

# *Managerial Economics & Business Strategy*

## Chapter 10

### Game Theory: Inside Oligopoly

Revised 2/12 by DF



# Overview

- I. Introduction to Game Theory
- II. Simultaneous-Move, One-Shot Games
- III. Infinitely Repeated Games
- IV. Finitely Repeated Games
- V. Multistage Games

# Normal Form Game

- A Normal Form Game consists of:
  - Players, at least 2.
  - Strategies or feasible actions: at least 2 for each player.
  - Payoffs for each player, for each strategy combination.

# A Normal Form Game

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

# Normal Form Game: Scenario Analysis

- Suppose 1 thinks 2 will choose “A”.

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

# Normal Form Game: Scenario Analysis

- Then 1 should choose “a”.
  - Player 1’s best response to “A” is “a”.

Player 2

Player 1	Strategy	A	B	C
	a	12,11	11,12	14,13
	b	11,10	10,11	12,12
	c	10,15	10,13	13,14

# Normal Form Game: Scenario Analysis

- Suppose 1 thinks 2 will choose “B”.

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

# Normal Form Game: Scenario Analysis

- Then 1 should choose “a”.
  - Player 1’s best response to “B” is “a”.

Player 1	Player 2			
	Strategy	A	B	C
	a	12,11	11,12	14,13
	b	11,10	10,11	12,12
	c	10,15	10,13	13,14



# Normal Form Game

## Scenario Analysis

- Similarly, if 1 thinks 2 will choose C...
  - Player 1's best response to "C" is "a".

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

# Dominant Strategy

- Regardless of whether Player 2 chooses A, B, or C, Player 1 is better off choosing “a”!
- “a” is Player 1’s Dominant Strategy!

Player 2

Player 1	Strategy	A	B	C
	a	12,11	11,12	14,13
	b	11,10	10,11	12,12
	c	10,15	10,13	13,14

# Putting Yourself in your Rival's Shoes

- What should player 2 do?
  - 2 has no dominant strategy!
  - But 2 should reason that 1 will play “a”.
  - Therefore 2 should choose “C”.

		Player 2		
Player 1	Strategy	A	B	C
	a	12,11	11,12	14,13
	b	11,10	10,11	12,12
	c	10,15	10,13	13,14

# The Outcome

		Player 2		
Player 1	Strategy	A	B	C
	a	12,11	11,12	14,13
	b	11,10	10,11	12,12
	c	10,15	10,13	13,14

- This outcome is called a Nash equilibrium:
  - “a” is player 1’s best response to “C”.
  - “C” is player 2’s best response to “a”.

# Key Insights

- Look for dominant strategies.
- Put yourself in your rival's shoes.
- At Nash equilibrium, every player is best responding to the other players' strategies.

# A Market-Share Game

- Managers of two rival firms want to maximize market share.
- Strategies are pricing decisions.
- Simultaneous moves.
- One-shot game.
  - [Owners might prefer for them to maximize profits, but the managers are empire builders...]

# The Market-Share Game in Normal Form

Manager 2

Manager 1

Strategy	P=\$10	P=\$5	P = \$1
P=\$10	.5, .5	.2, .8	.1, .9
P=\$5	.8, .2	.5, .5	.2, .8
P=\$1	.9, .1	.8, .2	.5, .5

# Market-Share Game

## Equilibrium

		Manager 2		
Manager 1	Strategy	P=\$10	P=\$5	P = \$1
	P=\$10	.5, .5	.2, .8	.1, .9
	P=\$5	.8, .2	.5, .5	.2, .8
	P=\$1	.9, .1	.8, .2	.5, .5

**Nash Equilibrium**





# Comment

- Game theory can be used to analyze situations where “payoffs” are non monetary
  - The bar scene in “A Beautiful Mind” is a (bad) example
- We will usually focus on situations where businesses want to maximize profits.
  - Hence, payoffs are measured in monetary units.
  - Expected NPV in \$millions, say.

# Examples of Coordination Games

- Industry standards
  - size of floppy disks.
  - size of CDs.
  - Etc.
- National standards
  - electric current.
  - traffic laws.
  - Etc.

# A Coordination Game in Normal Form

Player 1	Player 2			
	Strategy	A	B	C
	1	0,0	0,0	\$10,\$10
	2	\$10,\$10	0,0	0,0
	3	0,0	\$10,\$10	0,0

# A Coordination Problem: Three Nash Equilibria!

Player 2

Player 1

Strategy	A	B	C
1	0,0	0,0	\$10,\$10
2	\$10,\$10	0,0	0,0
3	0,0	\$10, \$10	0,0

# Comments.

- Not all games are games of conflict.
- Communication can help solve coordination problems.
- Sequential moves can help solve coordination problems.
- We'll play some games in class that are mainly coordination and others that involve conflicts of interest.

# An Advertising Game

- Two firms (Kellogg's & General Mills) managers want to maximize profits.
- Strategies consist of advertising campaigns.
- Simultaneous moves.
  - One-shot interaction.
  - Repeated interaction.

# A One-Shot Advertising Game

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12, 12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

# Equilibrium to the One-Shot Advertising Game

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12, 12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

Nash Equilibrium





# Can collusion work if the game is repeated 2 times?

## General Mills

Kellogg's

Strategy	None	Moderate	High
None	12, 12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

# **No (by backwards induction).**

- In period 2, the game is one-shot, so High Advertising is the equilibrium in the last period.
- This means period 1 is “really” the last period, since everyone knows what will happen in period 2.
- Equilibrium entails High Advertising by each firm in both periods.
- The same holds true if we repeat the game any known, finite number of times.

# Can collusion work if firms play the game each year, forever?

- Consider the following “trigger strategy” by each firm:
  - “Don’t advertise, provided the rival has not advertised in the past. If the rival ever advertises, “punish” it by engaging in a high level of advertising forever after.”
- In effect, each firm agrees to “cooperate” so long as the rival hasn’t “cheated” in the past. “Cheating” triggers punishment in all future periods.

# Suppose General Mills adopts this trigger strategy. Kellogg's profits?

$$\begin{aligned}\Pi_{\text{Cooperate}} &= 12 + 12/(1+i) + 12/(1+i)^2 + 12/(1+i)^3 + \dots \\ &= 12 + \boxed{12/i} \quad \leftarrow \text{Value of a perpetuity of \$12 paid at the end of every year}\end{aligned}$$

$$\begin{aligned}\Pi_{\text{Cheat}} &= 20 + 2/(1+i) + 2/(1+i)^2 + 2/(1+i)^3 + \dots \\ &= 20 + 2/i\end{aligned}$$

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12, 12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

# Kellogg's Gain to Cheating:

- $\Pi_{\text{Cheat}} - \Pi_{\text{Cooperate}} = 20 + 2/i - (12 + 12/i) = 8 - 10/i$ 
  - Suppose  $i = .05$
- $\Pi_{\text{Cheat}} - \Pi_{\text{Cooperate}} = 8 - 10/.05 = 8 - 200 = -192$
- It doesn't pay to deviate.
  - Collusion is a Nash equilibrium in the infinitely repeated game!

## General Mills

Kellogg's

Strategy	None	Moderate	High
None	12, 12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

# Benefits & Costs of Cheating

- $\Pi_{\text{Cheat}} - \Pi_{\text{Cooperate}} = 8 - 10/i$ 
  - 8 = Immediate Benefit (20 - 12 today)
  - $10/i$  = PV of Future Cost (12 - 2 forever after)
- If Immediate Benefit - PV of Future Cost > 0
  - Pays to “cheat”.
- If Immediate Benefit - PV of Future Cost ≤ 0
  - Doesn't pay to “cheat”.

## General Mills

Kellogg's

Strategy	None	Moderate	High
None	12, 12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

# Main Idea

- Cooperation can be sustained as a Nash Eq. even when there is a conflict of interest.
  - E.g., collusion in oligopoly
- Requires repeated interaction into the indefinite future.
  - Won't work if everyone knows the end date.
- Works better given:
  - Ability to monitor actions of rivals
  - Ability (and reputation for) punishing defectors
  - Low interest rate
  - High probability of future interaction

# Real World Examples of Collusion

- Garbage Collection Industry
- OPEC
- NASDAQ
- Airlines



# Garbage Collection Industry

- Homogeneous products
- Bertrand oligopoly
- Identity of customers is known
- Identity of competitors is known

# Normal Form Bertrand Game

**Firm 2**

**Firm 1**

Strategy	Low Price	High Price
Low Price	0,0	20,-1
High Price	-1, 20	15, 15

# One-Shot Bertrand (Nash) Equilibrium

**Firm 2**

**Firm 1**

Strategy	Low Price	High Price
Low Price	0,0	20,-1
High Price	-1, 20	15, 15

# Potential Repeated Game Equilibrium Outcome

**Firm 2**

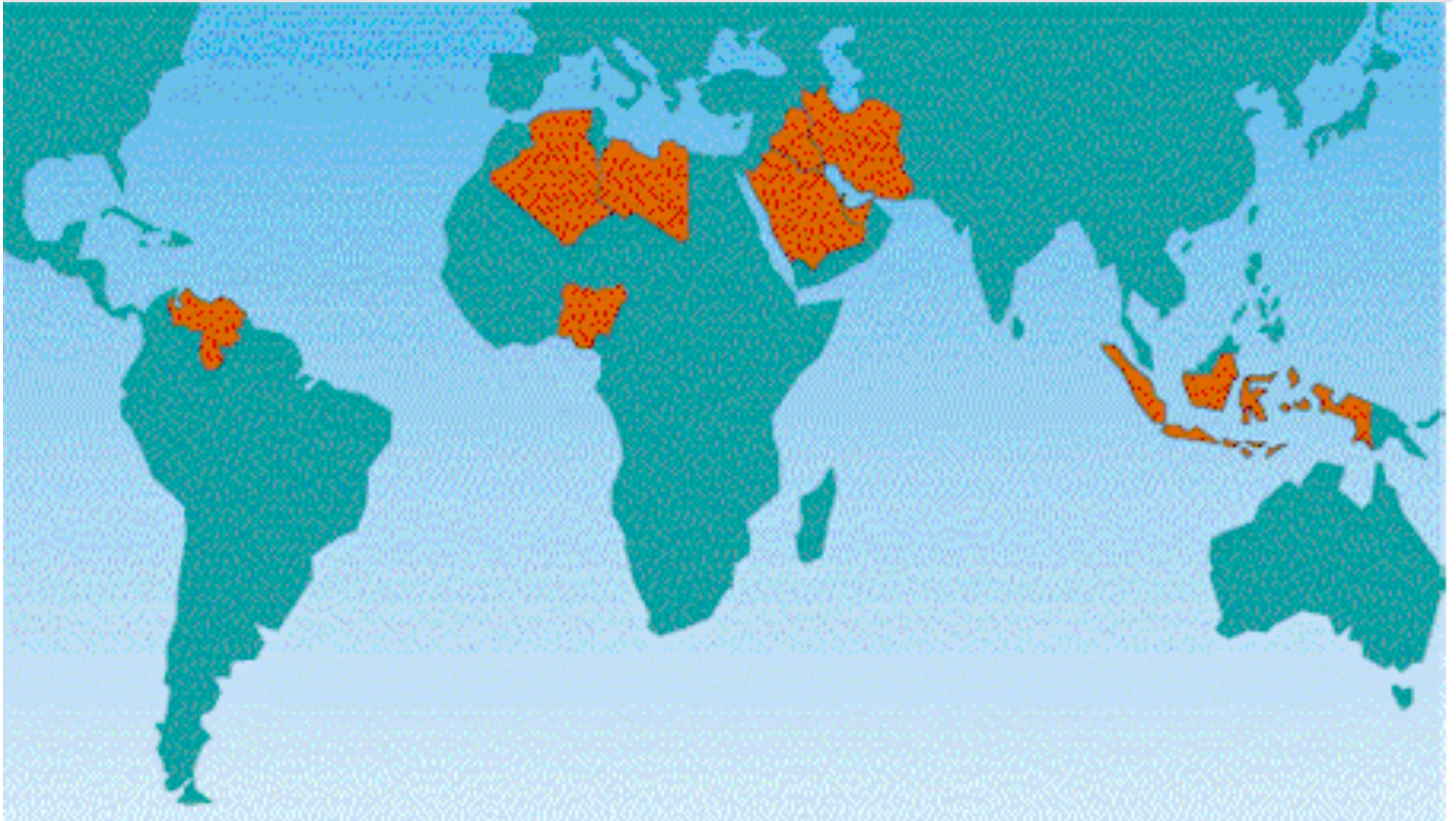
**Firm 1**

Strategy	Low Price	High Price
Low Price	0,0	20,-1
High Price	-1, 20	15, 15

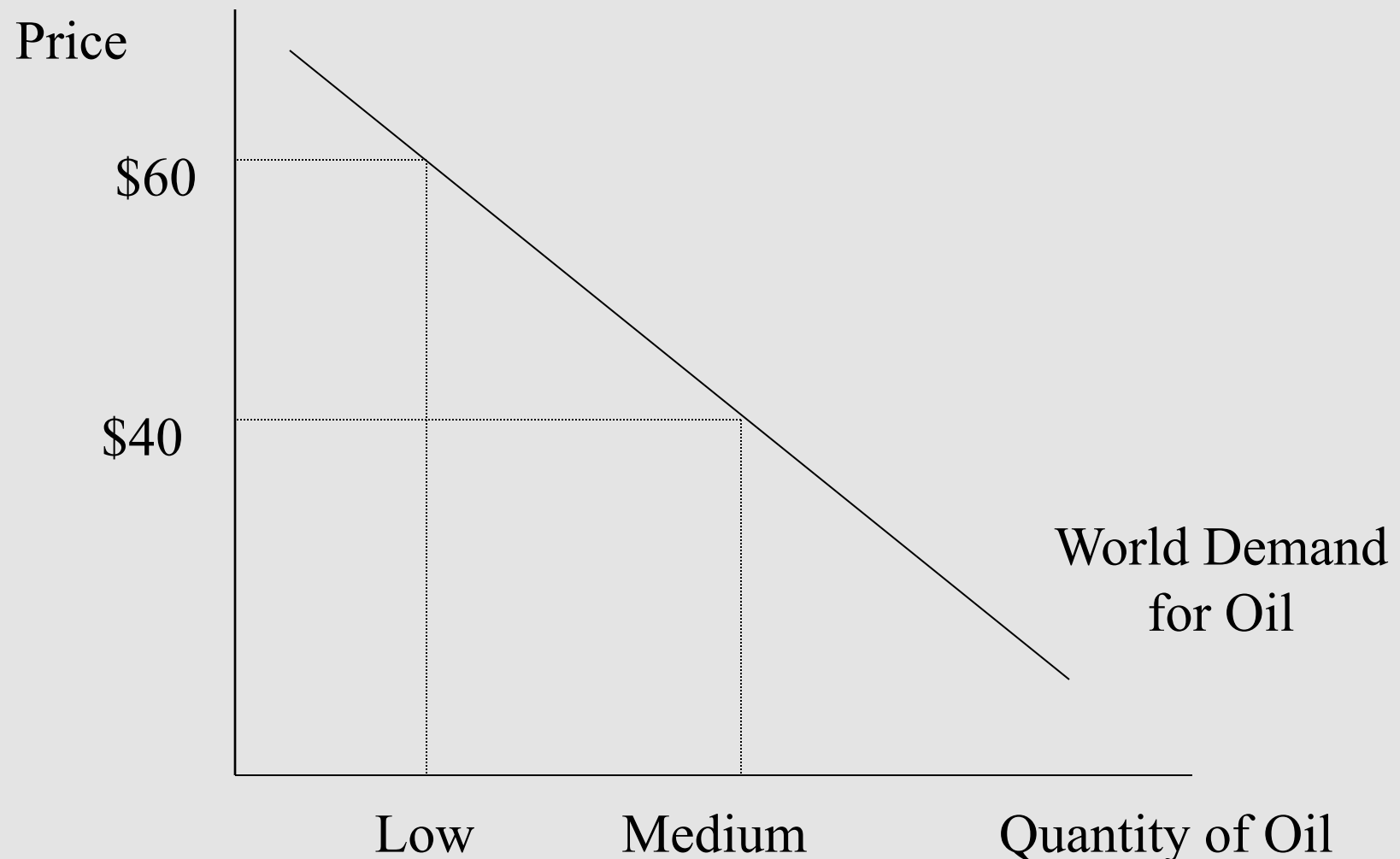
# OPEC

- Cartel founded in 1960 by Iran, Iraq, Kuwait, Saudi Arabia, and Venezuela
- Currently has 11 members
- *“OPEC’s objective is to co-ordinate and unify petroleum policies among Member Countries, in order to secure fair and stable prices for petroleum producers...”* (www.opec.com)
- Cournot oligopoly
- Absent collusion:  $P^{\text{Competition}} < P^{\text{Cournot}} < P^{\text{Monopoly}}$

# Current OPEC Members



# Effect of Collusion on Oil Prices



# Cournot Game in Normal Form

Venezuela

Saudi Arabia

Strategy	High Q	Med Q	Low Q
High Q	5, 3	9, 4	3, 6
Med Q	6, 7	12, 10	20, 8
Low Q	8, 1	10, 18	18, 15



# One-Shot Cournot (Nash) Equilibrium

Venezuela

Saudi Arabia

Strategy	High Q	Med Q	Low Q
High Q	5, 3	9, 4	3, 6
Med Q	6, 7	12, 10	20, 8
Low Q	8, 1	10, 18	18, 15

# Repeated Game Equilibrium\*

Venezuela

Saudi Arabia

Strategy	High Q	Med Q	Low Q
High Q	5, 3	9, 4	3, 6
Med Q	6, 7	12, 10	20, 8
Low Q	8, 1	10, 18	18, 15

\* *(Assuming a Low Interest Rate)*

# Caveat

- Collusion is a felony under Section 2 of the Sherman Antitrust Act.
- Conviction can result in both fines and jail-time (at the discretion of the court).
- Some NASDAQ dealers and airline companies have been charged with violations
- OPEC isn't illegal; US laws don't apply

# Simultaneous-Move Bargaining

- Management and a union are negotiating a wage increase.
- Strategies are wage offers & wage demands.
- Successful negotiations lead to \$600 million in surplus, to be split among the parties.
- Failure to reach an agreement results in a loss to the firm of \$100 million and a union loss of \$3 million.
- First consider simultaneous moves: only one last shot at making a deal due to impending deadline.

# The Bargaining Game in Normal Form

Union

Management

Strategy	W = \$10	W = \$5	W = \$1
W = \$10	100, 500	100, 500	100, 500
W = \$5	-100, -3	300, 300	300, 300
W = \$1	-100, -3	-100, -3	500, 100

# Three Nash Equilibria!

Union

Management

Strategy	W = \$10	W = \$5	W = \$1
W = \$10	100, 500	100, 500	100, 500
W=\$5	-100, -3	300, 300	300, 300
W=\$1	-100, -3	-100, -3	500, 100

# Fairness: The “Natural” Focal Point

Union

Management

Strategy	W = \$10	W = \$5	W = \$1
W = \$10	100, 500	100, 500	100, 500
W=\$5	-100, -3	300, 300	300, 300
W=\$1	-100, -3	-100, -3	500, 100

# Lessons in Simultaneous Bargaining

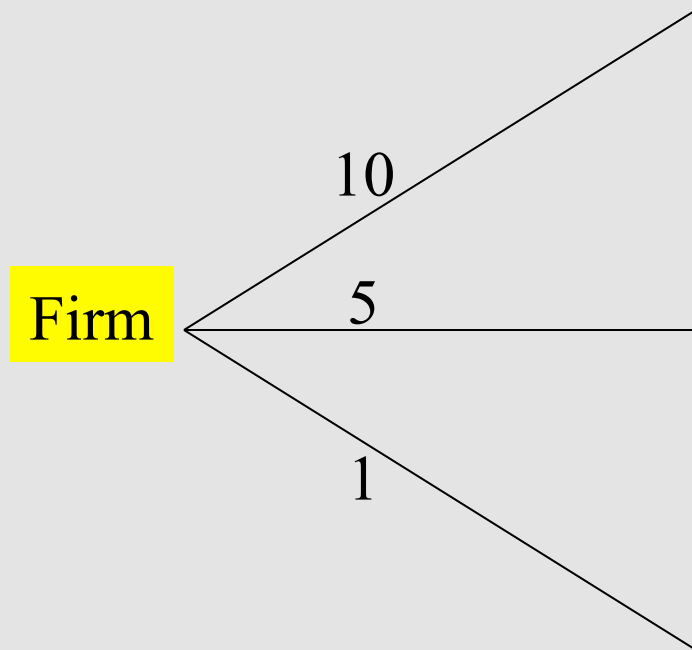
- Simultaneous-move bargaining results in a coordination problem.
- Experiments suggests that, in the absence of any “history,” real players typically coordinate on the “fair outcome.”
- When there is a “bargaining history,” other outcomes may prevail.



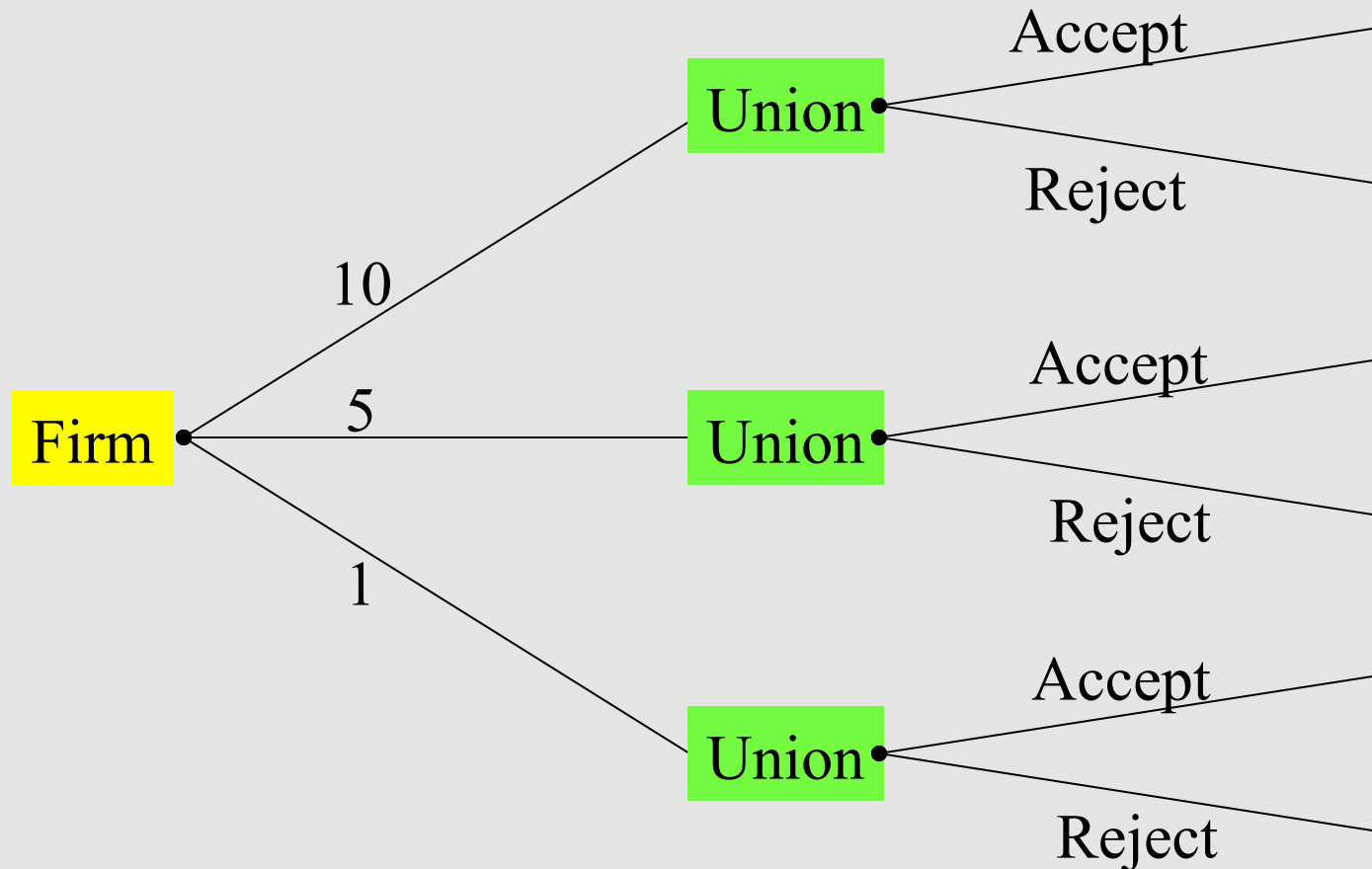
# Single Offer Bargaining

- Now suppose the game is sequential in nature, and management gets to make the union a “take-it-or-leave-it” offer.
- Analysis Tool: Write the game in extensive form
  - Summarize the players.
  - Their potential actions.
  - Their information at each decision point.
  - The sequence of moves.
  - Each player’s payoff.

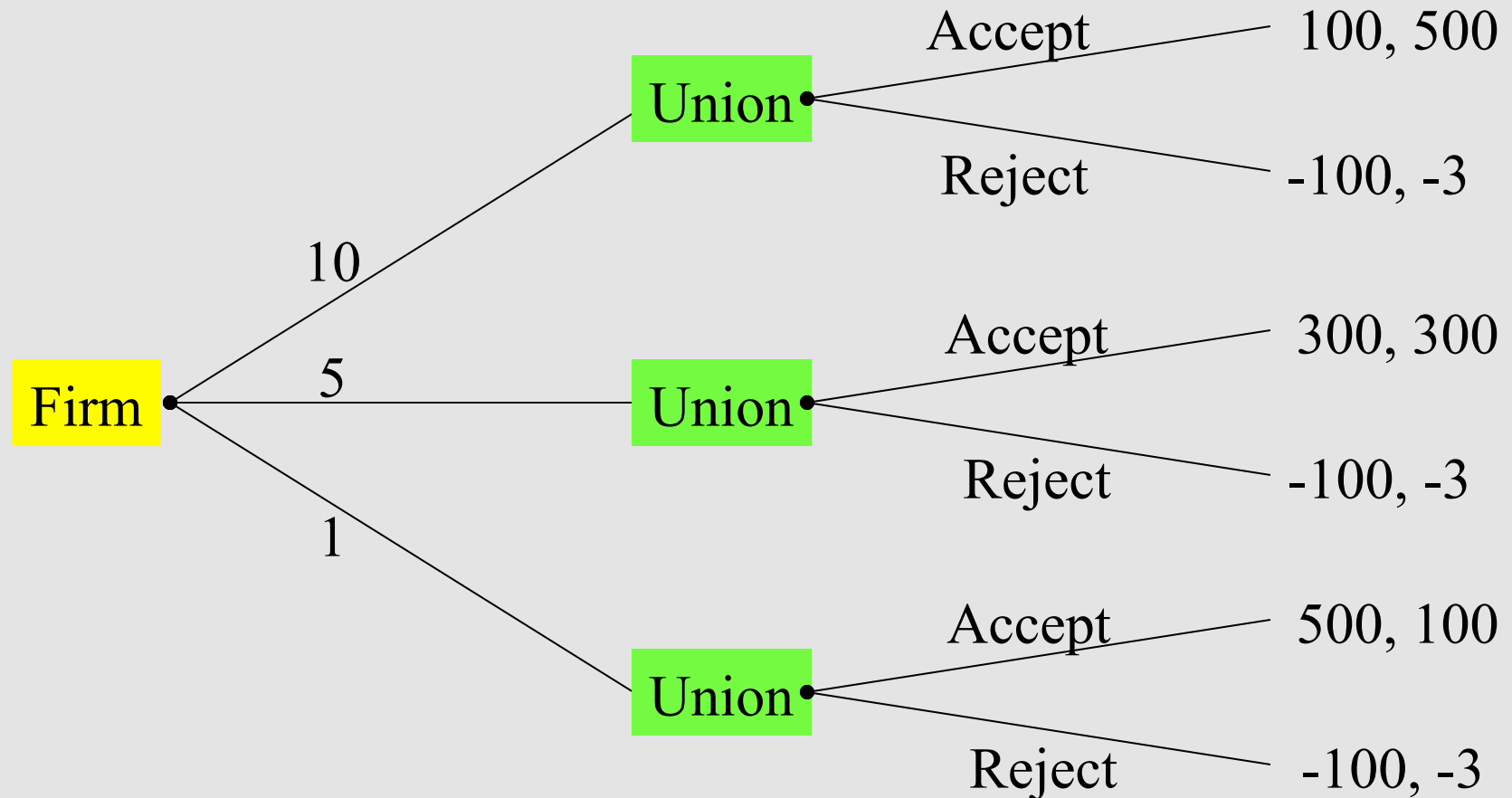
# Step 1: Management's Move



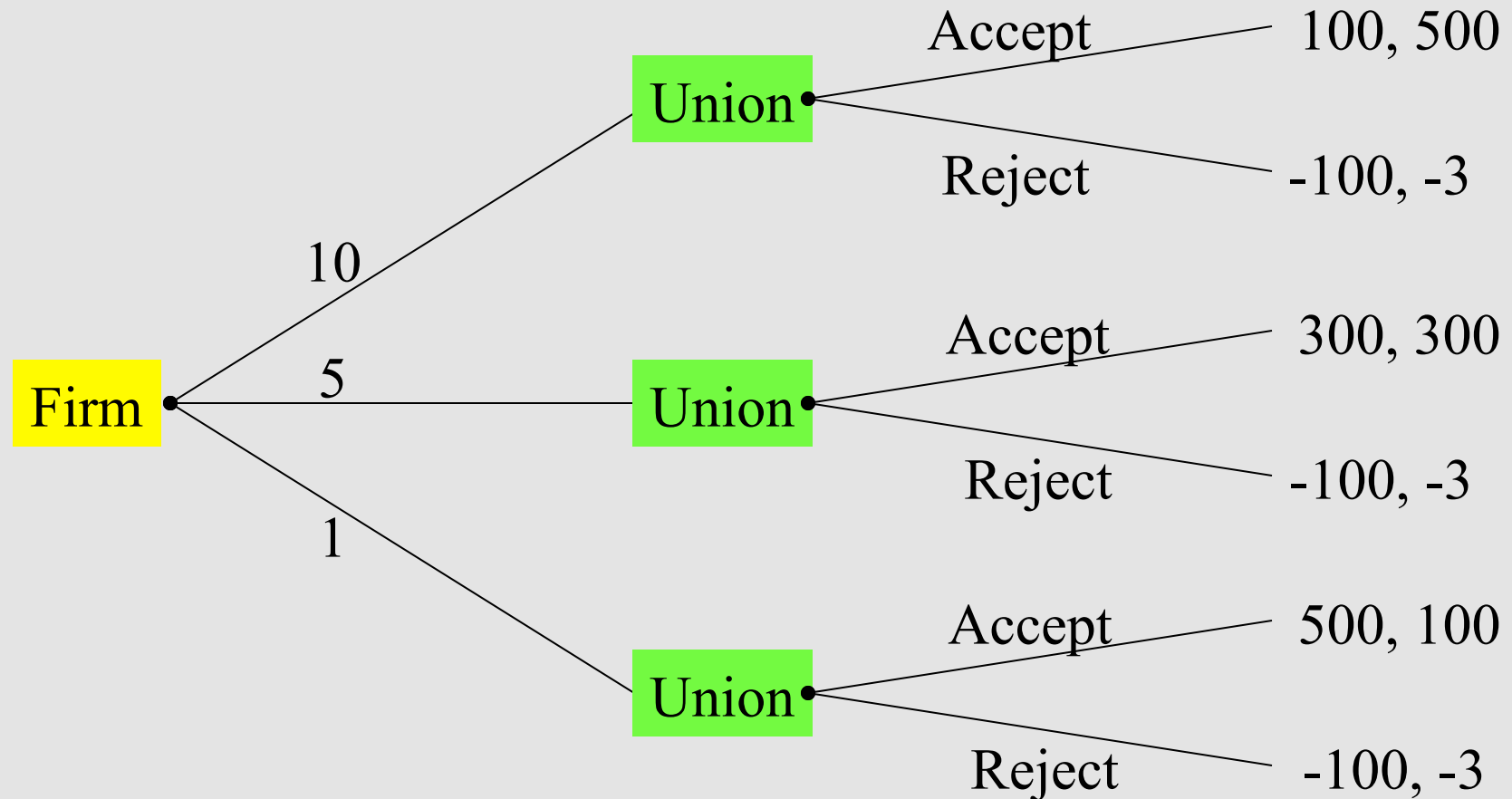
## Step 2: Add the Union's Move



# Step 3: Add the Payoffs



# The Game in Extensive Form



# Step 4: Identify the Firm's Feasible Strategies

- Management has one information set and thus three feasible strategies:
  - Offer \$10.
  - Offer \$5.
  - Offer \$1.

# Step 5: Identify the Union's Feasible Strategies

- The Union has three information sets and thus eight feasible strategies:
  - Accept \$10, Accept \$5, Accept \$1
  - Accept \$10, Accept \$5, Reject \$1
  - Accept \$10, Reject \$5, Accept \$1
  - Accept \$10, Reject \$5, Reject \$1
  - Reject \$10, Accept \$5, Accept \$1
  - Reject \$10, Accept \$5, Reject \$1
  - Reject \$10, Reject \$5, Accept \$1
  - Reject \$10, Reject \$5, Reject \$1

# Step 6: Identify Nash Equilibrium Outcomes

- Outcomes such that neither the firm nor the union has an incentive to change its strategy, given the strategy of the other.



# Finding Nash Equilibrium Outcomes

<b>Union's Strategy</b>	<b>Firm's Best Response</b>	<b>Mutual Best Response?</b>
<b>Accept \$10, Accept \$5, Accept \$1</b>	\$1	Yes
<b>Accept \$10, Accept \$5, Reject \$1</b>	\$5	Yes
<b>Accept \$10, Reject \$5, Accept \$1</b>	\$1	Yes
<b>Reject \$10, Accept \$5, Accept \$1</b>	\$1	Yes
<b>Accept \$10, Reject \$5, Reject \$1</b>	\$10	Yes
<b>Reject \$10, Accept \$5, Reject \$1</b>	\$5	Yes
<b>Reject \$10, Reject \$5, Accept \$1</b>	\$1	Yes
<b>Reject \$10, Reject \$5, Reject \$1</b>	\$10, \$5, \$1	No

# Step 7: Find the Subgame Perfect Nash Equilibrium Outcomes

- Outcomes where no player has an incentive to change its strategy, given the strategy of the rival, **and**
- The outcomes are based on “credible actions;” that is, they are not the result of “empty threats” by the rival.

# Checking for Credible Actions

<b>Union's Strategy</b>	<b>Are all Actions Credible?</b>
<b>Accept \$10, Accept \$5, Accept \$1</b>	Yes
<b>Accept \$10, Accept \$5, Reject \$1</b>	No
<b>Accept \$10, Reject \$5, Accept \$1</b>	No
<b>Reject \$10, Accept \$5, Accept \$1</b>	No
<b>Accept \$10, Reject \$5, Reject \$1</b>	No
<b>Reject \$10, Accept \$5, Reject \$1</b>	No
<b>Reject \$10, Reject \$5, Accept \$1</b>	No
<b>Reject \$10, Reject \$5, Reject \$1</b>	No

# The “Credible” Union Strategy

<b>Union's Strategy</b>	<b>Are all Actions Credible?</b>
<b>Accept \$10, Accept \$5, Accept \$1</b>	<b>Yes</b>
<b>Accept \$10, Accept \$5, Reject \$1</b>	No
<b>Accept \$10, Reject \$5, Accept \$1</b>	No
<b>Reject \$10, Accept \$5, Accept \$1</b>	No
<b>Accept \$10, Reject \$5, Reject \$1</b>	No
<b>Reject \$10, Accept \$5, Reject \$1</b>	No
<b>Reject \$10, Reject \$5, Accept \$1</b>	No
<b>Reject \$10, Reject \$5, Reject \$1</b>	No

# Finding Subgame Perfect Nash Equilibrium Strategies

Union's Strategy	Firm's Best Response	Mutual Best Response?
Accept \$10, Accept \$5, Accept \$1	\$1	Yes
Accept \$10, Accept \$5, Reject \$1	\$5	Yes
Accept \$10, Reject \$5, Accept \$1	\$1	Yes
Reject \$10, Accept \$5, Accept \$1	\$1	Yes
Accept \$10, Reject \$5, Reject \$1	\$10	Yes
Reject \$10, Accept \$5, Reject \$1	\$5	Yes
Reject \$10, Reject \$5, Accept \$1	\$1	Yes
Reject \$10, Reject \$5, Reject \$1	\$10, \$5, \$1	No

Nash and Credible

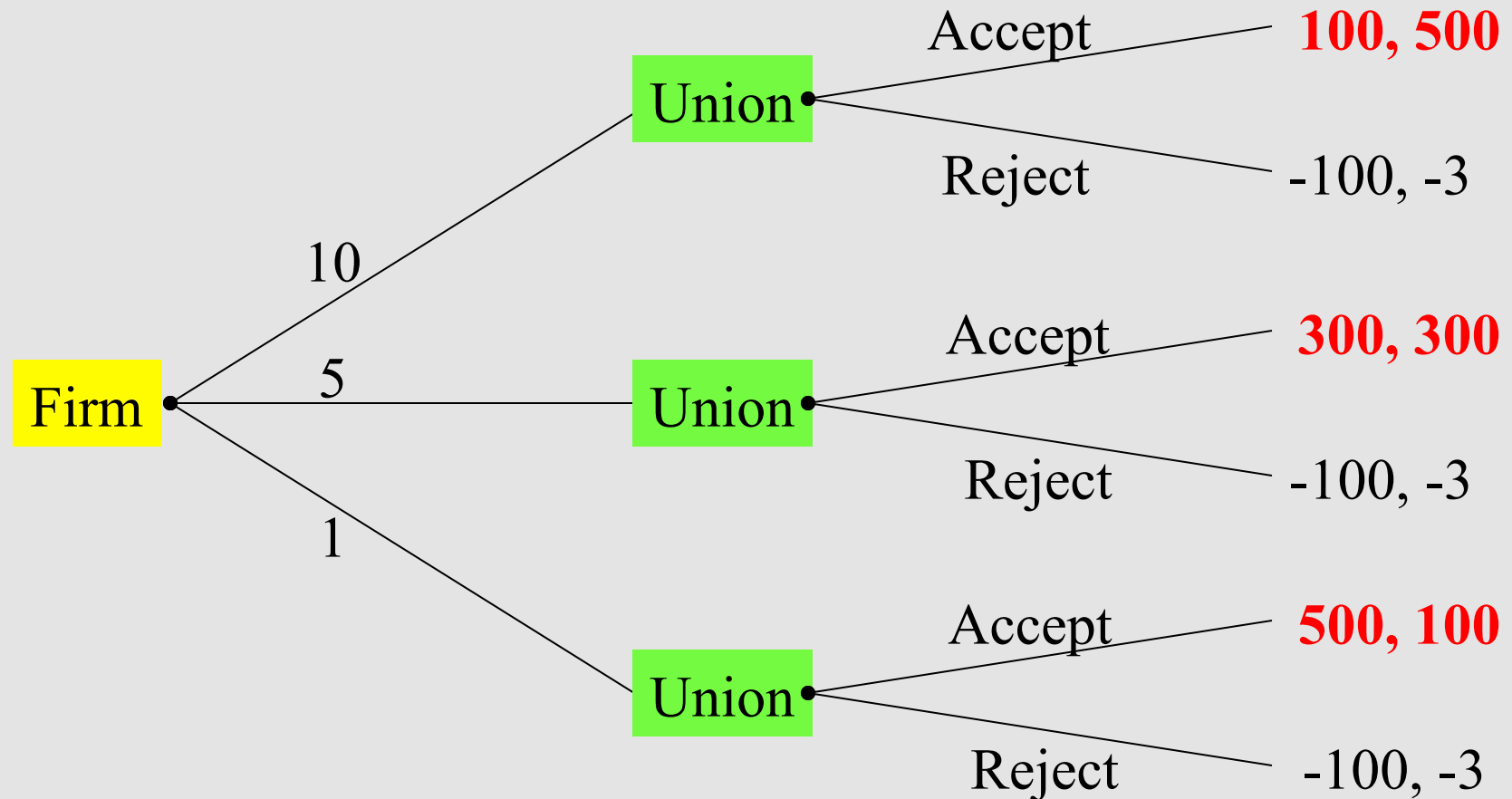
Nash Only

Neither Nash Nor Credible

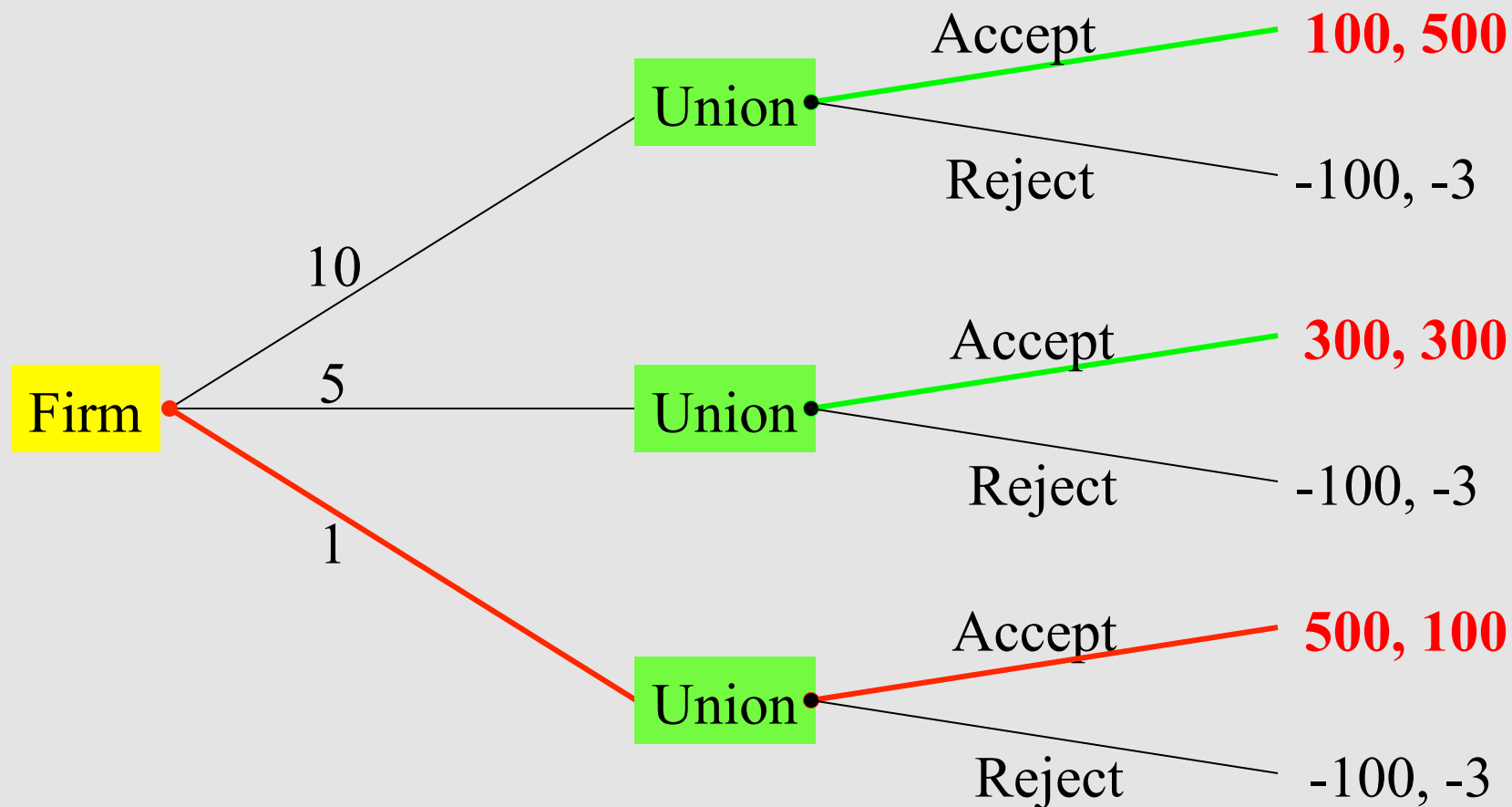
# To Summarize:

- We have identified many combinations of Nash equilibrium strategies.
- In all but one the union does something that isn't in its self interest (and thus entail threats that are not credible).
- Graphically:

# There are 3 Nash Equilibrium Outcomes!



# Only 1 Subgame-Perfect Nash Equilibrium Outcome!





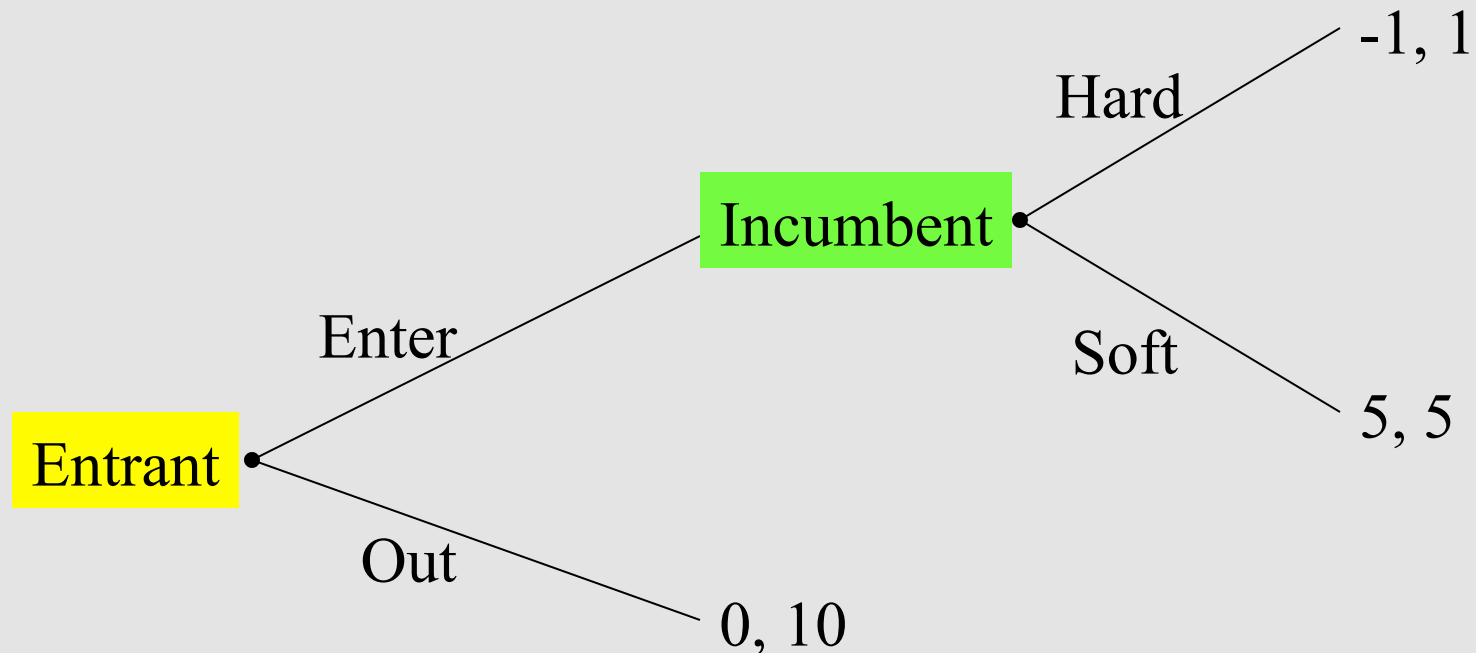
# Bargaining Re-Cap

- In take-it-or-leave-it bargaining, there is a first-mover advantage.
- Management can gain by making a take-it-or-leave-it offer to the union. But...
- Management should be careful; real world evidence suggests that people sometimes reject offers on the the basis of “principle” instead of cash considerations.

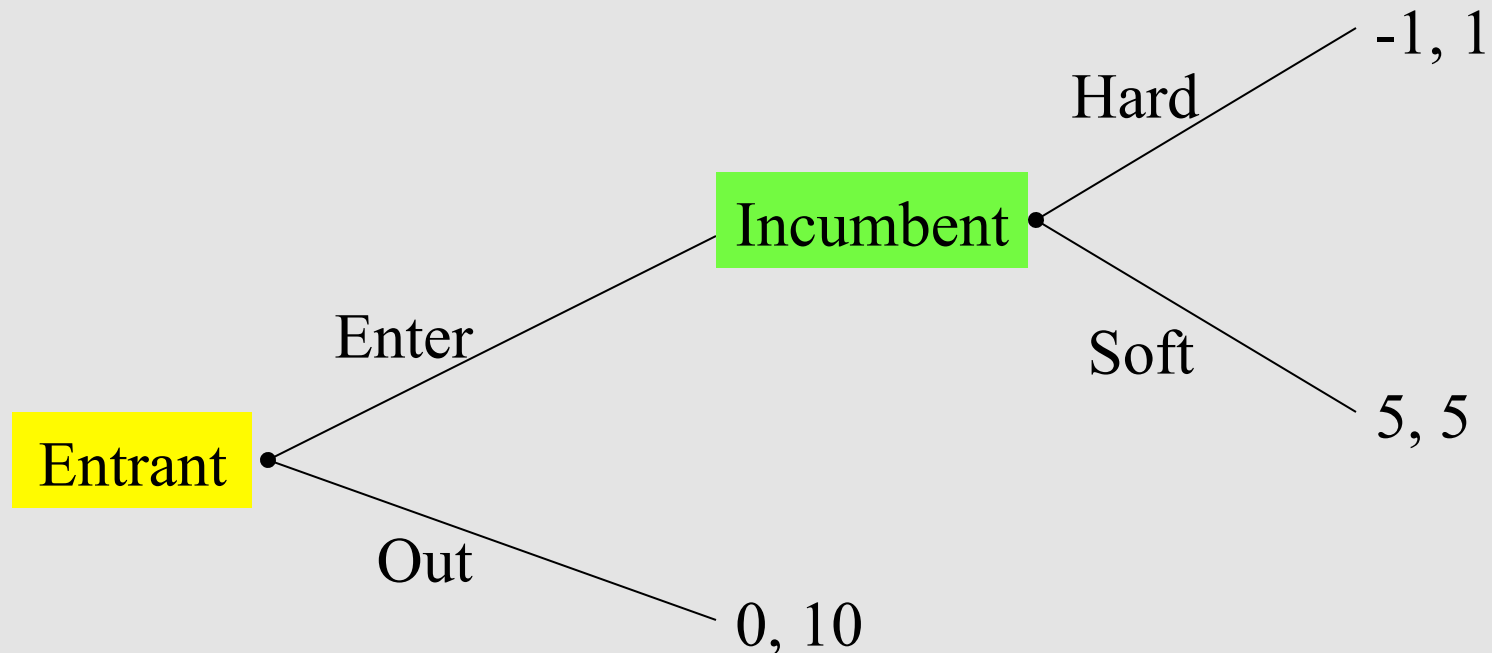
# Pricing to Prevent Entry: An Application of Game Theory

- Two firms: an incumbent and potential entrant.
- Potential entrant's strategies:
  - Enter.
  - Stay Out.
- Incumbent's strategies:
  - {if enter, play hard}.
  - {if enter, play soft}.
  - {if stay out, play hard}.
  - {if stay out, play soft}.
- Move Sequence:
  - Entrant moves first. Incumbent observes entrant's action and selects an action.

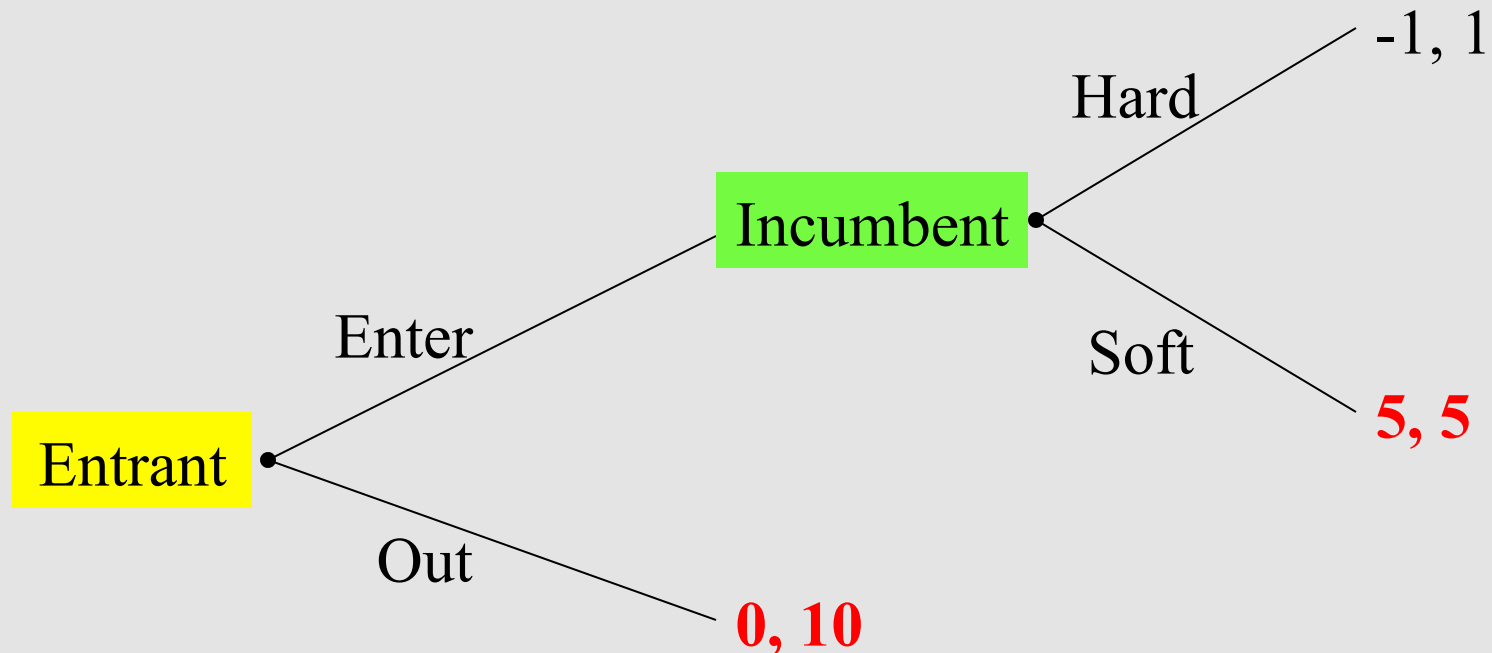
# The Pricing to Prevent Entry Game in Extensive Form



# Identify Nash and Subgame Perfect Equilibria

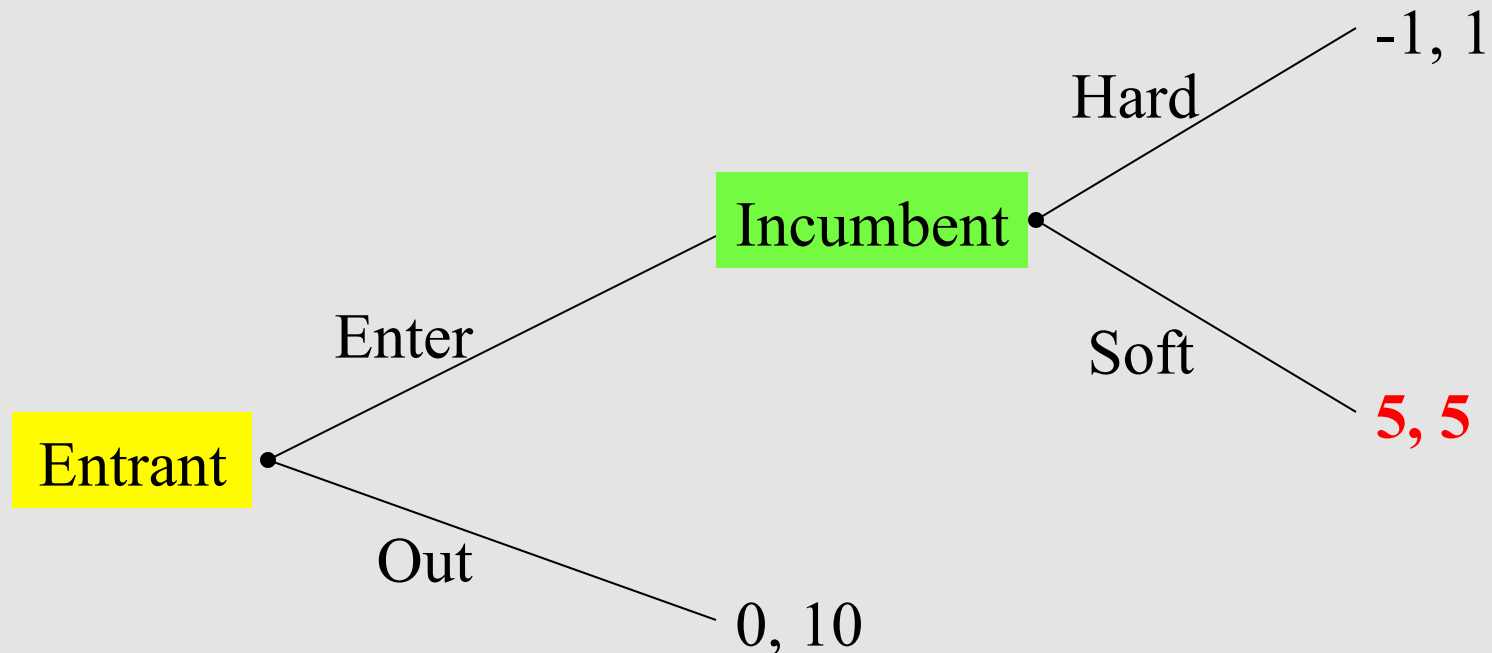


# Two Nash Equilibria



Nash Equilibria Strategies {player 1; player 2}:  
{enter; If enter, play soft}  
{stay out; If enter, play hard}

# One Subgame Perfect Equilibrium



Subgame Perfect Equilibrium Strategy:  
{enter; If enter, play soft}

# Insights

- Establishing a reputation for being unkind to entrants can enhance long-term profits.
- It is costly to do so in the short-term, so much so that it isn't optimal to do so in a one-shot game.

# Holdup Problem Revisited

- Sunk cost investments create quasi-rents
  - These can be appropriated
  - This would create a loss on the investment
- Hence the investment might not be made
  - And the opportunity is lost
- Examples include
  - UCSC buys enterprise software from PAS...
  - NASA contracts with Obing Corp..
  - Many Dilbert episodes



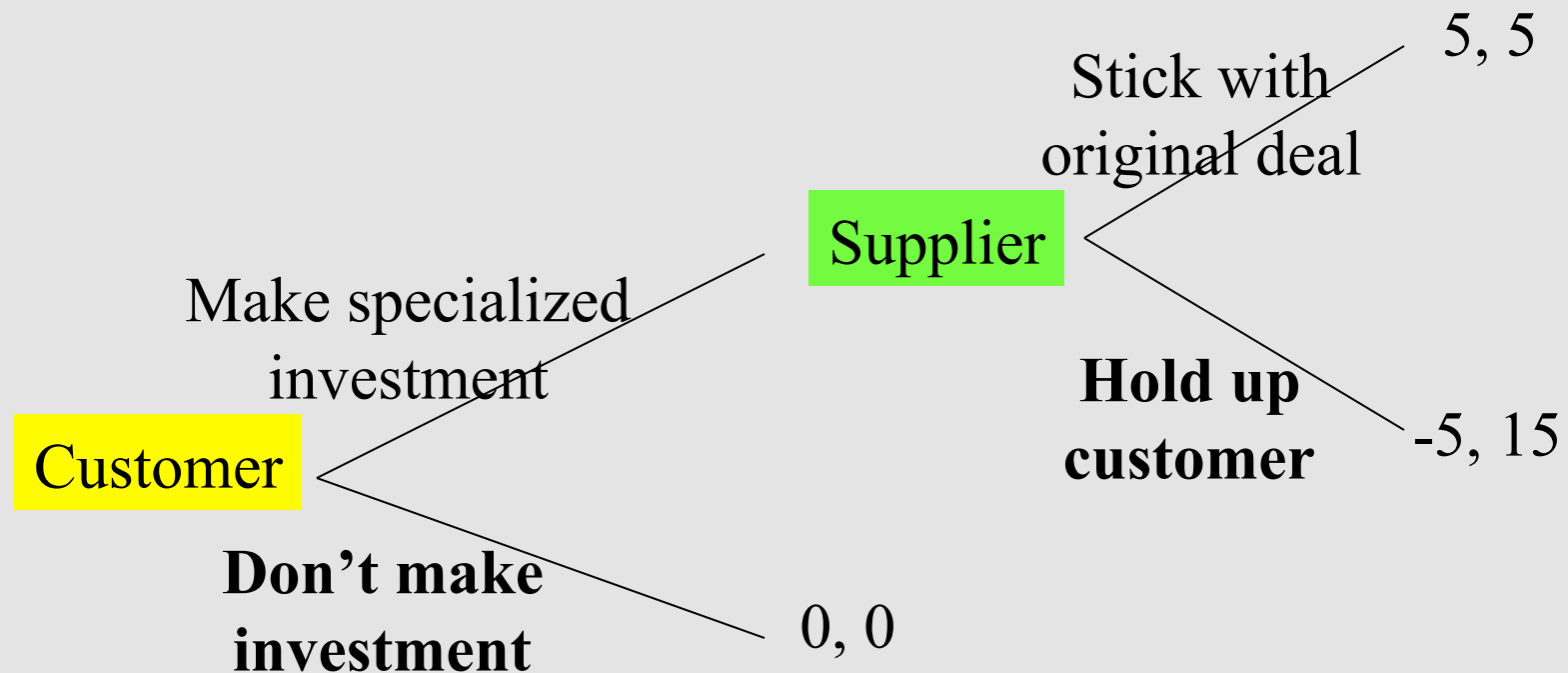
# A typical scenario

- Customer can make investment (cost=5) in specialized software that will enhance productivity (benefit=15)
- Original deal: customer keeps 10 of benefit and nets 5, supplier gets the other 5.

# A typical scenario

- Customer can make investment (cost=5) in specialized software that will enhance productivity (benefit=15)
- Original deal: customer keeps 10 of benefit and nets 5, supplier gets the other 5.
- **Hold up:** Supplier can later demand an extra amount (at most 10) to keep software working.

# The Holdup Problem in Extensive Form



# Missing piece of theory: mixed strategies

- It's third down and 4 yards to go for the NY Giants...should they run or pass? Should the NE Patriots stack the defense against the run or pass?
- No Nash equilibrium in pure strategies.
- The NE: mix it up!
- See text for short discussion, and any game theory book for a long discussion.
- Theorem: every “regular” game has at least one NE, but it may involve mixed strategies.