4. Competitive Firms

I. Maximizing Profit

A. Neoclassical theory assumes that all firms act so as to maximize profits.

B. This implies that the firm minimizes cost: whatever output it might choose, the firm can only maximize profit if it minimizes the cost of producing that output. Soon we'll mention the math supporting this intuitively obvious assertion.

C. As noted in Varian's nice discussion at the beginning of Ch 2, this immediately implies that for any activity the firm undertakes, the associated marginal revenue must equal the marginal cost for that activity. This insight alone is enough to guide some decisions (or to solve certain homework problems)!

D. A second assumption for *competitive* firms is that they are price takers, for output(s) as well as inputs.

1. Later we'll relax that second assumption to analyze firms with the power to influence output prices.
2. Even those firms want to minimize cost, so we will still use the cost functions developed earlier.

E. The competitive firm's SR problem.

1. For clarity, again we hold one factor f fixed at some level $\bar{x}_f$, and let the firm freely choose the level $x_v \geq 0$ of another factor v.
2. Use the symbols π, R, p to represent profit, revenue, output price, and keep y as output level and w's as input prices. Since profit = revenue - cost, the firm's problem is

$$\begin{aligned}
\max_{y, x_v \geq 0} \pi &= \max_{y \geq 0}[\max_{x_v \geq 0} R(y) - w_v x_v - w_f \bar{x}_f \text{ s.t. } y = f(x_v, \bar{x}_f)] \qquad (1) \\
&= \max_{y \geq 0}[R(y) - \min_{x_v \geq 0}[w_v x_v - w_f \bar{x}_f \text{ s.t. } y = f(x_v, \bar{x}_f)]] \\
&= \max_{y \geq 0}[R(y) - c(y)]
\end{aligned}$$

where $c(y)$ is shorthand for the SR cost function (which also depends on the w's and on $\bar{x}_f$).

3. If the firm is competitive (a price-taker), then $R(y) = py = pf(x_v, \bar{x}_f)$.
4. To analyze *un*conditional factor demand, write the competitive firm's problem as

$$\max_{x_v \geq 0} pf(x_v, \bar{x}_f) - w_v x_v - w_f \bar{x}_f. \tag{2}$$

5. The FOC is $p\frac{\partial f}{\partial x_v} = w_v$, i.e., the value of the marginal product must be equal to its price!

Ex: Graphical representation of problem (2).

Ex: Factor demand with Cobb-Douglas production function.

F. Price changes

1. Increasing the price of a factor will obviously decrease the firm's factor demand.

   Ex: Differentiating FOC by w.

   Ex: Graphical representation of factor price increase.

2. Increasing the price of output y however will increase the firm's factor demand.

   Ex: Differentiating FOC by p.

   Ex: Graphical representation of product price increase.

G. The long run version

1. In the long run the problem is exactly the same except that the firm chooses both factors.

$$\max_{x_v, x_f \geq 0} pf(x_v, x_f) - w_v x_v - w_f x_v \tag{3}$$

2. Values of each marginal product have to equal respective prices.

II. Profit functions, supply and (factor) demand

A. We can define a *profit function* as the solution to problem (3), i.e.,

$$\pi(p, \mathbf{w}) = \max_{\mathbf{w} \geq 0} pf(\mathbf{x}) - \mathbf{w} \cdot \mathbf{x}. \tag{4}$$

B. Warning: this profit function does not always exist – corners and kinks complicate its expression, and IRS or even CRS production functions cause problems. Cost functions exist under much milder assumptions than profit functions.

C. In applied work, one sometimes sees profit functions estimated directly from data. (E.g., in banking.) If so, it is helpful to know the general properties such a function must have, so they can be imposed as parameter restrictions in the estimation.

D. Using straightforward arguments, Varian shows that profit functions (when they exist) must be (a) (weakly) increasing in p and decreasing in each w_i, (b) homogeneous of degree 1 in $(p, \mathbf{w})$, (c) convex in p, and (d) continuous in p.

E. One great thing about cost functions is that they tell us conditional factor demands (via Shephard). Profit functions, when they exist, can tell us unconditional factor demands, and even the supply function.

F. Hotelling's Lemma tells us that the supply function is:

$$y^*(p, \mathbf{w}) = \frac{\partial \pi(p, \mathbf{w})}{\partial p} \tag{5}$$

and that unconditional factor demands are

$$x_i(p, \mathbf{w}) = -\frac{\partial \pi(p, \mathbf{w})}{\partial w_i} \tag{6}$$

G. The proof is analogous to that for Shehard's lemma, and involves the envelope theorem; see Varian p43-44 for self-contained proofs.

Ex: The one input, one output case.

III. The Firm's Supply

A. The firm's supply curve is $y(p) = y^*(p, \mathbf{w})$ holding constant the input price vector $\mathbf{w}$ while letting output price vary.

B. The supply curve turns out to be just selected portions of the marginal cost curve. To see this notice that

1. The competitive firm's profit maximization problem (taking both price and factor prices as given) can be expressed as.

   max $\pi = py - c(y)$

2. The FOC is $p = c'(y)$

3. The firm sets marginal cost equal to marginal revenue as all profit maximizing firms will.

4. However, the competitive firm takes price as given and therefore also sets marginal cost equal to price!

   a. So, at a given quantity, the price is equal to the marginal cost of that quantity.

   b. This is just the inverse supply curve that we regularly graph when doing competitive analysis.

C. Some qualifications

1. "U" shaped marginal cost curves

   a. If MC is U-shaped, then there may be two levels of output where price and marginal cost are equal.

      - In fact when MC is decreasing, the gap between revenue and cost is increasing as more is produced.
      - If this is so, the firm will always produce more.

- This is confirmed by the SOC $0 < \pi''(y) = -c''(y)$, associated with a relative minimum.

b. It is only upward sloping portions of MC (where $c''(y) > 0$ and thus the SOC $\pi''(y) < 0$ holds) that can be part of the inverse supply curve.

2. The Shutdown Condition

a. Firms incur (sunk) fixed costs even if they aren't producing anything, and this fact can encourage them to produce even when they can't turn a positive profit.

- Here's the algebra, where y denotes some positive level of output: $\pi(0) > \pi(y)$

$$\iff -F > py - c_v(y) - F$$
$$\iff AVC = \tfrac{c_v(y)}{y} > p$$

b. This implies that the firm is only willing to produce if the price is large enough – greater than average variable cost.

c. So only the MC greater than AVC can really be thought of as a supply curve.

3. Conclusion: the firm's supply function $y^*(p)$ follows the increasing portion of the MC curve above AVC; elsewhere $y^*(p) = 0$.

D. An Individual Firm's Producer Surplus

1. When we look at an entire market, producer surplus is the area to the left of the supply curve, below the price.

a. The same is true of the individual firm.

- Recall that the area under the marginal cost curve is the variable cost curve.
- Thus the producer surplus is the difference between total revenue and total variable cost!

Ex: Graphical representation of PS with the individual firm.

2. It shouldn't be surprising that profits have a natural relationship to producer surplus.

   a. Profits are just the difference between total revenue and total cost

   $$\pi = py - c_v(y) - F$$

   b. Surplus is the difference between total revenue and variable cost

   $$PS = py - c_v(y)$$

   c. Thus producer surplus is just the sum of profits and fixed costs.

   $$PS = \pi + F$$

IV. Competitive Industry

A. Short run supply

1. The industry supply curve is just the **horizontal** sum of individual market supply curves.

   a. Sum of quantities supplied.

   b. To the individual firms, prices are exogenous.

   c. Prices individually determine firms' output decisions.

2. In the short run,

   a. Sunk fixed costs F are incurred no matter what.

   b. As long as price is above its AVC, each firm is better off staying in business even if taking losses.

   c. No entry or exit on this time scale (starting a business usually involves incurring fixed costs).

B. Long run supply

1. In the long run, firms can adjust all of their productive inputs.

   a. Firms can enter

   b. Firms can exit without incurring fixed costs.

      - Now the supply curve is marginal cost in excess of *average cost*.

2. Entry

   a. When there are few firms in a market, they each may make relatively high profits.

      - Profits attract new firms.
      - Each new (identical) firm in a market causes the supply curve to become more elastic.
      - This in turn lowers prices and therefore the profits of individual firms.

   b. This process of attraction and entry only ends when there are so many firms in the market that an extra entrant would drive profits below zero.

      - Meaning that, in the long run, price will be driven close to minimum average cost.
      - The supply curve can therefore be modeled as the price at minimum average cost.

   c. The long run cost firm of a competitive industry is flat, exhibiting constant returns to scale.

   d. Profits are signals that mobilize entrepreneurs to focus attention on highly valued projects.

      Ex: Entry

C. Rent and rent seeking

1. Rent is the payment to a factor in excess of its opportunity cost.

   a. Determined by the market price for outputs and variable costs of production.
   b. Rent is really just producer surplus seen in a different light.
2. A competitive market for the factor will drive the price to the full amount of the rent.
   a. Drives firms' profits back down to zero.
3. Politics is a second way rent can be dissipated.
   a. Firms should be willing to spend all of the potential rents to control fixed factors.
   b. By creating or maintaining barriers to entry firms can create artificial scarcity.
   c. The rents from this scarcity are valuable and firms will pay to create it.
   d. This is pure waste and is called *rent seeking*.