Problem Set 2 - ECON 200 - Fall 2016
Answer Key

October 18, 2016

Part I
Short Case Study Problems

1 Supply in the Short and Long Run

A firm has production function $\ln y = \frac{1}{3}\ln x_1 + \frac{1}{2}\ln x_2$. Input 2 is unchangeable for now at the level $x_2 = 8$. Prices are \$8 and \$5 respectively for inputs 1 and 2.

1.1 a. What is the firm's variable cost? fixed cost (if any)? total cost? marginal cost?

$$\begin{aligned} y &= x_1^{1/3} x_2^{1/2} \\ y &= \sqrt{8} \cdot x_1^{1/3} \\ x_1 &= \frac{y^3}{8^{3/2}} \end{aligned}$$

$$\begin{aligned} \text{Cost} &= 8 \cdot x_1 + 5 \cdot 8 \\ C(y) &= 8 \cdot \frac{y^3}{8^{3/2}} + 40 \\ &= \frac{y^3}{8^{1/2}} + 40 \end{aligned}$$

The variable component is $\frac{y^3}{8^{1/2}}$, and the fixed component is 40. The equation above is total cost. Marginal cost:

$$\frac{dC(y)}{dy} = \frac{3}{8^{1/2}} y^2 \approx 1.06 \cdot y^2$$

1.2 b. What is the firm's supply function?

Above the minimum efficient scale, $s_i(P)$ satisfies $P = MC$

To find minimum efficient scale:

$$\begin{aligned} AVC &= MC \\ \frac{y^2}{8^{1/2}} &= \frac{3}{8^{1/2}} y^2 \end{aligned}$$

This is only satisfied for $y = 0$, after which $MC > AVC$ for all y. Thus:

$$P = \frac{3}{8^{1/2}} \cdot s_i^2$$
$$s_i = \left(\frac{8^{1/2}}{3} \cdot P\right)^{1/2} = \frac{2^{1/4}}{3^{1/2}} \cdot P^{1/2} \approx .6866 \cdot \sqrt{P}$$

1.3 c. Now assume that both inputs can be adjusted freely. What are the firm's conditional input demands? What is its average cost? Marginal cost? Supply function?

$$y = x_1^{1/3} x_2^{1/2}$$
$$x_1 = \frac{y^3}{x_2^{3/2}}$$

$$\frac{\mathrm{MP}_1}{\mathrm{MP}_2} = \frac{w_1}{w_2}$$
$$\frac{\frac{1}{3}\frac{x_2^{1/2}}{x_1^{2/3}}}{\frac{1}{2}\frac{x_1^{1/3}}{x_2^{1/2}}} = \frac{w_1}{w_2}$$
$$\frac{2}{3}\frac{x_2}{x_1} = \frac{w_1}{w_2}$$
$$x_2 = \frac{3}{2}\frac{w_1}{w_2} x_1 = \frac{3}{2}\frac{w_1}{w_2}\frac{y^3}{x_2^{3/2}}$$
$$x_2^{5/2} = \frac{3}{2}\frac{w_1}{w_2} y^3$$
$$x_2^\star(y; w_1, w_2) = \left(\frac{3}{2}\frac{w_1}{w_2} y^3\right)^{2/5} = \left(\frac{3}{2}\frac{w_1}{w_2}\right)^{2/5} y^{6/5}$$
$$x_1^\star(y; w_1, w_2) = \frac{2}{3}\frac{w_2}{w_1}\left(\frac{3}{2}\frac{w_1}{w_2} y^3\right)^{2/5} = \left(\frac{2}{3}\frac{w_2}{w_1}\right)^{3/5} y^{6/5}$$

The above are the conditional input demands.

$$C(y, w_1, w_2) = w_1 x_1^\star(y, w_1, w_2) + w_2 x_2^\star(y, w_1, w_2)$$
$$C(y; w_1, w_2) = w_1 \cdot \left(\frac{2}{3}\frac{w_2}{w_1}\right)^{3/5} y^{6/5} + w_2 \cdot \left(\frac{3}{2}\frac{w_1}{w_2}\right)^{2/5} y^{6/5}$$
$$= \left(\left(\frac{2}{3}\right)^{\frac{3}{5}} + \left(\frac{3}{2}\right)^{\frac{2}{5}}\right) \cdot w_1^{2/5} \cdot w_2^{3/5} \cdot y^{6/5}$$
$$= \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot \left(1 + \frac{3}{2}\right) \cdot w_1^{2/5} \cdot w_2^{3/5} \cdot y^{\frac{6}{5}}$$
$$= \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot \frac{5}{2} \cdot 8^{2/5} \cdot 5^{3/5} \cdot y^{\frac{6}{5}} \approx 5.15 \cdot y^{6/5}$$

$$AC = \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot \frac{5}{2} \cdot 8^{2/5} \cdot 5^{3/5} \cdot y^{\frac{1}{5}} \approx 5.15 \cdot y^{1/5}$$
$$MC = \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot 3 \cdot 8^{2/5} \cdot 5^{3/5} \cdot y^{\frac{1}{5}} \approx 15.59 \cdot y^{1/5}$$

Minimum efficient scale is zero, and the supply function satisfies $P = MC$

$$P = \left(\frac{2}{3}\right)^{\frac{3}{5}} \cdot 3 \cdot 8^{2/5} \cdot 5^{3/5} \cdot s_i^{\frac{1}{5}}$$

$$s_i(P) = \frac{1}{2^3 \cdot 3^2 \cdot 8^2 \cdot 5^3} \cdot P^5 = \frac{P^5}{72 \cdot w_1^2 \cdot w_2^3} = \frac{P^5}{576,000}$$

2 Long-Run Competitive Equilibrium and the Number of Firms

Suppose that each firm in an industry has long run total cost function $c(y_i) = y_i^3 - 9y_i^2 + 36y_i$, and they face industry demand curve $y_T = 200 - 10p$.

2.1 a. What is each firm's marginal cost function? Average cost function? Fixed cost? Supply curve?

$$\begin{aligned} MC_i &= 3y_i^2 - 18y_i + 36 \\ AC_i &= y_i^2 - 9y_i + 36 \\ FC_i &= 0 \end{aligned}$$

Individual firm supply satisfies $P = MC$ above minimum efficient scale

$$\begin{aligned} MC_i(y_{min}) &= AC_i(y_{min}) \\ 3y_{min}^2 - 18y_{min} + 36 &= y_{min}^2 - 9y_{min} + 36 \\ 2y_m^2 - 9y_m &= 0 \end{aligned}$$

The roots of the above are $y_m = 0$ and $y_m = 4.5$, so minimum efficient scale is 4.5, above which:

$$P = s_i^{-1}(y_i) = 3y_i^2 - 18y_i + 36$$

This means the minimum price is $4.5^2 - 9 \cdot 4.5 + 36 = 63/4 = 15.75$

To obtain the supply function, solve that equation for y_i in terms of p in the relevant range. Using the quadratic formula and simplifying, you get

$$s_i^2 - 6s_i + 12 - \frac{1}{3}P = 0$$

$$\begin{aligned} s_i &= \frac{1}{2} \cdot \left(6 + \sqrt{6^2 - 4 \cdot \left(12 - \frac{1}{3}P\right)}\right) \\ s_i(P) &= 3 + \frac{1}{2}\left(\frac{4}{3}P - 12\right)^{1/2} \\ s_i(P) &= 3 + \left(\frac{1}{3}P - 3\right)^{1/2} \end{aligned}$$

$$s_i(P) = \begin{cases} 3 + \left(\frac{1}{3}P - 3\right)^{1/2} & \text{if } p \geq \frac{63}{4}, \\ 0 & \text{if } p < \frac{63}{4}. \end{cases}$$

The total supply curve is zero below $p = 63/4$ and infinite (due to unlimited entry) above $p = 63/4$.

2.2 b. What is long run competitive equilibrium price? Output per firm? Number of firms?

In the long run, $P = MC = AVC$

$$P_{LR}^{\star} = \frac{63}{4}$$

Output per firm is a function of this price:

$$\begin{aligned} s_i^{\star}\left(P_{LR}^{\star}\right) &= 3 + \left(\frac{1}{3}\frac{63}{4} - 3\right)^{1/2} \\ &= 3 + \left(\frac{21}{4} - \frac{12}{4}\right)^{1/2} \\ &= 3 + \sqrt{\frac{9}{4}} = 3 + \frac{3}{2} = 4.5 \end{aligned}$$

As implied above by the minimum efficient scale. The number of firms matchesthis to total demand:

$$\begin{aligned} n_{LR} \cdot s_i^{\star}\left(P_{LR}^{\star}\right) &= y_T\left(P_{LR}^{\star}\right) \\ n_{LR} &= \frac{1}{4.5} \cdot \left(200 - 10 \cdot \frac{63}{4}\right) = \frac{1}{4.5}(200 - 157.5) \approx 9.44 \end{aligned}$$

There will be very small rents to firms from the slight excess demand that is too little to warrant a tenth firm. Moreover, firms will produce only at or above 4.5 units, no firm will enter and produce less.

2.3 c. Compute the producer surplus (PS), consumer surplus (CS) and total surplus (TS) at long run competitive equilibrium.

Ignoring the slight imperfection, we will calculate surplus assuming $P^{\star} = 15.75$ and $q^{\star} = 42.5$ exactly. Note that inverse demand is $20 - y$

$$CS = \frac{1}{2}(20 - 15.75) \cdot 42.5 = \frac{1445}{16} \approx 90.3125$$

$PS = 0$ note that the supply curve is horizontal and infinite at $P = 63/4$

$$TS = CS + PS = \frac{1445}{16} \approx 90.3125$$

3 Translog Cost and Profit Functions

You estimated a 3-factor translog cost function for a client. Write down a possible numerical function of this sort (about half of its coefficients can be zero).

3.1 Check that the coefficients satisfy the main required conditions (homogeneity, etc)

Demonstrate that you know the four conditions and how to verify that they are satisfied, at least locally for a reasonable set of prices.

1. Non-decreasing in input prices and output
2. Homogeneous of degree 1 in input prices
3. Concave in input prices (negative semi-definite Hessian)
4. Continuous and differentiable (any combination of logarithmic functions will satisfy this unless you put additional terms inside the logarithmic functions)

An example is

$$\begin{aligned}\ln \mathrm{Cost} =& .5 \cdot \ln (w_1) - .2 \cdot [\ln (w_1)]^2 \\ &+ .25 \cdot \ln (w_2) + .1 \cdot [\ln (w_2)]^2 \\ &+ .25 \cdot \ln (w_3) + .1 \cdot [\ln (w_3)]^2\end{aligned}$$

3.2 Write down the implied factor demand equations.

Use Shepherd's Lemma

$$\frac{\partial c (w_1, w_2, w_3, y)}{\partial w_i} = x_i^{\star} (w_1, w_2, w_3, y)$$

3.3 Suppose instead that you estimated a translog profit function. How would you modify your answers to parts a and b?

Demonstrate that you know the four conditions and how to verify that they are satisfied, at least locally for a reasonable set of prices.

1. Non-decreasing in output price and non-increasing in input prices
2. Homogeneous of degree 1 in output price and input prices
3. Convex in output price (can only be satisfied locally)
4. Continuous and differentiable (any combination of logarithmic functions will satisfy this unless you put additional terms inside the logarithmic functions)

An example is

$$\begin{aligned}\ln \mathrm{Cost} =& \ln (P) + \ln (P) \cdot \ln (w_1) - \ln (P) \cdot \ln (w_2) \\ & .5 \cdot \ln (w_1) - .2 \cdot [\ln (w_1)]^2 \\ &+ .25 \cdot \ln (w_2) + .1 \cdot [\ln (w_2)]^2 \\ &+ .25 \cdot \ln (w_3) + .1 \cdot [\ln (w_3)]^2\end{aligned}$$

Use Shepherd's Lemma for profit functions

$$\frac{\partial \pi (w_1, w_2, w_3, p)}{\partial w_i} = x_i^{\star} (w_1, w_2, w_3, p)$$

4 Varian questions

4.1 Question 4.6

Two-plant problem.

$$c (y) = \min \{4\sqrt{y_1} + 2\sqrt{y_2}; y_1 + y_2 \geq y\}$$

Since the cost is concave, rather than convex, the optimal solution will always occur at a boundary. If you derive optimality conditions without checked the second derivative, you will maximize cost (instead of minimizing). The solution is to produce entirely at the cheaper plant, 2. $c (y) = 2\sqrt{y_2}$.

4.2 Question 5.14

Use time series data to estimate marginal cost in each period.

Take a total derivative of the cost function to get the following:

$$
\begin{aligned}
dc &= \sum_{i=1}^{n} \frac{\partial c}{\partial w_i} dw_i + \frac{\partial c}{\partial y} dy \\
\frac{\partial c}{\partial y} dy &= dc - \sum_{i=1}^{n} \frac{\partial c}{\partial w_i} dw_i \\
\frac{\partial c}{\partial y} &= \frac{1}{dy} \cdot \left[dc - \sum_{i=1}^{n} \frac{\partial c}{\partial w_i} dw_i \right] \\
MC &= \frac{1}{\Delta y} \cdot \left[\Delta c - \sum_{i=1}^{n} x_i^{\star} \Delta w_i \right]
\end{aligned}
$$

Using Shepherd's Lemma to find marginal cost as a function of your known variables.

Part II
Short Essay

5 Short Essay 1

1. Your client, a specialty materials supplier, wants to plan how to respond to changes in key input prices (electricity, gadolinium ore, labor) to achieve given levels of output. What sort of data should you gather, and how might you analyze it? Write an overview memo of no more than 250 words that is intelligible to the client's non-technical managers as well as to its data scientists; please print it on a separate page.

This question is about estimating conditional factor demands, which probably is best done via estimating a cost function and using Shepard's lemma. Credit will be given for sensible ideas on how to do it, and for writing clearly.

6 Short Essay 2

2. What are the differences between (a) increasing returns to scale, (b) learning curve, and (c) decreasing (average) cost? What are the different implications of (a-c) for competitive equilibrium price in the long run? Write about 100 words intelligible to your TA.

- Increasing returns to scale is a production function where doubling inputs results in a more than doubling of output
- A learning curve relates to increasing efficiency with experience
- Decreasing average cost is related to increasing returns to scale; it implies that marginal cost is below average cost (potentially indefinitely, such as in an industry where fixed costs are the primary costs). This generally occurs because of increasing returns to scale with constant factor prices.

All these things cause problems for CE in the LR; a more likely industry structure in each case is oligopoly or even monopoly.