

Equations for Competitive Markets

Linear Demand: $q_d = a - bp$ **Linear Supply:** $q_s = x + yp$

Log-linear demand: $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$ **Log-Linear Supply:** $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$

Total Surplus=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost

Total Cost=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost

Quantity Tax (tax per unit): $p_d = p_s + t$; **Value Tax** (tax on percentage spent): $p_d = (1 + t)p_s$

Price Elasticity of Demand: $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p} \frac{p}{D}$; If $|\varepsilon| > 1$ then curve is elastic

Tax Incidence Formula: $p_s(t) = p^* - \frac{t|D'|}{S' + |D'|}$; $p_d = p^* + \frac{tS'}{S' + |D'|}$; If ε_d is constant: $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d| + \varepsilon_s}$

Equations for Consumer Choice and Demand

Marginal Utility: $MU_i = \frac{\partial u}{\partial x_i}$; **Marginal Rate of Substitution:** $MRS_{ij} = \frac{MU_j}{MU_i}$ and at interior optimum $= \frac{p_i}{p_j}$

Perfect Substitutes: $u(x_1, x_2) = x_1 + cx_2$; **Cobb-Douglas:** $u(x_1, x_2) = \ln(x_1) + c \ln(x_2)$

CES Utility: $u(x_1, x_2) = \frac{1}{\rho} \ln(x_1^\rho + x_2^\rho)$; $\rho \in (-\infty, 1]$; **Quasilinear:** $u(x_0, x_1) = x_0 + g(x_1)$

Marshallian Demand: $x_i^*(\mathbf{p}, m) : \mathcal{L} = u(x_1, x_2) + \lambda(m - \mathbf{p} \cdot \mathbf{x})$

Dual Problem; Hicksian Demand: $h_i^*(\mathbf{p}, u_0) : \min \mathbf{p} \cdot \mathbf{x} \text{ s.t. } u(\mathbf{x}) \geq u_0$

Roy's Identity: $x_i^*(\mathbf{p}, m) = -\frac{\partial v}{\partial p_i} / \frac{\partial v}{\partial m}$; **Shepard's Lemma:** $h_i^*(\mathbf{p}, u) = \frac{\partial e(\mathbf{p}, u)}{\partial p_i}$

Slutsky Equation: $\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, m))}{\partial p_i} - \frac{\partial x_i^*(\mathbf{p}, m)}{\partial m} x_i^*(\mathbf{p}, m)$; **(Elasticity Form):** $\varepsilon_i = \varepsilon_i^h - s_i \varepsilon_m$; $s_i = \frac{p_i x_i}{m}$

Demand Elasticity for product i, homogeneous of degree 0: $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j \neq i} \varepsilon_{ij} = 0$

Equations for Cost and Technology

Technical Rate of Substitution: $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j}$; **MC:** $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$ **MC to VC:** $\int MC = VC$

Factor Prices: $\mathbf{w} = (w_1, w_2, \dots, w_n)$; **Production Function:** $y = f(x_1, x_2)$

Cost Function with two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$
 $= \min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2 \text{ s.t. } y = f(x_1, x_2)$

Shepard's Lemma conditional factor demand: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w}, y)}{\partial w_i}$

Learning Curve: The typical specification is for $Y_t = \Sigma_{s \leq t} y_s$, AC falls proportionately, $\ln AC_t = AC_0 - b \ln Y_t$

Equations for Competitive Firms

SR Profit Maximization: $\max_{y, x_v \geq 0} \pi = \max_{y \geq 0} [\max_{x_v \geq 0} R(y) - w_v x_v - w_f x_f \text{ s.t. } y = f(x_v, \bar{x}_f)] = \max_{y \geq 0} [R(y) - c(y)]$

Revenue if firm is competitive: $R(y) = py = pf(x_v, \bar{x}_f)$ **FOC of unconditional factor demand:** $p \frac{\partial f(x_v, \bar{x}_f)}{\partial x_v} = w_v$

Hotelling's Lemma, Supply: $y^*(p, \mathbf{w}) = \frac{\partial \pi(p, \mathbf{w})}{\partial p}$; unconditional factor demands: $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p, \mathbf{w})}{\partial w_i}$

Shutdown Condition (Competitive Firms): $-F > py - c_v(y) - F \implies AVC = \frac{c_v(y)}{y} > p$

Equations for Monopolies

FOC for a monopolist: $p(y) + p'(y)y = c'(y)$ which can be rewritten as $p = \frac{1}{1 + \frac{1}{\varepsilon}} MC$; valid if $\varepsilon < -1$

Passing Along Costs: $\frac{\partial p}{\partial c} = \frac{1}{2 + yp''(y)/p'(y)}$

Price Discrimination. Third Degree: Monopolist's Problem: $\max p_1(x_1)x_1 - cx_1 + p_2(x_2)x_2 - cx_2$

$FOC_{x1} : c = p_1(x_1)[1 - \frac{1}{|\epsilon_1|}]$ Markup factor: $M_i = \frac{1}{1 - \frac{1}{\epsilon_i}} = \frac{|\epsilon_i|}{|\epsilon_i| - 1}$

Quasilinear utility: $\max u_i(x) + y$ s.t. $px + y = m$; FOC (inverse demand curve): $p = u'_i(x)$

Decision Theory. Probability Identities: $P(A|B) = \frac{P(A \cap B)}{P(B)}$; $P(A \cap B) = P(B \cap A)$; Given probability sets A,B,C,D:

$P(A) = P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D)$

Bayes Theorem: $P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$ and given that A is a binary variable $\frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$

Also: $\frac{p(s|m)}{p(t|m)} = \left[\frac{p(m|s)}{p(m|t)} \right] \left[\frac{p(s)}{p(t)} \right]$. Can also take logs to get linear expression.

Cournot. Given $D(Y) = a - bY$. $BR_i(Y_{-i}) = \arg\max_{y_i} \pi_i = P(\sum_{i=1}^n y_i)y_i - c(y_i) \implies P(\sum_{i=1}^n y_i) + P'(\sum_{i=1}^n y_i)y_i -$

$MC_i(y_i) = 0$

To solve for the Nash equilibrium, we want to find where the Best Response functions intersect. $\Rightarrow NE_{Cournot} : Y^* =$

$\frac{N}{(N+1)b}(a - c) \implies y_i^* = \frac{Y^*}{N} = \frac{a-c}{(N+1)b}$

Stackelberg. $\arg\max D(Y) = a - bY \rightarrow BR_L = \max_{y_L} \pi_L(y_L, BR_F(y_L)) = D(y_L + BR_F)y_L - cy_L$

Intertemporal Choice. Given $U(c_0, c_1)$, we have $\frac{\partial_0 U}{\partial_1 U} = \frac{\frac{\partial U}{\partial c_0}}{\frac{\partial U}{\partial c_1}} = MRS_{01} = 1 + MRT_P$

Given Initial Endowment $E = (e_0, e_1)$ and intertemp prod function $y = f(x)$,

the PPF is $\{(q_0, q_1) : q_0 = e_0 - x \leq 0, q_1 = e_1 + f(x) \leq 0\}$

$ROI = f(x) - x$; $ARO = \frac{f(x)}{x} - 1$; $MROI = f'(x) - 1$

Present Value: $PV_r(C) = c_0 + \frac{c_1}{1+r}$

Agent Optim: $\max_x w = PV(Q) = q_0 + \frac{q_1}{1+r} = e_0 - x + \frac{e_1 + f(x)}{1+r}$ FOC: $1 + r = f'(x) = 1 + MROI \implies r = MROI$

Optimal individual borrowing, consumption and lending: $\max_{c_0, c_1 \leq 0} U(c_0, c_1)$ s.t. $PV_r(Q) = PV_r(C) = w$,

given $c_0 = q_0 + b$ and $c_1 = q_1 - (1+r)b \Rightarrow \max_b U(q_0 + b, q_1 - (1+r)b)$

Fisher's Equation: $k \approx r + \pi$ (obtained from $1 + k = (1+r)(1+\pi)$)

Present Value of discrete cash stream $X = (x_0, x_1, \dots, x_T)$ or $[x(t) : t \in [0, T]]$

$$PV_k(X) = \sum_{t=0}^T \frac{X_t}{(1+k)^t} \text{ or } PV_k(X) = \int_{t=0}^T x_t e^{-kt} dt$$

General formula for interest rates and asset yields: $k_a = r^* + \pi^e + RP_a \pm T_a$,

where T_a = Transaction Costs and RP_a = Risk Premium for a specific asset a.

Risky Choice. Given a lottery with monetary outcomes m_1, \dots, m_n and corresponding probabilities p_1, \dots, p_n ,

its **expected value** is $Em = \sum_i p_i m_i$ and its **variance** is $\text{Var } m = E(m - Em)^2 = \sum_i p_i (m_i - Em)^2$.

Given **Bernoulli function** $u(m)$ — so $u' > 0$ and, if the person is risk-averse, $u'' < 0$ —

the **certainty equivalent** m^{CE} to the lottery solves $u(m^{CE}) = Eu(m) = \sum_i p_i u(m_i)$.

The **coefficient of absolute risk aversion** is $a(m) = -u'(m)/u''(m)$ and the **coefficient of relative risk aversion** is $r(m) = ma(m)$.