Econ 204b
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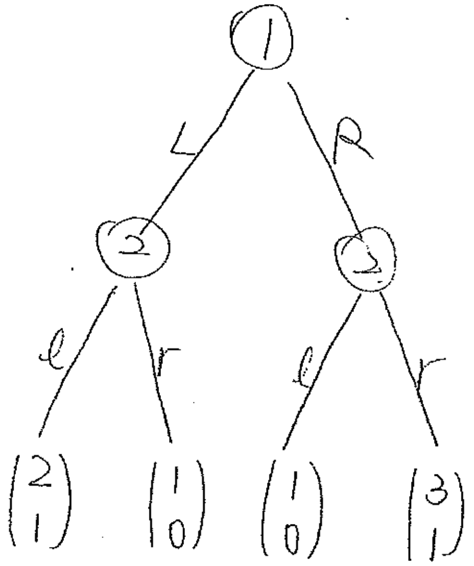
W2013

Answer Key for Midterm Exam Practice Problems

1a. Extensive Form Game:



Normal Form Game:

p1's strategy set: {L}, {R}

p2's strategy set: {Ll, Rl}, {Ll, Rr}
{Lr, Rl}, {Lr, Rr}

p2.

p1		Ll, Rl	Ll, Rr	Lr, Rl	Lr, Rr
	L	$\underline{2}, \underline{1}$	$2, \underline{1}$	$\underline{1}, 0$	$1, 0$
	R	$1, 0$	$\underline{3}, \underline{1}$	$\underline{1}, 0$	$\underline{3}, \underline{1}$

Here p2's strategy is a complete contingency plan for each information set he may encounter.

b. Pure NE:

From NFG,

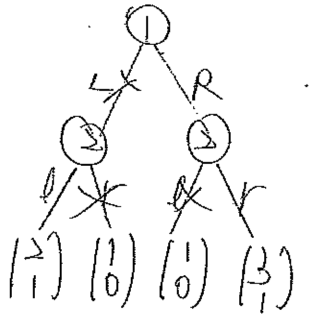
p1's best response for Ll, Rl is L
Ll, Rr is R
Lr, Rl is any
Lr, Rr is R.

p2's best response for R is {Ll, Rr} or {Lr, Rr}
— — — — — L is {Ll, Rl}.

Therefore there are three pure NE:
{L, (Ll, Rl)} with payoff (2, 1)
{R, (Ll, Rr)} — — — (3, 1)
{R, (Lr, Rr)} — — — (3, 1)

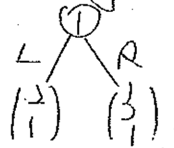
SGPNE:

By backward induction in each subgame, as show in the graph:



p2:
Given L, p2 will choose l
Given R, p2 will choose r

The reduced game is

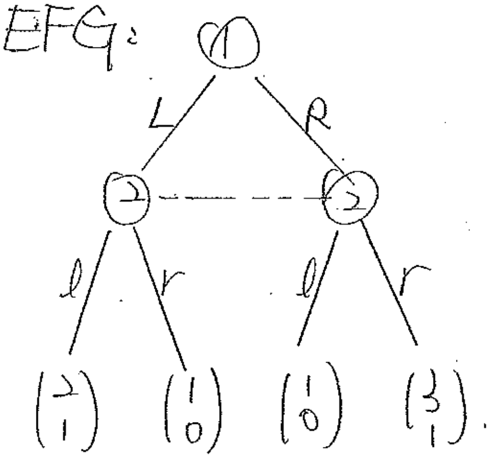


p1 will choose R.

The unique SGPNE is (R, r) with payoff (3, 1).

1.C. FTLR:

EFG:



✓

NFG:

p2

		q l	$1-q$ r
p1	p L	2, 1	1, 0
	1-p R	1, 0	3, 1

✓

d. NE: From EFG:

pure NE

p1's Best Response to r is R

------- l is L.

p2's ------- R is r

------- L is l

Two pure NE:

(L, l) with payoff (2, 1). ✓

(R, r) ------- (3, 1).

Mixed NE:

Suppose p1 chooses L with prob p, R $1-p$

p2 chooses l with prob q, r $1-q$

For $0<p<1$, p1 must be indifferent between L&R

$$2q + 1(1-q) = 1q + 3(1-q).$$

$$\Rightarrow \quad q = \frac{2}{3}$$

For $0<q<1$, p2 must be indifferent between l & r

$$1p + 0(1-p) = 0p + 1(1-p)$$

$$\Rightarrow \quad p = \frac{1}{2}.$$

Therefore the mixed NE is strategy

$\{\frac{1}{2}L + \frac{1}{2}R, \frac{2}{3}l + \frac{1}{3}r\}$ with payoff $(\frac{5}{3}, \frac{1}{2})$.

✓

~~E~~CON 204B Midterm Yuhan Xue

d, 1 continued).

SGPNE:

Since here there is no other subgames besides the whole game itself, there is not "subgame perfection".

Therefore SGPNE = NE! ✓

There are 2 pure SGPNEs:

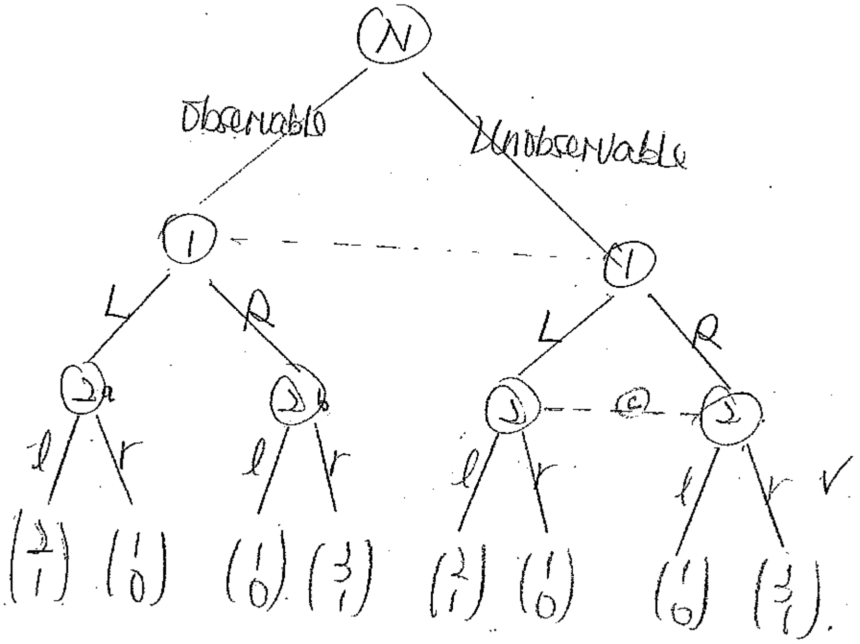
$\{L, l\}$ with payoff $(2, 1)$

$\{R, r\}$ with payoff $(3, 1)$.

There is 1 mixed SGPNE:

$\{\frac{1}{2}L+\frac{1}{2}R, \frac{2}{3}l+\frac{1}{3}r\}$ with payoff $(\frac{5}{3}, \frac{1}{2})$.

e. EFG.



NFG:

P1's strategy set: L, R.

P2's complete strategy set is the complete contingent plan for each information set he may encounter.

S1: Ll, Rl, Ul

S2: Ll, Rl, Ur

S3: Ll, Rr, Ul

S4: Ll, Rr, Ur → earier notation is $\overset{\uparrow}{l}\,\overset{\nwarrow}{r}\,r \leftarrow$ info set a, b, info set c

S5: Lr, Rl, Ul

S6: Lr, Rl, Ur

S7: Lr, Rr, Ul

S8: Lr, Rr, Ur

~~28~~/28

behavioral strategy:

		P2	
		l	r
P1	L	2, 1	1, 0
	R	1, 0	3, 1

f) → Now part of HW#3!

P2.

		S1	S2	S3	S4	S5	S6	S7	S8
P1	L	2, 1	depend on N. 4		[illegible]	[illegible]	[illegible]	[illegible]	1, 0
	R	1, 0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3, 1

✓

2. Assume: you are risk-neutral, each ~~way~~ method is worth trying at most once (won't work any better a second time), and that none of them damages the treasure. Then this problem is a special case of Problem #5 in HW #1. The decision tree there shows that the optimal rule is:

a) sort the methods i from highest p_i/c_i to lowest

b) try them in order until either success is achieved or $p_i/c_i < \left(\frac{1}{\text{treasure value}}\right)$. ↑ TV

Here $p_1/c_1 = .05/10 = .005$, so $TV \geq \frac{1}{.005} = 200$ for it to be worthwhile.

$p_2/c_2 = .90/360 = \cancel{.00}\frac{1}{400}$ so $TV \geq 400$ for it to be worthwhile.

Solution: ~~Try~~ If ETV < 200, forget it - don't try either method.

If ETV $\in [200, 400)$, try Method 1 first, give up if it fails.

≥ 400, try Method 1, then (if it fails) Method 2.

For the case ETV ≥ 400, the expected loss of trying Method 2 first, then method 1 is: $p_1c_2 - p_2c_1 = (.05)360 - (.9)10 = 18 - 9 = 9$ ✓

(~~as explan~~ again, see answer key for ~~PS#1~~ HW #1.

Name: Linh Bun

Problem 3. Bernoulli Function

$u(W) = \ln(W)$

ⓐ Coefficient of Relative Risk Aversion $r_r(W)$

$$r_r(W) = -W\left[\frac{u''(W)}{u'(W)}\right]$$

$$u'(W) = \frac{1}{W} \quad \text{and} \quad u''(W) = \frac{-1}{W^2}$$

$$r_r(W) = W\left[\frac{(1/W^2)}{(1/W)}\right] = 1$$

$$\boxed{r_r(W) = 1 \quad \forall W}$$ ✓

Coefficient of Absolute Risk Aversion $r_A(W)$

$$r_A(W) = -\left[\frac{u''(W)}{u'(W)}\right]$$

$$\boxed{r_A(W) = \frac{1}{W}}$$

$$r_A(W_0) = \frac{1}{100} \quad \text{and} \quad r_A(W_0 - 50) = \frac{1}{50}$$ ✓

ⓑ Certainty Equivalent

$$u[CE] = E[u(W)] = (0.2)u(50) + (0.8)u(100)$$

$$u[CE] = (0.2)\ln(50) + (0.8)\ln(100)$$

$$\Rightarrow \ln[CE] = (0.2)\ln(50) + (0.8)\ln(100)$$

I don't have a calculator so I will leave the solution with the Natural Logarithm

$$\boxed{CE = e^{(0.2)\ln(50)} \cdot e^{(0.8)\ln(100)}}$$

$$CE = (50^{0.2})(100^{0.8})$$ ✓ $= 100 \cdot 2^{-1/5} \approx 87$

Name: Linh Bun

Willing to pay $100-87=\underline{13}$ to completely insure!

ⓒ For risk averse individual, the Bernoulli utility function is concave. The certainty equivalent (CE) is by definition the amount of money that makes the individual indifferent between the gamble and the money for sure.

CE is then the amount of money that I need to ensure against risk

For a risk neutral individual, the Bernoulli utility function is Linear and

the CE is equal to the expected payoff from the gamble. Hence, the agent is indifferent between the CE and the gamble

The expected payoff from the gamble/risk

$E(W) = (0.2)(50) + (0.8)(100) = 10+80$

$E(W) = \underline{90}$

WTP for complete insurance $= 100-90$

$= \underline{10}$.

For risk neutral, $CE = 90$ ✓

for risk averse, $CE = (50^{0.2})((100)^{0.8}) < 90$

✓

i.e. $CE < E(W)$

d) Similarly

d) $u(CE) = 0.2\,u(150) + 0.8\,u(100)$

$\ln CE = 0.2 \ln 150 + 0.8 \ln 100 = 0.2 \ln 3/2 + 0.2 \ln 100 + 0.8 \ln 100$

$= 0.2 \ln 3/2 + \ln 100$

∴ $CE = 100\,(3/2)^{0.2} \approx 108.$

∴ willing to accept ≈ 8 to give up the opportunity

Risk-neutral person has $CE = EV = (0.2)(150) + (0.8)100 = 110$

∴ willing to accept ≥ 10 to give up the opportunity.

e) Behavioral economists emphasize that most people are loss-averse – would either pay more than \$13 to avoid the risk, or would accept less than \$8 for the opportunity, or both.

Problem 4.

	a	b
A	(10,10)	$(0,\underline{18})$
B	$(\underline{18},0)$	$\boxed{(\underline{8},\underline{8})}$

The NE in the Stage game {(B,b)}

ⓐ When the game is played 3 Times ✓

the NE of the repeated game is playing the NE in each Stage game, i.e. playing (B,b) in Stage ✓ game 1, 2 and 3.

In a finite repeated game, the NE of that game is playing the NE in the Stage game.

ⓑ Infinitely Repeated Game

The grim trigger strategy.

Player 1

{ Play A if player 2 plays a
Otherwise play B

Player 2:

{ Play a if player 1 plays A
Otherwise play b

- Suppose play 1 cooperates and plays A

Then the payoff is:

$$10 + 10\delta + 10\delta^2 \dots = \frac{10}{1-\delta}$$

- If player 1 deviates in the first period by playing B then the payoff is:

$$18 + 8\delta + 8\delta^2 \dots = 18 + \frac{8\delta}{1-\delta}$$

Hence, player 1 cooperates if

$$\frac{10}{1-\delta} \geq 18 + \frac{8\delta}{1-\delta}$$

$$\Rightarrow \quad 10 \geq 18(1-\delta) + 8\delta$$

$$10 \geq 18 - 10\delta \Rightarrow \delta \geq \frac{8}{10} = \frac{4}{5}$$

$\delta \geq 0.80$ ✓

Hence, the grim trigger strategy is NOT a NE when $\delta = 0.75$

The NE of the infinitely repeated game is playing the NE in each stage game.

✓

d) If δ increases or decreases for just *one* of the players, all-B is the only NE since the other player will still defect (play B). If *both* players have $\delta \geq 0.8$, then cooperation is possible in NE.