A Continuous Dilemma *

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Abstract

We study prisoners' dilemmas played in continuous time with flow payoffs accumulated over 60 seconds. In most cases, the median rate of mutual cooperation is about 90%. Control sessions with repeated matchings over 8 subperiods achieve less than half as much cooperation, and cooperation rates approach zero in one-shot control sessions. In follow-up sessions with a variable number of subperiods, cooperation rates increase nearly linearly as the grid size decreases and, with one-second subperiods, they approach the level seen in continuous sessions. Our data support a strand of theory that explains how the capacity to respond rapidly stabilizes cooperation and destablizes defection in the prisoner's dilemma.

Keywords: Prisoner's dilemma, game theory, laboratory experiment, continuous time game.

JEL codes: C73, C92, D74

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The computerized ATPCO system, introduced in the early 1990s, allowed airlines to rapidly adjust prices and monitor those of rivals. The result, according to the US Department of Justice, was anti-competitive behavior—firms were able to more easily cooperate to keep prices high, costing consumers up to 2 billion dollars (Severin Borenstein, 2003, Joel I. Klein, 1998).

Does the ability to rapidly adjust actions actually encourage cooperation? Compared to the one-shot or discrete time strategic interactions usually analyzed by game theorists, are outcomes much different when choices are made asynchronously, in continuous time? These questions are not merely of theoretical interest because many modern interactions, ranging from just-in-time team production to e-commerce pricing, involve asynchronous strategic decisions made in real time.

In this paper we take such questions to the laboratory. In a continuous time setting, we study variants of the prisoner's dilemma, the simplest and most famous example of a strategic tension between efficient cooperation and inefficient self-interest. Pairs of laboratory subjects are matched anonymously in 60-second periods, within which they can switch freely between cooperation and defection. They accrue flow payouts from one of four parametric variants of the prisoner's dilemma, and then are randomly rematched for the next period. Each session runs 32-36 periods. We also run control sessions with one shot periods and with repeated discrete time subperiods, using identical payoff matrices, period lengths and matching procedures.

Section I recalls some previous theoretical and experimental work related to our investigation, and section II lays out our experimental design. Section III reports first results: continuous time enables median mutual cooperation rates of 90% or more in most of the variants, more than double the rates seen in our discrete (8 stages per period) repeated games, while mutual cooperation becomes quite rare in the one shot sessions. The underlying strategies seem broadly consistent with some of the theoretical literature, particularly with Roy Radner (1986) and Leo K. Simon and Maxwell B. Stinchcombe (1989).

Adapting previous theory to a continuous time setting with fast but not instantaneous reactions, Section IV obtains predictions arising from a class of epsilon equilibria in cutoff strategies. It then reports a second wave of sessions that vary the number of discrete stages from 2 to 60 within each 60 second period. As predicted, the data show a negative, almost linear, relationship between the the cooperation rate and the length of the stage game. Indeed, the mutual cooperation rates with 2 stages per period are not far from zero, and those with 60 stages are not far from the rates seen in continuous time.

Section V offers a broader discussion of the findings and remaining questions, and an Appendix col-

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} (10,10) & (0,x) \\ (x,0) & (y,y) \end{pmatrix} \end{array}$$

Table 1: Generic form of prisoner's dilemma, with $20 > x > 10 > y > 0$.

lects mathematical details. Two online appendices provide additional data analysis and instructions to subjects.

I Some Previous Work

Table I parametrizes the standard prisoner's dilemma payoff bimatrix, first introduced by Merrill M. Flood (1952); see Anatol Rapoport and Albert M. Chammah (1965) for a related parametrization. With no loss of theoretical generality, the table normalizes the "cooperation" payoff at 10 and the "sucker" payoff at 0. Strategy B is strictly dominant, and so (B,B) is the unique Nash equilibrium, as long as the "tempation" payoff satisfies $x > 10$. The restriction $y < 10$ on the "punishment" payoff ensures that the Nash equilibrium is inefficient, and $x < 20$ ensures that the sucker-temptation profiles (A,B) and (B,A) also yield a lower payoff sum than the cooperation profile (A, A). Thus the dilemma: the unique equilibrium is inefficient.

A Theory

Legions of theorists have sought ways to evade the dilemma and to support cooperation. They soon discovered that finite repetition doesn't help: as explained in any modern game theory text, backward induction eliminates every equilibrium profile sequence except all-defect—(B,B) every period. Patient pairs of players rematched over an infinite sequence of stages can support cooperation, but by the Folk Theorem (e.g., Drew Fudenberg and Eric S.Maskin, 1986), they can just as easily support (as a Nash equilibrium of the repeated game) all-defect and a wide variety of other inefficient profile sequences.

What happens if the game is played over a continuous finite time interval, say $t \in [0, 1]$? Perhaps the most obvious approach is to specify a minimum reaction time τ and to formalize the game as a finitely repeated game with $1/\tau$ stages (rounding up to the nearest integer). The theoretical prediction again is that the dilemma persists, and only all-defect survives in Nash equilibrium.

Bernardo A. Huberman and Natalie S. Glance (1993) show that cooperation evaporates in spatial versions of the repeated prisoner's dilemma when players move asynchronously, in real time. According to the authors, the lesson is that coordination and cooperation can get an artificial boost when all players must move simultaneously at discrete time intervals. Clearly there is a need for more fully articulated models of games played in continuous time.

James Bergin and Bentley MacLeod (1993) develop one such model. They assume that actions cannot be reversed within time ϵ, look for ϵ-equilibria, and pass to the limit as ϵ goes to 0. For the prisoner's dilemma, they obtain a Folk Theorem result: virtually any profile sequence that gives each player an average payoff of at least $y < 10$ can be supported as a Nash equilibrium (indeed, one that is renegotiation-proof).

Radner (1986) had previously studied ϵ-equilibria of the finitely repeated prisoner's dilemma, and (although he did not emphasize it) obtained an insight that we find very useful. Assume that players seek to maximize the average payoff over $T < \infty$ repetitions of the game in Table I. Let C_k denote the strategy of playing Grim (i.e., choosing A until the other player plays B and choosing B thereafter) up through period k and playing B in the remaining periods $k+1, ..., T$. The usual unravelling argument notes that C_{k-1} is a best response to C_k for all $k = 1, ..., T$. However, for $\epsilon > (x+y-10)/T$, Radner's equations (17, 19) show that C_{T-1} yields a payoff within ϵ of the best response payoff against C_k for *every* $k = 1, ..., T$. The insight is that, when T is large, you lose considerably more than ϵ if your defection time n is much earlier than the other player's defection time k, but by choosing near-maximal n, you never lose more than ϵ, no matter how large or small is k. Thus waiting longer to defect is nearly dominant in Radner's setup. This suggests that mutual cooperation can prevail.

Simon and Stinchcombe (1989) propose a general model of games played in continuous time. They consider discrete grids in the time interval [0, 1) for games with finite numbers of players and actions. Given some technical conditions (e.g., the number of strategy switches remains uniformly bounded for each player), in the limit as the grid interval approaches zero they obtain well-defined games in continuous time. Subgame perfection is automatic in these games, but backward induction does not work because the real numbers are not well ordered (Robert M. Anderson, 1984). For example, time $t = 1$ has no immediate predecessor: for any previous time, say $t = 1-h$, there are an infinite number of later times that fall before $t = 1$, e.g., $t = 1-h/17$. Consequently some repeated game equilibria disappear in the continuous limit, while new equilibria can appear. Consistent with Radner's insight, Simon and Stinchcombe focus on Nash equilibria which survive iterated deletion of weakly dominated strategies. For games in continuous time similar to the prisoner's dilemma, they find a unique such equilibrium outcome: full cooperation at all times.

Thus existing theoretical literature offers three competing predictions for our continuous time experiment. The (naively extended) theory of finitely repeated games predicts that the all-defect profile (B,B) will predominate; Simon and Stinchcombe (and Radner) predict that the full cooperation profile (A,A) will predominate; and the extended Folk Theorem predicts virtually any profile sequence that gives each player at least the all-defect payoff y.

This complete information literature predicts no other role for payoff parameters x, y within their admissible range. The famous Gang of Four model introduced by David M. Kreps et al. (1982) incorporates a touch of incomplete information, and obtains the qualitative prediction that cooperation will decrease when either x or y increases. The same prediction arises from Quantal Response Equilibrium (e.g., Richard McKelvey and Thomas Palfrey, 1995), from the heuristics of Anatol Rapoport et al. (1965), and from most other models that include some sort of noise or imperfect information.

B Experiments

Rapoport et al. (1965) conducted laboratory experiments with variants of the prisoner's dilemma iterated over 350 stages with fixed pairs of subjects, changing the x, y parameters randomly between 50 stage blocks. They find mutual cooperation rates above 60 percent in blocks with lowest x, y parameters, and cooperation rates under 50 percent in blocks with highest x, y. Rapoport et al. (1976) fix $x = 15, y = 5$ in our normalization, and report that, over 300 stages, individual cooperation rates initially declined, then rose modestly and averaged about 55 percent overall. Unfortunately, this early work with the finitely repeated prisoner's dilemma did not provide subjects the opportunity to learn the logic of backward induction, because there was no stationary repetition of the repeated game.

More recent experiments — e.g., Reinhard Selten and Rolf Stoecker (1986), James Andreoni and John Miller (1993), Esther Hauk and Rosemarie Nagel (2001) and Yoella Bereby-Myer and Alvin E. Roth (2006) — feature stationary repetition of 10-stage repeated prisoner's dilemmas; that is, each subject plays a sequence of different 10-stage games against different opponents. These papers report that, after several repetitions of the repeated game, most subjects cooperate in early stages but cooperation begins to unravel around the fifth stage and is rare after the 8th stage. Thus, even with ample opportunity to learn, the unravelling process seems at best incomplete in the laboratory data.

Potential explanations include the sequential equilibria of Kreps et al. (which were motivated by the laboratory results), or widespread altruism. Russell W. Cooper et al. (1996) find both of these explanations inadequate. In their experiment, cooperation rates decline fairly steadily over periods, not all at once as in pure sequential equilibria, and remain positive in the last period. Moreover, contrary to the best calibrated mixed sequential equilibrium, there is an increasing, not decreasing, rate at which cooperation declines in later subperiods (their Fig. 3, p. 205). Incomplete unravelling remains a puzzle.

Other experiments study the "infinitely repeated" prisoner's dilemma, in which there is an announced probability q that the matching ends after the current stage. Roth and Keith J. Murningham (1978) produced mixed evidence for the theoretical prediction that cooperation is possible in such games. More recently, Pedro Dal Bo (2005) finds that individual cooperation rates respond sensitively to q, exceeding 50 percent in the most favorable case, while Masaki Aoyagi and Guillaume Frechette (2009) observe individual cooperation rates as high as 85% under a very high q of 0.9.

Dal Bo and Frechette (2008) show that experience in these repeated games does not necessarily lead to greater cooperation and that variation in parameters similar to our x and y have a significant impact on cooperation. Individual cooperation rates average roughly 35 percent and rise to 76 percent with parameters more conducive to cooperation than any used in our own study. These authors conclude that the "shadow of the future" seems pivotal to cooperation. [1]

There are several ways to extrapolate these empirical results to continuous time. In 60 second periods, the shadow of the future shrinks steadily to zero. Will cooperation also decline steadily to zero? With more than thirty stationary repetitions per session and continuous time, our experiments provide unusually good learning opportunities. Will subjects learn to unravel cooperation more completely? Or will the absence of a well-ordering, or asynchronous decisions or other aspects of continuous time, twist the strategic behavior in a different direction? Answering such questions clearly requires new experiments.

II Treatments and Experimental Design

We ran experiments using a new software package called ConG, for Continuous Games. Figure 1 shows the user interface. Each subject can freely switch between row actions A and B by clicking a radio button (or pressing an arrow key), causing the chosen row to be shaded. In our main

[1] Indeed, under very favorable payoff parameters and a very strong shadow of the future, Dal Bo and Frechette eventually observe individual cooperation rates as high as 96%.

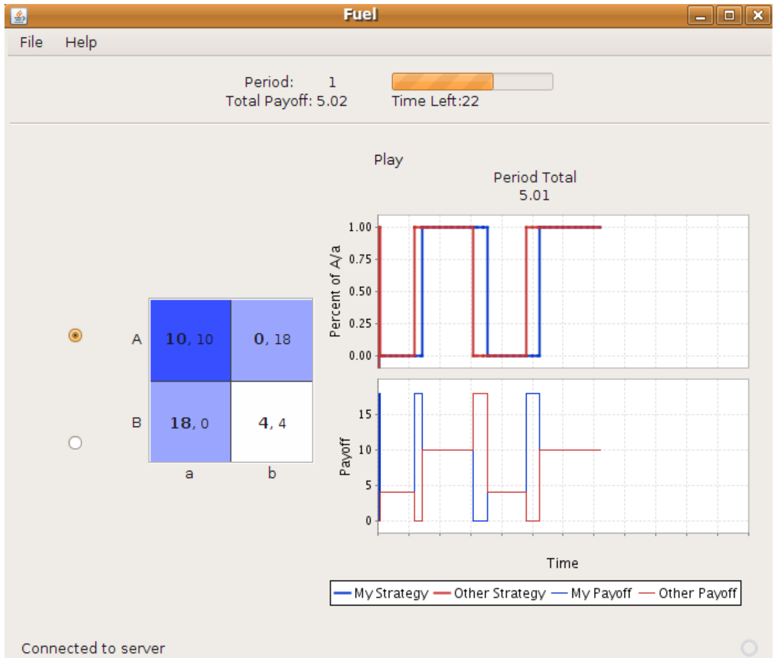

Figure 1: Screenshot of Continuous time display.

treatment (Continuous time) the other player's current choice is shown as a shaded column, and the intersection is doubly shaded. The computer response time to action switches is less than 50 milliseconds, giving players the experience of continuous action. The screen also shows the time series of actions (coded here as 1 for A and 0 for B) for the player and her counterpart in the upper right graph, while flow payoffs for each player are shown in the lower right graph. The top of the screen also shows the time remaining and the accumulated flow payoff.

We study three treatments of time: Continuous, One-Shot, and Grid. In all treatments, each period lasts 60 seconds, during which subjects are allowed to change their actions at will. In Continuous time, subjects observe the unfolding history of actions and payoffs, and at the end of the period they earn the integral of the flow payoffs shown in the lower right hand graph of Figure 1.

In One-Shot time, subjects do not observe their counterpart's action until the period's end. They earn the lump sum payoffs for the action profile chosen at that point.

Grid time divides each sixty second period into n equal subperiods. Payoffs in each subperiod are determined only by the last action profile chosen in that subperiod. Only at the end of the subperiod does a player see her counterpart's choice, and that last profile becomes the initial profile of the next subperiod. Payoffs for the entire period are the average of the lump sum subperiod payoffs or, equivalently, the integral across subperiods of the piecewise constant flow payoffs. Thus One-Shot time is the same as Grid time with $n = 1$, and Continuous time is closely approximated by Grid time with $n > 300$.

Our other treatment variable, payoff parameters, examines four different configurations of (x, y), one from each quadrant of the admissible domain $(10, 20) \times (0, 10)$. They are Easy = $(14, 4)$, Mix-a = $(18, 4)$, Mix-b = $(14, 8)$, and Hard = $(18, 8)$. The names reflect the presumption that cooperation will be more difficult given either a larger temptation x or a larger punishment payoff y. [2]

In all treatments, subjects are randomly rematched with a new counterpart each period. At period's beginning, each of the four possible initial profiles is chosen independently with probability 0.25. Within period, profiles are automatically extended forward in time (to the next subperiod in Grid) until at least one player changes her action.

We first ran 4 sessions for Continuous and 3 parallel sessions for One-Shot.[3] Ten subjects participated in each session (except for one Continuous session with only eight subjects), which consisted of 32 periods divided into 8 blocks. Each of the four parameter sets appears once, in random order, in each block, and the sequences are matched across the two time treatments. Then we ran another 4 matched sessions, again using the same sequences, under the Grid treatment with $n = 8$ subperiods (hereafter called Grid-8 sessions). This treatment is comparable to the 10-stage repeated games featured in previous laboratory studies. We also ran three additional Grid sessions, to be described later, that varied n within session.

A key aspect of our design is that period lengths and potential payoffs are kept constant across Continuous, One-Shot and Grid treatments. The only difference between these treatments is the frequency with which subjects can adjust their payoff-relevant choices.

Subjects in all sessions were randomly selected using online recruiting software at the University of California, Santa Cruz from our pool of volunteers, undergraduates from all major disciplines. They were all inexperienced, i.e., had never participated in a prisoner's dilemma experiment in our lab. On arrival, subjects received written instructions (available in the online Appendix) which also were read out loud. Sessions lasted on average 75 minutes, subjects were paid 5 cents per point each period, and average earnings were roughly US$17.50 per subject.

[2] After normalizing, the payoff parameters used in the studies mentioned in the previous section mostly are in the neighborhood of our Easy parameters, and only a few are more challenging than our Mix parameters.

[3] A coding error garbled several periods in a single One-Shot session. The data analysis to follow drops these periods, but all results are robust to, instead, using the entire dataset or dropping the entire offending session.

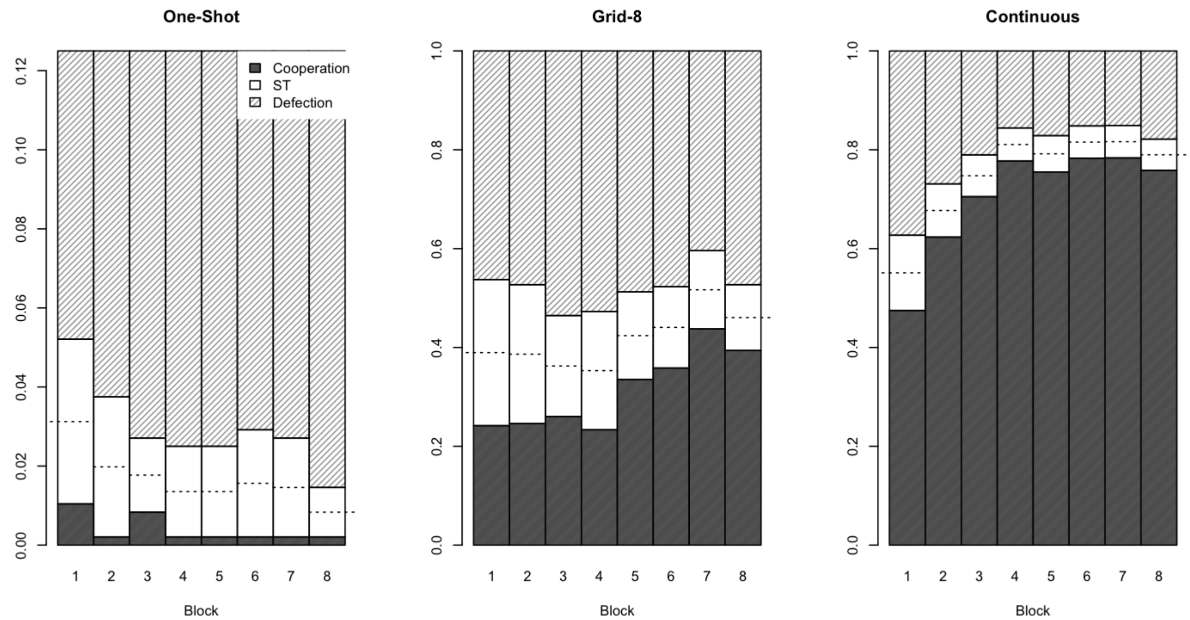

Figure 2: Outcomes over blocks by treatment.

III Main Results

To provide an overview, we compile the fraction ρ_{ipk} of time spent in each profile ρ by player i and her counterpart $j(i,p,k)$ in period p of session k. Due to the symmetry of the game, the four action profiles reduce to three player-pair profiles:

- Mutual Cooperation ($\rho = c$): Profile (A,A).
- Mutual Defection ($\rho = d$): Profile (B,B).
- Sucker-Temptation ($\rho = s$): Profile (B,A) or (A,B).

For example, if player 2 spent equal time in each of the four action profiles in period 3 of session 4, the player-pair profile data would be $c_{234} = d_{234} = 0.25$ and $s_{234} = 0.50$. By definition, $\rho_{ipk} \in [0,1]$ and $\sum_\rho \rho_{ipk} = c_{ipk} + d_{ipk} + s_{ipk} = 1$. Of course, $\rho_{ipk} = 0$ or 1 in the One-Shot treatment, and $\rho_{ipk} \in \{m/n : m = 0,1,2,...,n\}$ in the Grid-n treatment.

Figure 2 shows mean rates of the three player-pair profiles $\rho = c, d, s$ over successive 4-period blocks,

$$\rho_{bT} = \frac{\sum_{k \in T} \sum_{p \in b} \sum_i \rho_{ipk}}{\sum_\rho \sum_{k \in T} \sum_{p \in b} \sum_i \rho_{ipk}}. \tag{1}$$

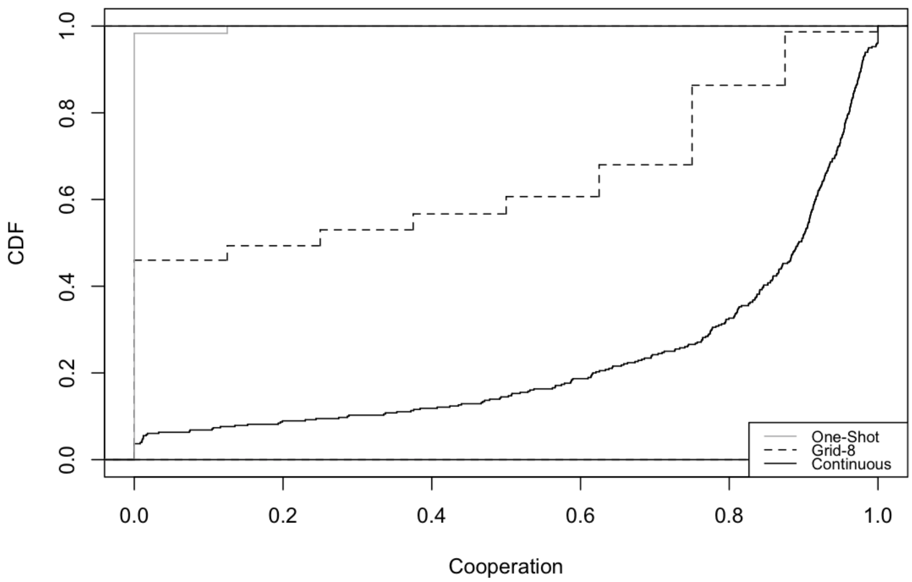

Figure 3: CDFs of individual subject median cooperation rates.

The randomized block design ensures that each block $b = 1, ..., 8$ includes an equal sample of each of the four parameter sets for each time treatment $T =$ One-Shot, Grid-8, Continuous.

Behavior seems fairly settled after block 3 (period 12). From this point onward, mean mutual cooperation rates are zero in One-Shot, but approach 80% in Continuous. The mutual cooperation rate in Grid-8 is intermediate at about 30 percent. Mutual defection rates have the opposite pattern, since mean s rates are low in One-Shot and Grid-8, and are even lower in Continuous.

Most previous authors report *individual* cooperation rates $\kappa = c + 0.5s$, which can be seen in Figure 2 as horizontal dotted lines that bisect the ST bar. Our subsequent analysis will focus on settled behavior; unless otherwise noted the data will be drawn from blocks 4-8 (periods 13-32). None of the broad conclusions is altered by including the noisier data from blocks 1-3, but in some cases the statistical significance is lower.

Cumulative distribution functions (CDFs) reveal heterogeneous behavior. For each subject in each time treatment, we take the median mutual cooperation rate and plot the resulting CDFs in Figure 3. In the Continuous treatment, this cooperation rate exceeds 80 percent for about two-thirds of the subjects, while in One-Shot it is zero for almost everyone. In Grid-8, a plurality (not quite a majority) of subjects have a median cooperation rate of zero, but a substantial minority (about a third) have rates of 75 percent or more.

Parameters	x	y	Continuous	Grid	One-Shot
Easy	14	4	0.931	0.750	0.000
			(0.014)	(0.066)	(0.000)
Mix-a	18	4	0.890	0.500	0.000
			(0.012)	(0.118)	(0.000)
Mix-b	14	8	0.905	0.000	0.000
			(0.013)	(0.028)	(0.000)
Hard	18	8	0.811	0.000	0.000
			(0.028)	(0.005)	(0.000)
All			0.893	0.250	0.000
			(0.009)	(0.105)	(0.000)

Table 2: Median cooperation rates (and bootstrapped standard errors).

For none of these treatments is behavior clustered symmetrically around the mean. The median therefore provides a more reliable measure of central tendency, and it will be our focus for the remainder of the data analysis. Table 2 shows the median cooperation rate in each treatment cell (again, for periods 13-32). Results are striking. For each parameter set, these rates are all zero in One-Shot, but in Continuous they range from 81 percent (in Hard) to over 93 percent (in Easy). The cooperation rates in Grid-8 are far more heterogeneous, ranging from zero (in Hard) to 75 percent (in Easy). Overall, as can be seen in the bottom row, there is a strong increase in cooperation as we move from One-Shot to Grid-8 to Continuous, and pairwise Mann-Whitney tests applied to by-subject median cooperation rates confirm this ordering at the one percent level.

To summarize,

Result 1 *Cooperation prevails in the Continuous treatment, is less than half as common in Grid-8 and is quite rare in One-Shot.*

Table 2 also suggests that the impact of parameters differs across the time treatments. In Grid-8, cooperation never takes hold when $y = 8$ (Mix-b and Hard) but it is substantial when $y = 4$. In this last case, the x parameter also seems to have an impact. Parameters have no visible impact in One-Shot and a relatively small impact in Continuous, probably because cooperation rates are already so extreme in those treatments.

To follow up on these impressions, we ran quantile regressions of the form

$$c_{ij} = \beta_0 + \beta_x X_{ij} + \beta_y Y_{ij} + \beta_{xy} X_{ij} \times Y_{ij} + \epsilon_{ij} \quad (2)$$

where c_{ij} is subject i's median rate of cooperation (over periods) under parameter set j; X and Y are indicator variables taking a value of 1 when x and y take their high values and ϵ_{ij} is a normally

Variable	Continuous	Grid
Intercept	0.919***	0.75***
	(0.018)	(0.052)
X	−0.032	−0.25**
	(0.021)	(0.119)
Y	−0.013	−0.75***
	(0.024)	(0.102)
$X \times Y$	−0.057	0.25*
	(0.043)	(0.149)

Table 3: Quantile regression coefficient estimates (and bootstrapped standard errors) for equation (2). One, two and three stars signify statistical significance at the ten, five and one percent levels.

distributed error term. Table 3 reports separate estimates for Continuous data and Grid data; there is insufficient variation in the One-Shot data to estimate the model. The intercept estimates the cooperation rate in the Easy treatment. Increasing either x or y does not significantly change cooperation rates in Continuous, but their joint effect (found by adding β_x, β_y and β_{xy}) in the Hard treatment is significant at the five percent level. In Grid-8 both parameters are highly significant, both statistically and economically.

Result 2 *Parameters have large negative effects on cooperation rates in the Grid-8 treatment, but have little or no effect in Continuous and One-Shot.*

A Behavior within Continuous Periods

What forces support the remarkably high rate of mutual cooperation in the Continuous treatment? Some clues can be gleaned from the trends within periods. The top panel of Figure 4 plots median[4] rates of mutual cooperation $c_{0.5}(t)$ at each second for each parameterization. The median initial rate $c_{0.5}(0)$ is zero due to the random assignments of initial actions $\rho_{ipk}(0)$, only a quarter of which are mutually cooperative. Strikingly, $c_{0.5}(t)$ rises to 100 percent by $t=$ 5 seconds, and remains there until only a few seconds remain. Then it falls rapidly, all the way to zero except in the Easy treatment.

The lower panel of 4 aggregates across parameter sets but shows other quantiles $c_Q(t)$. The behavior at $Q = 0.85$, the 85th percentile, is mutual cooperation at each second, and $c_{0.75}(t)$ also is 1.0 except during the first two seconds. Lower quantiles indicate that cooperation ceases in the last few seconds for the majority of players. The graph of $c_{0.15}(t)$ shows that the cooperation ceases for more than

[4]Mean rates are similar but less extreme—they rise more gradually, reach a lower plateau and begin to decline a few seconds earlier—because $c(t)$ is bounded above at 100 percent and choices are dichotomous.

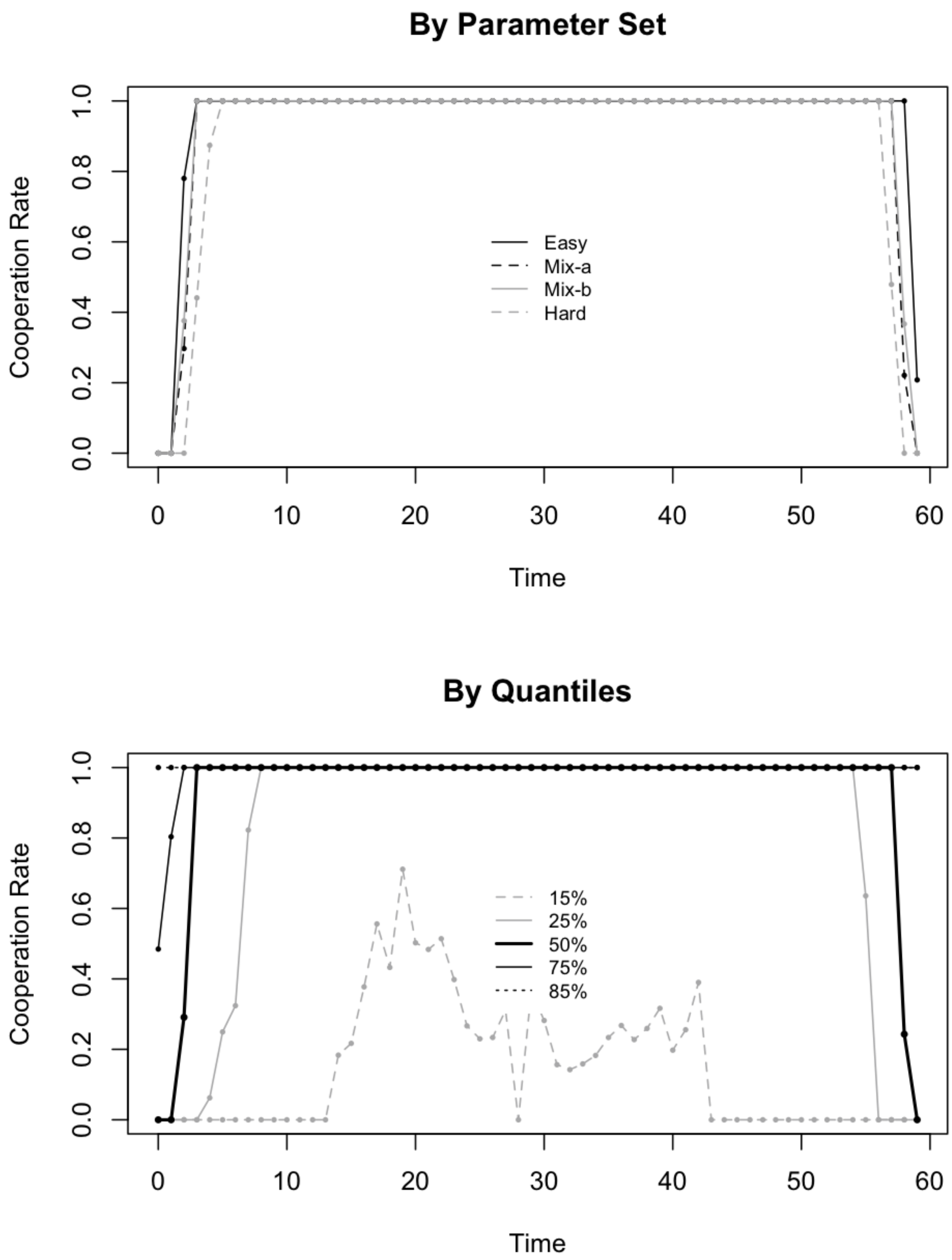

Figure 4: The top panel shows median rates of cooperation on a one-second grid in Continuous. The bottom panel shows cooperation rates at various quantiles of the distribution on the same one-second grid.

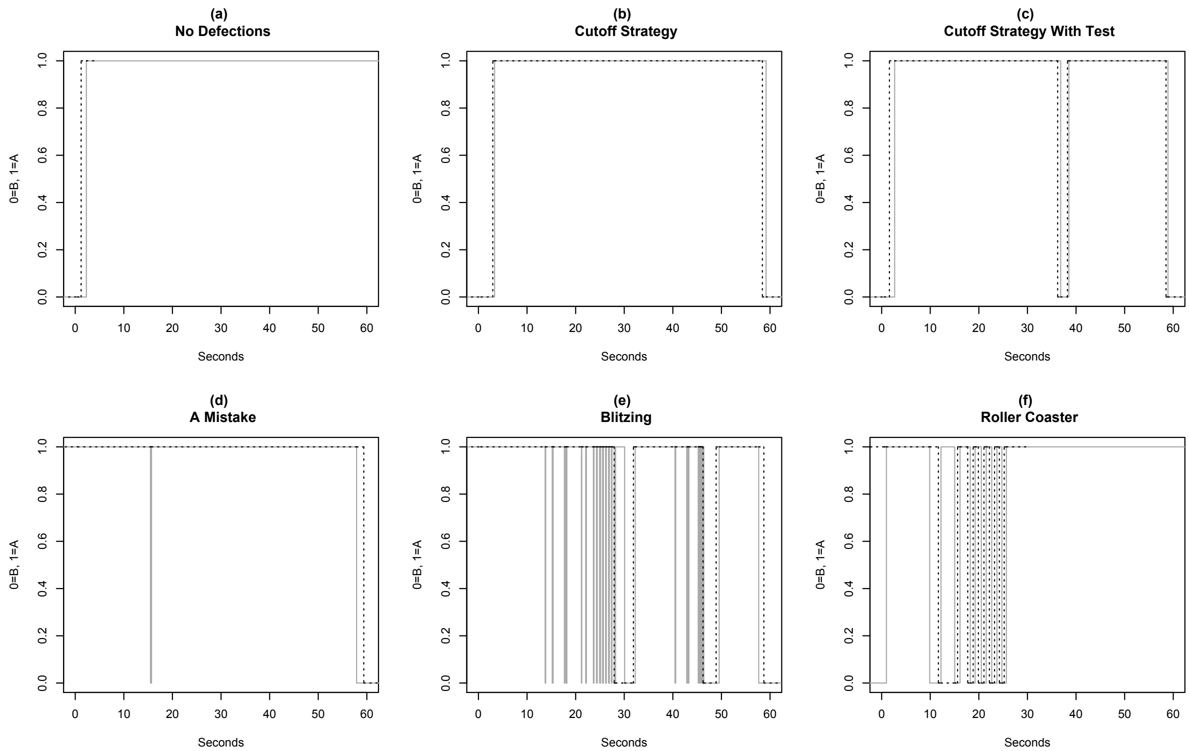

Figure 5: Examples of Within-Period Behavior

15 percent of the subjects when about 16 seconds remain (and doesn't begin for this fraction until 14 seconds have elapsed). The figure also shows that cooperation level falls below 75 percent only when 5 seconds remain, and below 50 percent only when one second remains.

Figure 5 collects examples of the underlying individual behavior. Panel (a) shows one pair of players randomly initialized at the mutual defection profile $\rho = d$. At about $t = 2$ seconds, one of the players (marked in Dotted heavy lines) switches to action A, putting the pair in Sucker-Temptation profile $\rho = s$. The other player (Gray line) follows about a second later, and the pair remains in profile c the rest of the period. Player Gray earns 9.95 points, very close to the full cooperation payoff of 10, and player Dot is close behind at 9.7 points. Similar behavior by another pair is shown in panel (b), except that Dot defects with about 3 seconds left in the period, and Gray follows within a second. Again, they earn just a bit under 10 points each (Gray 9.77, Dot 9.86).

Such behavior is quite typical. Ninety-five percent of player pairs initially assigned to profile $\rho = d$ or s sooner or later move to $\rho = c$, and the median time it takes to get there is only 2.89 seconds. As in panel (b), one of the players usually cuts off cooperation near the end of the period and the other player quickly follows.

# Breaks	Share	Cutoff time
0	0.479	60.0
1	0.305	57.6
2	0.089	57.9
3	0.057	57.3
4	0.022	57.6
5+	0.047	57.3

Table 4: Number of breaks from mutual cooperation, corresponding share of players, and corresponding median time (in seconds) of last mutual cooperation.

Table 4 shows that in the 20*38=760 player-periods observed in the settled final 5 blocks (20 periods) of the Continuous treatment, almost half of the observations include no defection from the mutual cooperation profile, and another 30 percent include only a single defection. About 9 percent defect twice, as in Panel (c) of Figure 5. In this example, from an initial d profile Player Dot switches within a couple of seconds to cooperate and player Gray quickly follows. As in the previous panels, profile c prevails for a time, but in this case, at about $t = 35$ seconds, Dot defects. Less than a second later, Gray follows suit. The resulting profile d doesn't last long. Dot soon switches back to A and Gray again follows quickly, restoring cooperation that lasts until almost the end of the period. With 2 or 3 seconds remaining (about the median, as shown in the last column of Table 4) Dot defects from mutual cooperation for the last time and Gray again quickly follows. In this period, Dot earned 9.50 and, despite being twice suckered, Gray earned 9.60.

B Cutoff Strategies, Simple and Augmented

Such behavior seems analogous to that observed in many repeated games settings, including Aoyagi and Frechette (2009), Dal Bo and Frechette (2010), and Jim Engle-Warnick and Robert L. Slonim (2006). To describe it in our setting, consider the following idealized strategy. Having achieved the mutual cooperation profile early in the period, a player using a *simple cutoff strategy* will unilaterally cut off cooperation only in the last few seconds but will match any prior defection as soon as possible. The median cutoff time in our data is with less than 3 seconds left, as shown in the last column of Table 4. The median response time to a defection is $\tau = 0.621$ seconds or about one percent of the period.[5]

Augmenting such simple cutoff strategies, players occasionally "test" the other player's reciprocity reflexes (or attention), as Dot does at $t = 35$ seconds in Panel (c). We refer to Dot's subsequent

[5]To be more precise, after filtering out the blitzes and roller coasters described below, the median duration of all profiles $\rho = s$ that follow $\rho = c$ is 0.621 and the mean is 0.91 seconds.

return to cooperation as "repentance," which is quickly accepted by Gray. Sometimes following a defection from cooperation, we see the other player offer "forgiveness" by switching briefly to action A; the defector usually follows quickly, restoring mutual cooperation. In an *augmented cutoff strategy*, a player will accept repentance, and may offer forgiveness, as long as his switch to A occurs before the chosen cutoff time and after sufficient time has passed that the other player does not profit from his defection.[6]

Almost 12 percent of the observations in Table 4 involve three or more defections, which suggests that not all observed behavior is consistent with these augmented cutoff strategies. Panel (d) of Figure 5 shows what seems to be a simple mistake: a player defected but returned almost instantly to the cooperation profile and remains there. In Panel (e), player Gray pulses briefly to defection at about $t = 13$ seconds, and returns before Dot reacts. This earns her a fraction of a penny, and she tries it again, and then many times again in the time interval (22, 28) seconds. At that point, Dot defects and Gray stops "blitzing." A similar blitzing episode plays out more rapidly in the time interval (45, 49). A final pattern we call "rollercoastering"—both players blitz as in panel (f), and earn average flow payoffs $(10 + x + y)/4$, which is only 1 to 3 points short of the cooperative rate of 10 points.

Since very few subjects consistently exhibit any such behavior, we regard these deviations from cutoff strategies mainly as either brief errors or inexpensive escapes from boredom. Perhaps a stronger piece of evidence is the fact that following defection from mutual cooperation prior to the last 10 seconds, the defector repents in 82 percent of cases, and the other player offers forgiveness in another 16 percent. In only 1.7 percent of these cases do players simply stay in defection, and they eventually resume mutual cooperation in 85 percent of cases. Thus most deviations from cooperation before the last few seconds are quite transitory.

Result 3 *Subjects in the Continuous treatment tend to use simple or augmented cutoff strategies, defecting near the end of the period. Earlier deviations from mutual cooperation are typically short lived and are followed by a return to mutual cooperation.*

Observed behavior in the Grid-8 treatment can also be interpreted as arising from cutoff strategies. Conditional on reaching a cooperative profile, the median Grid-8 subject defects during a cooperative profile only once.

Are cutoff strategies an expensive luxury? In the Continuous treatment, defections from mutual

[6] Our working paper discusses this last point at greater length. A figure analogous to Figure 6 below, but including earnings in the subsequent profile, shows that in fact such defections are not profitable on average.

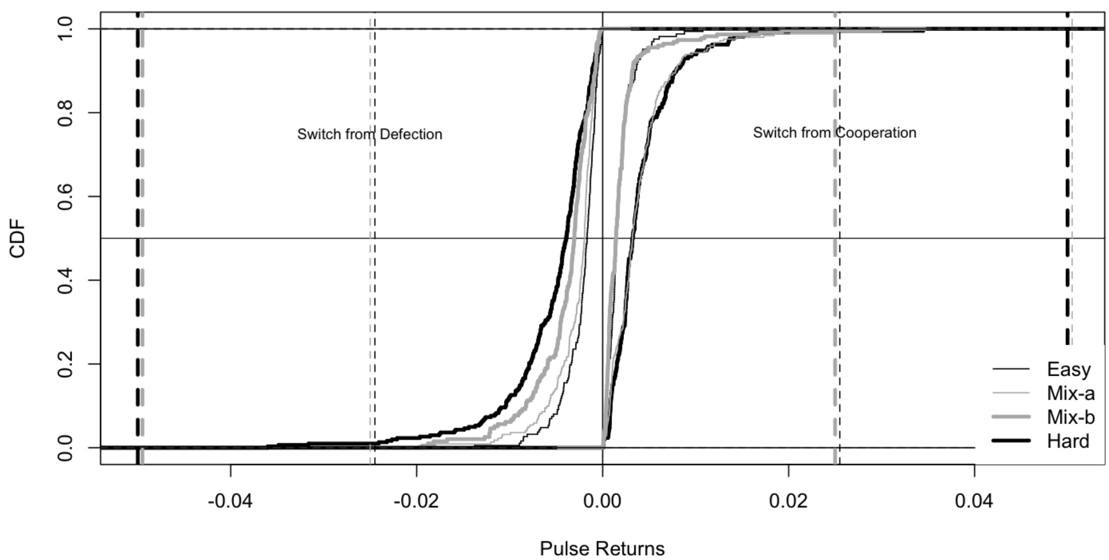

Figure 6: Empirical cumulative distributions of returns from switching from mutual defection and from mutual cooperation. Return is the flow payoff in dollars accumulated by the switcher until the next switch by either player, less the flow payoff that would have been accumulated in the original profile over the same time interval.

cooperation are usually matched so quickly that, prior to the last few seconds, they do not seem to improve expected earnings. Likewise, unilateral moves to cooperation can be matched (or retracted) so quickly that they cost very little relative to staying in d. Figure 6 plots CDFs of the direct payoff impact of switching from mutual cooperation and from mutual defection, relative to the counterfactual of staying put. It shows that the costs of initiating cooperation and the benefits of initiating defection are nearly always below one cent and do not much vary across parameters. The vertical dashed lines in the figure show that, by contrast, the payoff effects in Grid-8 are much larger and differ substantially across parameters.

IV Second Round Predictions and Experimental Results

The last section distilled from the data an intuitively appealing explanation for the high rates of mutual cooperation. In continuous time, the temptation to defect from $\rho = c$ vanishes when a player expects her counterparty to reciprocate immediately. Likewise, at $\rho = d$, signalling a willingness to cooperate has negligible opportunity cost. Augmented cutoff strategies therefore are quite economical. They seem quite prevalent in our data, and clearly are capable of supporting very high rates of cooperation. In our Grid-8 treatment, and in previous experiments with the finitely repeated prisoner's dilemma, the same forces are attenuated but still might support the observed moderate rates of cooperation and heterogeneity across parameter sets.

Intuitive appeal is a good start, but the explanation begs several questions. Why don't the cutoff strategies unravel? Could they constitute some sort of equilibrium? Does the explanation have any testable implications? Are there connections to existing theory?

Appendix A presents a mathematical model that is intended to answer these questions. Here we discuss the underlying assumptions, sketch the logic, and compare the model's predictions to the the data.

The model describes strategic behavior in a continuous time interval normalized to [0,1]. To analyze the impact of reaction lags, it assumes that all players share minimal reaction time $\tau \geq 0$, and restricts the strategy space of each player to the one-parameter family of simple cutoff strategies.

An informal theoretical justification for the restricted strategy space begins with Simon and Stinchcombe (1989), who obtain their equilibrium with $\tau = 0$ via iterated deletion of weakly dominated strategies in the full strategy space. We described our augmented cutoff strategies in such a way that, when τ is negligible, they dominate all the other strategies that we detected. The augmentation "testing" looks for exploitable lags in the opponent's strategy, and the augmentations "repentance" and "forgiveness" render the opponent's defection from $\rho = c$ unprofitable by requiring sufficient lags before returning to action A. Deleting all other strategies in the first iteration, we then look among augmented cutoff strategies for dominance relations. Although inexpensive when τ is small, both testing and blitzing clearly lose money against a simple cutoff strategy. Also, against any cutoff strategy, forgiveness is redundant. Thus in the second iteration we eliminate the augmentations, leaving us with the desired strategy space of simple cutoff strategies.

Some readers may find an empirical justification more convincing. Early cooperation and simple cutoff strategies describe the majority of our data; subjects almost always quickly achieve cooperation and defect no more than once in nearly 80 percent of cases. A final justification is that the restricted strategy space allows us to focus cleanly on the central question of unravelling.

Radner (1989) simply postulates simple cutoff strategies, and shows that it is almost dominant to wait until the end before unilaterally cutting off mutual cooperation. To see that this key insight remains valid in our continuous time setting, consider the following thought experiment.

Suppose that your opponent plays a simple cutoff strategy $K(u)$ with cutoff time $u \in (\tau, 1)$. Your best response, of course, is to play $K(s)$ with $s = u - \tau$, but suppose that you instead cut off cooperation too early, at time $s = u - z > 0$, with $z \geq \tau$. Relative to the best response, you lose $(10 - y)(z - \tau)$ because over a time interval of length $z - \tau$ you get the punishment payoff y instead of the cooperation payoff 10. This loss represents a substantial fraction of potential earnings when z is substantially greater than τ.

On the other hand, suppose that you plan to cut off cooperation too late, say at time $s = u + z \leq 1$ for some $z \geq 0$. Then relative to the best response you earn the sucker payoff 0 instead of the defection payoff y until you react to your opponent's defection, and you also forego the temptation payoff. Hence your loss is $(x + y - 10)\tau$. Crucially, this loss is independent of z; it depends only on the reaction speed and the payoff parameters. It follows that, no matter which cutoff time u your opponent selects, it is an ϵ-best response to set $s = 1$ as your cutoff time, for any $\epsilon \geq (x + y - 10)\tau$. That is, delaying your cutoff until the end of the period is nearly a dominant strategy.

Appendix A extends this sort of argument to the case where a player is uncertain of his opponent's cutoff time u but knows the distribution from which it is drawn. It shows that 100% cooperation is the unique outcome that survives in the limit as $\tau \to 0$. Thus Radner's insight allows us to replicate Simon and Stinchcombe's result in the relevant limit. Of course, at the other limit where $\tau \geq 1$, the game reduces to one-shot, and there the argument shows that no cooperation is the unique equilibrium outcome.

Appendix A then looks for ϵ-equilibria when τ is fixed at a small but positive value. To leverage the insight that waiting until the end is nearly dominant, we impose the condition (a) $\epsilon \geq (x+y-10)\tau$. To keep the equilibrium set from getting too large, we impose a second condition (b) strategies $K(s)$ that have lower expected profit than simply setting $s = 1$ are excluded from the equilibrium distribution.

Any ϵ-equilibrium consistent with conditions (a) and (b) is described by a distribution $F(s)$ of cutoff times between $s = 1$ and some earliest cutoff time $s_L < 1$. Appendix A notes that it is realistic empirically and sensible theoretically to assume that F is not negatively skewed, and to assume that (except perhaps for mass points at $s = 1$ and $s = 0$) it has a density f. For this case it derives the lower bound

$$s_L = 1 - \frac{2x}{10 - y}\tau \tag{3}$$

on cutoff times. Under the stronger assumption that cutoff times are uniformly distributed on $[s_L, 1]$ with $s_L > 0$, the Appendix derives the prediction that the median overall fraction of cooperation

Parameters	x	y	Predicted	Cooperation Median Rate	Final Time
				Continuous	
Easy	14	4	0.967	0.931	0.994
				(0.014)	(0.009)
Mixa	18	4	0.958	0.890	0.973
				(0.012)	(0.007)
Mixb	14	8	0.901	0.905	0.974
				(0.013)	(0.006)
Hard	18	8	0.873	0.811	0.959
				(0.028)	(0.006)
				Grid-8	
Easy	14	4	0.588	0.750	0.750
				(0.066)	(0.054)
Mixa	18	4	0.470	0.500	0.625
				(0.118)	(0.092)
Mixb	14	8	0.000	0.000	0.000
				(0.028)	(0.032)
Hard	18	8	0.000	0.000	0.000
				(0.005)	(0.006)

Table 5: Rates of cooperation predicted by equation (4), observed median rates and observed final times of mutual cooperation (with bootstrapped standard errors).

will be

$$c_{0.5} = 1 - \frac{\sqrt{2}x}{10 - y}\tau. \tag{4}$$

The Appendix also shows that the predicted level of cooperation is arguably zero when the expression for s_L is negative in (3), and is surely zero when the expression for $c_{0.5}$ is negative in (4).

In our Continuous treatment, τ depends on subjects' endogenous reaction time and we estimate it to be 0.01 minutes. With τ that small, equations (3-4) predict that cutoff times occur very near the end of the period for all (x, y) parameters that we used. The intuition is that the temptation to defect and the risks from suffering defection are quite small in this case. By contrast, in the Grid-8 treatment, τ is exogenous: subjects are forced to wait $\tau = 0.125$ of the period to react to a unilateral defection by a counterpart. The equations then predict considerably earlier cutoff times that vary substantially with the (x, y) parameters. Of course, with $\tau = 1.0$ in the One-shot treatment, the equations give negative values and the prediction is no cooperation.

These predictions match well the patterns observed in the data. Table 5 calculates (4) under each parameter set using $\tau = 0.01$ and $\tau = 1/8$ for Continuous and Grid-8 respectively. It also

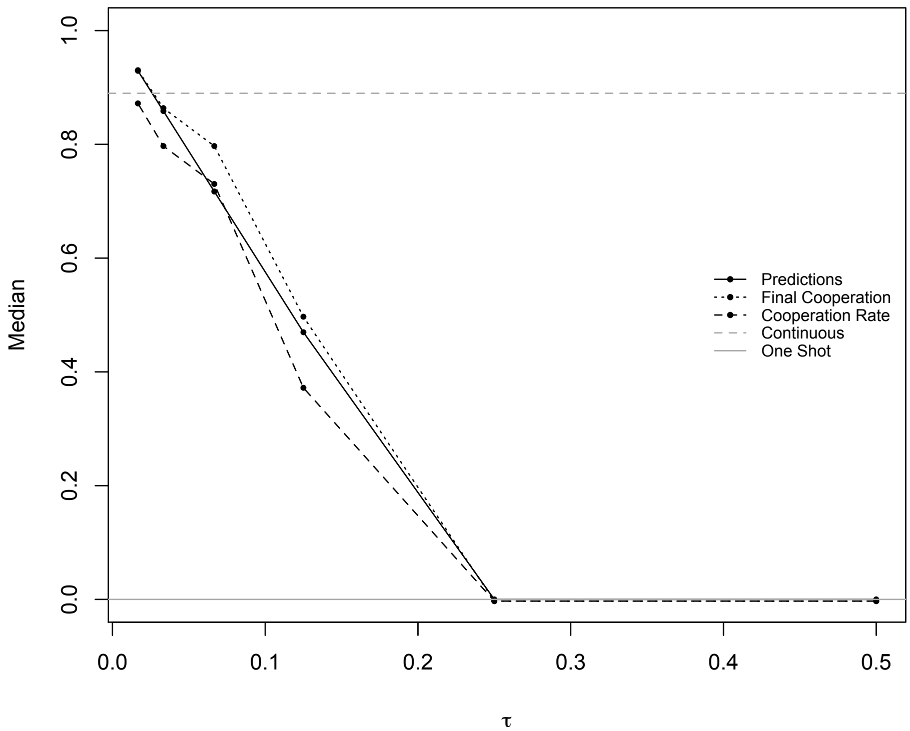

Figure 7: Grid-n predictions and data. Predictions are from equation (4) with $x = 18, y = 4$ and $\tau = 1/n$ for $n = 2, 4, 8, 15, 30, 60$. Data are median times of final mutual cooperation and median rates of mutual cooperation. Horizontal lines show Mix-a data for One-Shot and Continuous sessions.

reproduces median final cooperation times (a literal interpretation of the prediction) and median overall cooperation rates for Continuous and Grid-8. The predictions capture the comparative statics reported in Result 2—the impact of parameters (especially of y) is large in Grid-8 but is modest in Continuous. Even the point predictions are quite close to the data, except that the Grid-8 / Easy observations are about 16 percentage points (a bit more than one subperiod) above the prediction.

Result 4 *Parameter effects in Continuous and Grid-8 are rather well explained by a class of cutoff epsilon equilibria.*

A Grid-n Sessions and Results

Equilibrium cutoff strategies governed by reaction lags $\tau > 0$ seem to account fairly well for the data so far. Of course, the true test of any explanation lies in its excess predictive power—in verified implications beyond the facts it was constructed to explain. The key prediction of the model is that cooperation rates rise from zero (observed in one-shot) to about 90% (observed in Continuous) as the forced reaction lag, τ, shrinks to its minimal feasible value for humans. To better test this prediction, we conducted additional laboratory sessions that exogenously controlled τ at multiple levels. In a Grid-n treatment, the number n of subperiods exogenously imposes the minimum reaction time $\tau = 1/n$. Equation (4) predicts a monotonic (indeed linear) decrease in cooperation as τ rises from zero.

To illustrate, Figure 7 plots the theoretical predictions for Mix-a parameters and $\tau = 1/n$ as a solid black lines connecting the dots for $n = 60, 30, 15, 8, 4, 2$ subperiods. For reference, the horizontal gray lines plot median rates of cooperation observed in Mix-a Continuous and One-Shot ($\tau = 1$) periods. Epsilon cutoff equilibria predict that as the grid gets coarser, cutoff times fall from nearly continuous levels at $n = 60$ to one shot levels at $n \leq 4$.

We ran 3 sessions of Grid-n, each lasting 36 periods and using only the Mix-a parameters. In each of three 12 period blocks, we ran each n twice in consecutive periods and varied n in the sequence Incr = (2,4,8,15,30,60) or Decr = (60,30,15,8,4,2) or Random. (In two sessions the blocks were sequenced Incr-Decr-Random and in the other session the blocks were sequenced Decr-Incr-Random.) Note that the within session variation of n allows us to observe each subject's behavior at each value of τ, generating a particularly stringent test. To focus on settled behavior, we once again examine data after the first 12 periods (after subjects have experienced each n twice); the results are similar (though a bit noisier) if we include all data.

The median subject in the Grid-n sessions employed a cutoff strategy, departing from the c profile only once per period, just as in Continuous and Grid-8. As predicted, the observed timing of cutoffs depended negatively on n. Figure 7 plots the median final cooperation time and the median rate of cooperation as a function of $\tau = 1/n$. When periods are divided into 2 or 4 subperiods ($\tau = 0.5$ or 0.25) cooperation never gets off the ground. However as the grid gets finer and reaction lags become smaller, cooperation rates start to rise towards Continuous levels at $n = 60$ ($\tau = 0.016$). Most strikingly, the observed median rates of cooperation and cutoff times tightly bracket the point predictions of the model.

Result 5 *As the grid becomes finer in Grid-n sessions, the threshold times and cooperation rates approach those observed in Continuous. Moreover, both median cutoff times and cooperation rates fall nearly linearly with τ and closely track the point predictions given by equation (4).*

V Discussion

Our principal findings can be summarized briefly. First and foremost, in the Continuous time treatment, we found remarkably high levels of mutual cooperation in all four parameterizations of the prisoner's dilemma. Even with "hard" parameters (maximal temptation and minimal efficiency loss), the all-cooperate profile was played 81 percent of the time by the median pair of subjects in later periods. The other parameterizations led to median mutual cooperation rates of 89 to over 93 percent. By contrast, in the Grid-8 control treatment, with rapid repeat pairings over 8 subperiods, defection was more prevalent than cooperation, and cooperation was rare in the One-Shot control treatment.

Second, the parameterization had a considerably stronger impact (in the predicted direction) in the Grid-8 treatment than in Continuous time. In the One-Shot treatment, parameters had negligible impact because cooperation was always rare.

Third, within the 60 second Continuous periods, median rates of cooperation quickly reached 100 percent and remained there until the last few seconds of the period, when they dropped off abruptly.

Inspired by a strand of existing theoretical literature, we developed a mathematical model tuned to our experimental setting, and obtained predictions of how cooperation rates respond to adjustment lags and to payoff parameters. These predictions accounted well for the Continuous, Grid-8 and (trivially) One-Shot data. They also nicely explained a set of second-round data from Grid-n sessions, which varied the number of subperiods from 2 to 60. The theory thus explains defection in one shot games, cooperation in continuous time and intermediate results on the path between the two.

The underlying intuition is simple. When your opponent can react very quickly, defecting from mutual cooperation is likely to earn you the temptation payoff only briefly and may cost you the cooperation payoff for the rest of the period. Likewise, briefly switching from mutual defection is cheap for you, and may catalyze a sustained move to a higher payoff profile. Hence rapid reactions tend to stabilize mutual cooperation and destabilize mutual defection, at least until late in the period.

Conventional wisdom is that cooperation is susceptible to unravelling when the time horizon is finite. Sufficiently late in the period, your incentive is to defect. Your opponent's incentive is to defect before you do, and yours is to defect before she does, so backward induction might seem to unravel cooperation. However, experimentalists since Selten and Stoecker (1986) have found that, even given good learning opportunities, unravelling typically doesn't actually go very far.

Our work shows that the unravelling argument rests on a knife edge when players can react quickly. The faster she can react, the less incentive you have to pre-empt your opponent, and the earlier you defect, the more you stand to lose from preempting her. Extending the ideas of Radner (1986) and Simon and Stinchcombe (1989), we took some first steps towards formalizing this argument in terms of epsilon equilibrium. Unravelling is quite limited when players are willing to sacrifice a small part of their potential payoff and they can react sufficiently rapidly. The faster they can react, the smaller the potential sacrifice and the greater the level of cooperation.

Our results set the stage for new theoretical advances. Additional insights may be gained by formalizing the epsilon equilibrium argument more fully than in Appendix A, or by considering alternative approaches. More broadly, as noted in Section I, strategic interaction in continuous time may be affected by asynchronicity and by the fact that the real numbers are not well-ordered. These features of continuous time play only a minor part in Appendix A, but new theoretical analysis may find more substantive roles for them.

Much empirical work also remains. Future laboratory studies could test robustness of our predictions to different payoff parameters, to longer or shorter periods or more periods, and to variations on near continuous time, e.g., alternating moves, or perceptible lags in implementing action switches, or temporary action lock-ins. More generally, further studies might examine whether continuous time can ever reduce efficiency,[7] and seek additional practical insights into the forces that encourage or discourage efficient cooperation.

[7]See Jenny Anderson (2009) for a possible practical example. In terms of matrix games, consider $x > 20$ in Table 1. Unlike in a true prisoner's dilemma games, the sucker-temptation profile is now efficient. In discrete time, efficiency might be achieved by alternating the two ST cells, and such coordination might be more difficult in continuous time.

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Appendix A

This Appendix derives equations (3-4) of the text, and tightens connections to the theoretical work of Simon and Stinchcombe (1989) and Radner (1986).

We consider continuous play of the Prisoner's dilemma game with flow payoff shown in Table I. Recall the restriction $20 > x > 10 > y > 0$ on the temptation and punishment parameters. The time interval is normalized to [0,1]; in terms of the experiment, we analyze a single period measured in minutes. Recall that the median reaction time τ in the experiment was approximately 0.67 seconds, or 0.01 minutes.

Given a known reaction time $\tau > 0$, a *simple cutoff strategy*, denoted $K(s)$, specifies a time $s \in [0,1)$ with unconditional defection (choosing action B) at all later times $t \geq s \in [0,1]$ and conditional cooperation at all earlier times $t < s$. Conditional cooperation means that you choose action A until the other player first chooses B, and then you switch to B as soon as you can react, i.e., with lag τ, and remain at B thereafter. To focus on the main theoretical issues, we assume away the process of initially reaching the mutual cooperation profile; i.e., we assume (counterfactually in most periods) that both players initially play A.

Suppose that you face a player following strategy $K(u)$ whose cutoff time $u \in [0,1]$ is drawn randomly from a known distribution $F(u)$. Since choice is asynchronous, and there is no way for players to coordinate on specific times $t \in (0,1)$, we can assume safely that the distribution has a smooth density $f(u)$ on (0,1).[8] If you follow cutoff strategy $K(s)$ then your payoff is $[10u + 0\tau + y(1-u-\tau)]$ if $u < s - \tau$, or is $[10s + x\tau + y(1-s-\tau)]$ if $u > s + \tau$. For $u \in [s-\tau, s]$ your payoff is $10u + 0(s-u) + y(1-s)$, and for $u \in [s, s+\tau]$ it is $10s + x(u-s) + y(1-u)$.

Thus by cutting off cooperation at time $s \leq 1 - \tau$, your expected payoff is

$$\begin{aligned}
\pi(s|F) \;=\; & \int_0^{s-\tau} [10u + y(1-u-\tau)]f(u)du + \int_{s-\tau}^{s} [10u + y(1-s)]f(u)du \\
& + \int_s^{s+\tau} [10s + x(u-s) + y(1-u)]f(u)du + \int_{s+\tau}^{1} [10s + x\tau + y(1-s-\tau)]f(u)du
\end{aligned} \tag{5}$$

If instead you chose $s = 1$, your payoff would be the same over the region $u < s$, but would differ over the region $s < u < 1$. Here, instead of the integrands in (5), you would get $10u + y(1-u-\tau)$.

[8] Of course, players can coordinate on the endpoints $t = 0, 1$, so mass points may appear there. The reader can check that the results obtained below still hold even when there is a "Dirac delta" in the density f at $t = 1$ and/or $t = 0$.

Figure 8: Plot of $\psi(s)$ and the determination of s_L.

Thus the difference in expected payoff is

$$\begin{aligned}\pi(s|F) \quad - \quad \pi(1|F) &= \int_s^{s+\tau} [10(s-u) + x(u-s) + y\tau] f(u)du + \int_{s+\tau}^1 [10(s-u) + x\tau - y(s-u)] f(u)du \\ &= -(10-y)\int_s^1 (u-s)f(u)du + x\tau[1-F(s)] + (x-y)\int_s^{s+\tau}(u-s-\tau)f(u)du.\end{aligned} \tag{6}$$

In the limit as τ goes to zero, the first term in the last line of (6) remains negative and the other two terms go to zero for all $s < 1$. Thus, within the set of simple cutoff strategies, never defecting first is weakly dominant in the limit, consistent with the results of Simon and Stinchcombe.

Equilibrium Predictions

The next task is to characterize a set of ϵ-equilibrium distributions $F(u)$ of cutoff times when τ is positive but small. The definitive feature of this equlibrium set is that all of the cutoff times in the support[9] of F yield at least as much expected payoff as $s = 1$, i.e., as never defecting first. In view of the limiting result as $\tau \to 0$, we will call such distributions Nearly Dominant Cutoff Equilibria, or NDCE. They describe ϵ-equilibrium for any $\epsilon \geq \epsilon_F \equiv sup_s \pi(s|F) - \pi(1|F)$. The paragraphs in the text preceding equation (3) establish the uniform bound $\epsilon_F \leq (x + y - 10)\tau$. Thus any NDCE is an $(x + y - 10)\tau$-equilibrium, and also is an ϵ-equilibrium for somewhat smaller values of ϵ.

The support of an NDCE distribution F necessarily includes $s = 1$, and so is contained in an interval of the form $[s_L, 1]$, where $s_L < 1$ is the smallest solution to $\pi(s_L|F) = \pi(1|F)$. Early cutoff times $s < s_L$ will yield a lesser expected payoff and therefore will not be part of this ϵ-equilibrium.

We now compute s_L. First note that in the last term in the last line of (6), the integrand expression $(u-s-\tau)$ is negative and has absolute value less than τ over the relevant interval, which has length τ. Thus this last term is negative and is second order small in τ. We shall ignore this term below in order to obtain a simple and conservative estimate of s_L.

We have $F(s) = 0$ for $s \leq s_L$, so over this range we can rewrite the payoff advantage (6) as

$$\pi(s) - \pi(1) = -(10-y)\psi(s|F) + x\tau, \tag{7}$$

[9] Recall that $SuppF$ is the smallest closed set containing all points of increase of F, i.e., containing all points with positive density or positive mass. Given some $\epsilon > 0$, we say that a distribution F is an *ϵ-equilibrium* if for every $s \in SuppF$, we have $\pi(s|F) \geq \max_{x\in[0,1]} \pi(x|F) - \epsilon$, i.e., if every cutoff actually used yields near-maximal expected payoff.

where $\psi(s|F) = \int_s^1 (u-s)f(u)du \in (0, 1-s)$ has value $\bar{u}$ at $s = 0$ and is a decreasing function with derivative $F(s) - 1$. The payoff advantage is clearly negative for such s as long as

$$\psi(s) > \frac{x\tau}{10-y},$$

i.e., for $s < \psi^{-1}(\frac{x\tau}{10-y})$. For $s \in [0, s_L)$, the function ψ has slope -1 and hence is simply $\psi(s) = \psi(0) - s$. As illustrated in Figure 8, we can now characterize the lower support point s_L by the equation $\pi(s_L) - \pi(1) = 0$ or, using (7),

$$\frac{x\tau}{10-y} = \psi(s_L) = \psi(0) - s_L = \bar{u} - s_L. \tag{8}$$

Of course, the shape of the distribution F determines the position of $\bar{u}$ within $[s_L, 1]$. We have $\bar{u} = \alpha s_L + (1-\alpha)1$, where $\alpha \in (0, 0.5)$ represents an upward skew and $\alpha \in (0.5, 1)$ represents a downward skew. Substituting $\bar{u} - s_L = (1-\alpha)(1-s_L)$ into (8) and solving for s_L, we obtain

$$s_L = 1 - \frac{x\tau}{(1-\alpha)(10-y)}. \tag{9}$$

As asserted in the text, equation (9) reduces to (3) in the unskewed case $\alpha = 0.5$ and this case represents a lower bound for $\alpha \in (0, 0.5)$, the upward skewed case.

We regard downward skew as less relevant, theoretically as well as empirically. Theoretically, the Radner argument shows that cutoffs below the midpoint are less robust to deviations than those above the midpoint. Empirically, we find that observed cutoffs have considerable upward skew. Nevertheless, the downward skew case is pedagogically useful. For any fixed positive values of τ, x, y, one can find a value of α sufficiently close to 1 so that the expression for s_L is zero (or negative). The interpretation is unravelling: when the distribution is sufficiently downward skewed, it pays to cut off cooperation earlier than the modal time, and so cutoff times unravel all the way down to zero, as in the traditional analysis.

The next task is to predict the median fraction $c_{0.5}$ of time in mutual cooperation. Given the [0,1] time normalization, and the assumption of simple cutoff strategies initially in mutual cooperation, the fraction of cooperation coincides with the time of first defection. Hence it is the minimum y of two independent draws from the NDCE distribution F. Classic work on order statistics, e.g., Robert V. Hogg and Allen Craig (1970), shows that this minimum has density $g(y) = 2(1 - F(y))f(y)$. The median value $m = c_{0.5}$ of y therefore is the root of the equation $0.5 = \int_0^m g(y)dy$. Inserting the expression for g and dividing by 2, the equation becomes

$$0.25 = \int_0^m f(y)dy - \int_0^m F(y)f(y)dy = F(m) - 0.5[F(m)]^2.$$

Multiplying through by 4, we see that $z = F(m)$ satifies the quadratic equation $2z^2 - 4z + 1 = 0$, whose relevant root is $z = \frac{4-\sqrt{16-8}}{4} = 1 - \frac{1}{\sqrt{2}}$. Hence the median time of first defection, and thus the predicted median mutual cooperation rate, is

$$c_{0.5} = m = F^{-1}(z), \text{ where } 1 - z = \frac{1}{\sqrt{2}}. \tag{10}$$

When F is the uniform distribution on the interval $[s_L, 1]$ for $s_L \in (0, 1)$, then m can be written out explicitly. In this case, $z = F(m) = \frac{m-s_L}{1-s_L}$. Multiplying by the denominator $1-s_L$ and recalling from (3) that s_L takes the form $1 - a\tau$, we obtain

$$m = z(1 - s_L) + s_L = za\tau + 1 - a\tau = 1 - (1 - z)a\tau = 1 - \frac{a\tau}{\sqrt{2}}.$$

Inserting $a = \frac{2x}{10-y}$ we obtain equation (4).

Thus, as a function of reaction time τ, the fraction of time in mutual cooperation is predicted to be linearly decreasing, with slope $-\frac{a}{\sqrt{2}}$, with a the given increasing function of the parameters x, y. For example, for the Mix-a parameters $x = 18, y = 4$, the slope is predicted to be $-\frac{36}{6\sqrt{2}} \approx -4.24$.

Discrete Time Predictions

To predict mutual cooperation in Grid-n, we can just set $\tau = 1/n$, ignoring the subtle distinctions between discrete time and continuous time with reaction lags. Alternatively, we can translate Radner's discrete time results directly into our setting. His payoff parameters a, b, c in our notation are respectively $y, x - y, 10 - y$, and his trigger strategies C_j are our cutoff strategies $K(j/n)$. Radner's equation (16) shows that $\hat{L} = \frac{a+b-c}{T} = \frac{x+y-10}{n}$ is the maximum loss of $K(1)$ against $K(j/n)$ for any $j = 0, ..., n$, relative to the ex post best response. Note that substituting $1/n = \tau$ in $\hat{L}$ yields exactly the same expression for ϵ as in the paragraph preceding equation (3).

We now seek a bound on the first cutoff period j_L, and check whether $s_L \approx j_L/n$. Let F be the (discrete) uniform distribution over the $n-j_L$ cutoff times $\{j/n : j = j_L, ..., n-1, n\}$. The expected payoff for waiting until the end (i.e., playing strategy $K(n/n) = K(1)$) is

$$\pi(1|F) = 10 - [(10-0)\frac{n-j_L-1}{n-j_L} + (10-y)(n-j_L)/2]/n, \tag{11}$$

because it earns ($1/n$ times) 10 in every subperiod until the other player cuts off cooperation, at which point it earns 0 instead of 10 (in each realization except $j = n$) and thereafter earns y instead of 10 (on average in half of the $(n-j_L)$ realizations). Against the same uniform distribution, strategy $K(j_L/n)$ obtains payoff

$$\pi(j_L/n|F) = 10 - [(10-x)\frac{n-j_L-1}{n-j_L} + (10-y)(n-j_L-1+\frac{1}{n-j_L})]/n, \tag{12}$$

because, except in one realization ($j = j_L$, when it earns y), this strategy earns the temptation payoff x instead of 10 in one period, and it always earns the punishment payoff y instead of 10 in period $j_L + 1$ and after.

One obtains j_L by equating (11) to (12) and solving. Straightforward algebra yields

$$x\frac{n-j_L-1}{n-j_L} = (10-y)([n-j_L]/2 - 1 + \frac{1}{n-j_L}). \tag{13}$$

Using the approximations, valid for large $n - j_L$, that $\frac{n-j_L-1}{n-j_L} \approx 1$ and $[n-j_L]/2 - 1 + \frac{1}{n-j_L} \approx [n-j_L]/2$, we can solve (13) to obtain $j_L \approx n - \frac{2x}{10-y}$ and $s_L = j_L/n \approx 1 - \frac{2x}{10-y}/n$. The last expression is exactly the same as the continuous time expression (3) when $\tau = 1/n$.

Predictions when $s_L \leq 0$

The expressions obtained so far are valid if $s_L \geq 0$ in (3). When $s_L \leq 0$, the expressions can be extended mechanically by treating s_L in (3) as a latent variable, observed as 0 when the expression is negative. The effect on (4) turns out to be simple—one just trucates (4) below at zero. To confirm this, note that in the present case the distribution F is a specific mixture of $U[0,1]$ and the degenerate (Heaviside) distribution at 0. The mixture weight on the uniform distribution is $w = \frac{10-y}{2x\tau} < 1$ since that is the fraction of the line segment $[s_L, 1]$ that lies above zero when $s_L < 0$ in (3). Hence $F(y) = 1 - w + wy$. Applying (10), we solve $z = F(m)$ to obtain $m = \frac{z+w-1}{w} = \frac{w-1/\sqrt{2}}{w} = 1 - 1/(\sqrt{2}w) = 1 - \frac{\sqrt{2}x\tau}{10-y}$ when this expression is positive, and otherwise $m = 0$.

On the other hand, $s_L \leq 0$ overstretches the concept of NDCE, because in this case, $K(0)$ (never cooperating) yields a higher expected payoff than $K(1)$ (never being the first to cut off cooperation).

Thus condition (b) in the third paragraph of Section IV is violated so no NDCE exists. Little or no cooperation therefore should be expected in this case, which is potentially relevant for Grid-8 when $x = 18$ and for Grid-4 and Grid-2 with all parameter values. However, it turns out that both NDCE conventions yield the same numerical predictions in all cases we study.