Zeinab Golmohamadian

PS # 3 Submission: 2/28

By myself - 2 hours

1) one population case:

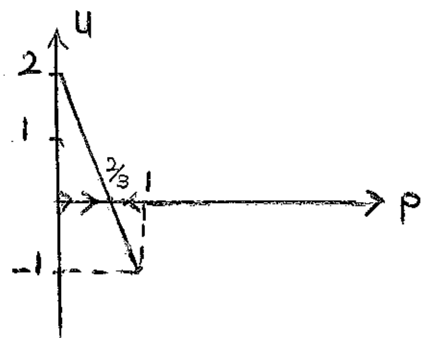
| | U | D |
|---|---|---|
| $\bar{U}$ | 0 | 4 |
| $\bar{D}$ | 1 | 2 |
| | $p$ | $1-p$ |

$u(\bar{U}, p) = 0(p) + 4(1-p) = 4-4p$

$u(\bar{D}, p) = p + 2(1-p) = 2-p$

$\Delta p = -3p+2$

$-3p+2=0 \Rightarrow p=\frac{2}{3}$ : Steady state




if $p > 2/3 \Rightarrow \Delta(p) < 0$ ($p$ decreases)

if $p < 2/3 \Rightarrow \Delta(p) > 0$ ($p$ increases)

if $p = \frac{2}{3} \Rightarrow \Delta(p) = 0$ ($p$ does not change)

$\Rightarrow p^* = 2/3$ is an stable evolutionary equilibria. (one player game)

b) Row players and column players have seperable populations.

| | | $p$ | $(1-p)$ |
|---|---|---|---|
| | | U | D |
| $q$ | U | (0,0) | (4,1) |
| $(1-q)$ | D | (1,4) | (2,2) |

$$\begin{cases} \Delta p = -3p+2=0 \\ \Delta q = -3q+2=0 \end{cases}$$

$p=\frac{2}{3} \quad q=\frac{2}{3}$

$0q + 4(1-q) = 4-4q$

$q + 2(1-q) = -q+2$

$\Delta(q) = -3q+2=0 \quad q=\frac{2}{3}$

If $p > 2/3 \Rightarrow \Delta(p) < 0 \Rightarrow \dot{q} < 0 \Rightarrow q\downarrow$

If $p < 2/3 \Rightarrow \Delta(p) > 0 \Rightarrow \dot{q} > 0 \Rightarrow q\uparrow$

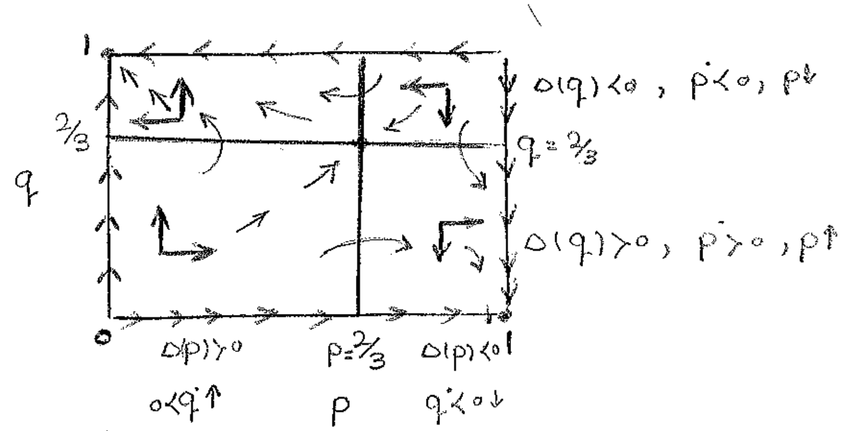
If $p^* = \frac{2}{3} \Rightarrow \Delta(p) = 0 \Rightarrow \bar{q}$ (does not change)

If $q^* > 2/3 \Rightarrow \Delta(q) < 0 \Rightarrow \dot{p} < 0$ , $p\downarrow$

If $q^* < 2/3 \Rightarrow \Delta(q) > 0 \Rightarrow \dot{p} > 0$ , $p\uparrow$

If $q^* = \frac{2}{3} \Rightarrow \Delta(q) = 0 \Rightarrow \bar{p}$ (does not change)

Phase Portrait

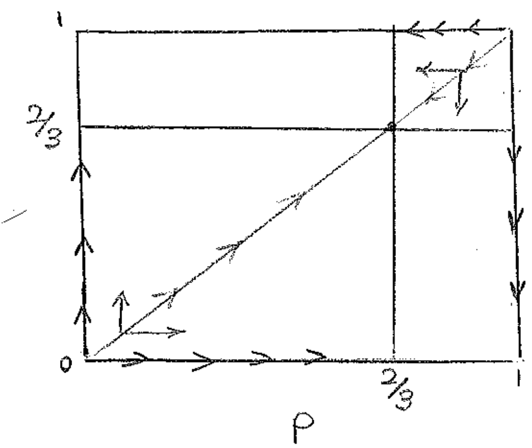



There are three evolutionary equilibria in this game

Stable equilibria : $(p_1, q_1) = (1, 0)$ , $(p_2, q_2) = (0, 1)$

✓ Saddle (unstable) equilibria : $(p_3, q_3) = (2/3, 2/3)$

c)




Since both axes are p, the only valid points are on the diagnol.

Stable evolutionary equilibria is $p^* = 2/3$

perfect!

Question #2

By myself - 4 hours

a) $u^* = \arg\max \prod_{i=1}^{2} (u_i - \bar{u}_i) = \max_{\{u_1, u_2\}} (u_1 - \bar{u}_1)(u_2 - \bar{u}_2)$

Subject to:

$u_1 + u_2 = 10$

$u_1, u_2$ : gains of country 1 and 2 if both trade

$\bar{u}_1, \bar{u}_2$ : threat points

Replacing constraint on the objective function:

$\max (u_1 - \bar{u}_1)(10 - u_1 - \bar{u}_2) = 10u_1 - u_1^2 - u_1\bar{u}_2 - 10\bar{u}_1 + u_1\bar{u}_1 + \bar{u}_1\bar{u}_2$

F.O.C wrt $u_1$ $\quad 10 - 2u_1 - \bar{u}_2 + \bar{u}_1 = 0 \Rightarrow u_1 = \dfrac{\bar{u}_2 - \bar{u}_1 - 10}{-2} = \dfrac{\bar{u}_1 + 10 - \bar{u}_2}{2}$

Nash Bargaining Solution : $u_1 = \dfrac{\bar{u}_1 - \bar{u}_2}{2} + 5$

for country 1

Asumming $\bar{u}_1 = \bar{u}_2 = 0$. $\Rightarrow u_1 = 5$ ✓

$$\text{Max } (10 - u_2 - \bar{u}_1)(u_2 - \bar{u}_2) = 10u_2 - 10\bar{u}_2 - u_2^2 + u_2\bar{u}_2 - \bar{u}_1 u_2 + \bar{u}_1\bar{u}_2$$

F.o.c w.r.t $u_2$:

$$10 - 2u_2 + \bar{u}_2 - \bar{u}_1 = 0$$

$$u_2 = \frac{\bar{u}_2 - \bar{u}_1}{10} + 5$$ : Nash Bargaining Solution for country 2

$\Rightarrow$ Nash Bargaining Solution for both countries is $(5,5)$ (if $\bar{u}_1 = \bar{u}_2 = 0$) ✓

b) $v(\{A\}) = v(\{B\}) = v(\{C\}) = 0$ : (No trade, No mutual gain)

$v(\{A, C\}) = 15$

$v(\{B, C\}) = 5$

$v(\{A, B, C\}) = 10 + 5 = 15$ ✓

C) According to MWG, the condition for convex function is:

If $S \subset T$ and $i \in N \setminus T$ $\quad v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$

$\gamma(\{A,C\}) - \gamma(\{A\}) = 15-0 = 15 \geq \gamma(\{A,B,C\}) - \gamma(\{A,B\}) = 15-10=5$

$\gamma(\{B,C\}) - \gamma(\{B\}) = 5-0=5 \geq \gamma(\{A,B,C\}) - \gamma(\{A,B\}) = 5$

$\gamma(\{A,B\}) - \gamma(\{B\}) = 10-0=10 = \gamma(\{A,B,C\}) - \gamma(\{B,C\} = 15-5=10$

$\gamma(\{A,C\} - \gamma(\{C\}) = 15-0=15 \geq \gamma(\{A,B,C\}) - \gamma(\{B,C\}) = 15-5=10$

$\Rightarrow$ This game is not a convex game. ✓

d) Dividing all functions by 15, to find normalized functions. ($\gamma\{A,B,C\} = 1$ :

$\gamma(\{A,B,C\}) = 1$

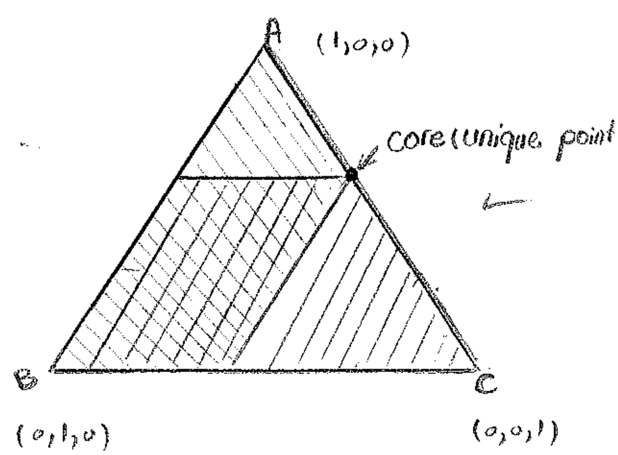
$\gamma(A) = \gamma(\{B\}) = \gamma(\{C\}) = 0$

$\gamma(\{A,C\}) = 1 \qquad U_A + U_C \geq 1$

$\gamma(\{B,C\}) = \frac{5}{15} = \frac{1}{3} \qquad U_B + U_C \geq \frac{1}{3}$

$\gamma(\{A,B\}) = \frac{10}{15} = \frac{2}{3} \qquad U_A + U_B \geq \frac{2}{3}$

Otherwise blocked!




/// $U_B + U_C \geq \frac{1}{3}$ Feasible Area

//// $U_A + U_B \geq 2/3$ " "

— $U_A + U_C = 1$ " "

Core : $(2/3, 0, 1/3) = (U_A, U_B, U_C)$

e)

| | $MC_A$ | $MC_B$ | $MC_C$ |
|---|---|---|---|
| $\{A, B, C\}$ | $v(\{A\})=0$ | $v(\{A,B\}) - v\{A\} = 2/3$ | $v(\{A,B,C\}) - v\{A,B\} = \frac{1}{3}$ |
| $\{A, C, B\}$ | $v(\{A\})=0$ | $v(\{A,C,B\}) - v\{A,C\} = 0$ | $v(\{A,C\}) - v(A) = 1$ |
| $\{B, A, C\}$ | $v(B,A) - v(B) = 2/3$ | $v(B)=0$ | $v(\{A,B,C\}) - v(\{A,B\}) = 1/3$ |
| $\{B, C, A\}$ | $v(\{A,B,C\}) - v(\{B,C\}) = 2/3$ | $v(B)=0$ | $v(\{B,C\}) - v(B) = 1/3$ |
| $\{C, A, B\}$ | $v(\{A,C\}) - v(\{C\}) = 1$ | $v(\{A,B,C\}) - v(\{A,C\}) = 0$ | $v(\{C\})=0$ |
| $\{C, B, A\}$ | $v(\{A,B,C\}) - v(\{B,C\}) = 2/3$ | $v(\{B,C\}) - v(\{C\}) = 1/3$ | $v(\{C\})=0$ |
| $\Sigma$ | 2 | 1 | 2 |
| $\frac{\Sigma}{3!}$ | $\frac{2}{6} = \frac{1}{3}$ | $\frac{1}{3!} = \frac{1}{6}$ | $\frac{2}{6} = \frac{1}{3}$ |

Shapley value: $\left(\frac{1}{3}, \frac{1}{6}, \frac{1}{3}\right)$ ✓

perfect!

3) a-

6 hours - I brain stormed with Alireza.

$mc = 10 \qquad mb = 210 - q$

$\Pi_{seller} = p.q - 10q$

$\Pi_{Buyer} = [(210-1) - p(1)] + [(210-2) - p(1)] + \cdots + [(210-q) - p(1)]$

$= \sum_{t=1}^{q} [(210-t) - p] = 210q - pq - \sum_{t=1}^{q} t = 210q - pq - \frac{q(q+1)}{2}$

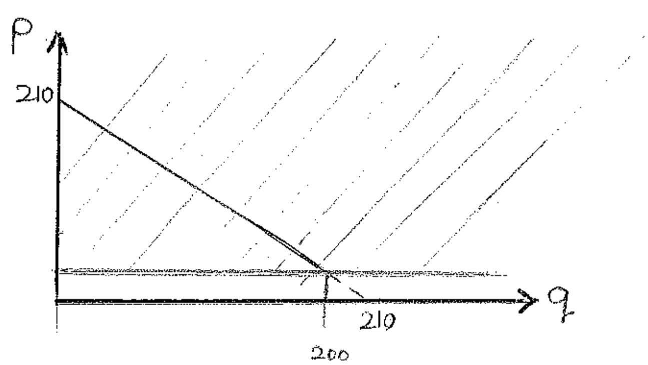
$\Pi_T = \Pi_S + \Pi_B = \cancel{p.q} - 10q + 210q - \cancel{pq} - \frac{q^2}{2} - \frac{q}{2} = 200q - \frac{q}{2} - \frac{q^2}{2} \simeq 200q - \frac{q^2}{2}$ ✓

b-

$\text{Max } \Pi_T = \text{Max}_q \left(200q - \frac{q^2}{2}\right)$

F.O.C w.r.t $q$ : $200 - q = 0 \Rightarrow q = 200 \qquad \text{Max } \Pi_T = 200$

$\text{Max } \Pi_T = 200(200) - \frac{(200)^2}{2} = 40,000 - 20,000 = 20,000$

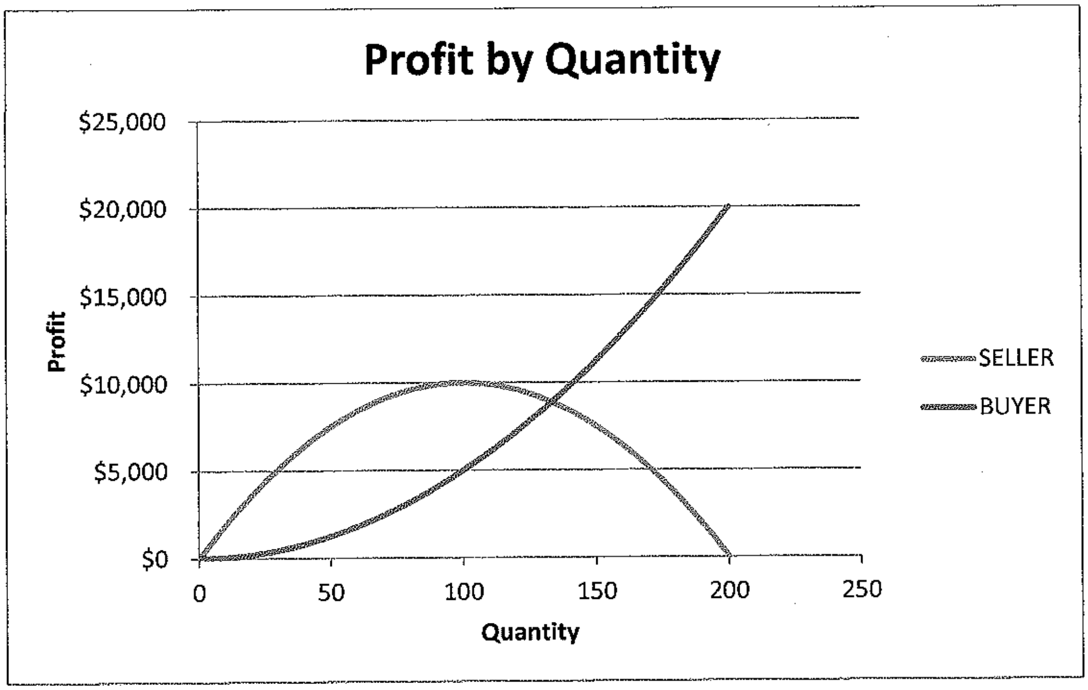


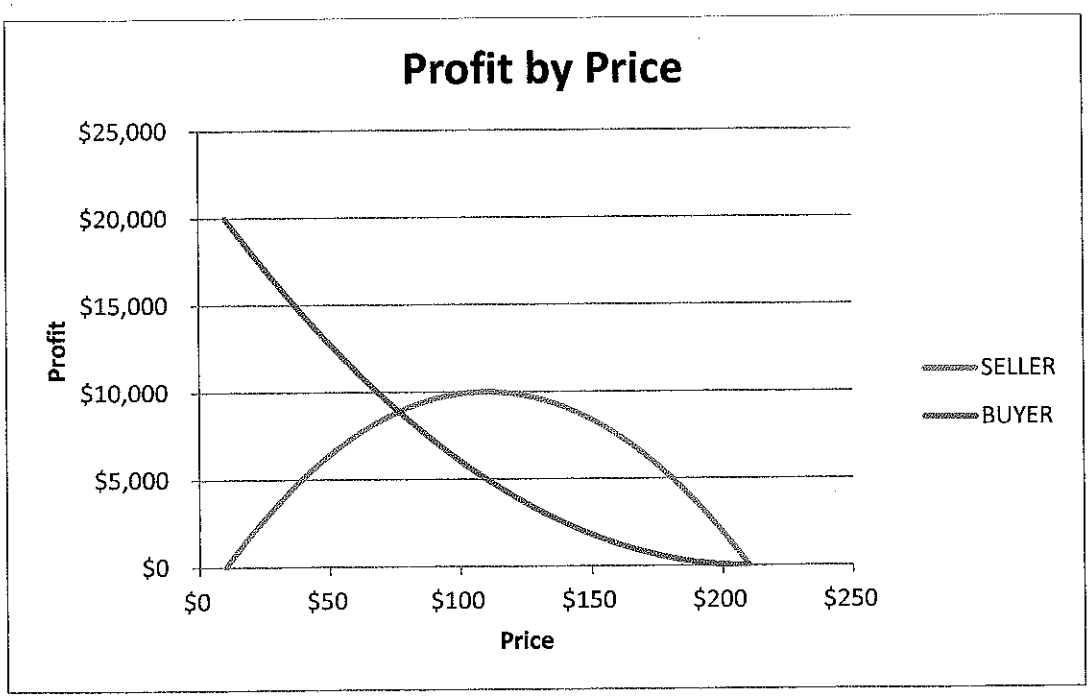

$\Pi_S / \partial q = p - 10 = 0 \Rightarrow p = 10$

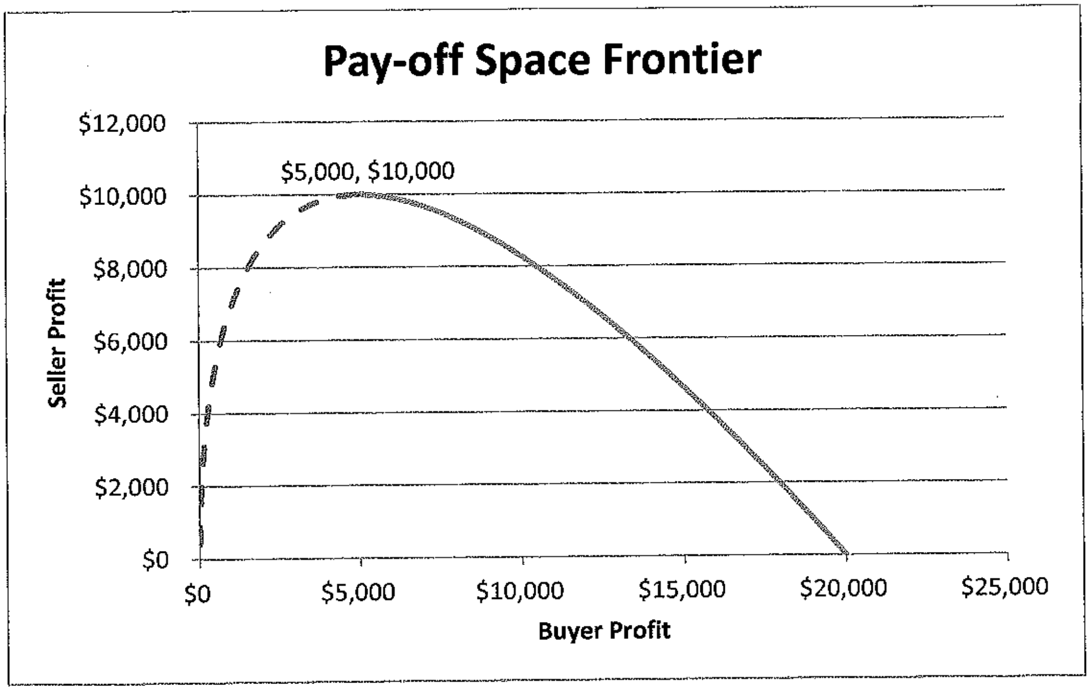
For seller, p must be greater than 10 to sell his product.

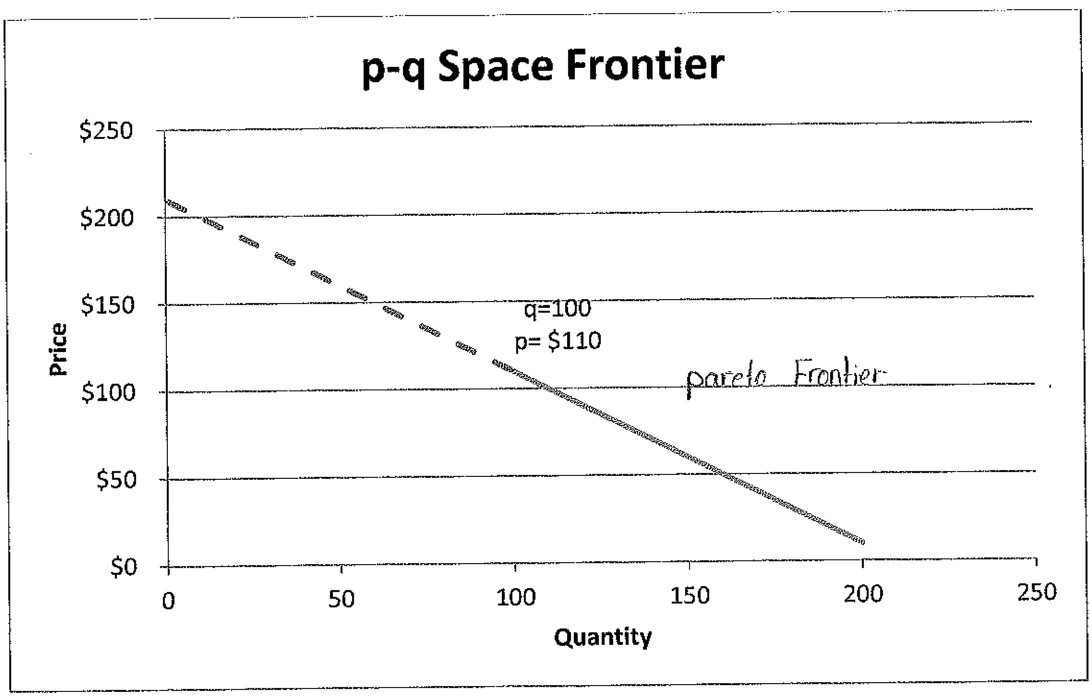
Marginal cost for seller $= p - 10 = 0 \Rightarrow p = 10$

Marginal " " Buyer $= 210 - q - p = 0 \Rightarrow 210 - p = q$


Profit by Quantity
$25,000
$20,000
$15,000
$10,000
$5,000
$0
Profit
0
50
100
150
200
250
Quantity
SELLER
BUYER


Profit by Price
$25,000
$20,000
$15,000
$10,000
$5,000
$0
Profit
$0
$50
$100
$150
$200
$250
Price
SELLER
BUYER


Pay-off Space Frontier
$12,000
$10,000
$8,000
$6,000
$4,000
$2,000
$0
Seller Profit
$5,000, $10,000
$0
$5,000
$10,000
$15,000
$20,000
$25,000
Buyer Profit


p-q Space Frontier
$250
$200
$150
$100
$50
$0
Price
q=100
p= $110
pareto Frontier
0
50
100
150
200
250
Quantity

3-c) The Seller chooses price and buyer adjust his q based on seller's price. If the seller puts a high price, the buyer will decrease his quantity purchased. $\Rightarrow$ Seller maximized his $\Pi_s$ subject to MB of buyer.

Seller $\quad \max \Pi_s = p.q - 10q$

subject to $210 - q = p$

$$\mathcal{L} = pq - 10q + \lambda(210 - q - p)$$

F.O.C : $\partial\mathcal{L}/\partial p = q - \lambda = 0 \Rightarrow q = \lambda$

$\Rightarrow q = p - 10$

$\partial\mathcal{L}/\partial q = p - 10 - \lambda = 0 \Rightarrow p - 10 = \lambda$

$\partial\mathcal{L}/\partial \lambda = 210 - q - p = 0 \Rightarrow p + q = 210$

$\Rightarrow p + q = 210 \Rightarrow p + p - 10 = 210 \Rightarrow 2p = 220 \Rightarrow p = 110$

$110 + q = 210 \Rightarrow q = 100$

$\Pi_s = p.q - 10q = 110(100) - 10(100) = 10{,}000$

$\Pi_B + \Pi_s = 15000$

$$\Pi_B = 210q - pq - \frac{q^2}{2} = \overset{21000}{210(100)} - \overset{11000}{110(100)} - \overset{5000}{\frac{10000}{2}} = 5000$$

d) Buyer maximizes his $\Pi_B$ (profit) subject to MB of the

Seller when $p \geq 10$

$$\text{Max } 210q - pq - \frac{q^2}{2}$$

Subject to $p \geq 10$

$$\mathcal{L} = 210q - pq - \frac{q^2}{2} + \lambda(p-10)$$

$$\partial\mathcal{L}/\partial q = 210 - p - q = 0 \Rightarrow 210 = p + q$$

$$\partial\mathcal{L}/\partial p = -q + \lambda = 0 \Rightarrow \lambda = q$$

$$\partial\mathcal{L}/\partial \lambda = p - 10 = 0 \Rightarrow p = 10$$ when buyer chooses price.

$$p + q = 210 \Rightarrow 10 + q = 210 \Rightarrow q = 200$$ ✓

$$\Pi_B = 210q - pq - \frac{q^2}{2} = 210\cdot(200) - 10(200) - \frac{(200)^2}{2} = 42000 - 2000 - 20000$$
$$= 20{,}000$$

$$\Pi_S = p\cdot q - 10q = 10\cdot(200) - 10(200) = 0$$

$$\Pi_B + \Pi_S = 20{,}000 + 0 = 20{,}000$$

3-e) $\max_{\Pi_B, \Pi_S} (\Pi_B - \bar{\Pi}_B)(\Pi_S - \bar{\Pi}_S)$

S.t $\quad \text{Max } \Pi_T = \Pi_B + \Pi_S = 20000$ ✓

$(\bar{\Pi}_B, \bar{\Pi}_S) = (0,0)$ treat point : when both firms don't participate in trade

$\Pi_S = 20{,}000 - \Pi_B \qquad \bar{\Pi}_B = 0 \qquad \bar{\Pi}_S = 0$

$\Rightarrow \max \quad \Pi_B(20{,}000 - \Pi_B) = 20{,}000\,\Pi_B - \Pi_B^2$

$\partial(\ )/\partial\Pi_B = 20{,}000 - 2\Pi_B = 0 \implies \Pi_B = 10{,}000$

$\Pi_T = \Pi_B + \Pi_S = 20000 \implies 10{,}000 + \Pi_S = 20{,}000 \Rightarrow \Pi_S = 10{,}000$

$\Rightarrow$ Nash Bargainning solution : $(10{,}000, 10{,}000) : (\Pi_S, \Pi_B)$ ✓

perfect!!

**PROBLEM 4**

NORMAL FORM GAME

| | $ll'l''$ | $ll'r''$ | $lr'l''$ | $lr'r''$ |
|---|---|---|---|---|
| $L$ | $(2,1)$<br>$(2,1)$ | (p+1,p)<br>$(1.7,.7)$ | $(2,1)$<br>$(2,1)$ | (p+1,p)<br>(1.7,.7) |
| $R$ | $(1,0)$<br>$(1,0)$ | (3-2p,1-p)<br>$(.6,.3)$ | (2p+1,0)<br>$(2.4,0)$ | $(3,1)$<br>$(3,1)$ |

*Values in parenthesis are for p=0.7

| | $rl'l''$ | $rl'r''$ | $rr'l''$ | $rr'r''$ |
|---|---|---|---|---|
| $L$ | (2-p, 1-p)<br>$(1.3,.3)$ | $(1,0)$<br>$(1,0)$ | (2-p, 1-p)<br>$(1.3,.3)$ | $(1,0)$<br>$(1,0)$ |
| $R$ | $(1,0)$<br>$(1,0)$ | (3-2p,1-p)<br>$(.6,.3)$ | (2p+1,0)<br>$(2.4,0)$ | $(3,1)$<br>$(3,1)$ |

*Values in parenthesis are for p=0.7

BNE: $(L, ll'l'')$, $(R, lr'r'')$, $(R, rr'r'')$, $(L, l, \frac{2}{7}l', \frac{2}{7}r', l'')$

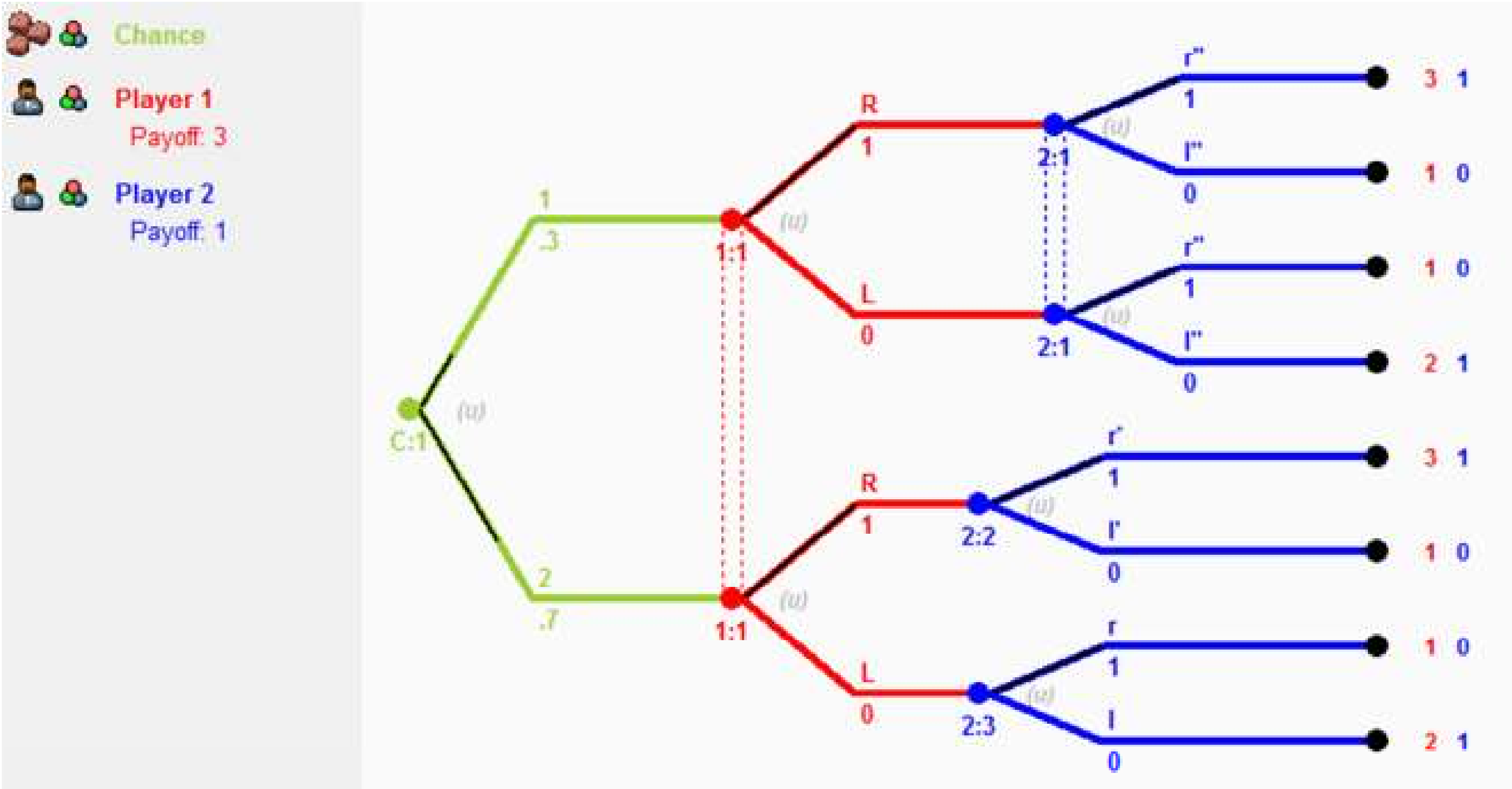
$(R, lr'r'')$ is the only PBE as equilibriums containing $r$ or $l'$ are not subgame perfect. Looking at the 2 subgames (ignoring the entire game) that occur under the "observed" tree, we see that $l$ and $r'$ are the best responses. Therefore, any strategy sets that contain $r$ or $l'$ (the opposite choices) cannot be subgame perfect.

Beliefs
Assume that in the unknown situation player 2 believes there is probability $q$ that player 1 has moved left and probability $1-q$ that player 1 has moved right.

Beliefs for each BNE/PBE for player 1 are unchanged, i.e. $p$ that player 2 observes and $(1-p)$ that player 2 doesn't observe.

Example: Say $q$ is the probability that player 1 moves left. Player 1, however, always knows $q$. Then for $(R, lr'r'')$:

- Known state: $p * (1-q) = p * 1 = p$
- Unknown state: $(1-p) * (1-q) = (1-p) * 1 = (1-p)$



Chance
Player 1
Payoff: 3
Player 2
Payoff: 1
C:1
1
.3
2
.7
1:1
1:1
R
1
L
0
R
1
L
0
2:1
2:1
2:2
2:3
r''
1
l''
0
r''
1
l''
0
r'
1
l'
0
r
1
l
0
3 1
1 0
1 0
2 1
3 1
1 0
1 0
2 1

STACKELBERG VS. COURNOT COMPARISON

**Setup**

$p = 30 - q_T$
$q_T = q_L + q_F$
$MC = c_L = c_F = 0$

**Cournot** *Note that individual L picks q simultaneously with individual F here*

Payoff function $u_L(q_L, q_F) = (p(q_T) - c)q_L$
$u_L(q_L, q_F) = (30 - q_L - q_F - 0)q_L$

Maximize with respect to $q_L$

$\frac{du_L}{dq_L} = p^{'}(q_T)q_L + p(q_T) - c = -q_L + 30 - q_T = 0$

This gives $q_L = 30 - q_T = p$, where $q_F$ is the same by symmetry.

Then $q_T^{NE} = 20$, $p^{NE} = 10$, and $\pi_L^{NE} = 100 = \pi_F^{NE}$.

**Stackelberg** *Note that individual L picks first here*

Payoff function $u_L(q_L, BR_F(q_L)) = (p(q_T) - c)q_L$
We can get the best response by solving for

$u_F(q_F, q_L) = 30q_F - q_F^2 - q_L q_F$

Differentiating with respect to $q_F$ (FOC) gives

$$BR_F(q_L) = q_F = \frac{30-q_L}{2}$$

We can then plug this into the leader's payoff function:
$u_L(q_L, BR_F(q_L)) = (30 - q_L - \frac{30-q_L}{2} - 0)q_L$

Maximize with respect to $q_L$

$\frac{du_L}{dq_L} = 15 - q_L = 0$

This gives $q_L = 15$ and $q_F = 7.5$

Then $q_T^{NE} = 22.5$, $p^{NE} = 7.5$, $\pi_F^{NE} = 56.25$, and $\pi_L^{NE} = 112.5$

As mentioned in class, the leader profits more in this case, but total profits are down.

MCW 12.B.1

a) Monopoly maximizes: $q(p^m)p^m - c(q(p^m))$

FOC: $q'(p^m)p^m + q(p^m) - q'(p^m)c'(q(p^m)) = 0$

$q'(p^m)[p^m - c'(q(p^m))] = q(p^m)$

$[p^m - c'(q(p^m))] = \frac{q(p^m)}{q'(p^m)}$

$\frac{[p^m - c'(q(p^m))]}{p^m} = \frac{q(p^m)}{q'(p^m)p^m} = \frac{1}{\epsilon}$

b) With $mc > 0$, $\frac{[p^m - c'(q(p^m))]}{p^m} < \frac{p^m}{p^m} = 1$

Therefore, $\frac{1}{\epsilon} < 1 \Rightarrow \epsilon > 1$

Liam Rose
3/5/13

12.B.3 FROM MWG

**Setup** – A monopolist faces demand function $x(p, \theta)$ and cost function $c(x(p, \theta), \phi)$
Let's assume that cost is concave in $x$ ($c_x > 0$ and $c_{xx} \geq 0$)and that demand is downward sloping in $p$ ($x_p < 0$).
Let's also make the conventions that $\theta$ shifts demand upward, so $x_\theta > 0$, and that $\phi$ increases total and marginal cost, so $c_\phi, c_{x\phi} > 0$.

**Maximize and Do Comparative Statics** Now we want to maximize with respect to p:

$max\ x(p, \theta)p - c(x(p, \theta), \phi)$

This gives FOC:

$$x_p(p, \theta)p + x(p, \theta) - c_x(x(p, \theta), \phi)x_p(p, \theta) = 0$$

Statics (arguments excluded for brevity):

Totally differentiate FOC with respect to $\theta$, collect terms involving $\frac{dp}{d\theta}$ and solve to obtain

$$\frac{dp}{d\theta} = \frac{-(p - c_p)x_{p\theta} - x_\theta + x_\theta x_p c_{xx}}{2x_p + (p - c_x)x_{pp} - x_p^2 c_{xx}} \tag{1}$$

Then this is greater than 0 (price increasing in $\theta$) if $x_{p\theta}$ is positive.

Totally differentiate FOC with respect to $\phi$, collect terms involving $\frac{dp}{d\phi}$ and solve to obtain

$$\frac{dp}{d\phi} = \frac{x_p c_{x\phi}}{2x_p + (p - c_x)x_{pp} - x_p^2 c_{xx}} \tag{2}$$

We know the signs of all the terms in (2), and $\frac{dp}{d\phi}$ is positive. Therefore, price is increasing in $\phi$.

12.B.6. If gov't tax or subsidize firm on per unit of outcome, ($t>0$: tax, $t<0$: subsidy)

$$\max_{q} \; p(q)q - c(q) - tq.$$

$$\Leftrightarrow p'(q)q + p(q) - c'(q) - t = 0$$

For efficient outcome requires $p(q) = c'(q)$.

So $p'(q) \cdot q = t$. Since demand curve is downward sloping, $p'(q) < 0$

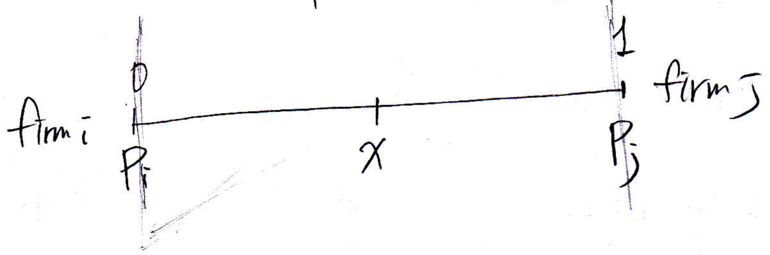
$$t = p'(q) \cdot q < 0$$

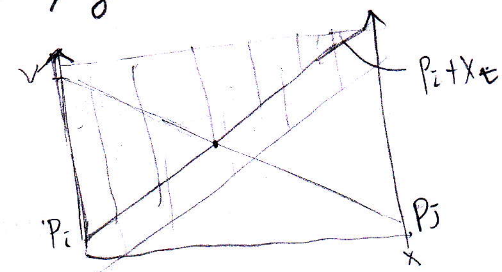
So goverment subsidize the monopoly to have efficient outcome.

12.C.14

Let's assume there is no difference in product and only difference is distance

① The consumer prefers buying to not buying




$$P_i + xt = P_j + (1-x)t \Leftrightarrow xt + (x-1)t = P_j - P_i$$

$$\Leftrightarrow 2tx - t = P_j - P_i, \Leftrightarrow 2tx = P_j - P_i + t \Leftrightarrow x = \frac{P_j - P_i + t}{2t}$$

The utility of this consumer from buying from firm $i$

is $v - P_i - xt = v - P_i - \frac{(P_j - P_i + t)}{2} = v + \frac{-P_j - P_i - t}{2}$

So the consumer prefer buying to not buying is

$v > \frac{P_i + P_j + t}{2}$ → demand is affected by other firm al

② The consumer prefers not buying to buying.

Similarly $v < \frac{P_i + P_j + t}{2}$

③ The consumer is indifferent b/w buying and not buying

$v = \frac{P_i + P_j + t}{2}$ → price competition

12.C.14

(a) M: costomers uniformly distributed on segment.

$$x_i(p_i, p_j) \begin{cases} 0 & \text{if } p_i > p_j + t \\ \dfrac{(t+p_2-p_1)M}{2t} & \text{if } p_i \in [p_j - t, p_j + t] \\ M & \text{if } p_i < p_j - t \end{cases}$$

若 $p_i$ change its price little bit from $p_1$ to $p_1^*$,

then $p_i^* + x_t = p_j + (1-x)t$.

so $x_i(p_1^*, p_j) = \dfrac{(t+p_j-p_i^*)M}{2t}$.

$$\underset{p_i^*}{\text{Max}}\ (p_i^*-c)x_i(p_i^*, p_j) = \frac{(p_i^*-c)(t+p_j-p_i^*)M}{2t}$$

$$\frac{\partial \pi_{p_i^*}}{\partial p_i^*} = \frac{(t+p_j-p_i^*)M}{2t} + \frac{-(p_i^*-c)M}{2t}$$

$$= \frac{M(t+p_j-2p_i^*+c)}{2t} = 0, \quad \frac{t+p_j+c}{2} = p_i^*$$

$$\boxed{p_i^* = p_j^*}$$

Similarly $\dfrac{\partial \pi_{p_j^*}}{\partial p_j^*} = \dfrac{M(t+p_i-2p_j^*+c)}{2t} = 0$, $\dfrac{t+p_i+c}{2} = p_j^*$

$$\Leftrightarrow t+p_i^*+c = 2p_i^* \Leftrightarrow t+c = p_i^* = p_j^*$$

looks good!

(b) ① case

∴ $v > \dfrac{p_i+p_j+t}{2} = \dfrac{3t+2c}{2} = c + \dfrac{3}{2}t$

∴ $v \in \left(c+\frac{3}{2}t, c+2t\right)$

12.C.14 (c) In case of ②,

If firm $i$ changes its price little bit $p_i \to p_i^*$

$p_i + xt = v$ the demand will be determind by $x < \frac{1}{2}$ costomer.

therefore $X_i(p_i^*, p_j) = \frac{(v - p_i^*)M}{t}$

$$\max_{p_i^*} (p_i^* - c)(x_i(p_i, p^*)) = (p_i^* - c)\frac{(v - p_i^*)M}{t}$$

$$\frac{\partial \pi_{p_i}^*}{\partial p_i^*} = \frac{M(v - p_i^* - p_i^* + c)}{t} = \frac{M}{t}(v - 2p_i^* + c) = 0$$

$$\therefore \underbrace{p_i^* = \frac{v+c}{2}}$$ Similarly $\underbrace{p_j^* = \frac{v+c}{2}}$

from ②

$$\therefore v < \frac{p_i + p_j + t}{2} = \frac{v + c + t}{2} \qquad 2v < v + c + t$$

Some costomer not buying $\rightarrow$ $\boxed{\therefore v < c + t}$

(d) In case of ③

If firm $i$ change (raise) price $p_i = \frac{2v - t}{2} = v - \frac{t}{2}$ to $p_i^*$

then the demand will be determined by indifferent purchase costomer.

$$p_i + xt = v$$

$$X_i(p_i^*, p_j) = \frac{(v - p_i^*)M}{t}$$

12.C.14.

(d) continued.

$$\max \frac{(p_i^* - c)(v - p_i^*)M}{t}$$

Since firm i pick $p_i$ to max profits

so Increase in $p_i$ is not increasing profits.

$$\frac{\partial \Pi_{p_i}^*}{\partial p_i} = \frac{M}{t}(v - p_i^* - p_i^* + c) \leq 0 \rightarrow$$

$$v - 2p_i^* + c \leq 0, \quad \underline{v \leq 2p_i^* - c}$$

Similary, $\underline{v \leq 2p_j^* - c}$

Also, lowering price $p_i$ make constomer who is indifferent between buying from firm i and firm j determine demand.

Therefore $x_i(p_i^*, p_j) = \frac{(t + p_j - p_i^*)M}{2t}, \quad p_i^* \leq p_i$

Since firm i chooses $p_i$, decrease in $p_i$ will not increase profits.

$$\max \; (p_i^* - c)x_i(p_i^*, p_j) = \frac{(p_i^* - c)(t + p_j - p_i^*)M}{2t}$$

$$\frac{\partial \Pi_{p_i}^*}{\partial p_i} = \frac{(t + p_j - p_i^* - p_i^* + c)M}{2t} = \frac{(t + p_j - 2p_i^* + c)M}{2t} \geq 0$$

$$\therefore t + p_j - 2p_i^* + c \geq 0$$

Similarly, $t + p_i - 2p_j^* + c \geq 0$

In case of 3

$$v = \frac{p_i + p_j + t}{2}, \quad 2v - t = p_i + p_j, \quad \underline{p_i = v - \frac{t}{2} + \varepsilon}, \quad \underline{p_j = v - \frac{t}{2} - \varepsilon}$$

12.C.14 (e)

In case of 1 a reduction in t decrease prices and profits. But in case of 3, reduction in t increases prices and profits. However, t fall sufficiently, then 3 is no longe possible and game switches to 1 case.

you seem to have the basic idea!

12.D.4.

(a) The most profitable price that can be sustained for $\delta \in [\frac{1}{2},1)$ is the monopoly price. To verify this, just compare (i) PV of $0.5\,\Pi(c)$ each period to (ii) PV of $\Pi(c)$ now and 0 thereafter.

$$\frac{1}{1-\delta}[0.5(p(c)-c)] \geq (p(c)-c) \Leftrightarrow \delta \geq \frac{1}{2}$$

We see that the PV's are equal if delta=1/2 and (i) is larger if delta>1/2. This comparison is what is relevant because the payoff streams come from not defecting or defecting against Grim strategy of equal split of monopoly profit.

The monopoly price is increasing in cost of production. To verify this, I will show that if demand elasticity is unchanged, the monopoly price increases proportionately with MC. Suppose $c_2 \geq c_1$, and $p_2, p_1$ are the monopoly prices given MC $c_2, c_1$ respectively. By revealed preference of the monopolist we have

$$(p_1 - c_1)x(p_1) \geq (p_2 - c_1)x(p_2)$$

&

$$(p_2 - c_2)x(p_2) \geq (p_1 - c_2)x(p_1).$$

where $x(p_i)$ is a demand function. Adding above two equalities and rearranging yields

$$(c_1 - c_2)(x(p_2) - x(p_1)) \geq 0.$$

Because $c_2 \geq c_1$ this implies that the following inequality must be satisfied:

$$x(p_2) \leq x(p_1).$$

Therefore we must have that $p_2 \geq p_1$, verifying the claim.

(b) Let $p(c)$= optimal monopoly price, $\Pi(c)$= monopoly profit, $c_1$= marginal cost in period 1, and $c_2$= marginal cost starting from period 2. By assumption, $c_2 \geq c_1$.

( i ) When $c_2 = c_1$, the monopoly price and profits can be sustained in period 1 because this strategy constitutes a subgame perfect Nash equilibrium of infinitely repeated Bertrand game if and only if $\delta \geq \frac{1}{2}$ as explained in part (a).

( ii ) When $c_2 > c_1$, the gain from deviating in initial period stays the same, but the future payoff from complying falls. For this reason, only lower profits can be supported in the period.

To be precise, the highest profits in period 1 can be supported by the strategies which result in the best collusive equilibrium after compliance and revert to the Bertrand punishment after a deviation. The best collusive equilibrium starting from period 2 brings the firms $\Pi(c_2)$ in every period. The highest supportable joint profits $\pi$ in period 1 therefore are such that the gain to deviating is $\pi/2$ and the loss is $\Pi(c_2)/2$ starting next period must satisfy

$$\pi/2 \leq \pi + \delta/(1-\delta)\Pi(c_2)/2 \Leftrightarrow \pi \leq \delta/(1-\delta)\Pi(c_2).$$

Observe that by assumption $\delta/(1-\delta) \geq 1/2$. We need to consider two cases:

<u>Case 1</u>

$\Pi(c_1) \leq \delta/(1-\delta)\Pi(c_2)$, i.e., a small cost increase. Then monopoly profits can still be sustained in initial period, and the most profitable price will still be $p(c_1)$. The highest sustainable price may be higher than that, but an increase in marginal cost starting from period 2 reduces it.

<u>Case 2</u>

$\Pi(c_1) > \delta/(1-\delta)\Pi(c_2)$, a large cost increase. Then monopoly profits in period 1 can no longer be sustained. Therefore, the monopoly price can no longer be sustained in that period, nor any higher price. The most profitable price will now be the highest sustainable price, and it will be lower than $p(c_1)$. Intuitively, the players can successfully collude if the temptation to defect is not too large in the potentially more profitable first period.

12.E.4.

Since the firms form cartel no matter how many firms enter the market, price will be unchanged by the # of firms. Only the aggregate fixed costs increase as the # of firms increase. Thus, the socially optimal # of firms is 1.

If the planner cannot control entry, the equilibrium # of firms will be

$$\frac{monopoly\_profit\_level}{K}.$$

In terms of welfare this means that free entry leads to a complete dissipation of monopoly profits, without any benefit to consumers.