UCSC

ECON 204B W13 Final Exam **Answer Key**

1.

	a	b
A	5, 5	0, 4
B	4, 0	1, 1

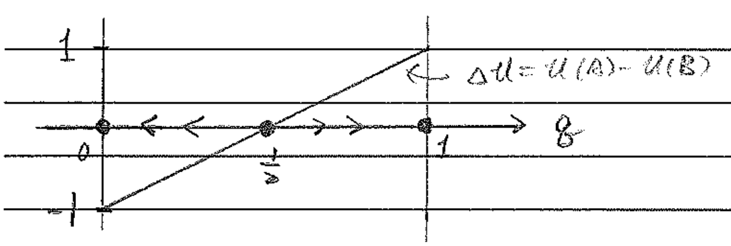
(a)

The pure NE is (A, a) which yields (5,5) and (B, b) which yields (1, 1). And the mixed NE is $(\frac{1}{2}A+\frac{1}{2}B, \frac{1}{2}a+\frac{1}{2}b)$ which yields $(\frac{5}{2}, \frac{5}{2})$. ✓

In real life, since it is just a 2x2 one-shot game, I'd say (A, a) is most likely to be played by human players. This is because ① ~~it~~ is more likely ~~that~~ people consider pure strategy rather than mixed strategy in such a small game; ② since the game is played once and the payoff is easy to calculate, the probability of a trumble-hand is very very small.

(b) Denote g as the proportition of the population that plays a, then for a row player he has

$$\left.\begin{aligned} u(A) &= 5g + 0\cdot(1-g) = 5g \\ u(B) &= 4g + (1-g) = 3g+1 \end{aligned}\right\} \Rightarrow \Delta u = u(A) - u(B) = 2g - 1 \begin{cases} \geq 0 & \text{if } g \geq \frac{1}{2} \\ < 0 & \text{if } g < \frac{1}{2} \end{cases}$$



So the EE is (A, a) and (B, b)

And the basin of attraction of (**B**, **b**) is $\{g : 0 \leq g < \frac{1}{2}\}$,

and the basin of attraction of (A, a) is $\{g : \frac{1}{2} < g \leq 1\}$.

(c) The EE's are the ~~two~~ pure NE in part (a). The mixed NE is not a EE in part (b) because it is unstable. A $g = \frac{1}{2} + \varepsilon$, for any ε close to 0, would lead the whole population to either (A, a) or (B, b) though start from $(\frac{1}{2}A+\frac{1}{2}B, \frac{1}{2}a+\frac{1}{2}b)$

So my guess of human players' choice is quite consistent with the result in part (c), if we set that the prior belief is at least slightly above ~~1/2~~. $g = \frac{1}{2}$. ✓ And it is reasonable to believe so since (A, a) generates (5,5) which is higher than (1, 1), the result of (B, b). However, if ~~for~~ some reason that the prior is slightly below $g = \frac{1}{2}$, then the population would eventually choose (B, b). But I think it is more reasonable to have a prior above $g = \frac{1}{2}$.

18/18

1)

		II	
		a	b
I	A	5,5	0,6
	B	6,0	1,1

18

a) For II, a is dominated by b. Then II will always play b. I knows this, and does better by playing B.

$\Rightarrow (B,b)$ is the pure NE by IDDS ✓ indeed, DS

Because II's strategy is dominant, a human player would probably play, especially in a one-shot game. I would figure this out and play B.

Perhaps (A,a) is possible with more periods, but not in the one-shot game.

The reverse argument is true as well, as B is dominant for I ✓ so DS

b)

	p	$1-p$
	5	0
	6	1

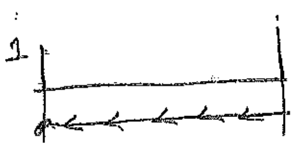
one population

$5p + 0(1-p) = 5p$

$6p + 0(1-p) = 6p+1-p = 5p+1$

$1 = 1$?

$p = 0$

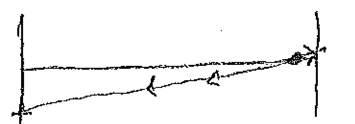


✓

To me, this implies everyone plays B. Say we had 5.1 instead of 5

$\Rightarrow 5.1p = 5p+1$

$.1p - 1$ $\quad p = \frac{1}{1.02} < 1 \Rightarrow$

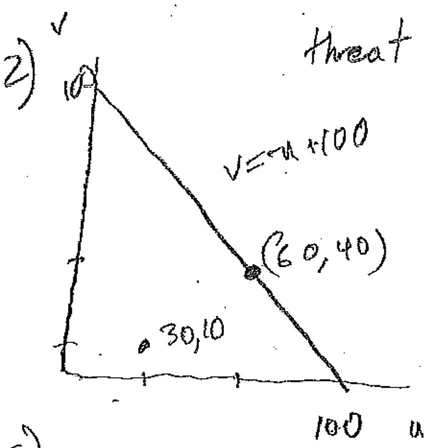


$\Rightarrow$ Then there is an unstable eqm. at .98

But in our question, the slope is flat. ✓

c) These are consistent, B is a dominant strategy for both players, and as such, each has incentive to cheat if the other plays A. With sufficiently patient players and an indefinite game, this can be remedied, but there is no indication that this is the case in evolutionary game in part (b).

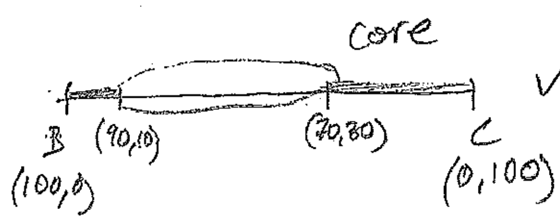
2) threat $\bar{u},\bar{v}=30,10$ resp?



a)

$$g=(u-\bar{u})(v-\bar{v})$$
$$(u-30)(-u+100-10)$$
$$(u-30)(-u+90)$$
$$-u^2+90u+30u-30\cdot 90$$
$$g=-u^2+120u-2700$$
$$\frac{\partial g}{\partial u}=-2u+120 \quad \boxed{u=60} \Rightarrow \boxed{v=40} \quad \text{NBS}$$

b) $v(B)=30$
$v(C)=10$
$v(BC)=100$



core is between (90,10) and (70,30)

	B	C
BC	30	70
CB	90	10
$\phi(v)$	120/2	80/2

$\phi(v)=60,40$ → same as NBS

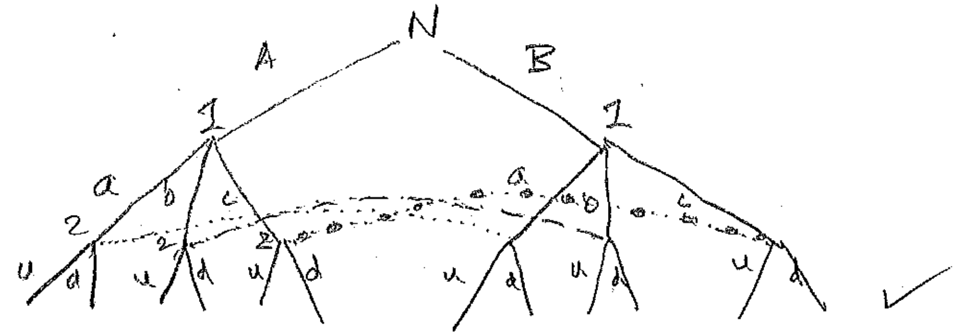
#3 continued

- Partial pooling is not possible with only two Nr states
- Hybrid is a mixed messaging strategy, which may be possible here, although I am not sure.

d) This is a signalling game. Nature moves, then the informed player, then the uninformed player.

3)

Liam Re



a) We also need a common belief about the probabilities of Nature move t.

b) Because of the info sets, we can't really break apart any subgames. For PBE, we need μ to be the posterior given a common prior (not defined here) and each player's strategy profile. We also need each component of the profiles to be a BR given μ. For PBE, this needs to hold in every subgame.

Then we would have $[m^*, x^*, \mu]$ s.t.

1) $m^* \in \arg\max u(m, x^*; t) \ \forall t$ → P1 sends the message that maxs utility given P2's BR to that m, right... what does that mean for this game?

2) $x^* \in \arg\max \sum_t \mu \cdot u(x, m; t)$ → pick action that max expected utility.

3) μ is consistent with Bayes rule given nature move and m^* etc.

c) For separating $m^*(A) \neq m^*(B)$

⇒ P1 has a different optimal message in each state of nature

⇒ We also need that sending a false message is too costly for P1

For pooling $m^*(A) = m^*(B)$

⇒ P1 sends the same message no matter the state of nature & that is BR...

Wei Xu

4.(a) In the basic Bertrand model, the NE is $p_1 = p_2 = p_3 = MC = 2$, and the corresponding payoffs is $\pi_1 = \pi_2 = \pi_3 = 0$. ✓

(b) In the basic Cournot model, we have

$$p = 38 - Q = 38 - q_1 - q_2 - q_3 = 38 - q_i - q_{-i}$$

$$\Rightarrow \text{firm } i \quad \max_{q_i} (p-c) q_i = (38 - q_i - q_{-i} - 2) q_i$$

$$\Rightarrow \text{Best response of firm } i \text{ is } q_i = \frac{36 - q_{-i}}{2}.$$

Since this a symmetric game, we have $q_1 = q_2 = q_3 = q$

$$\Rightarrow q = \frac{36 - 2q}{2} \Rightarrow q = 9 \quad ✓$$

$$\Rightarrow \pi_i = (p-c) q = (38 - 3q - 2) q = 81 \quad \forall i = 1,2,3$$

So the NE is $q_1 = q_2 = q_3 = 9$, and $\pi_1 = \pi_2 = \pi_3 = 81$ is the corresponding payoffs. ✓

(c) If the incumbent is the monopolist, then it seeks to

$$\max_{q} (p-c) q = (36 - q) q$$

$$\Rightarrow q^* = 18, \quad \pi^* = (p^* - c) q^* = (36 - q^*) q^* = 324. \quad ✓$$

So if it is going to play Bertrand as in part (a), then the incumbent is willing to pay up to 324 to prevent entry; ✓

if it is going to play Cournot as in part (b), then the incumbent is willing to pay up to $324 - 81 = 243$ to prevent entry. ✓

(d) If post-entry competition is Bertrand, then the profits to each firm will be drawn down to zero if more than one firm enter, so only ONE firm will enter in the SPNE. ✓

If post-entry competition is Cournot, then the profits to each firm is $\frac{(p-MC)^2}{(J+1)^2} = \frac{36^2}{(J+1)^2} = \frac{1296}{(J+1)^2}$ when J firms enter. So in the SPNE ✓

only $J^* = \left[\sqrt{\frac{1296}{k}} - 1\right] = [7.2 - 1] = [6.2] = 6$ firms will enter

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Wei Xu

5(a)

The consumer's reservation price is $r(\theta) = \theta + \frac{1}{5}\cdot\frac{\theta}{2} = \frac{5}{4}\theta$. ✓

(b)

At price p, only the consumers whose reservation prices are above the price would like to purchase. So the range of θ is

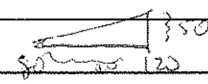
$$\begin{cases} [\frac{4}{5}p, 120] & \text{if } 0 \le p \le 150 \\ \phi & \text{if } p > 150 \end{cases}$$ ✓

(c) In the competitive equil. firms are making zero profits, therefore we must have

$\cancel{E(\theta|\theta \ge \frac{4}{5}p)\ p^* = \frac{1}{2}(\frac{4}{5}p+120) \Rightarrow p^*}$

$$E(\theta|\theta \ge \tfrac{4}{5}p) = \tfrac{1}{2}(\tfrac{4}{5}p + 120) = p \Rightarrow p^* = 100.$$ ✓

And the gains are



$$\int_{\frac{4}{5}p^*}^{120} r(\theta) - p^* \, d\theta = \int_{80}^{120} \tfrac{5}{4}\theta - 100 \, d\theta = \tfrac{5}{8}\theta^2 - 100\theta \Big|_{80}^{120}$$

$= 1000$. (in \$ thousands per year) ✓

(d) Absent information constraints, then ~~the~~ consumer whose expected loss is θ should buy the insurance at price $p = \theta$, so the gains is

$$\int_0^{120} r(\theta) - \theta \, d\theta = \int_0^{120} \tfrac{1}{4}\theta \, d\theta = \tfrac{1}{8}\theta^2 \Big|_0^{120} = 1800$$ ✓

(in \$ thousands per year)

(e) Firms could use screening strategies by offering different insurance contracts with different coverages (or deductables) at different prices. ✓

Or firms may require consumers to provide health reports before purchasing the insurance and set the prices according to the health reports. Here, the health reports give a ~~signal~~ message / information about the unobservable θ. (though the message / information is noisy) ✓

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