Econ 204b | Winter 2017
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Problem Set #1

Instructions. Due in class January 21. You are encouraged to discuss the problems with your classmates, but: (a) write up your answers separately-**do not copy**; and (b) at the end of **each** problem, **give credit** to anyone who helped and state how much of your own time you spent on it. For example, you might say "Jaehyun helped me when I was stuck; 2 hours," or "Eilin and I solved it together in 45 minutes; the writeup took me another 10 minutes."

Part I. Problems. When insufficient information is provided, write down a plausible specific assumption and proceed to a solution.

1. Your payoff function is $u(a, s) = 1 - (a - s)^2$, where the true state s is uniformly distributed on $[0, 1]$ independent of your actions. (Remark: u is sometimes called "the quadratic scoring rule.") You act before observing s, and your feasible actions a are constrained to the interval $[-10, 20]$.
   (a) What is your optimal action, and what is the maximal expected payoff?
   (b) How would your answers change if the distribution for s had an arbitrary cdf F?

2. Consider your own preferences over lotteries (L, M, ...) where outcomes are in the range [\$800, \$1200].
   (a) Suppose that you are an expected utility maximizer with a Bernoulli function in the CARA family. What value of the parameter a seems best to represent your true preferences? Hint: what is the largest amount x you pay for lottery L = [\$800 w/ prob .5, and \$1200 w/ prob .5]? Then solve the relevant equation (using x and L) for a.
   (b) Find a value of $c > 0$ for a mean-variance approximation of your preferences over the relevant range, that is, $\mu_L - c\sigma_L^2 > \mu_M - c\sigma_M^2$ whenever you prefer lottery L and to lottery M.
   (c) Suppose you could win \$1000 with probability 0.001, and otherwise get 0. Compute the mean and variance of this lottery. Compute its exact expected utility for your CARA preferences, and its approximate mean-variance utility using the value c obtained in part (b).
   (d) Should you accept this lottery if the alternative is 0 for sure? What does first order stochastic dominance say? What does the approximate mean-variance utility say? If there is a discrepancy, please explain.

3. A patient has either disease A or disease B. Diagnostic test 1 is positive with probability 0.7 (and negative with probability 0.3) with disease A, and is positive with probability 0.4 with disease B. Similarly, diagnostic test 2 is positive with probability 0.2 when the disease is A, and with probability 0.5 when the disease is B. Overall, the relative incidence of the two diseases is 60% A and 40% B.
   (a) Compute the joint probabilities $p(d, t_1, t_2)$ for each disease $d =$A,B and each test result $t_1 =$pos,neg and $t_2 =$pos,neg.
   (b) Compute the prior probabilities of each test result and each disease.
   (c) Calculate the posterior probability of disease A when t_2 is not performed and t_1 is positive (and also t_1 negative). Similarly, when t_1 is not performed and t_2 is positive (and also t_2 negative).
   (d) Calculate and put into a table the posterior probability of disease A for each possible combination of the test results (t_1, t_2) when both tests are performed. [Hint: you might want to do this problem on a spreadsheet! If so, please show the key cell formulas somewhere on your printout.]

4. A car dealership is considering whether to hire a new salesman. Based on previous experience, it classifies salesmen as great (bringing an average net value increment, i.e. payoff, of 20), good (payoff = 5) and poor (payoff = -10). The prior probabilities are 0.1, 0.5 and 0.4 respectively for great, good, and poor salesmen. Daily car sales follow a Poisson process with $\lambda = \frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively for great, good, and poor salesmen. [Recall that in a Poisson process the probability of n hits (cars sold) in a period of length t (days) is $p(n|\lambda t) = e^{-\lambda t}\left(\frac{(\lambda t)^n}{n!}\right)$]

(a) Find the (gross) value of perfect information regarding the salesman's type, for each realized type, and the expected value of such information. (Hint: what is the optimal uninformed decision?)

(b) In reality, the only way the owner can get information on salesman type is to hire for a week at a time (4 days at this dealership) and to observe how many cars he sells. The hiring cost is 0.04 per week. Find gross and net (less cost of sampling, the hiring cost) expected value of the sample information obtained from hiring for one week.

(c) Suppose the owner hires a new salesman for one week, and he sells 2 cars. At the end of the week, the owner has three choices: fire, hire for another week, or hire permanently. Draw the decision tree and solve it. [Hint: a spreadsheet again might help.]

5. The prior probability that your rival has new technology is 0.1. Their reported productivity is Normally distributed with variance 1.0 and mean 10 (respectively 12) with the old (respectively new) technology. What ranges of reported productivity would lead you to conclude (with posterior probability at least 0.5) that the rival has the new technology?

6. You must decide whether your pfissle firm has adequate or substandard quality control, the prior probabilities being 0.5 for adequate and 0.5 for substandard. Testing pfissles costs 1 per unit, and the test outcomes are iid normal with variance 9 and mean $+1$ if adequate (-1 if substandard). Your utility is -1000 when you err (deciding adequate when it isn't, or inadequate when it is adequate), and is 0 when you are correct.

(a) You must decide in advance how many units to test. Sketch the decision tree, calculate the optimal decision (# units to test, adequate/inadequate choice) and the minimized expected loss. [Hint: assume that you will decide quality control is adequate iff the mean test result is positive. For completeness, verify the assumption when you are done. Warning: This problem is challenging, even with the hint.]

(b) Write out the objective function for the same problem except that (a) the prior probability of substandard quality control is 0.2, and (b) you lose 400 if you decide control is inadequate when it actually is adequate. (For extra credit, solve the problem.)

(c) For the same cost you can test 1 unit at a time. Sketch the decision tree and outline a procedure for solving it. (Extra credit for actually solving for the optimal-sequential procedure.)

7. To achieve a particular outcome (payoff normalized to 1.0), alternative methods $k = 1, \ldots, K$ are available, each with known cost $c_k > 0$ and known success probability $p_k > 0$. You can only try each method once, and you can only achieve the outcome once. Which method should you try first? When should you give up? Sketch a decision tree for this decision problem, conjecture the full solution, and give a plausibility argument for your conjecture. For extra credit, write out a formal proof of your conjecture. [Hint: to get started, let $K = 2$, pick a few examples of c_k and p_k, try both orders, and see which order gives a higher expected net payoff. Then guess the general pattern, and use inequalities in arbitrary p's and c's to verify your guess.]

II. Textbook problems. Skim all the problems in MWG Chapter 6. Write up and turn in solutions to 6.C.1, 6.C.2, 6.C.18 and 6.C.20.

III. Short essay. Write briefly (about 100 words) for an audience whose technical background is similar to yours, on **one** of the following two topics.

1. Bayesian decision theory describes how an idealized decision maker would behave. It doesn't resemble at all the process that normal humans go through when making decisions. Why is there such a large gap between theory and reality? What are the implications (if any) of your analysis?

2. A sports fan wants badly for the home team to win, and receives a large negative payoff when it loses. Write out a simple model of the fan's decision of how much to bet for (or against) the home team at fair odds. Try to reconcile the prediction of your model (optimal betting) with the fact that bets in favor of the home team come disproportionately from local fans.