Problem Set 3

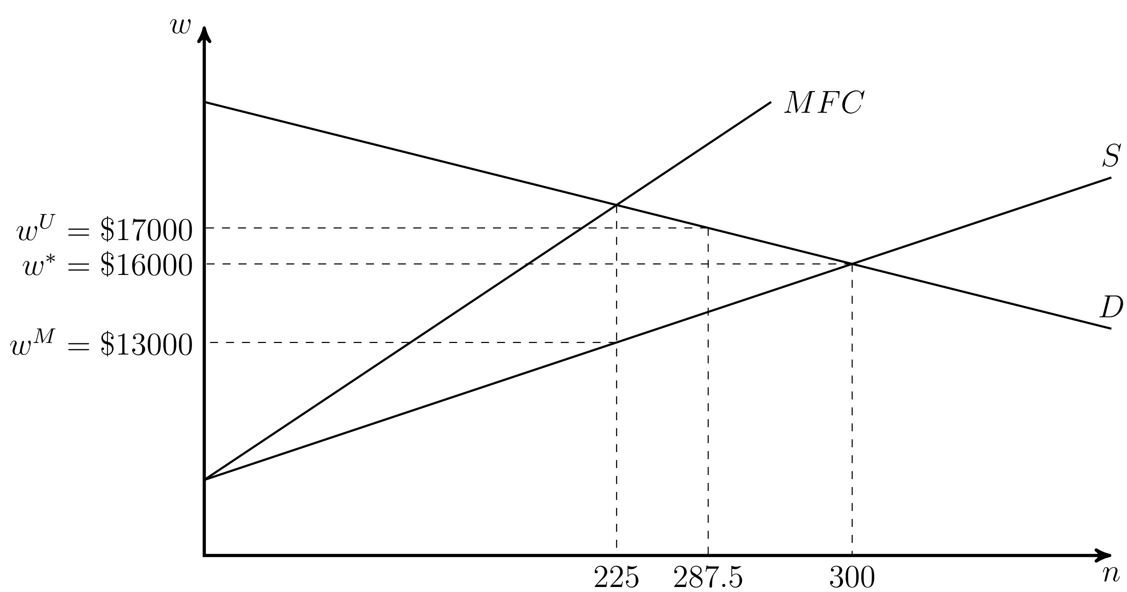
Econ 200

I. Short Case Study Problems

1. The demand for TAs at the University of Chico is approximated by the inverse demand function $w = 40000 - 80n$, where w is the annualized wage and n is the number of TAs hired. The supply is approximated by $w = 4000 + 40n$. If U Chico uses its market power fully, what wage will it pay and how many TAs will it hire? If TAs unionize and enforce $w = \$17,000$, how many TAs will be hired? Comment briefly on the efficiency (TS=CS+PS) implications of the union wage.

Solution: The university chooses the number of TAs to maximizes its objective function, which is the benefit from TAs minus cost to hire. First-order condition is marginal benefit equals marginal factor cost. Total cost is given by $(4000 + 40n)n$, so marginal factor cost is $w = 4000 + 80n$. Setting marginal factor cost equal to inverse demand function, the university hires $n^M = 225$ TAs and sets wages equal to $w^M = \$13,000$ when it acts as a monopsonist. When students unionize and demand wages of $w^U = \$17,000$, the university will hire $n^U = 287.5$ TAs.

When the TAs unionize, the effect is that there is a price floor on wages. When we looked at price floors earlier in the quarter, we saw that they resulted in deadweight loss. But in this problem, the price floor actually *reduces* deadweight loss since it brings us closer to competitive equilibrium. This is because the monopsonist is already inefficient, so introducing a second distortion such as a price floor can actually reduce inefficiency. If the union compromises and sets the wage $\$16,000$, this achieves the most efficient allocation.



2. Lost Lake CA is an isolated community with 20 households, each of whose demand for electricity is well approximated by $p = 60 - q$. The total cost of electricity generation and distribution there is $c(Q) = 900 + Q$. The local city council regulates electricity suppliers.

(a) If the city council wants to avoid deadweight loss, what price will they set for electricity? What is the corresponding CS and PS? What profit does the electrical power supplier earn?

Solution: To avoid deadweight loss, the council will have to set the price to what it would be in competitive equilibrium, p^*. In this problem, since marginal cost is constant and equal to 1, the supply curve is just a flat line given by $p = 1$. Therefore, in competitive equilibrium the price is $p^* = 1$.

Producer surplus is equal to zero because the supply curve is flat. To find consumer surplus, we first aggregate household demand q into demand for the entire town: $Q = 20q = 1200 - 20p$. At a price of $p^* = 1$, $Q^* = 1180$. Consumer surplus is then $\frac{1}{2}(1180)(60 - 1) = 34810$.

The power company's profit is $p^*Q^* - c(Q^*) = -\$900$, so it actually is losing \$900.

(b) Disappointing profits provoke the supplier to threaten to shut down. What price can the city council set to ensure nonnegative economic profit?

Solution: In general, the supplier's profit $p(Q)Q - c(Q)$ is a function of Q. Therefore, we can set this equation equal to zero and find its roots in order to find which values of Q generate zero profits. The equation is

$$\frac{1}{20}Q^2 - 59Q + 900 = 0$$

Applying the Quadratic Formula, we find that the roots are $Q = \{15.5, 1164.5\}$. The profit is nonnegative when $1.77 \leq p \leq 59.2$.

(c) An imaginative city council member suggests charging each household a fixed annual fee plus a per-unit price for electricity consumption. Is there a fee+price combination that would maximize efficiency, and also induce participation by the supplier and all households? If so, compute it; if not, explain why none exists.

Solution: The only way to maximize efficiency (and avoid deadweight loss) is to set the per-unit price $p = p^* = 1$. However, we know that the supplier is making a loss at that price so we will need to charge households a fixed fee sufficient to make up for the supplier's loss in order to guarantee its participation. Since the supplier's loss is \$900, we would need to charge a fixed fee of at least $\$900/20 = \$45$ per household. However, if we charge too high of a fee then households may not wish to participate. In general, households will choose to participate if their surplus is nonnegative. In part (a), we calculated the surplus of all households when $p^* = 1$ and there is no fixed fee. Therefore, a household's surplus will be nonnegative as long as the fixed fee does not exceed \$34810/20 = \$1740.5.

To summarize, the per-unit price must be equal to p^* in order to achieve maximum efficiency. When $p = p^*$, the fixed fee for each household must be between \$45 and \$1740.5. If the fee is lower than \$45, then the supplier would rather shut down than continue to make a loss. If the fee is higher than \$1740.5, then households would rather avoid using electicity altogether. This way of pricing is called 2-part tarrif.

3. Two firms selling related products face demand functions $y_1 = 4 - 2p_1 + p_2$ and $y_2 = 6 + p_1 - 3p_2$. Both firms have constant marginal cost $c = 2$.

(a) Write down the payoff functions of both firms, assuming they choose price simultaneously.

Solution: Profit is revenue minus cost.

$$\pi_1 = p_1(4 - 2p_1 + p_2) - 2(4 - 2p_1 + p_2),$$
$$\pi_2 = p_2(6 + p_1 - 3p_2) - 2(6 + p_1 - 3p_2).$$

(b) Compute and sketch the best response functions of each firm.

Solution: We can find best response function by taking deriving of profit with respect to price and rearrange it.

$$BR_1(p_2) = \frac{p_2 + 8}{4},$$
$$BR_2(p_1) = \frac{p_1 + 12}{6}.$$

(c) Compute Nash equilibrium (NE) prices and payoffs.

Solution: Simply solving the system of best response functions yields $p_1^B = 2.608$ and $p_2^B = 2.434$.

(d) Sketch what happens to BR functions and NE, when the goods become closer substitutes.

Solution: Let's solve for a simple but general case. Inverse demand functions are given as follows,

$$p_1 = \alpha - \beta y_1 - \gamma y_2,$$
$$p_2 = \alpha - \beta y_2 - \gamma y_1.$$

When γ approaches β, the products are closer substitute.
We rewrite the demand system as a system of demand functions,

$$y_1 = \alpha(\beta + \gamma) - \frac{\beta}{\beta^2 - \gamma^2} p_1 + \frac{\gamma}{\beta^2 - \gamma^2} p_2,$$
$$y_2 = \alpha(\beta + \gamma) - \frac{\beta}{\beta^2 - \gamma^2} p_2 + \frac{\gamma}{\beta^2 - \gamma^2} p_1.$$

Therefore, we can observe that when the products are closer substitute, coefficient of the cross price approaches the coefficient of own price, and coefficient of own price also increases.
Assuming marginal cost equals zero, first-order conditions are,

$$p_2 = -\frac{\alpha(\beta + \gamma)(\beta^2 - \gamma^2)}{\gamma} + \frac{2\beta}{\gamma} p_1^*,$$
$$p_2^* = \frac{\alpha(\beta + \gamma)(\beta^2 - \gamma^2)}{2\beta} + \frac{\gamma}{2\beta} p_1.$$

When the products are closer substitute, the best response function for firm 1 becomes flatter and for firm 2 becomes steeper. This means that when the rival firms decrease price by one, the firm has to decrease price more than the original case when products are closer substitute. Also, We can see that when we have perfect substitute ($\gamma = \beta$), equilibrium price goes down to zero.

4. Baytech sells gizmos in the home market where it faces the demand function $q_H = 100 - 3p_H$. It also sells the same product in a foreign market where it faces the demand function $q_F = 200 - 7p_F$. Its cost function is $c(q_T) = 30 + 12q_T + q_T^2$, where $q_T = q_H + q_F$.
(a) What output and price choices maximize Baytech's profit?

Solution: Write profit as

$$\mathrm{TR}_H(q_H) + \mathrm{TR}_F(q_F) - c(q_H + q_F)$$

where TR_H and TR_F are total revenues in the home and foreign markets, respectively. By taking derivatives, we see that Baytech's profit-maximizing conditions are

$$\mathrm{MR}_H(q_H) = c'(q_H + q_F) = \mathrm{MR}_F(q_F)$$

This says that marginal revenue at home equals marginal revenue abroad equals the marginal cost of producing for the combined market. (By the way, $\frac{dc(q_T)}{dq_H} = \frac{dc(q_T)}{dq_T} \cdot \frac{dq_T}{dq_H} = \frac{dc(q_T)}{dq_T} = c'(q_H + q_F)$ since $\frac{dq_T}{dq_H} = 1$. Repeat this to show that $\frac{dc(q_T)}{dq_F} = c'(q_H + q_F)$.)

Substituting in the marginal revenues (obtained by taking the derivative of $\mathrm{TR}_H(q_H) = p_H(q_H) \cdot q_H$ and $\mathrm{TR}_F(q_F) = p_F(q_F) \cdot q_F$) and marginal cost,

$$\frac{100}{3} - \frac{2}{3}q_H = 12 + 2(q_H + q_F)$$
$$\frac{200}{7} - \frac{2}{7}q_F = 12 + 2(q_H + q_F)$$

This system of 2 equations in 2 unknowns can be solved to obtain $q_H = 7.45$ and $q_F = 0.73$ which gives the prices $p_H = 30.85$ and $p_F = 28.47$.

(b) Antidumping regulations force Baytech to unify its prices. What choice of $p = p_H = p_F$ now maximizes profit? What is the maximum Baytech would rationally spend to eliminate the antidumping regulation?

Solution: With $p = p_H = p_F$, we now have $q_T(p) = q_H(p) = q_F(p) = 300 - 10p$ which gives inverse demand $p(q_T) = 30 - \frac{1}{10}q_T$. Baytech's optimization problem is now

$$\max_{q_T \geq 0} \quad \left(30 - \frac{1}{10}q_T\right) q_T - (30 + 12q_T + q_T^2) \quad \text{such that } q_H \geq 0, q_F \geq 0, q_H + q_F = q_T$$

The first-order condition gives $q_T = 8.18$ and $p = 29.18$. Remember, though, that an interior solution must also satisfy $q_H \geq 0$ and $q_F \geq 0$. It turns out that $q_F(29.18)$ is negative. Therefore, the first-order condition does not gives us an interior solution so we have to check corner solutions.

First, find the solution when $q_F = 0$. The problem is

$$\max_{q_H \geq 0} \quad (\frac{100}{3} - \frac{1}{3}q_H)q_H - (30 + 12q_H + q_H^2)$$

which gives $q_H = 8$, $p = 30.67$, and profits of 55.33. The other possible corner solution is when $q_H = 0$. In this case, the problem is

$$\max_{q_F \geq 0} \quad (\frac{200}{7} - \frac{1}{7}q_F)q_F - (30 + 12q_F + q_F^2)$$

which gives $q_F = 7.25$, $p = 27.535$, and profits of \$30.07.

Therefore, Baytech will choose to only its gizmos in the home market and make a profit of \$55.33. Baytech's unregulated profit in part (a) is \$251.88, implying that Baytech should be willing to spend up to $55.54 - 55.33 = 0.21$, \$0.21 per period to overturn the regulation.

5. Varian 16.10. Consider an industry with 2 firms, each having marginal costs equal to zero. The (inverse) demand curve facing this industry is

$$p(Y) = 100 - Y,$$

where $Y = y_1 + y_2$ is total output.

(a) What is the competitive equilibrium level of industry output?

Solution: In the perfectly competitive market, the pricing condition is $P = MC$. Since marginal cost is zero, $p^* = 0$. From demand function, $Y^* = 100$.

(b) If each firm behaves as a Cournot competitor, what is firm 1's optimal choice given firm 2's output?

Solution: Each firm solves the profit maximization problem,

$$\max_{y_i} (100 - Y) y_i.$$

The first-order condition with respect to output is,

$$BR_1(y_2) = 50 - \frac{y_2}{2},$$
$$BR_2(y_1) = 50 - \frac{y_1}{2}.$$

(c) Calculate the Cournot equilibrium amount of output for each firm.

Solution: Simply solving the system of best response functions yields $y_1^c = y_2^c = 100/3$.

(d) Calculate the cartel amount of output for the industry.

Solution: The cartel maximize the joint profit by choosing y_1 and y_2,

$$\max_{y_1,y_2}(100 - Y)Y. \tag{1}$$

First-order conditions are both,

$$100 - 2(y_1 + y_2) = 0. \tag{2}$$

Therefore, $Y^{cartel} = 50$ and in this simple case, any combination of (y_1, y_2) which satisfies $y_1 + y_2 = 50$ maximizes the cartel's profit.

(e) If firm 1 behaves as a follower and firm 2 behaves as a leader, calculate the Stackelberg equilibrium output of each firm.

Solution: We solve this problem by backward induction. Firm 1 maximizes profit given the output of the leader. We already solved this in part a),

$$BR_1(y_2) = 50 - \frac{y_2}{2}.$$

The leader maximize profit given the best response of firm 1,

$$\max_{y_2}(100 - BR_1(y_2) - y_2)y_2. \tag{3}$$

The first-order condition yields $y_2^s = 50$. From the best response function of firm 1, $y_1^s = 25$.

6. Econometric analysis of air travel demand yields the following elasticity estimates for (first class, unrestricted coach, discount) demand: income = (1.8, 1.2, 1.1), and own-price = (-0.9, -1.2, -2.7). Predict the fares that a profit-oriented airline would pick for a route with no serious competition and marginal cost \$150 per passenger.

Solution: This is an example of third-degree price discrimination. Applying the mark-up formula $p_i = \frac{|\epsilon_i|}{|\epsilon_i|-1}c$ from the notes, we get

$$p_F = \frac{0.9}{0.9-1}(100) = -\$1350$$
$$p_U = \frac{1.2}{1.2-1}(100) = \$900$$
$$p_D = \frac{2.7}{2.7-1}(100) = \$205.56$$

Plainly, the formula does not give a valid price for first-class tickets, p_F. This is because demand for these tickets is inelastic ($|-0.9| < 1$). Faced with inelastic demand for first-class tickets, the profit-maximizing firm will raise p_F to an arbitrarily high level until demand is no longer inelastic. (As long as demand is inelastic, when a firm raises its price total revenue will increase and total costs will fall.)