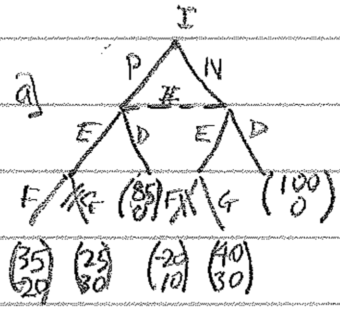
Answer Key: 204B Final Exam. W17

(1) a]



b] $E \in BR_{II} \iff p(-20)+(1-p)30 \geq 0$

$\iff 30 \geq 50p \iff p \leq 3/5$.

c] First step in BI shown, Remaining NFG is

	E	D
PF_pG_N	35, -20	85, 0
NF_pG_N	40, 30	100, 0

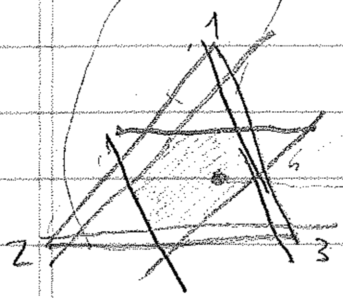
So SPNE is (NF_pG_N, E) with $p=0$.
That is, it is a (weakly) dominant strategy, and SGP, for I to Not prepare, to Fight if prepared (not on eq. path) and to Go easy in not prepared, and entrant's BR is to Enter.

(2) a) Simple BI gives $(Out_{PLCL}, Out_{PHCL}, In_{PHCH}, In_{PLCH})$ for entrant,
thus $(P_H|C_H, P_L|C_L)$, with expected payoffs $(1,1)_{C_H}(.2)+(3,1)_{C_L}(.8) = \binom{2.6}{1}$.

b) Try $(P_H|C_H, P_L|C_L)$, so $\mu(C_H|P_H)=1=\mu(C_L|P_L)$. Then
(*) [$BR_2(P_H) = In$, $BR_2(P_L) = Out$]. But BR_1 to (*) includes $P_L|C_H$, breaking this eq
Try $(P_L|C_H, P_H|C_L)$ so $\mu(C_H|P_L)=1=\mu(C_L|P_H)$. Then
(**) [$BR_2(P_H) = Out$, $BR_2(P_L) = In$]. But BR_1 to (**) includes $P_H|C_H$, breaking the eq
Thus neither possible pooling strategy is part of a PBE.

c) Try $(P_H|C_H, C_L)$. So $\mu(C_H|P_H) = .2$ (the prior) and $\mu(C_H|P_L) = q \in [0,1]$ arbitrary.
$BR_2(P_H) = Out$, $BR_2(P_L) = In$ iff $q \geq .5$. Then
$BR_1(C_H) = P_H$ ✓, $BR_1(C_L) = P_H$ if $q \geq .5$ ✓ So a pooling PBE is
$\sigma^* = (P_H | C_H, C_L)$ $\mu(\cdot|P_H) =$ prior, $\mu(C_H|P_L) = q \geq 0.5$.
Try $(P_L|C_H, C_L)$. So $\mu(C_H|P_L) =$ prior, $\mu(C_H|P_H) = q \in [0,1]$ arbitrary
$BR_2(P_L) = Out$, $BR_2(P_H) = In$ iff $q \geq 0.5$. Then
$BR_1(C_L) = P_L$ ✓, $BR_1(C_H) = P_L$ iff $q \geq 0.5$. So again we have a pooling PBE
$\sigma^* = (P_L | C_H, C_L)$; $\mu(\cdot|P_L) =$ prior, $\mu(C_H|P_H) \geq 0.5$; $\alpha^*(P_L) = Out$, $\alpha^*(P_H) = In$

③ $w(1)=1,\ w(2)=2,\ w(3)=3,\ w(12)=6,\ w(13)=8,\ w(23)=10,\ w(123)=18.$



Core, e.g. $(6,6,6)\in$ Core.

$x_1 \in [1,8]$
$x_2 \in [2,10]$
$x_3 \in [3,12]$

b)

ρ	MC_1	MC_2	MC_3
123	1	5	12
132	1	10	7
213	4	2	12
231	8	2	8
312	5	10	3
321	8	7	3
Σ	27	36	45
ϕ = SV.	$\boxed{9/2}$	$\boxed{6}$	$\boxed{15/2}$
normalized	1/4	1/3	5/12

c. Yes, since w is convex (supermodular),
$\phi(w)\in \text{Core}(w)$

d. NBS: $\max\limits_{x}\ (x_1-1)(x_2-2)(x_3-3)$ s.t. $x_1+x_2+x_3=18$
$\Leftrightarrow \max\limits_{y}\ y_1y_2y_3$ s.t. $y_1+y_2+y_3=18-1-2-3=12$
$\Rightarrow y_i=4 \Rightarrow \boxed{x_1=4+1=5,\ x_2=4+2=6,\ x_3=4+3=7}$

④ a) $W_i = CE_i = M_i + 0.2\,Var_i = \begin{cases} 1+(0.2)1^2 = \underline{1.2},\ i=L \\ 2+(0.2)2^2=\underline{2.8},\ i=H \end{cases}$

b) $E\,loss = (.4)2+(.6)1 = \boxed{1.4 \text{ K/yr} = P}$

c) At $P=1.4$, low risk people refuse $(1.2<1.4)$, so only H-types accept
$E\,\text{profit} = 4000(P - E\,loss|H) = 4000(1.4-2) = -2400\text{K}$ or $\boxed{\$2.4\text{M loss}}$

d) Assuming a uniform price, insurers will serve only H-types (as just seen),
at $P = \overset{\text{cost}}{2} + \overset{\text{allowed margin}}{.4} = \underline{2.4 \text{ K/yr}}$.

e) With free entry, P gets bid down to 0-π level, $\underline{P=2}$.

f) Use screening model, find IC's & PC to separate contracts aimed at H, L types.
PC's imply that an upper bound on profit for each H customer is $(0.2)\,2^2 = 0.8$
and is $(0.2)1^2 = 0.2$ for each L-customer, or $(0.2)6000+(0.8)4000 = 44{,}000\text{K} =$ $\underline{\$44\text{M}}$

g) U is not equivalent to Eu, as explained in the Notes1 p24+.
It is equivalent up to second order. Over a limited range, the function $u(x)=x-cx^2$ works.
See also PS1 problem #2.

5) a) Yes, it is symmetric in that Col's payoff matrix is the transpose of Row's.

b) For $x \in (-2,0)$, we have $p^* = \frac{a_2}{a_1+a_2} = \frac{x}{-2+x} \in (0,1)$, e.g., $p^* = \frac{1}{3}$ for $x=-1$.

It is a downcrossing since $0 > a_1 = 3-5$, $0 > a_2 = x-0$, hence a unique NE & stable.

c) For $x \in (0,10)$, $a_2 = x > 0 > a_1 = -2$, hence s_2 is a dominant strategy

the Pure NE s_2 is ∴ globally stable.

d) Since $a_1 = -2 < 0$, the CO case with 2 pure NE is not possible.

e) With $x = 1$, (s_2, s_2) is the stage game NE. To sustain cooperation,

consider trigger strategy: play s_1 until someone first plays s_2, then play s_2 ever after.

Playing s_1 (or trigger) against trigger yields stream $3, 3, 3, \ldots$ (*)

s_2 " " " " $5, 1, 1, 1, \ldots$ (**)

∴ * is BR (hence (trigger, trigger) ∈ NE) iff $PV(*) \geq PV(**) \Leftrightarrow \frac{3}{1-\delta} \geq 4 + \frac{1}{1-\delta}$

$\Leftrightarrow 2 \geq 4(1-\delta) \Leftrightarrow \delta \geq \frac{1}{2}$ ∴

If $\delta = \frac{q}{1+r}$, then the condition is $q \geq \frac{1}{2}(1+r)$.

Economics 204 B
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UCSC

Final Examination

Instructions. In class, closed book, three hours, only official double-sided page of notes allowed. When insufficient information is provided, please write down a plausible specific assumption and proceed to the solution. Partial credit will be awarded for partial solutions and for brief, relevant remarks, but not for rambling. Points (pts) are marked total is 100.

1. [Entry deterrence version 1] An incumbent firm can prepare to fight (P) at cost c = 15, or not prepare (N). In either case, a potential entrant firm then can enter (E) or not (D). If it enters the incumbent firm can fight (F) or go easy (G). Payoffs to (incumbent, entrant) are (50-c, -20) for PG, E ~~(PF, E)~~ and (-20, 10) for (NF, E). The payoff vector is (100-c, 0) for (PF, D) or (PG, D), and is ~~(40-c, 30) for (PE, G)~~; the corresponding payoffs for (N*, *) are the same except that c=0 instead of c=15. The entrant assesses the probability that the incumbent is prepared as p. All this is common knowledge. P-E-F

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(a) Draw the extensive form game where the incumbent moves first. Be sure to label the nodes and branches with the notation and payoffs given above. (4 pts)

(b) For what values of p does the entrant want to enter? (4 pts) $p \le 3/5$

(c) Find all subgame perfect Nash equilibrium strategies and payoffs. Be sure to specify entire strategies for both players, not just strategy fragments. (8 pts)

(PFG, D) p=1

2. [Entry deterrence version 2]A monopoly incumbent(player1) faces a potential entrant(player2). The monopoly has private information about its costs which may be either high (c_H) or low (c_L). The monopoly chooses either a high price (P_H) or a low price (P_L). The price is then observed by the potential entrant who then decides to enter (in) or to stay out of (out) the market. The potential entrant's prior belief is that there is an 80% chance that the monopoly is a low cost type (c_L). The payoffs are as follows (don't worry if they seem odd, just take them as given).

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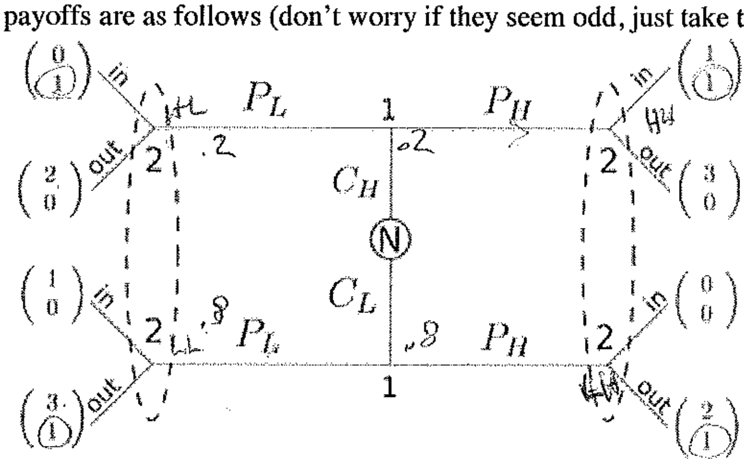

HL

(a) Find a subgame perfect Nash equilibrium if, contrary to the figure above, there were perfect information (so the potential entrant knew whether the monopoly had high or low cost). Be sure to clearly state the complete equilibrium strategies and expected payoffs. (6 pts)

(b) In the game shown, the monopolist's cost is private information. For this game, find a separating perfect Bayesian equilibrium (PBE) or show that none exists, completely specifying the equilibrium strategies and beliefs. (8 pts) both break

(c) In the game shown, find a pooling PBE or show that none exists, completely specifying the equilibrium strategies and beliefs both on and off the equilibrium path. (8 pts)

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3. A three-player game (players labelled i = 1,2,3) has ChF (or worth) $w(S) = |S| \sum_{i \in S} i$, where $|S|$ is the number of members of the coalition S. For example, if S = {1, 3} then w(S) =2(1+3) = 8.

(a) Find the core and identify a point in the core. (5 pts)

Please turn over