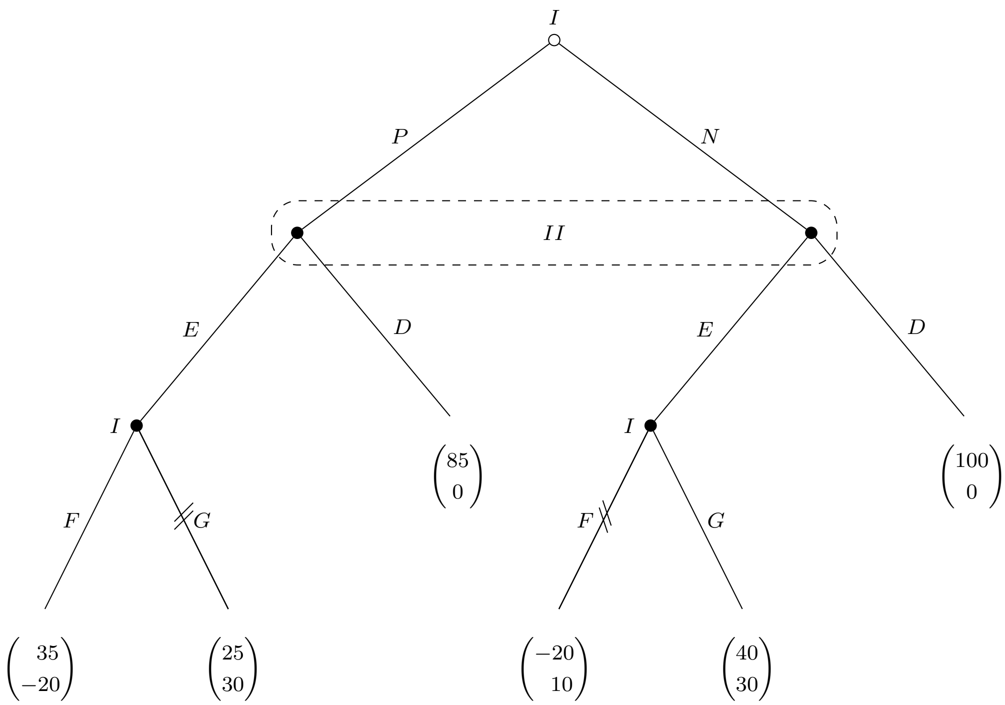
# Problem 1

## (a)




Player $I$ is incumbent and Player $II$ is potential entrant.

## (b)

$$
\begin{aligned}
E \in BR_{II} &\Leftrightarrow p(-20) + (1-p)30 \geq 0 \\
&\Leftrightarrow 30 \geq 50p \\
&\Leftrightarrow p \leq \frac{3}{5}
\end{aligned}
$$

That is, Player $II$ (potential entrant) will choose to enter if $p \leq \frac{3}{5}$.

## (c)

The first step of backward induction (BI) is shown in the game tree in part (a). The remaining normal form game (NFG) is

| | | II | |
|---|---|---|---|
| | | $E$ | $D$ |
| $I$ | $PF_PG_N$ | $35, -20$ | $85, \underline{0}$ |
| | $NF_PG_N$ | $\underline{40}, \underline{30}$ | $\underline{100}, 0$ |

So the SPNE is $(NF_PG_N, E)$ with $p = 0$.

That is, for the incumbent, it is a (weakly) dominant strategy and subgame perfect (SGP) to not prepare ($N$), to fight if prepared ($F_P$, not on the equilibrium path) and to go easy if not prepared ($G_N$) and the entrant's best response is to enter ($E$).

# Problem 2

## (a)

Simple BI gives $(Out_{P_L C_L}, Out_{P_H C_L}, In_{P_H C_H}, In_{P_L C_H})$ for entrant, thus $(P_H|C_H, P_L|C_L)$ for incumbent, with expected payoffs $(1,1)_{C_H} \cdot (.2) + (3,1)_{C_L} \cdot (.8) = (2.6, 1)$.

## (b)

1) Try $(P_H|C_H, P_L|C_L)$.

   So the beliefs can be updated as $\mu(C_H|P_H) = 1$ and $\mu(C_L|P_L)$.

   Then $\{BR_2(P_H) = In, BR_2(P_L) = Out\}^{\cdots(*)}$, but $BR_1$ to $(*)$ includes $P_L|C_H$, breaking this candidate equilibrium.

2) Try $(P_L|C_H, P_H|C_L)$.

   So the beliefs can be updated as $\mu(C_H|P_L) = 1$ and $\mu(C_L|P_H)$.

   Then $\{BR_2(P_H) = Out, BR_2(P_L) = In\}^{\cdots(**)}$, but $BR_1$ to $(**)$ includes $P_H|C_H$, breaking this equilibrium.

Thus, neither possible separating strategy is part of a PBE.

## (c)

1) Try $(P_H|C_H, C_L)$.

   So the beliefs are $\mu(C_H|P_H) = .2$ (the prior) and $\mu(C_H|P_L) = q \in [0,1]$ (i.e. arbitrary).

   Then, $BR_2(P_H) = Out$ and $BR_2(P_L) = In$ iff $q \geq .5$.

   Then, $BR_1(C_H) = P_H$ and $BR_1(C_L) = P_H$ if $q \geq .5$.

   So a pooling PBE is

$$\big\{m^* = (P_H|C_H, C_L); \mu(\cdot|P_H) = prior, \mu(C_H|P_L) = q \geq 0.5; a^*(P_L) = In, a^*(P_H) = Out\big\}.$$

2) Try $(P_L|C_H, C_L)$.

   So the beliefs are $\mu(C_H|P_L) = prior$ and $\mu(C_H|P_H) = q \in [0,1]$ (i.e. arbitrary).

   Then, $BR_2(P_L) = Out$ and $BR_2(P_H) = In$ iff $q \geq .5$.

   Then, $BR_1(C_L) = P_L$ and $BR_1(C_H) = P_L$ if $q \geq .5$.

   So we have another pooling PBE:

$$\big\{m^* = (P_L|C_H, C_L); \mu(\cdot|P_L) = prior, \mu(C_H|P_H) = q \geq 0.5; a^*(P_L) = Out, a^*(P_H) = In\big\}.$$

# Problem 3

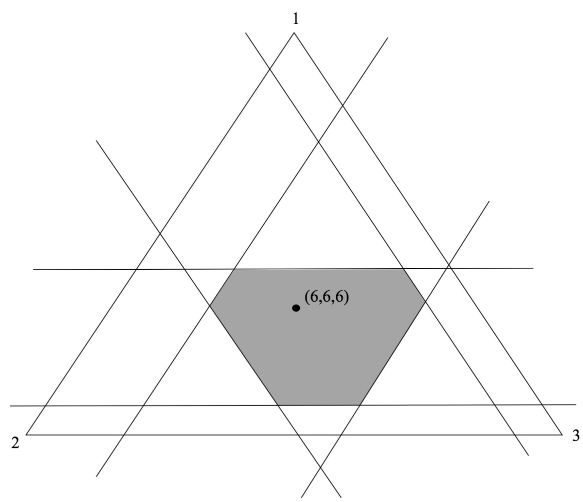
## (a)

$$w(\phi) = 0 \qquad w(\{1\}) = 1 \qquad w(\{1,2\}) = 6 \qquad w(\{1,2,3\}) = 18$$
$$w(\{2\}) = 2 \qquad w(\{2,3\}) = 10$$
$$w(\{3\}) = 3 \qquad w(\{1,3\}) = 8$$

The core is

$$x_1 \in [1,8], \quad x_2 \in [2,10], \quad x_3 \in [3,12],$$
$$x_1 + x_2 + x_3 = 18.$$

An example is $(6,6,6)$.




## (b)

| $\rho$ | $MC_1$ | $MC_2$ | $MC_3$ |
|---|---|---|---|
| 123 | 1 | 5 | 12 |
| 132 | 1 | 10 | 7 |
| 213 | 4 | 2 | 12 |
| 231 | 8 | 2 | 8 |
| 312 | 5 | 10 | 3 |
| 321 | 8 | 7 | 3 |
| $\sum$ | 27 | 36 | 45 |
| $\phi_i$ | 9/2 | 6 | 15/2 |

The normalized Shapley values are $(1/4, 1/3, 5/12)$.

### (c)

Yes, since $w$ is convex (supermodular), $\phi(w) \in Core(w)$.

### (d)

The NBS solves

$$\begin{aligned} & \max_{x_1,x_2,x_3} (x_1-1)(x_2-2)(x_3-3) \quad \text{s.t.} \quad x_1+x_2+x_3=18 \\ \Leftrightarrow & \max_{y_1,y_2,y_3} y_1y_2y_3 \quad \text{s.t.} \quad y_1+y_2+y_3 = 18-1-2-3=12 \\ \Rightarrow & \quad y_i = 4 \quad \text{for } i=1,2,3 \\ \Leftrightarrow & \quad x_1=5, x_2=6, x_3=7 \end{aligned}$$

where we set $y_i = x_i - i$.

## Problem 4

### (a)

$$w_i = CE_i = \mu_i + 0.2\sigma_i^2 = \begin{cases} 1+0.2\cdot 1^2 = 1.2 & (i=L) \\ 2+0.2\cdot 2^2 = 2.8 & (i=H) \end{cases}$$

### (b)

$$E(loss) = (.4)2 + (.6)1 = 1.4 = P$$

### (c)

At $P = 1.4$, low risk people refuse $(1.2 < 1.4)$, so only $H$-type people accept. Then,

$$E(profit) = 4000(P - E(loss|H) = 4000(1.4-2) = -2400,$$

which is \$ 2.4 million loss.

### (d)

Assuming a uniform price, insurers will serve only $H$ types (as just seen) at the price $P = 2 + .4 = 2.4$.

### (e)

With free entry, $P$ gets down to zero-profit level, so $P = 2$.

### (f)

Use screening model and find the insurance company's participation constraint (PC) to separate contracts aimed at $H$-types and $L$-types. The PC's imply that an upper bound in profit for each $H$-type customer is

$(0.2)2^2 = 0.8$ and for each $L$-type customer is $(0.2)1^2 = 0.2$, or $(0.2)6,000 + (0.8)4,000 = 44,000$, which is \$ 44 million.

### (g)

$U$ is not equivalent to $Eu$, as explained in the Notes 1 (p.24+). It is equivalent up to second order. Over a limited range, the function $u(x) = x - cx^2$ works. See also Problem 2 of Problem Set 1.

## Problem 5

### (a)

Yes, it is symmetric, since the column player's payoff matrix is the transpose of the row player's.

### (b)

For $x \in (-2, 0)$, we have $p^* = \frac{a_2}{a_1+a_2} = \frac{x}{-2+x} \in (0, 1)$, e.g., $p^* = 1/3$ for $x = -1$. It is downcrossing since $0 > a_1 = 3 - 5$ and $0 > a_2$, hence a unique, stable NE.

### (c)

For $x \in (0, 10)$, $a_2 = x > 0 > a_1 = -2$, hence $s_2$ is a dominant strategy. Therefore, the pure NE $s_2$ is globally stable.

### (d)

Since $a_1 = -2 < 0$, the CO case with two pure NE is not possible.

### (e)

With $x = 1$, $(s_2, s_2)$ is the stage game NE. To sustain cooperation, consider trigger strategy: play $s_1$ until someone first plays $s_2$, then play $s_2$ ever after.
Playing $s_1$ (or trigger) against trigger yields stream $3, 3, 3, ...$ $\quad (*)$.
Playing $s_2$ against trigger yields stream $5, 1, 1, ...$ $\quad (**)$.
$(*)$ is BR ($\therefore$ $(trigger, trigger) \in NE$) iff

$$\begin{aligned} PV(*) \geq PV(**) &\Leftrightarrow \frac{3}{1-\delta} \geq 5 + \frac{\delta}{1-\delta} \\ &\Leftrightarrow 3 \geq 5(1-\delta) + \delta \\ &\Leftrightarrow \delta \geq \frac{1}{2}. \end{aligned}$$

If $\delta = \frac{q}{1+r}$, the the condition is $q \geq \frac{1+r}{2}$.