Equations for Competitive Markets

Linear Demand: $q_d = a - bp$ **Linear Supply**: $q_s = x + yp$
Log-linear demand: $\ln(q_d) = \ln(a) + \varepsilon_d \ln p$ **Log-Linear Supply**: $\ln(q_s) = \ln(x) + \varepsilon_s \ln p$
Total Surplus=Consumer Surplus+Producer Surplus; **Revenue**=Producer Surplus + Variable Cost
Total Cost=Fixed Cost + Variable Cost; **Profit**=Revenue-Total Cost=Producer Surplus-Fixed Cost
Quantity Tax (tax per unit): $p_d = p_s + t$; **Value Tax** (tax on percentage spent): $p_d = (1+t)p_s$
Price Elasticity of Demand: $\varepsilon_d = \frac{\partial \ln(D)}{\partial \ln(p)} = \frac{\partial D}{\partial p}\frac{p}{D}$; If $|\varepsilon| > 1$ then curve is elastic
Tax Incidence Formula: $p_s(t) = p^* - \frac{t|D'|}{S'+|D'|}$; $p_d = p^* + \frac{tS'}{S'+|D'|}$; If ε_d is constant: $\frac{\partial p_d}{\partial t} = \frac{\varepsilon_s}{|\varepsilon_d|+\varepsilon_s}$

Equations for Consumer Choice and Demand

Marginal Utility: $MU_i = \frac{\partial u}{\partial x_i}$; **Marginal Rate of Substitution**: $MRS_{ij} = \frac{\frac{\partial u}{\partial x_j}}{\frac{\partial u}{\partial x_i}}$ and at interior optimum $= \frac{p_i}{p_j}$
Perfect Substitutes: $u(x_1, x_2) = x_1 + cx_2$; **Cobb-Douglas**: $u(x_1, x_2) = \ln(x_1) + c\ln(x_2)$
CES Utility: $u(x_1, x_2) = \frac{1}{\rho}\ln(x_1^\rho + x_2^\rho)$; $\rho \in (-\infty, 1]$; **Quasilinear**: $u(x_0, x_1) = x_0 + g(x_1)$
Marshallian Demand: $x_i^*(\mathbf{p}, m) : \mathcal{L} = u(x_1, x_2) + \lambda(m - \mathbf{p}\cdot \mathbf{x})$
Dual Problem; Hicksian Demand: $h_i^*(\mathbf{p}, u_0) : \min_x \mathbf{p}\cdot \mathbf{x} s.t. u(\mathbf{x}) \geq u_0$
Roy's Identity: $x_i^*(\mathbf{p}, m) = -\frac{\partial v}{\partial p_i}/\frac{\partial v}{\partial m}$; **Shepard's Lemma**: $h_i^*(\mathbf{p}, u) = \frac{\partial e(\mathbf{p},u)}{\partial p_i}$
Slutsky Equation: $\frac{\partial x_i(\mathbf{p},m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p},v(\mathbf{p},m))}{\partial p_i} - \frac{\partial x_i^*}{\partial m}x_i^*(\mathbf{p}, m)$; (**Elasticity Form**): $\varepsilon_i = \varepsilon_i^h - s_i\varepsilon_m$; $s_i = \frac{p_i x_i}{m}$
Demand Elasticity for product i, homogeneous of degree 0: $\varepsilon_{im} + \varepsilon_{ii} + \sum_{j\neq i}\varepsilon_{ij} = 0$

Equations for Cost and Technology

Technical Rate of Substitution: $TRS_{ij} = -\frac{\frac{\partial f}{\partial x_j}}{\frac{\partial f}{\partial x_i}} = -\frac{mp_i}{mp_j}$; **MC**: $MC(y) = \frac{\partial c}{\partial y} = \frac{\partial c_v}{\partial y}$ **MC to VC**: $\int MC = VC$
Factor Prices: $\mathbf{w} = (w_1, w_2, ..., w_n)$; **Production Function**: $y = f(x_1, y_2)$
Cost Function with two factors: $c(\mathbf{w}, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$
$= \min_{x_1,x_2\geq 0} w_1x_1 + w_2x_2$ s.t. $y = f(x_1, x_2)$
Shepard's Lemma conditional factor demand: $x_i^*(\mathbf{w}, y) = \frac{\partial c(\mathbf{w},y)}{\partial w_i}$
Learning Curve: The typical specification is for $Y_t = \Sigma_{s\leq t} y_s$, AC falls proportionately, $\ln AC_t = AC_0 - b\ln Y_t$

Equations for Competitive Firms

SR Profit Maximization: $\max_{y,x_v\geq 0} \pi = \max_{y\geq 0}[\max_{x_v\geq 0} R(y) - w_v x_v - w_f x_f s.t. y = f(x_v, \bar{x}_f)] = \max_{y\geq 0}[R(y) - c(y)]$
Revenue if firm is competitive: $R(y) = py = f(x_v, \bar{x}_f)$ **FOC of *un*conditional factor demand**: $p\frac{\partial f(x_v,\bar{x}_f)}{\partial x_v} = w_v$
Hotelling's Lemma, Supply: $y^*(p, \mathbf{w}) = \frac{\partial \pi(p,\mathbf{w})}{\partial p}$; unconditional factor demands: $x_i(p, \mathbf{w}) = -\frac{\partial \pi(p,\mathbf{w})}{\partial w_i}$
Shutdown Condition (Competitve Firms): $-F > py - c_v(y) - F \rightarrow AVC = \frac{c_v(y)}{y} > p$

Equations for Monopolies

FOC for a monopolist: $p(y) + p'(y)y = c'(y)$ which can be rewritten as $p = \frac{1}{1+\frac{1}{\varepsilon}}MC$; valid if $\varepsilon < -1$
Passing Along Costs: $\frac{\partial p}{\partial c} = \frac{1}{2+yp''(y)/p'(y)}$