Final Exam

Econ 200

1. Business is good at Acme Products. George, the owner, estimates that he can earn an additional 12 per year indefinitely starting next year if he invests in expansion now, George is risk-neutral: he is not averse to risk, but neither does he seek it. The investment will be sunk once George makes the decision.

(a) (4 points) If George can finance at rate $k = 0.10$, what is his maximum willingness to pay for that investment?

Solution: Maximum willingness to pay for this investment is the present value of the expansion,

$$\begin{aligned} \frac{12}{(1+0.1)} + \frac{12}{(1+0.1)^2} + \cdots &= \left(\frac{12}{1-\frac{1}{1.1}}\right)\left(\frac{1}{1+0.1}\right) \\ &= 120. \end{aligned} \tag{1}$$

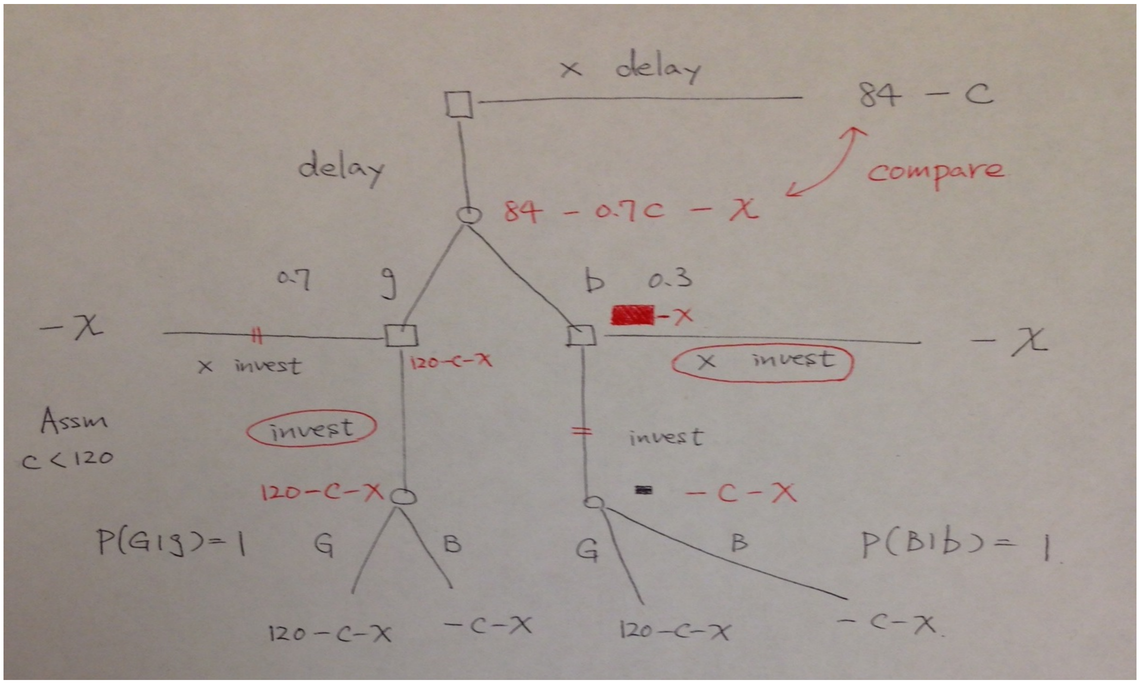
(b) (6 points) Suddenly George realizes that things might go wrong during the next year, and if so the additional earnings will be zero. He estimate the probability if 0.3 that things will go wrong. Now how much is he willing to pay for that investment.

Solution: Expected present value of the investment with possible failure is,

$$0.7(120) + 0.3(0) = 84. \tag{2}$$

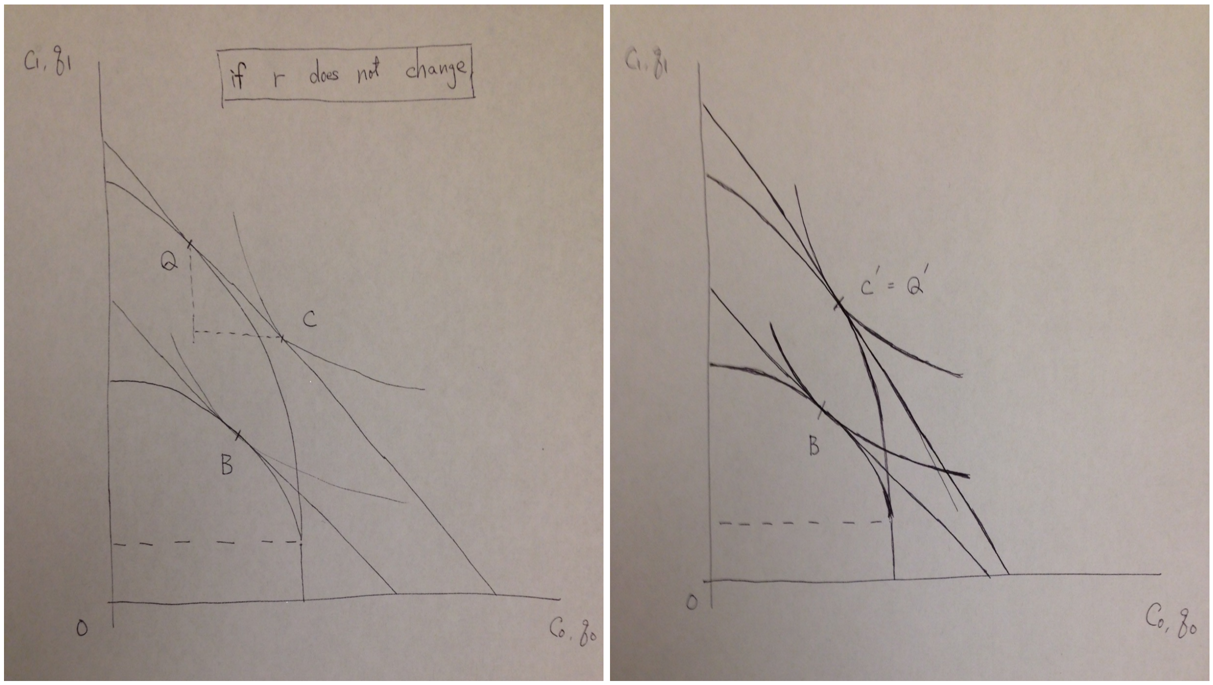
(c) (8 points) George then realizes that, at some cost, he can delay making the investment until he knows whether or not things will go wrong. How much cost of delay is he willing to incur?

Solution: Let c be the investment cost and x be the cost to delay. The decision tree is as follows. (”Not delay” is marked ”x delay” and similarly for Not invest.) Therefore, willingness to pay for delaying is $(84 - 0.7c) - (84 - c) = 0.3c$.



2. Developments in Silicon Valley increases productivity worldwide. Other things (i.e., thrift unaffected), what should be the impact on the real interest rate? Nominal interest rates? Current and future real consumption? Wealth? (20pts)

Solution: The increase in productivity shifts PPF upward as in the left panel, where the slope is steeper above each q_0. Therefore, at the old interest rate, the optimal q_0 decreases (this is the production effect) and the optimal c_0 increases. As shown in the left panel of the figure below, the upshot is that borrowing exceeds lending.
In equilibrium, therefore, the real interest rate must rise as in the right panel. The substitution effect decreases c_0 and increases c_1. Income effect (via the production effect in the left panel) increases both c_0 and c_1. Overall, c_1 increases but the effect on c_0 is ambiguous. Although real interest rate increases, q_1 also increases and thus effect on real wealth is ambiguous. In the graph, c_0 barely increases and real wealth also barely increases. If the inflation rate unaffected, then the nominal interest rate increases by the same amount as the real interest rate.



3. Suppose that Abe makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = \ln(m + 1)$. He is faced with a situation in which he will receive either 0 or 24; the outcomes are equally likely.

(a) (4 points) What is the expected outcome? Variance of outcome?

Solution: Expected value and the variance of the situation is,

$$\begin{aligned} E[m] &= \frac{1}{2}(0) + \frac{1}{2}(24) = 12, \\ V[m] &= E[(m - E(m))^2] = \frac{1}{2}(0 - 12)^2 + \frac{1}{2}(24 - 12)^2 = 144. \end{aligned} \tag{3}$$

(b) (6 points) What is Abe's certainty equivalent of the risky outcome? What is the maximum amount he would be willing to pay an insurer to get the mean outcome for sure?

Solution: Certainty equivalence, m^{CE}, satisfies,

$$\begin{aligned} \ln(m^{CE} + 1) &= \frac{1}{2}\ln(0 + 1) + \frac{1}{2}\ln(24 + 1), \\ &= \ln(24 + 1)^{\frac{1}{2}} = \ln 5 = \ln(4 + 1). \end{aligned} \tag{4}$$

Thus, $m^{CE} = 4$. Max insurance payment = RP = $E[m] - m^{CE} = 12 - 4 = 8$.

(c) (4 points) What is Abe's coefficient of relative risk aversion at the mean outcome?

> **Solution:** Since $u'(m) = (m+1)^{-1}$ and $u''(m) = -(m+1)^{-2}$, the coefficient of relative risk aversion is,
>
> $$r(m) = -\frac{-(m+1)^{-2}}{(m+1)^{-1}}(m) = \frac{m}{m+1}. \tag{5}$$
>
> Thus, $r(E[m]) = \frac{12}{13}$.

4. Ajax Inc and Bestco produce imperfectly substitutable products, each at per unit marginal costs of 1. Inverse demand for Ajax is $p_A = 6 - q_A - 0.5q_B$ and symmetrically Bestco's inverse demand is $p_B = 6 - q_B - 0.5q_A$, where q_A and q_B denote the quantities of the two firms. Find the Nash equilibria of the following games, where payoffs are profits.

(a) (6 points) Both firms choose quantity simultaneously and independently.

> **Solution:** (Cournot competition) Firm A maximizes profit with respect to q_A,
>
> $$\max_{q_A}(6 - q_A - \frac{1}{2}q_B)q_A - q_A. \tag{6}$$
>
> First-order condition is,
>
> $$5 - \frac{1}{2}q_B - 2q_A^* = 0. \tag{7}$$
>
> Since firms are symmetric,
>
> $$5 - \frac{1}{2}q_A - 2q_B^* = 0. \tag{8}$$
>
> Thus, in Nash equilibrium, $q_A^{NE} = q_B^{NE} = 2$.

(b) (4 points) Ajax chooses quantity first, then Bestco observes it and then chooses its own quantity.

> **Solution:** (Stackelberg) A moves first and B moves second. We solve this problem by backward induction. The best response function of firm B is the same as Cournot competition case,
>
> $$\begin{aligned} 5 - \frac{1}{2}q_A - 2q_B^* &= 0, \\ \Leftrightarrow \quad BR_B(q_A) &= \frac{5}{2} - \frac{1}{4}q_A. \end{aligned} \tag{9}$$
>
> Firm A maximizes profit taking this best response function of firm B into consider-

ation,

$$\max_{q_A} \quad \left(6 - q_A - \frac{1}{2}\left(\frac{5}{2} - \frac{1}{4}q_A\right)\right) q_A - q_A. \tag{10}$$

First-order condition is,

$$\frac{15}{4} - \frac{7}{4}q_A^* = 0. \tag{11}$$

Thus, $q_A^{NE} = \frac{15}{7}$ and $q_B^{NE} = \frac{55}{28}$.

(c) (6 points) Both firms choose price simultaneously and independently.

Solution: (Bertrand) By rewriting the demand system, we obtain,

$$q_A = 4 - \frac{4}{3}p_A + \frac{2}{3}p_B, \tag{12}$$
$$q_B = 4 - \frac{4}{3}p_B + \frac{2}{3}p_B. \tag{13}$$

Firm A maximizes its profit with respect to p_A,

$$\max_{p_A} \quad p_A(4 - \frac{4}{3}p_A + \frac{2}{3}p_B) - (4 - \frac{4}{3}p_A + \frac{2}{3}p_B). \tag{14}$$

First-order condition is,

$$\frac{16}{3} - \frac{8}{3}p_A^* + \frac{2}{3}p_B = 0. \tag{15}$$

Since firms are symmetric,

$$\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0. \tag{16}$$

Thus, $p_A^{NE} = p_B^{NE} = \frac{8}{3}$.

(d) (4 points) Ajax chooses price first, then Bestco observes it and then chooses its own price.

Solution: Again, we solve this problem by backward induction. The best response function of firm B is the same as Bertrand case,

$$\begin{aligned} &\frac{16}{3} - \frac{8}{3}p_B^* + \frac{2}{3}p_A = 0 \\ \Leftrightarrow \quad &BR_B(p_A) = 2 + \frac{1}{4}p_A. \end{aligned} \tag{17}$$

Firm A maximizes profit taking this best response function of firm B into consideration,

$$\max_{p_A} \quad (p_A - 1)\left(4 - \frac{4}{3}p_A + \frac{2}{3}\left(2 + \frac{1}{4}p_A\right)\right). \tag{18}$$

First-order condition is,

$$-\frac{7}{3}p_A^* + \frac{39}{6} = 0. \tag{19}$$

Thus, $p_A^{NE} = \frac{39}{14}$ and $p_B^{NE} = \frac{151}{56}$.

5. Name at least three techniques of price discrimination commonly used by airlines. How does each technique overcome the main obstacles to price discrimination? Comment very briefly on the efficiency implications of each technique. (18pts)

Solution: Laws against one customer reselling an airline ticket to another customer virtually eliminate the arbitrage obstacle for any form of price discrimination. Self-selection, or incentive compatibility, helps cope with the unobservability of WTP obstacle, as noted below.

- Timing of purchase: Charge higher prices for those who buy tickets on closer dates to departure dates. Incentive compatible because last minute customers tend to have less price-elastic demand. Can be regarded as Third-degree PD.

- First-class vs Economy-class: Price way higher prices for consumers with high willingness to pay. Incentive compatible because of the difference in service and quality. Can be regarded as Second-degree PD.

- Mileage: Give bonuses to customers with low willingness to pay. Incentive compatible because customers with high willingness to pay does not have incentive to earn mileage because of opportunity cost. Similar to Second-degree/ quantity discount, and a little like a 2-part tariff.

- Payment history: This is similar to the example from one of the clip in class. Charges different prices to different customers based on histories of purchase, which reflect willingness to pay. Time will tell us whether this new way of price discrimination will work or not. Might approximate First-degree.

6. The price of crude oil decreased about 60% over the last 1.5 years. Suppose that a sector of the economy can be approximated reasonably well by a production function with crude oil as one input and labor and capital as the other inputs. If that production function is Leontieff, what impact would you expect to see on the sector's input demands, cost and output? How would you answer change if the production function were CES with substitution elasticity between 0 and 1? Which production function seems a more reasonable description in the short run of a month or two? (14pts)

Solution:

- When the relative price of input changes, ratios of input quantities do not change if production function is Leontieff. One way to see this is to draw isoquant. Isoquant has a kink and regardless of the input price ratio, cost is minimized at the kink. Still, the overall marginal cost is lower, and thus total cost. With lower marginal cost, the profit-maximizing output should increase.
- When the elasticity of substitution is not zero, there is a substitution toward the lowered priced goods. This additional effect even more lowers the total cost.
- In the short-run, it is hard to switch to different input ratios. As a result, production function with low elasticity of substitution is suitable.