3. Cost and Technology

Varian, Chapters 1-5

I. Describing the Firm

A. The neoclassical description of the firm is really just a description of the firm's production possibilities.

1. Which outputs can be obtained from given inputs
2. How much has to be expended to get those inputs
3. How these two factors generate cost curves for the firm.
4. These cost curves themselves completely describe *everything we need to know about the firm*, if we are neoclassical.

B. Input/Output

1. The firm produces a vector $\mathbf{y}$ of product quantities.
   a. We'll usually focus on a firm with a single product with quantity y.
2. The firm has a set of inputs it can use to create these products.
   a. We describe these inputs as a vector (or a bundle) $\mathbf{x} = (x_1, x_2, ..., x_n)$.
      - The vector components are the quantities of each input being utilized in production.

C. Technology

1. What output y can the firm produce with a bundle $\mathbf{x}$?
2. This is described by the firm's technology.
3. The **input requirement set** V(y) consists of all of the bundles $\mathbf{x}$ that can produce output quantity y.

   Ex: Activity analysis and production plans.
   Basically, recipes. For spaghetti sauce, for 16Gb memory chips, for 100 rides to SFO, ...

4. The **production function** $y = f(\mathbf{x})$ describes the maximum output that can be produced with any input bundle .

   Ex: Cobb-Douglas technology, $y = a_o x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$.

   Ex: Leontief technology , $y = \min\{a_1 x_1, a_2 x_2, ..., a_n x_n\}$.

5. The **isoquant** for given output level y^* is the set of input bundles that can produce y^* .

   Analogous to an indifference curve, but the label here (y^*) is meaningful.

D. Common assumptions about technology

1. Monotone
   a. More input enables at least as much output.
   b. Say this using V's: if you can produce y' with $\mathbf{x}$ you can still produce y' with a bigger bundle $\mathbf{x}' > \mathbf{x}$.
   c. This is innocuous if extra inputs can be thrown away, "free disposal."
2. Convex
   a. If plans $\mathbf{x}$ and $\mathbf{x}'$ are in $V(y)$ (i.e. can produce y), then so is the mixture $\alpha\mathbf{x} + (1 - \alpha)\mathbf{x}'$, for any mixing proportion $0 < \alpha < 1$.
   b. If a production plan can be replicated, then it is reasonable to say that the technology is convex.

      Ex: Replicating two production plans to create any convex sets.
3. Non-empty
   a. With enough of and the right kinds of inputs, you can create any level of output y.
4. Closed: a boring technical condition.

E. Trade Offs in Production Plans

1. Assume we have a "smooth" production technology.

2. At what rate can we substitute one of our inputs for another in producing a particular output, y?

   a. Called the **technical rate of substitution**.

   b. With a smooth technology with two inputs, it is just the slope of the isoquant.

   c. TRS is a direct analogue of MRS.

   Ex: Using the production function and the **implicit function theorem** to find the technical rate of substitution.

$$TRS_{ij} = \frac{dx_j}{dx_i}|_{[f(\cdot)=y^*]} = -\frac{mp_i}{mp_j} = -\frac{\partial f}{\partial x_j} / \frac{\partial f}{\partial x_j} \tag{1}$$

   Ex: A Cobb-Douglas example.

3. Elasticity of substitution σ is elasticity of $[x_j/x_i]$ wrt $|TRS|$. It is a measure of isoquant curvature. See Varian for ugly details (optional).

F. Returns to Scale

1. The returns to scale tells us what happens when we try to scale up a production plan.

2. If we multiply $\mathbf{x}$ by t, what happens to y?

3. Three cases:

   a. Constant returns to scale.

      - If $y = f(x_1, x_2)$, then for $f(tx_1, tx_2) = ty$
      - Output is proportional to the inputs.

   b. Increasing returns to scale.

      - If $y = f(x_1, x_2)$, then $f(tx_1, tx_2) > ty$ for $t > 1$.
      - We get more bang for our buck (at fixed prices of course) at higher scales of production.

- May be inherent in a technology or possibly related to learning from doing.

c. Decreasing returns to scale.

- If $y = f(x_1, x_2)$, then $f(tx_1, tx_2) < ty$ for $t >$.
- We get diminishing returns from scaling our plans up.
- A major reason for DRS: there is some fixed input (not in the list), such as CEO attention, or planetary resources, or ...

Ex: Cobb-Douglas and returns to scale.

4. Homogeneity degree 1 as CRS. Homogeneous functions of degree $d = 0, 1, ...$

G. Long run and short run

1. Short run means that at least one component of the input bundle $\mathbf{x}$ is fixed.
   a. Note that (for standard production functions) this implies DRS at large scale in the short run.
2. Long run means that all inputs are choice variables for the firm.

II. Cost Minimization

A. Behavior of the firm

1. We assume that firms economize in production.
2. Assume they choose technologies which minimize the cost of producing their output.
3. When is this reasonable to assume?

B. The firm's problem.

1. To derive cost function, take as given the desired output quantity y, and the input price vector $\mathbf{w} = (w_1, w_2, ..., w_n)$. Sometimes also called factor prices.
2. Firms choose an input bundle $\mathbf{x}$.

a. For convenience we will often write $\mathbf{x} = (x_1, x_2)$ and $\mathbf{w} = (w_1, w_2)$, but the reasoning extends to any finite vector of inputs.

3. The firm's main constraint (aside from factor prices) is technological.

   a. Can be summarized with the production function: $y = f(x_1, y_2)$.

4. So the firm's problem is simply:

$$c(\mathbf{w}, y) = \min w_1 x_1 + w_2 x_2 \text{ s.t. } f(x_1, x_2) = y \tag{2}$$

5. The first order condition for this problem says that the technical rate of substitution is equal to the ratio of the factor prices.

C. Conditional factor demand

1. The firm's cost minimizing problem yields the firm's demand for each input as a function of prices and the scale of output.

2. Conditional factor demand for input i is $x_i^*(w_1, w_2, y)$.

D. The cost function

1. Cost functions give the lowest cost of production available to a firm at a given set of factor prices. So we can rewrite equation (2) as

$$c(\mathbf{w}, y) = \mathbf{w} \cdot \mathbf{x}(\mathbf{w}, y) \tag{3}$$

2. With only two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$

Ex: Constant Elasticity technology.

Special cases: Cobb-Douglas technology, Leontief technology, Linear technology

E. Relationship between cost and conditional factor demand

1. If the cost function is differentiable, then you can use it to recover the input (or factor) demand functions.

2. This is known as **Shephard's lemma**: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w},y))}{\partial w_i}$

3. To verify, just differentiate equation (3), remembering that the FOC tells us that $\frac{\partial x_i^*(\mathbf{w},y))}{\partial w_i} = 0$. (An example of the envelope theorem.)