Answer Key for Final Exam Practice Problems

Econ 200, Fall 2015

1. Demand in an industry can reasonably be approximated by $y = 400 - 5p$. A single firm has access to technology summarized in the cost function $c = 5(w_1 w_2)^{1/2} y$ and faces input prices $w_1 = 4$ and $w_2 = 16$.

(a) (2 points) Are returns to scale constant, increasing or decreasing?

> **Solution:** Since costs scale directly with output y, returns to scale must be constant.

(b) (6 points) Compute our firm's total cost and marginal cost of producing any given level of output $y > 0$.

> **Solution:** Total costs are given by the cost function: $c(y) = 5(4 \cdot 16)^{1/2} y = 40y$. Marginal costs are $c'(y) = 40$.

(c) (8 points) Compute our firm's profit-maximizing level of output y^*, and its corresponding profit π^*. What is the corresponding price and consumer surplus?

> **Solution:** The firm's profit-maximizing condition is MR=MC. Since inverse demand is $p = 80 - \frac{y}{5}$, marginal revenue is $80 - \frac{2y}{5}$. Therefore, the firm maximizes output when $80 - \frac{2y}{5} = 40 \Rightarrow y^* = 100$ and $p^* = 60$. Corresponding profit is $\pi^* = py - c = 60 \cdot 100 - 40 \cdot 100 = 2000$. Consumer surplus is $\frac{1}{2}(100)(80-60) = 1000$.

(d) (4 points) So far, all quantities are annual in millions of units. If the annual interest rate is $k = 0.10$, what is the fundamental value of the firm?

> **Solution:** The fundamental value of the firm is the present value of current and future profits. This is the sum of the infinite geometric series (in billions):
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> $$2 + \frac{2}{1.1} + \frac{2}{1.1^2} + \ldots = 2\left(\frac{1}{1 - \frac{1}{1.1}}\right) = \$22 \text{ billion}$$

(e) (6 points) Suppose that pending legislation would allow many other firms gain access to the same technology as our firm. Predict the resulting price and our firm's profit.

> **Solution:** With free entry, prices should fall to the competitive equilibrium level where price is equal to marginal cost, 40. At this price, profits are zero.

(f) (2 points) What is the maximum amount our firm would pay to permanently defeat that legislation? Explain your reasoning very clearly.

> **Solution:** The legislation would decrease the value of the firm from \$22 billion to \$0 (since profits would be zero each period). Therefore, the firm would be willing to pay up to the difference, \$22 billion.

2. Two weeks ago the business press reported major declines in telecom stocks triggered by Sprint's large price cut for wireless data plan. The other two large firms in this industry are ATT and Verizon.

(a) (5 points) Use the standard static Bertrand model to predict the result of direct price competition in this industry.

> **Solution:** The standard static Bertrand model predicts that the price would drop to just below the second-lowest marginal cost among the three firms and the firm with the lowest marginal cost would take the whole market. In the real world, the data plans are not perfect substitutes so we would expect we would not expect the price to fall as sharply or for one firm to take the whole market.

(b) (2 points) Use anything covered in this course (plus your economic intuition) to connect the static (one period) result to stock price.

> **Solution:** One theory of finance commonly used as a benchmark says that stock prices reflect the present value of firms' net cash flows. Applied to this problem, lower prices would reduce current and future net cash flows resulting in lower stock prices. Of course, this need not be the outcome. For example, it may be the case that one of the three firms can successfully capture the entire market and end up increasing its net cash flows.

3. Your eccentric uncle left you a locked treasure chest, and you believe that the contents are worth about \$2000. Despite its unusually sturdy design, a locksmith offers to open the chest for \$100 (nonrefundable), and you estimate an 80% probability of success. Your friend offers to open it for \$20 (for supplies, also not refundable) and you estimate a 10% probability of success.

(a) (8 points) What is the expected payoff if you try your friend first, then (if he doesn't succeed) the locksmith? Be sure to mention any sensible assumptions you need to make to solve the decision tree.

> **Solution:** We'll assume conditional independence. That is, the success probability of each method is independent of the order in which it appears (2 points). Therefore,
>
> $$E(\text{payoff of trying friend first}) = 0.1(2000 - 20) + 0.9E(\text{payoff of trying locksmith})$$

Note that if we did not assume conditional independence, then the expectation on the right-hand side would be $E(\text{payoff of trying locksmith}|\text{friend failed})$. In any case,

$$E(\text{payoff of trying friend first}) = 0.1 \cdot 1980 + 0.9 \cdot 1480 = \$1530$$

since

$$E(\text{payoff of trying locksmith}) = 0.8(2000 - 20 - 100) + 0.2(-20 - 100) = \$1480$$

(b) (6 points) What is the expected payoff if you try the locksmith first, then (if she doesn't succeed) your friend?

Solution: Similar to the previous part,

$$\begin{aligned} E(\text{payoff of trying locksmith first}) &= 0.8(2000 - 100) + 0.2E(\text{payoff of trying friend}) \\ &= 0.8 \cdot 1900 + 0.2 \cdot 80 = \$1536 \end{aligned}$$

since

$$E(\text{payoff of trying friend}) = 0.1(2000 - 100 - 20) + 0.9(-100 - 20) = \$80$$

(c) (Extra credit) If time permits, write the optimal rule for sequencing methods $i = 1, \ldots, nn$ each with its own cost c_i and success probability p_i, of achieving a goal of value V.

Solution: The rule is to try methods in order of descending p_i/c_i, stopping when successful or when $p_i V < c_i$, whichever comes first.

4. Owners of a local movie theater hired you to advise on pricing. They now price general admission tickets at \$10, and feel that works well for them. You estimate own price elasticity at -2.5.

(a) (6 points) What marginal cost level would rationalize the current price?

Solution: Using the price mark-up formula,

$$\text{MC} = p(1 + \frac{1}{\varepsilon}) = 10(1 + \frac{1}{-2.5}) = \$6$$

(b) (6 points) The theater owners ask you to recommend a discount price for students, and you estimate students' own price elasticity at -4. What do you recommend?

Solution:

$$p = \text{MC}(\frac{1}{1+\frac{1}{\varepsilon}}) = 6(\frac{1}{1+\frac{1}{-4}}) = \$8$$

(c) (4 points) Suppose you found the same student and general public elasticities for cameras. Would you recommend the same percentage student discount? Explain briefly. (Hint: what are the main constraints on price discrimination?)

Solution: One of the main constraints on price discrimination is that arbitrage cannot be feasible. This may be true for movie tickets since they are difficult to resell, but considerably less true for cameras.

5. (8 points) Your demand study for a local restaurant chain yields preliminary estimates of own price elasticity = -2.0 and income elasticity = 0.8. Now you want to include related goods in the estimating equation. Do you expect to find mainly substitutes or complements? What total value can you expect for cross price elasticities?

Solution: Recall that for any good i,

$$\varepsilon_{i,m} + \varepsilon_{i,p_i} + \sum_{j \neq i} \varepsilon_{i,p_j} = 0$$

where $\varepsilon_{i,m}$ is the income elasticity, ε_{i,p_i} is the own-price elasticity, and the ε_{i,p_j}'s ($j \neq i$) are the cross-price elasticities. Because of this relationship, the cross-price elasticities must sum to $-(-2 + 0.8) = 1.2$. If prices for all related goods go up by 1%, then we expect demand for the local restaurant chain to go up as well. Therefore, we would expect to find mainly substitutes.

6. (20 points) In the late 20th century, "world" financial markets mainly comprised of North America, Japan and Western Europe - Developed for short. Then, around the beginning of the 21st century, world financial markets integrated China, India, Russia and Eastern Europe, Brazil, and a number of other economies - Emergent for shot. Suppose that thrift is higher in Emergent (i.e., MRTP is lower at any given point) than in Developed, and productivity (i.e., MROI) is about the same. Predict the impact of integration with Emerging on the real interest rate, current and future consumption, and wealth in Developed.

Solution: For simplicity, assume that Developed and Emergent have economies of the same size. Since they also have the same productivity, this implies that the two regions have identical PPF's. The only difference between the two regions is the difference in thrift. As the more thrifty region, Emergent values future consumption more highly relative to current consumption than Developed. Thus, in "autarky", the real interest rate is higher in Developed than in Emergent since agents in Developed need more inducement to save compared to their counterparts in Emergent.

When the two regions are integrated, the single resulting real interest rate must be in between the autarky rates, so the real interest rate must fall in Developed. This also implies that wealth increases in Developed (that is, the x-intercept of the line representing the real interest rates shifts out). It is unclear what happens to c_0^* and c_1^* relative to their autarky levels. (However, we know that by borrowing or lending at the integrated interest rate, agents in Developed are able to achieve a higher level of utility.)

7. Referring to an October 31, 2014 article in the Wall Street Journal, an academic commentator posed the following question. "Suppose the cost of majoring in economics at university X is \$200,000, which is paid at commencement. The annual income in the year following commencement, which is on May 15, is \$60,000 and then increases 5% per year for 10 years. The annual discount rate is 3%. Assume that the annual salary is paid on May 15 of each year. What is the present value at commencement of the earnings stream? What is the return on investment?

(a) (2 points) What significant opportunity costs of going to college are not mentioned here?

Solution: The most significant opportunity cost is probably the foregone earnings while attending school.

(b) (3 points) What is the present value (today) of \$200,000 paid 4 years from now if the interest rate is $k = 0.03$?

Solution:

$$\text{PV} = \frac{200000}{1.03^4} \approx \$178,000$$

(c) (2 points) If you had all the relevant data, how would you compute the economic value of majoring in economics at university X?

Solution: You would want to calculate $\text{PV}(Y - C)$, taking into account lifetime earnings as well as the cost of attending university, the foregone earnings while attending, and the opportunity cost of the lifetime earnings you could have received from the "next best" job you would have had without the degree.

Additional Problems

1. The short (one period) spot interest rate is $k_{01} = 0.035$, and the per-period long spot rate is $k_{02} = 0.045$; that is, depositors receive $(1 + k_{02})^2$ dollars two periods from now for each 1 dollar they deposit now. What is the forward rate k_{12}?

> **Solution:** No arbitrage condition requires that the return from two-period spot and one-period spot and one-period future from period 1 must be equal,
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> $$(1 + 0.045)^2 = (1 + 0.035)(1 + k_{12}). \tag{1}$$
>
> Thus, the forward rate $k_{12} = 0.0551$.

2. A person is willing to pay at most \$4 to play a lottery that pays \$10 with probability 0.5 and pay 0 otherwise.

(a) What is that person's certain equivalent? Risk premium?

> **Solution:** Certainty equivalent of the lottery is the amount of money for which the individual is indifferent between the lottery and the certain amount, thus it is \$4. Since the expected value of the lottery is \$5, the risk premium, which is defined as the difference between the two, is \$1.

(b) Assuming that the person has constant absolute risk aversion, what is their coefficient of risk aversion?

> **Solution:** If preference is constant absolute risk aversion, utility takes a form of $u(w) = 1 - e^{-aw}$. Since the person is indifferent between the lottery and \$4,
>
> $$\frac{1}{2}u(0) + \frac{1}{2}u(10) = 0 + \frac{1}{2} - \frac{1}{2}e^{-10a} = 1 - e^{-4a} = u(4). \tag{2}$$
>
> Thus, the coefficient of risk aversion is implicitly determined by,
>
> $$2e^{-4a} - e^{-10a} = 1. \tag{3}$$
>
> There are two solutions for this equation, $a_1 = 0$, and $a_2 \approx 0.08$. (Use a spreadsheet or other numerical methods to find a_2.) However, if $a = 0$, we do not have an increasing function, so this solution is bogus. Thus, the unique solution to this problem is $a \approx 0.0822$.

(c) Assuming that the Bernoulli function is such that $E[u(x)] = E(x) - (1/2)bVar(x)$, compute the parameter b for this person.

Solution: The expected value of the lottery is 5 and the Variance of the lottery is 25. The expected value of getting \$4 for sure is 4 and the variance is 0. And again, the person is indifferent between the two. Thus,

$$5 - \frac{1}{2}b(25) = 4, \tag{4}$$

and, $b = 2/25$.

(d) Extra credit. Find a utility function u that works in part c.

Solution: Quadratic utility function.