi_L(y_L, BR_F(y_L)) = D(y_L + BR_F)y_L - cy_L$

Intertemporal Choice. Given $U(c_0, c_1)$, we have$\frac{\partial_0 U}{\partial_1 U} = \frac{\frac{\partial U}{\partial c_0}}{\frac{\partial U}{\partial c_1}} = MRS_{01} = 1 + MRTP$

Given Initial Endowment $E = (e_0, e_1)$ and intertemp prod function $y = f(x)$,
the PPF is $\{(q_0, q_1) : q_0 = e_0 - x \geq 0, q_1 = e_1 + f(x) \geq 0\}$
ROI= $f(x) - x$; AROI= $\frac{f(x)}{x} - 1$; MROI= $f'(x) - 1$
Present Value: $PV_r(C) = c_0 + \frac{c_1}{1+r}$
Agent Optim: $\max_x w = PV(Q) = q_0 + \frac{q_1}{1+r} = e_0 - x + \frac{e_1+f(x)}{1+r}$ FOC: $1 + r = f'(x) = 1 + MROI \implies r = MROI$
Optimal individual borrowing, consumption and lending: $\max_{c_0,c_1 \leq 0} U(c_0, c_1)$ s.t. $PV_r(Q) = PV_r(C) = w$,
given $c_0 = q_0 + b$ and $c_1 = q_1 - (1 + r)b \Rightarrow \max_b U(q_0 + b, q_1 - (1 + r)b)$
Fisher's Equation: $k \approx r + \pi$ (obtained from $1 + k = (1 + r)(1 + \pi)$)
Present Value of discrete cash stream $X = (x_0, x_1, ..., x_T)$ or $[x(t) : t \in [0, T]]$

$$PV_k(X) = \sum_{t=0}^{T} \frac{X_t}{(1+k)^t} \text{ or } PV_k(X) = \int_{t=0}^{T} x_t e^{-kt} dt$$

General formula for interest rates and asset yields: $k_a = r^* + \pi^e + RP_a \pm T_a$,
where T_a = Transaction Costs and RP_a = Risk Premium for a specific asset a.

Risky Choice. Given a lottery with monetary outcomes $m_1, ..., m_n$ and corresponding probabilities $p_1, ..., p_n$,
its **expected value** is $Em = \sum_i p_i m_i$ and its **variance** is Var $m = E(m - Em)^2 = \sum_i p_i(m_i - Em)^2$.

Given **Bernoulli function** $u(m)$ — so $u' > 0$ and, if the person is risk-averse, $u'' < 0$ —
the **certainty equivalent** m^{CE} to the lottery solves $u(m^{CE}) = Eu(m) = \sum_i p_i u(m_i)$.
The **coefficient of absolute risk aversion** is $a(m) = -u''(m)/u'(m)$ and
the **coefficient of relative risk aversion** is $r(m) = ma(m)$.
The **risk premium** is RP = $Em - m^{CE}$. It is also given by the second term of the Taylor expansion of u around Em.