6. Price Discrimination

I. Overview

A. Price discrimination is the sale of identical units of a good at different prices.

B. By price discriminating, a firm can capture some of what would be consumer surplus. PS ↑.

1. In so doing, a monopolist may also increase output, leading to a more efficient outcome, i.e., we may have PS ↑.

C. Constraints on price discrimination

1. The firm must have market power, otherwise it will just be a price-taker.
2. Arbitrage must somehow be limited; otherwise low price units could be resold and undercut the higher priced units.
3. The firm must somehow be able to detect WTP differences across consumers (or across quantities purchased by a single customer).
4. Legal or moral constraints.

D. The three classical types of price discrimination are methods of coping with the constraints and sorting consumers according to their WTP.

II. A Basic Model (from Varian)

A. A simple quasilinear model helps explain several varieties of price discrimination.

B. Consumers, $i = 1, 2$ have utility $u_i(x) + y$. Think of y as money left over for everything other than x.

1. Consumers are willing to pay up to $r_i(x) = u_i(x)$ for x units of the good.
2. Hence i's marginal WTP is $u_i'(x)$.

C. The inverse demand curve for the individual consumer is therefore found by solving the consumer's problem

1. $\max u_i(x) + y$
   s.t. $px + y = m$

2. FOC is $p = u_i'(x)$ – the inverse demand curve.
   a. The firm can charge a price of at most p to induce consumption of x.

D. From now on we'll assume that consumer 2 has higher WTP than consumer 1.

1. $u_2'(x) > u_1'(x)$, so by integration,
2. $u_2(x) > u_1(x)$.
3. This implies the **single crossing property**.
   a. Implies indifference curves of two consumers only cross once.

E. The monopolist has a cost of $c(x) = cx$

III. First Degree Price Discrimination

A. The monopolist is able to charge a different price for each unit sold.

1. Sometimes called perfect price discrimination.

B. Suppose the monopolist makes an offer to each buyer of a lump sum payment of r for x units.

$$\max r - cx$$
$$\text{s.t. } u_i(x) \geq r$$

C. The FOC is $u'(x^*) = c$ and therefore $r^* = u(x^*)$.... which is Pareto efficient!

1. Note that this x^* is the same level of output as a competitive firm.
2. $u'(x) = p(x)$ and so we are setting $p(x) = c$.

D. This is equivalent to charging a different price (for marginal willingness to pay) for each unit of the good.

E. Constraints: all of them are problematic here.

F. Colleges attempt to approximate this for families who apply for financial aid.

IV. Second Degree Price Discrimination

A. The monopolist charges prices that are not simple per-unit prices.

1. Sometimes called nonlinear pricing.
2. Includes quantity discounts etc.

B. Simplest version: a monopolist offers two different price/quantity *bundles* (r_i, x_i)

1. Bundle i is designed for consumers of type i.
2. The monopolist doesn't know whether a given consumer is type 1 or type 2.
3. The consumer *sorts* him or herself.

C. In order to get consumers i to always choose type i the monopolist has to design the bundles to satisfy two types of constraints:

1. Individual Rationality:

$$u_1(x_1) - r_1 \geq 0$$
$$u_2(x_2) - r_2 \geq 0$$

2. Self selection (aka. Incentive constraints)

$$u_1(x_1) - r_1 \geq u_1(x_2) - r_2$$
$$u_2(x_2) - r_2 \geq u_2(x_1) - r_1$$

D. The monopolist has to satisfy these constraints in order to effectively discriminate.

1. Because the monopolist wants to make as much as possible, she wants to set r_1 and r_2 as high as she can while satisfying the constraints.
2. This fact combined with the single crossing property guarantees that some of the constraints above bind.

$$r_2 = u_2(x_2) - u_2(x_1) + r_1$$
$$r_1 = u_1(x_1)$$

E. The monopolist's problem

1. The monopolist's problem is simply the sum of the profits from the two consumer types.

$$\pi = r_1 - cx_1 + r_2 - cx_2$$

2. All of our hard work above gives us constraints to substitute into this equation:

$$\pi = u_1(x_1) - cx_1 + u_2(x_2) - u_2(x_1) + r_1 - cx_2$$

3. We can maximize this with respect to outputs x_1 and x_2 just as we have before!

F. Welfare

1. First, the per unit price charged to the low value consumer is above marginal cost, implying inefficient production.

$$u_1'(x_1) = p(x_1) = c + u_2'(x_1) - u_1'(x_1)$$

2. Second, our first order conditions tell us that the per unit price charged to the high value consumer is equal to marginal cost, implying efficient production.

$$u_2'(x_2) = p(x_1) = c$$

V. Third Degree Price Discrimination

A. The monopolist is once again able to charge different prices to different groups but not different prices on different units within a group.

1. Think senior citizen discounts.

B. We start with an assumption that the prices posed to a group has no effect on the quantity demand in the other group.

1. The monopolist's problem is

   $\max p_1(x_1)x_1 - cx_1 + p_2(x_2)x_2 - cx_2$

2. The FOCs from this problem can be written as:

   $p_1(x_1)[1 - \frac{1}{|\epsilon_1|}] = c$
   $p_2(x_2)[1 - \frac{1}{|\epsilon_2|}] = c$

3. Since this implies the left hand side expressions are equal to one another, it implies that $p_1(x_1) > p_2(x_2)$ only when $|\epsilon_1| < |\epsilon_2|$
   a. It turns out (though it is more complicated to show) that the same thing holds if we relax our assumption about the independence of the two discriminatory markets!

C. Welfare

1. Does the ability to price discriminate in the third degree help or hurt social welfare.
2. This depends on the effects on output. A few basic facts can be established theoretically.
   a. First, the only way welfare can be *improved* is if output increases due to the discrimination.
   b. Second, as long as $(p_1 - c)\Delta x_1 + (p_2 - c)\Delta x_2$ (where the prices are the prices after the discrimination is instituted), welfare has to improve!
   c. Third, if a whole new market is served due to the discrimination, welfare has to improve.

   Ex: Third degree price discrimination with linear demand.