2014 204b Cheat Sheet

Formulas:

Normal Distribution Function

$$f(x;\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

Message Probability

$$p(m)=\sum_{t\in S}p(m,t)$$

Prior Probability

$$p(s)=\sum_{m\in M}p(m,s)$$

Likelihood $p(m|s)=\frac{p(m,s)}{p(s)}$

$$p(m,s)=p(m|s)p(s)$$

Posterior $p(s|m)=\frac{p(m,s)}{p(m)}$

Bayes

$$p(s|m)=\frac{p(m|s)p(s)}{p(m)}$$

$$p(s|m)=\frac{p(m|s)p(s)}{\sum_{t\in s}p(m|t)p(t)}$$

P(m, s)=P(m | s)*P(s)=P(s | m)*P(m)

Absolute Risk Aversion

$$A(c)=-\frac{u''(c)}{u'(c)}$$

Relative Risk Aversion

$$R(c)=cA(c)=\frac{-cu''(c)}{u'(c)}$$

Poisson Process

$$p(n|\lambda)=\frac{\lambda^n e^{-\lambda}}{n!}\ ,$$

$$\lambda=rate\quad n=number$$

Certainty Equivalence

$$u\big(c(F,u)\big)=\int u(x)dF(x)$$

$$P*U(a)+(1-P)U(b)=U(c)$$

1st Order Stochastic Dominance

$F(s)\geq G(s)\ \forall s$ (F and G are CDF's)

2nd Order Stochastic Dominance

$$\int_{-\infty}^{s}F(t)dt\geq\int_{-\infty}^{s}G(t)dt\quad\forall s$$

iff F is a mean-preserving spread of G

Gross Value of Information

$$\sum_{m\in M}p(m)\sum_{s\in S}p(s|m)(w(a^*(m),s)-w(\hat{a},s))\geq 0$$

Best Response

$s_i\in S_i$ is a best response to $s_{-i}\in S_{-i}$ if

$$u_i(s_i,s_{-i})\geq u_i(t_i,s_{-i})\ \forall t_i\in S_i$$

Nash Equilibrium

$s^*=(s_1^*\ldots s_n^*)$ is a Nash Equilibrium if $s_i{}^*$ is a Best Response to s_{-i}^* $\forall i$

Generalized BI

1. Find all NE at each minimal terminal sub-game
2. Write out reduced game with a NE payoff vector replacing sub-game
3. Iterate until start, always at least one SPNE

Incomplete info (Harsanyi)

1. Specify types and connect with N-move, drawing relevant info sets
2. Assume common prior for N-move
3. Solve for NE (BNE) and SPNE (PBE) by normal methods.

Repeated Game

1 T finite: Only stage game NE are equilibria of the repeated game.
2 T infinite: cooperation can be sustained as a NE of the repeated game if d≥d* (discount factor)

FolkThm any of stage game feasible payoff vector that dominates the NE is achievable as average payoff in a SPNE of the infinitely repeated game if players are sufficiently patient.

Evolutionary Games

It includes transitory dynamics and higher payoff strategies become more prevalent over time in a population "survival of the fittest"

One population case

	p	1-p
A	0	4
B	1	2

u(A, p) =0*p+4*(1-p)=4-4p
u(B, p) =1*p + 2*(1-p)=2-p
Δ{row}=2-3p, Δ{row}=0 if p=2/3, Δ{row} > 0 if p>2/3 Δ{row} <0 if p<2/3 and then draw the basin of attraction graph to find mix equilibrium.

Separate population case

2 / 1	p	1-p
q	0,0	4,1
1-q	1,4	2,2

Calculate the mix NE for Δ(row)= 2-3p, p* = 2/3[Δ(row)<0 if p*>2/3, Δ(row)>0 if p*<2/3]
For Δ (col) = 2-3q, q*=2/3 [Δ(col)<0 if q*>2/3, Δ(col)>0 if q*<2/3]
Then draw phase portrait based on analysis above.

Cooperative games

Cooperative games are specified by a characteristic function defined on subsets (coalitions) (K⊂N).

Coalition K blocks allocation u if $\sum_{i\in K}u_i<v(K)$. That means they can do better by themselves. Core is all allocations unblocked by any K⊂N

Shapley Value is based on marginal contribution of each player to every k. Method: list all possible player sequences, if n=3 then n!=6 (6 possibilities). Ex: V(i)=0, V(ab)=1 V(ac)=2 V(bc)=0 V(abc)=2

	MCa	MCb	MCc
abc	0	1	1
acb	0	0	2
bac	1	0	1
bca	2	0	0
cab	2	0	0
cba	2	0	0
sum	7	1	4
S.V.	7/6	1/6	4/6

Convex Game

v(S)+v(T)⩽v(S∩T)+v(S∪T)
SV∈ Core if game is convex.

NBS allocation maximizes the product of players utility gains relative to a threat point. Simple example, max g(u, v)=(u-$\underline{u}$)(v-$\underline{v}$) if feasible utility fn is u+v=10,($\underline{u}$, $\underline{v}$) = (0,0) then g=(u)(10-u) FOC w.r.t u, get u =5, then v = 5

Monopolistic general problem max qp(q)-c(q). Take derivative w.r.t q and solve.

Bertrand Duopoly (symmetric case)

max $(p_1-c)\ q_1\ (p_1,p_2)$
$q_1\ (p_1,p_2)$=x (p_1), if $p_1<p_2$
$q_1\ (p_1,p_2)$=0.5 x (p_1), if $p_1=p_2$
$q_1\ (p_1,p_2)$=0, if $p_1>p_2$
NE: $p_1=p_2=MC,\ \pi_1=\pi_2=0$

Cournot Duopoly (symmetric case)

max $p\ (q_1+q_2)q_1-cq_1$ w.r.t q_1
NE: $q_1=\ q_2\&\pi_1=\pi_2$
(Asymmetric case): max $(p-c_i)\ q_i$

Stack. Duopoly

Calculate firm 2's max profit. w.r.t q_2 and then plug into firm 1's max profit to get q_1.
$q^m<q^c<q^s<q^b=<q^{CE}$

Entry game: Stage 1: [in with cost K, out]. Stage 2: K is sunk, J entrants. Stage 2: get$\pi_j^{NE} - K$ if in, 0 if out.

Formula: K ≈$\frac{(a-c)^2}{b(J+1)^2}$ if price = a-bq and cost = cq

Asymmetric Info: BNE is (σ, ρ) s.t. $\forall i$, σ_i maximizes $E_\rho u_i(\sigma_i, \sigma_{-i})$ and ρ_i is consistent with priors, σ, and Bayesian Theorem. In PBE, the same is true in every subgame.

Adverse Selection: Ex: Seller knows quality θ = value to buyer. Seller values at r(θ). Θ(p) = $\{\theta\colon r(p) \leq p\}$ is the subset of sellers willing to sell at price p.

Then a competitive eqm. in a market with asymmetric info is (P*, Θ(p*) s.t. p* = E(θ|θ∈Θ*) and Θ(P*) = {θ: r(p) ≤ P*}(i.e expected quality among those that are selling is the price). Used car example: θ = [2, 3]. r(θ) = θ-0.1 and Θ(p) = {θ-0.1 ≤ p}. Then p* = $\frac{2+(p+0.1)}{2}$ solving for p gives p* = 2.1, and Θ* = [2, 2.2]. Only 20% of market sold.

Signaling: N-move first, θ; Informed (sender) player send message m(θ) and Uninformed (receiver) player picks action a(m) after forming beliefs μ(θ|m). PBE is [m*(θ), a*(θ), μ(θ|m)] s.t. 1. m*∈ argmax $u_s(m, a^*(m), \theta)\forall$ θ (for every possible state, send m that max u given U's BR to m). 2. a*(m)∈argmax $E_\mu \mathrm{U_r(a)}$ (pick a max Expected payoff) 3. μ(θ|m) is consistent with Bayes given N-move (given priors) and m*(θ) (likelihood).

Types of PBE in Signaling: 1. Separating (each state θ a different m* beware the offer or bid's interval has to be different) 2. Pooling (m* constant) 3. partial pooling (not 1:1) 4. hybrid (mixed).

Screening: U-N-I, usually uninformed players offered menu to informed players. For example, buyers offer deferred contingent payment; self-selection of insurance customers to reveal more personal information to get premium reduction.

P/A: $\max_{e^*}$ E profit (e^*) = E([π|e^*] – cost(e^*)) where cost (e^*) = min **E(w)** s.t. PC & IC

Case 0: If e is observable, P: $min_{w(\pi)}\, \boldsymbol{E}(\boldsymbol{w}) = \int_{\underline{\pi}}^{\overline{\pi}} w(\pi)\, f(\pi|e^*) d\pi$ s.t. PC = $\int_{\underline{\pi}}^{\overline{\pi}} v\big(w(\pi)\big) f(\pi|e^*) d\pi - g(e) \geq \overline{u}$
A: U(e, w) =v(w) - g(e); v (w*) = $\overline{u}$ + g (e) then w*= $v^{-1}(\overline{u} + g(e^*))$

Case 1: If e is unobservable and agent is the risk neural, P: $min_{w(\pi)}\, \boldsymbol{E}(\boldsymbol{w}) = \int_{\underline{\pi}}^{\overline{\pi}} w(\pi)\, f(\pi|e^*) d\pi$ s.t. PC = $\int_{\underline{\pi}}^{\overline{\pi}} v(\pi - B) f(\pi|e^*) d\pi - g(e) \geq \overline{u}$ Note: the owner's payoff under the optimal compensation scheme is exactly B (the agent gets w(π) = π-B; the principal gets B); B is franchisee fee.

Case 2: If e is unobservable and agent is risk averse,

P: $min_{w(\pi)}\, \boldsymbol{E}(\boldsymbol{w}) = \int_{\underline{\pi}}^{\overline{\pi}} w(\pi)\, f(\pi|e^*) d\pi$ s.t. PC = $\int_{\underline{\pi}}^{\overline{\pi}} v\big(w(\pi)\big) f(\pi|e^*) d\pi - g(e) \geq \overline{u}$ & s.t.

IC = $\int_{\underline{\pi}}^{\overline{\pi}} v\big(w(\pi)\big)\, f(\pi|e^*)\, d\pi - g(e^*) \;\geq\; \int_{\underline{\pi}}^{\overline{\pi}} v\big(w(\pi)\big)\, f(\pi|e)\, d\pi - g(e)$

FOC w.r.t w (π) we get $\frac{1}{v'(w(\pi))} = \gamma + \mu\, [1 - \frac{f(\pi|e)}{f(\pi|e^*)}]$, $e^* \in \{e_H, e_L\}$