Problem Set 4

Econ 200

I. Short Case Study Problems

1. Direct demand is $Y = 86 - p$, where Y is the sum of output across all firms, and each firm has cost function $c(y) = 14y$.

(a) Firms operate in the market by setting quantities. What are outputs, prices, profits and deadweight losses for monopoly, duopoly, triopoly and perfect competition markets? Show all work, but then collect your answers into a table, with columns for market structure and rows for performance measures. Which market is most efficient and WHY?

Solution: With perfect competition, the profit maximization condition $p = \text{MC}$ implies that $86 - Y = 14$, or $Y^* = 72$, $p^* = 14$, and $\pi^* = 0$. Of course, with perfect competition there is no deadweight loss.

Rather than solving the cases with 1 firm, 2 firms, and 3 firms separately, it's easier to just solve for N firms. Then, firm i maximizes

$$\max_{y_i} \quad [86 - (y_1 + \ldots + y_N)]y_i - 14y_i$$

which has the first-order condition

$$72 - (y_1 + \ldots + y_{i-1} + 2y_i + y_{i+1} + \ldots + y_N) = 0$$

which can be re-arranged as

$$y_i = 72 - Y$$

Note that this condition holds for each of the N identical firms. If we add all these conditions together, we get

$$\sum_{i=1}^{N} y_i = N(72 - Y)$$

Of course, the sum on the left-hand side is just equal to Y, so we can solve the above for Y to get $Y = 72\frac{N}{N+1}$. Note that when we let N go to infinity, we get $\lim_{N\to\infty} Y = Y^*$.

For the monopolist case where $N = 1$, we get $Y^M = 36$, $p^M = 50$, $\pi^M = 1296$, and deadweight loss of $\frac{1}{2}(72 - 36)(50 - 14) = 648$. Note that to find the deadweight loss, we first had to find industry supply. Since each firm has inelastic supply at $p = 14$, inverse industry supply is also the horizontal line $p = 14$.

With a duopoly ($N = 2$), we get $Y^D = 48$, $p^D = 38$, $\sum_{i=1}^{N} \pi_i^D = 1152$, and deadweight loss of $\frac{1}{2}(72 - 48)(38 - 14) = 288$. With a triopoly ($N = 3$), we get $Y^T = 54$,

$p^T = 32$, $\sum_{i=1}^{N} \pi_i^T = 972$, and deadweight loss of $\frac{1}{2}(72-48)(38-14) = 162$.

To summarize the results:

	Y	p	$\sum \pi_i$	DWL
Monopoly	36	50	1296	648
Duopoly	48	38	1152	288
Triopoly	54	32	972	162
Perf. Comp.	72	14	0	0

Not surprisingly, perfect competition comes out as the most efficient market structure. With the other market structures, the firm(s) restrict output in order to raise prices and earn profits. This behavior, however, generates deadweight loss.

(b) Suppose firms set price instead of quantity. Again prepare a table of the same size and shape, and compare it to part a. How does your answer on efficiency change?

Solution: When firms compete by choosing prices, then in (short run) Bertrand equilibrium, the firm with lowest cost can capture the whole market by choosing a price just below the marginal cost of the firm with the next-lowest cost. If all firms are identical, then they lower prices until they reach marginal cost; they divide up the market and all earn zero profits. Therefore, the results are:

	Y	p	$\sum \pi_i$	DWL
Monopoly	36	50	1296	648
Duopoly	72	14	0	0
Triopoly	72	14	0	0
Perf. Comp.	72	14	0	0

Compared to part (a), the monopolist's behavior is still the same but now, as soon as we introduce *any* competition, we go immediately to the efficient (zero-profit) outcome.

2. National demand for LED bulbs is approximated by $Q = 100 - p$, where Q is the industry's annual sales in millions of boxes and p is the per box price. The two producers, Everglow (E) and Dimwit (D) have identical cost functions $c_i = 10q_i + 0.5q_i^2$ for $i = E, D$. Of course, $Q = q_E + q_D$.

(a) Not recognizing their possible market power, the two firms act as price takers. What are the equilibrium prices, output quantities, and profits?

Solution: When the two firms act as price takers, $p = \text{MC} = 10 + q_i$ gives firm demand $q_i = p - 10$. Substituting this into inverse industry demand,

$$p = 100 - (q_D + q_E) = 100 - (2p - 20)$$

which can be solved to get $p^* = 40$, $q_D^* = q_E^* = 30$, and $\pi_D^* = \pi_E^* = 450$.

(b) New management takes over in each company and independently chooses output each period. What now are the equilibrium prices, output quantities, and profits?

Solution: The phrase "independently chooses output" is a clue that we can apply the Cournot model to this problem. In that case, Dimwit's profit maximization problem is

$$\max_{q_D} \quad [100 - (q_D + q_E)]q_D - (10q_D + \frac{1}{2}q_D^2)$$

which has the first-order condition

$$100 - 2q_D - q_E - 10 - q_D = 0$$

Solving this for q_D gives Dimwit's best response to q_E:

$$\text{BR}_D(q_E) = 30 - \frac{1}{3}q_E$$

In general, the next step would be to solve Everglow's profit maximization problem to get Everglow's best-response function. However, in this case we can take a short cut by observing that the two companies are identical. Exploiting symmetry, we get $\text{BR}_E(q_D) = 30 - \frac{1}{3}q_D$.

The Nash equilibrium (q_D^C, q_E^C) is given by the intersection of these two best-response functions:

$$q_E = \text{BR}_E(\text{BR}_D(q_E)) = 30 - \frac{1}{3}\left(30 - \frac{1}{3}q_E\right)$$

which gives $q_D^C = q_E^C = 22.5$, $p^C = 55$, and $\pi_D^C = \pi_E^C = 759.375$. Not surprisingly, output is lower and profits are higher since the firms are now exploiting their oligopoly power.

(c) Everglow's manager (but not Dimwit's) finds how to commit to a particular value of q_E and make Dimwit aware of it before Dimwit chooses q_D. Again, predict equilibrium prices, output quantities, and profits.

Solution: For this part we will apply the Stackelberg model; here Everglow no longer takes q_D as given. Instead, Everglow takes into account its ability to influence q_D since it knows Dimwit will choose $q_D = \text{BR}_D(q_E)$. Everglow's profit maximization problem becomes

$$\max_{q_E} \quad [100 - (\text{BR}_D(q_E) + q_E)]q_E - (10q_E + \frac{1}{2}q_E^2)$$

Plugging in the $\text{BR}_D(q_E)$ that we found in part (b), we get $q_E^S = \frac{180}{7}$, $q_D^S = \frac{150}{7}$, $p = \frac{370}{7}$, $\pi_E^S = 771.43$, and $\pi_D^S = 688.78$.

Two things are worth pointing out. First, industry profits are still higher than under perfect competition ($\pi_D^S + \pi_E^S > \pi_D^* + \pi_E^*$) since the firms are still behaving as oligopolists. Second, compared to the outcome in part (b), Everglow makes a higher profit while Dimwit makes less of a profit.

(d) What gains could the two companies make under collusion?

Solution: Now the firms are cooperating to maximize joint profits:

$$\max_{q_D, q_E} \quad [100 - (q_D + q_E)](q_D + q_E) - (10q_D + \frac{1}{2}q_D^2) - (10q_E + \frac{1}{2}q_E^2)$$

which has the first-order conditions

$$\begin{aligned} 90 - 3q_D - 2q_E &= 0 \\ 90 - 2q_D - 3q_E &= 0 \end{aligned}$$

which can be solved to get $q_D^M = q_E^M = 18$, $p^M = 64$, and $\pi_D^M = \pi_E^M = 810$. As this is identical to the case where the two firms merge into a monopolist, it is not surprising that output is lower and profits are higher than in the previous parts.

3. About 20 years ago, a new way of delivering pesticide to combat gypsy moths was developed in the form of tape. Each of 3 firms was able to produce the product in slightly differentiated form, each at constant marginal cost approximately \$1.00 per unit; fixed costs were sunk and amounted to about \$500,000 per firm. A study estimated demand for the first term as $\ln q_1 = \ln(3,325,000) - 3.5 \ln p_1 + 0.25 \ln p_2 + 0.25 \ln p_3$ and similarly for the other two firms. That is, all three firms have the same intercept and the same own (-3.5) and cross price (0.25) elasticities.

(a) Find the (differentiated good) Bertrand equilibrium prices, outputs and profits for the gypsy moth tape industry. (First order conditions could help here, since goods are not perfect subs.)

Solution: Here's a trick to make life easier. Since adding a constant or otherwise taking an increasing transformation of a function doesn't affect where the maximum occurs, the value of p_1 which solves

$$\max_{p_1} \quad p_1 q_1 - (500000 + q_1)$$

(that is, the argmax) is the same value which solves

$$\max_{p_1} \quad p_1 q_1 - q_1 = (p_1 - 1) q_1$$

and the same value which solves

$$\max_{p_1} \quad \ln(p_1 - 1) + \ln q_1$$

To solve the last problem, the first-order condition is simply

$$\frac{1}{p_1 - 1} - \frac{3.5}{p_1} = 0$$

which gives $p_1 = \frac{7}{5}$ ($= p_2 = p_3$ due to symmetry), $q_1 = q_2 = q_3 = 1212000$, and $\pi_1 = \pi_2 = \pi_3 = -15600$. We have a sunk fixed cost, so we must be in the short run. The firms will choose to produce despite the negative profits since they are selling at a price above minimum average variable cost (which is \$1).

(b) Suppose the three firms merge. What is the profit-maximizing choice now for prices and outputs, and what is the maximum profit?

Solution: The maximization problem is now

$$\max_{p_1, p_2, p_3} \quad p_1 q_1 + p_2 q_2 + p_3 q_3 - (1500000 + q_1 + q_2 + q_3)$$

Unfortunately, the trick we used in part (a) can not be used here. Letting $C = 3325000$, the first-order condition with respect to p_1 is

$$\begin{aligned} &Cp_1^{-3.5} p_2^{0.25} p_3^{0.25} - 3.5C(p_1 - 1) p_1^{-4.5} p_2^{0.25} p_3^{0.25} + 0.25Cp_1^{-0.75}(p_2 - 1) p_2^{-3.5} p_3^{0.25} \\ &+ 0.25Cp_1^{-0.75} p_2^{0.25} (p_3 - 1) p_3^{-3.5} = 0 \end{aligned}$$

We could take the other two first-order conditions and solve the system of equations, but that is *painful*. Instead, let's again exploit symmetry since the firms are identical. Letting $p_1 = p_2 = p_3$, the above first-order condition can be solved to get $p_1 = p_2 = p_3 = \frac{3}{2}$, $q_1 = q_2 = q_3 = 985000$, and $\pi_1 = \pi_2 = \pi_3 = -7000$. By merging, the firms can reduce their short-run losses.

(c) Briefly discuss realistic practices that might increase profits for the three separate firms beyond those you calculate in part a, but short of those in part b.

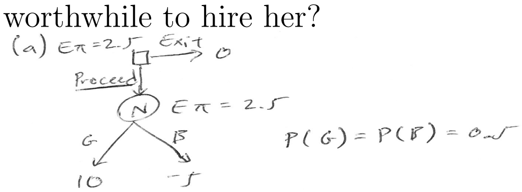
Solution: Barring anti-trust concerns, the firms can form a cartel and fix prices. This will increase profits beyond part (a) but will likely fall short of profits in part (b) since each firm would have an incentive to cheat and charge a slightly lower price to maximize its own profit at the expense of its cartel partners.

Another possibility is that firms could spend money on advertising to try to differentiate their product from their competitors' so that demand for their product would be less sensitive to the prices of their competitors. That is, they would like the cross price elasticities of demand for their product to be closer to zero.

4. If the market for your new product turns out well (G) you will gain incremental profit of 10 (millions of $), but otherwise (B) you will lose 5 if you bring out the product. You can assure an incremental profit of 0 if you cancel now. You believe the probability of G is 0.5.

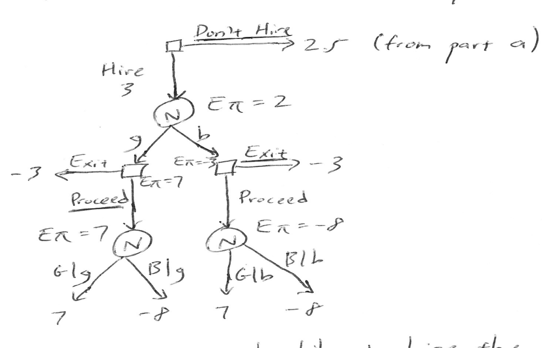
(a) Draw the decision tree and solve it; state whether or not you should bring out the product.

(b) Suppose a consultant can tell you now whether to expect G or B. Her fee is 3. Is it worthwhile to hire her?



Bring out the product since the expected payoff of doing so (2.5) is greater than exiting (0).

(b) If the consultant is always correct, $P(g|G)=P(b|B)=1$:



It is not worthwhile to hire the consultant. Even if the consultant is completely accurate, the informed tree has value 2, less than the uninformed tree (2.5).

5. Three candidates – A, B, and C – are running for President, and so far the evidence (e.g., prediction markets) indicates that their probabilities of winning are 0.6, 0.3 and 0.1 respectively. A crucial debate looms, and your analysis indicates that the probability that the eventual winner of the election wins such a debate with probability 0.5, while candidates who eventually lose the election are equally likely ($p = 0.25$) to win the debate. Use the spreadsheet template to compute:

(a) The posterior probabilities that each candidates will win the election given each possible winner of the debate.

(b) The "message probabilities" that each candidate will win the debate.

Solution: First, let's summarize the information given in the question. Prior probabilities are,

$$P(A) = 0.6, \quad P(B) = 0.3, \quad P(C) = 0.1,$$

and likelihood functions are,

$$\begin{aligned} &P(a|A) = 0.3, && P(b|A) = 0.25, && P(c|A) = 0.25, \\ &P(a|B) = 0.25, && P(b|B) = 0.5, && P(c|B) = 0.25, \\ &P(a|C) = 0.25, && P(b|C) = 0.25, && P(c|C) = 0.5. \end{aligned}$$

Then we can calculate joint probabilities as the product of prior and likelihood (ex. $P(a, A) = P(a|A)P(A)$),

$$\begin{aligned} &P(a, A) = 0.3, && P(b, A) = 0.15, && P(c, A) = 0.15, \\ &P(a, B) = 0.075, && P(b, B) = 0.15, && P(c, B) = 0.075, \\ &P(a, C) = 0.025, && P(b, C) = 0.025, && P(c, C) = 0.05. \end{aligned}$$

Message probabilities are the sum of corresponding joint probabilities $\left(\text{ex. } P(a) = \sum_{s=A,B,C} P(a, s)\right)$,

$$P(a) = 0.4, \quad P(b) = 0.325, \quad P(c) = 0.275.$$

By Bayes' theorem, posterior probabilities are joint probability divided by message probability,

$$\begin{aligned} &P(A|a) = 0.75, && P(A|b) = 0.46, && P(A|c) = 0.55, \\ &P(B|a) = 0.1875, && P(B|b) = 0.46, && P(B|c) = 0.27, \\ &P(C|a) = 0.0625, && P(C|b) = 0.08, && P(C|c) = 0.18. \end{aligned}$$

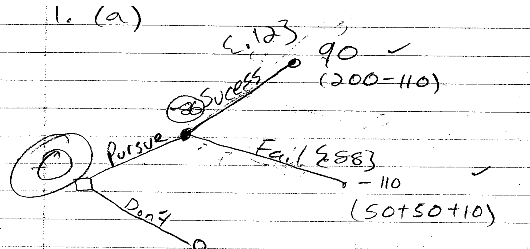
6. As a financial analyst for a biotech startup company, you are to advise on whether to pursue a line of research that will cost \$10 ("R&D") now and with probability 0.3 will lead to a potential product. It costs \$ 50 to test such a potential product, and the test leads to FDA approval ("validation") with probability 0.4. At that point the product is marketable, and at a further cost of \$50 ("production") it can be sold, bringing on average \$200 in revenue (net of other costs not mentioned). Assume that all dollar figures are present values in \$millions. Revenue come only from marketable products.

(a) Draw and solve the decision tree, assuming that you must sink all mentioned costs (R&D, validation and production) immediately?

(b) Redraw and solve the decision tree assuming that you can wait to see the R&D outcome before sinking the validation cost, and can wait for FDA approval before sinking the production cost.

(c) Now assume that the situation is as in b. above except that with probability 0.2 a marketable product is a "blockbuster". At an additional cost of \$100 (expansion) a blockbuster generates net revenue of 1000 instead of 200. Once mroe, draw and solve the decision tree. Also, note the probability that the line of research will ultimately produce a blockbuster.

1. (a)



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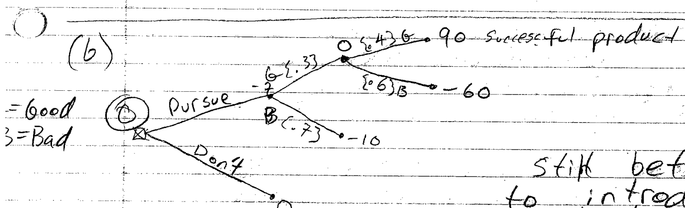
Don't Pursue -86 < 0

(b)



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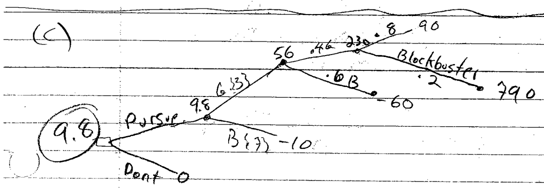
still better not to introduce the product

-7 < 0

(c)



you should pursue the product since expected payoff is now 9.8

chance of blockbuster = .3 · .4 · .2 = .024 = 2.4%