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# International trade and the internal organization of firms: An evolutionary approach

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## Abstract

Masahiko Aoki and others have distinguished two alternative modes for a firm's internal organization. We argue that the profitability of each mode depends on the distribution of firms across modes and on the general economic environment. We characterize the evolutionary equilibria in both a parametric and a general model, and argue that corner equilibria predominate. We analyze the effects of trade between the two countries in (a) outputs only and (b) inputs (factors) as well as outputs. Our most striking conclusion is that in case (b) a less efficient mode can displace a more efficient mode when trade barriers are sufficiently low.

*Key words:* Japanese corporation; Evolutionary games; Internal organization of firms; Factor movement

*JEL classification:* C73; C92; D23; F12; F20; L22

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## 1. Introduction

The huge success of post World War II Japanese manufacturing spawned numerous articles and books, popular as well as academic, on the internal organization of the firm and on international trade. The discussions of internal organization emphasize the contrast between the new flexible “Japanese” mode and the traditional hierarchical “American” mode. The

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literature often focuses on how the Japanese mode affects the balance of trade, but seldom looks at the reciprocal influence of trade on internal organization. Much of the existing literature lacks a coherent framework for sorting out the influences on and the consequences of firms' choice of organizational mode. Our first goal in this paper is to provide such a framework.

We point out that the profitability of each mode depends on the modes currently employed by other firms as well as on the general economic environment. The relative prevalence of each mode (i.e. the state) will evolve over time as firms expand or shrink and attempt to increase profit by switching modes or by entering and exiting the industry. Reciprocally, the profitability of switching (or expansion or entry and exit) will change as the state evolves. Evolutionary game theory provides an appropriate analytical framework for working out the consequences. The secondary goal of our paper is to demonstrate by example that evolutionary games, recently fashionable among pure theorists, can be useful in economic applications.

Recent policy debates suggest that international trade may affect the viability of an internal organizational mode. The final goal of our paper is to analyze the interaction between international trade in outputs (goods and services) and in inputs (factors of production) on the one hand and firms' choice of internal organization on the other. It turns out that the trade regime is a crucial aspect of the economic environment.

We begin in Section 2 with a verbal characterization of the two modes, drawing heavily on Aoki (1988) and Aoki (1990). Section 3 illustrates the main ideas and builds intuition by working with a concrete, parametric model featuring Cournot competition. General definitions, models and results appear in Section 4. We show that, in long-run or "evolutionary" equilibrium, one mode can dominate in isolated countries, and that trade only in outputs can lead to different modes dominating the different trading partners. We also show that trade in inputs as well as outputs can lead to dominance of the less efficient (higher cost) mode in both countries. Broader discussion is reserved for the final section. Appendix A contains technical details and proofs.

Our results extend some standard conclusions in the trade literature. Ethier (1982) shows that when one traded good is produced with increasing returns to scale at the national level, then the larger country will tend to specialize in that good; see also Helpman (1984) and Markusen (1983). In our model, it is not necessarily the larger country that specializes in the mode with increasing returns (or decreasing costs). Ethier (1979) considers international increasing returns in a model that roughly corresponds to our case of free trade in inputs as well as outputs, and obtains standard results on the pattern of trade according to comparative advantage. Again, our results point to other possibilities.

Our model draws inspiration from the burgeoning theoretical literature on evolutionary games and adjustment dynamics; see, for example, Weibull (1994) and the 1993 double special issue of *Games and Economic Behavior*. Unfortunately there still are relatively few economic applications; Benabou (1993) is perhaps the most relevant. Using a model more complex but in some respects analogous to our Cournot parametric model, Benabou argues that choice of residential locational (analogous to choice of production mode in our model) may produce inefficient stratification. Arthur (1989) makes the general point that given increasing returns in an essentially single-country setting and given sequential choices by firms, historical accidents can lock-in an inferior technology.

## 2. Organizational modes

The first mode of internal organization, denoted *A*, is popularly identified with American firms in a traditional manufacturing sector such as auto or steel. In the *A* mode, workers are narrowly classified and they tend to specialize in a small set of tasks. Buffer inventories are used to meet fluctuations in demand. Decisions are planned and coordinated through the central office. Information flows from the top of the hierarchy to the shop floor. In the *A* firm, intrashop as well as intershop coordination has become the specialized function of supervisors and managers. The separation of coordinating tasks and operating tasks facilitates hierarchical control by management.

We consider a single alternative mode, denoted *B*, popularly identified with Japanese corporations. In the *B* mode, workers typically practice job rotation. Within some limits, regular workers can be thought of as having lifetime employment. In addition to merit ratings, wages are heavily influenced by seniority. The just-in-time system is used to minimize the cost of inventories. Market demands come from dealers directly to the head of the assembly line. Decisions are made and coordinated by shop floor units, often without the direct involvement of the central office.

Another feature of the *B* firms is that often the major stockholders include banks and other corporations. Long-term intercorporate stockholding gives rise to corporate groupings. One type is the main bank group, with a nucleus bank serving as a stockholder, a lender, a monitoring agent and a rescuer in times of distress. Another type is the grouping of a major manufacturer and its layers of suppliers and distributors. These groups of *B* firms tend to be exclusive, stable and long-term oriented.<sup>1</sup>

<sup>1</sup> For a more detailed discussion, see Aoki (1988, chapters 2–4). See Fung (1991) for an empirical study of how the Japanese industrial groups affect trade between the US and Japan.

Milgrom and Roberts (1990) argue that intermediate organizational modes between *A* and *B* are inefficient because of complementarities and non-convexities. We offer no independent analysis on this point, and just take as given the two alternative modes of organization.

The economic environment affects the relative efficiencies of the two organizational modes. For example, if demand favors small batch production of many varieties, if external shocks are moderate and frequent, if the workers learn rapidly and if the costs of holding inventories are high, then the *B* mode will be relatively efficient. However, if demand favors high volume, homogeneous products, if shocks are either miniscule or drastic, if the workers are slow to adapt and if the inventory costs are low, then the *A* mode of organization will work relatively well.<sup>2</sup>

Efficiency also depends on the relative prevalence of the two modes. In particular, average production cost for *B* firms declines as their prevalence increases. This externality has two sources that we refer to as the skimming effect and the network effect. In the *B* mode, workers generally have lifetime employment and rotate among jobs. Consequently firms invest heavily in training, especially early in a worker's career. On the other hand, wages are closely tied to seniority, more so than in *A* firms. As a result, workers of equivalent productivity may receive higher wages in *A* firms, especially in early and mid career. Thus the greater the prevalence of *A* firms, the greater the threat that *B* firms will find their best workers lured away or "skimmed". Threatened or actual skimming raises costs for *B* firms relative to *A* firms.<sup>3</sup>

A second effect arises from closer supplier/customer and governance interdependencies among *B* firms. Firms using the *B* mode are often tied to each other via cross-shareholding. The *B* firms also have ties to suppliers and distributors that depend on the *B* manufacturers for long-term business. These satellite firms will have greater bargaining power and the *B* firms' costs will increase when *A* firms become more prevalent and offer alternative opportunities to the *B* satellites. This is the network effect.

Due to network and skimming effects, the production cost in the *B* mode will decrease in  $s$ , the fraction of firms adopting the *B* mode. To the extent that customers do not regard the output of *A* and *B* firms as perfect substitutes, there will also be a demand-side or "glut" effect on output price. Cost and demand interdependence imply that the profitability of all

<sup>2</sup> Aoki (1990) for more references and details.

<sup>3</sup> We will not offer a formal model to explain why *B* firms do not skim workers from each other. We should point out, however, that such skimming would upset norms of interfirm behavior as well as upset the intrafirm wage structure. See Hashimoto (1990) for a detailed description of Japanese labor market practices.

firms will depend on  $s$  as well as on the general environmental variables discussed earlier.

### 3. A parametric example

In this section we develop a two-country parametric model in some ways reminiscent of Brander and Krugman (1983). The main purpose is to build intuition for the general results in Section 4. The conclusions of our paper do not depend on the parameter values or the functional forms; these are chosen for illustration only.

We begin with a single isolated country with a fixed number of *a priori* identical firms. Each firm chooses between two alternative modes of internal organization. Then firms simultaneously choose output quantity, yielding a short-run equilibrium à la Cournot and Nash. In the long run, firms achieve a steady-state equilibrium when no firm can increase its short-run profits by unilaterally switching modes. Later subsections extend the analysis to two countries and analyze the impact of international trade in goods (firms' output) and in factors (firms' inputs).

#### 3.1. Autarky

Consider a single isolated country with  $N \geq 2$  firms. Suppose for the moment that  $sN$  of the firms employ the  $B$  mode and  $(1-s)N$  employ the  $A$  mode for some fraction  $s \in [0,1]$ .<sup>4</sup> Fixed costs are zero. Mode  $A$  firms have constant unit cost  $c_A > 0$ . Mode  $B$  firms have a constant unit cost  $c_B - bs > 0$ . The parameter  $b \in [0, c_B)$  captures the network and skimming externalities in a simple linear fashion: mode  $B$  firms' costs decrease at rate  $b$  as their prevalence  $s$  increases. We will usually focus on the case  $c_A > c_B - b$ , i.e. the  $B$  mode firms have lower unit costs when they are sufficiently prevalent.

Demand is linear and consumers regard the outputs of  $A$  and  $B$  firms as close but imperfect substitutes. (Think of US and Japanese cars, for example.) Specifically, let  $P_A$  and  $P_B$  be the prices of  $A$  and  $B$  firms' output and let  $X_A = \sum_{i \in A} x_i$  and  $X_B = \sum_{i \in B} x_i$  be the total outputs of mode  $A$  and

<sup>4</sup> The Cournot model assumption that  $sN$  is an integer imposes restrictions on the set of feasible  $s \in [0,1]$ , restrictions that shift with the industry size parameter  $N$ . These restrictions have no substantial consequences in the model so we ignore them for simplicity. Later we implicitly assume that  $N$  is large enough that firms' choice of organizational mode is not intended to influence other firms' later choice of mode; in our view relatively small numbers, say  $N = 3$ , will suffice for this purpose.

mode  $B$  firms. Then the (inverse) demand functions are  $P_A = \alpha_A - \beta X_A - \gamma X_B$  and  $P_B = \alpha_B - \beta X_B - \gamma X_A$ , for given positive parameters  $\alpha_A$ ,  $\alpha_B$  and  $\gamma < \beta$ .

Short-run equilibrium is Nash–Cournot for the  $sN$  mode  $B$  firms and the  $(1-s)N$  mode  $A$  firms. Each  $A$  firm chooses  $x_A \geq 0$  to maximize its profit  $(\alpha_A - \beta(x_A + \hat{X}_A) - \gamma X_B)x_A - c_A x_A$ , where  $\hat{X}_A = X_A - x_A$  is the combined output of all other  $A$  firms. Each  $B$  firm chooses a non-negative output level  $x_B$  to maximize an analogous expression for its profit. The simultaneous solution to the first-order conditions defines a unique symmetric equilibrium with output levels

$$x_B = [(1 + (1-s)N)\beta(\theta_B + bs) - \gamma(1-s)N\theta_A]/\Delta \quad (1)$$

and

$$x_A = [(1 + sN)\beta\theta_A - \gamma sN(\theta_B + bs)]/\Delta$$

where

$$\Delta = (1 + sN)(1 + (1-s)N)\beta^2 - sN(1-s)N\gamma^2$$

and  $\theta_A = \alpha_A - c_A > 0$  and  $\theta_B = \alpha_B - c_B > 0$ . The short-run equilibrium profits are  $\pi_A = (x_A)^2\beta$  and  $\pi_B = (x_B)^2\beta$ .

Fig. 1 graphs these short-run profits as a function of the fraction  $s$  of mode  $B$  firms for specified values of the industry size, demand and cost parameters. Firms' profits depend on the state  $s$  for two different reasons. First, the network and skimming effects (captured in the  $b$  parameter) decrease costs for  $B$  firms as  $s$  increases. Second, the glut effect (that is, to the extent that the parameter  $\gamma$  is less than  $\beta$ ) implies that an increase in the fraction  $s$  of  $B$  firms will lower the price of  $B$  output relative to  $A$ , decreasing  $\pi_B$  and increasing  $\pi_A$ . The profit function for  $B$  firms is upward sloping (and steeper than the profit function for  $A$  firms) when the network/skiming effects dominate the glut effect.

At state  $\hat{s} = 0.5$  in Fig. 1 the  $A$  and  $B$  modes are equally profitable, so there is no reason for firms to switch modes. Thus  $\hat{s}$  is a steady state. However, the  $A$  mode is more profitable at states  $s$  even slightly below  $\hat{s}$  and the  $B$  mode is profitable at states even slightly above  $\hat{s}$ . Hence  $\hat{s}$  is unstable, and we are unlikely to observe it after the state has a chance to evolve if there is even a tiny amount of noise in the system. The *stable* steady states in Fig. 1 are  $s = 0$  and  $s = 1$ ; these are the evolutionary equilibria, the states we expect to observe in the long run. In this case all firms in the country eventually will adopt the same mode: either all  $A$  or all  $B$ , depending on historically given initial conditions.

Other parameter values can produce different qualitative behavior. Consider, for example, a decrease in unit cost for  $B$  firms from  $c_B = 14$  to  $c_B = 10$ . *Ceteris paribus*, this parameter change makes  $B$  firms more

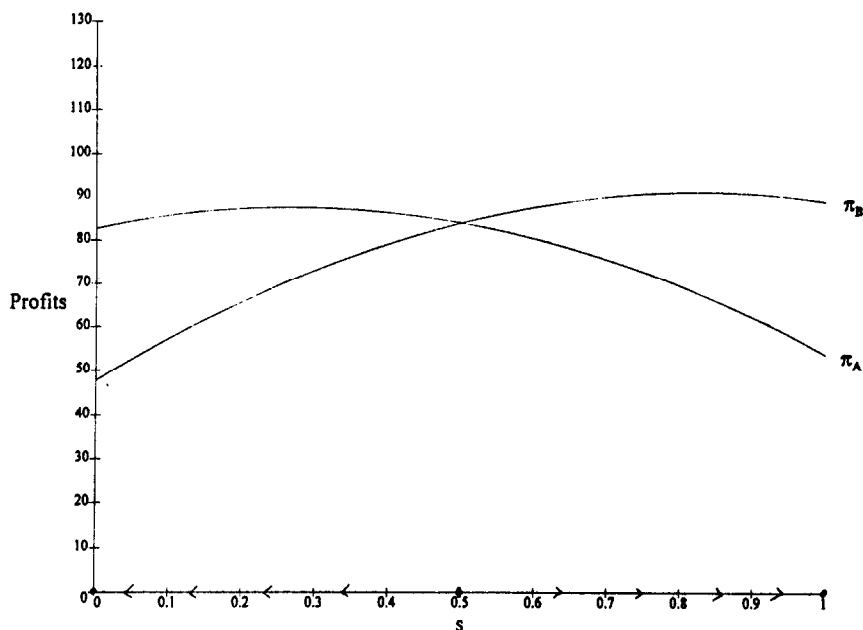


Fig. 1.

profitable than  $A$  firms at all states, so the only evolutionary equilibrium here is  $s = 1$ . In other words, eventually all firms adopt the  $B$  mode in this case. Similarly, a ceteris paribus change of  $c_B$  to 18 makes the  $A$  mode more profitable than the  $B$  mode at all states. Now the unique evolutionary equilibrium is  $s = 0$ , so all firms eventually adopt the  $A$  mode. Finally, consider a ceteris paribus decrease in the parameter  $\gamma$  from 0.98 (highly substitutable goods) to 0.95 (fairly substitutable goods). Now the glut effect is stronger than the network/skimming effect, so the slope of the  $A$  profit function exceeds that of the  $B$  profit function. The stability argument used above now leads to the conclusion that the interior equal profit point  $\hat{s} = 0.5$  is the unique evolutionary equilibrium. In other words, there is a stable mixture of  $100\hat{s}\%$   $B$  firms and  $100(1 - \hat{s})\%$   $A$  firms in this case.

### 3.2. Trade

Now consider a “foreign” country with  $N^*$  firms. Assume  $s^*N^*$  of the foreign firms employ mode  $B$ , as do  $sN$  home country firms, for some fractions  $s, s^* \in [0, 1]$  that are fixed in the short run. Using asterisks to denote foreign country variables and the superscripts  $d$  and  $e$  to denote production for the domestic market and the export market respectively, we

write the demand function faced by domestic mode  $A$  firms as  $P_A = \alpha_A - \beta(X_A^d + X_A^{*e}) - \gamma(X_B^d + X_B^{*e})$ . The demand for  $B$  output is the same with  $A$  and  $B$  subscripts interchanged. Foreign demand functions  $P_A^*$  and  $P_B^*$  are the same with asterisks and no-asterisks interchanged on all subscripted variables.<sup>5</sup> Home country  $A$  firms choose  $x_A^d \geq 0$  and  $x_A^e \geq 0$  to maximize profits

$$(\alpha_A - \beta(\hat{X}_A^d + x_A^d + X_A^{*e}) - \gamma(X_B^d + X_B^{*e}))x_A^d + (\alpha_A^* - \beta(X_A^{*d} + \hat{X}_A^e + x_A^e) - \gamma(X_B^{*d} + X_B^e))x_A^e - c_A(x_A^d + x_A^e) - tx_A^e.$$

The parameter  $t \geq 0$  in the last term represents trade barriers, the excess unit cost for selling in the export market rather than in the domestic market. Home country  $B$  firms and foreign firms of both types maximize analogous expressions. Foreign  $B$  firms have unit production costs  $c_B^* - b^*s^* > 0$  that decline with domestic prevalence at a rate  $b^*$  that may differ from the home country rate  $b$ . In this section, the other slope parameters ( $\beta, \gamma$  and  $t$ ) are assumed for simplicity not to differ across countries and organizational modes.

Even so, the short-run Nash–Cournot equilibrium is not easy to compute because we have a system of eight messy Kuhn–Tucker conditions, each with a non-negativity constraint, and the non-negativity constraints bind on some export quantities for some interesting states and parameter values. Appendix A reproduces expressions for short-run equilibrium outputs  $x_i^j(s, s^*)$ , where  $i = A, B$  and  $j = d, e$ , for states where the constraints do not bind. The expressions for foreign output  $x_i^{*j}(s, s^*)$  again are the same with asterisks and no-asterisks interchanged on all subscripted variables.

However complex the output formulas, the short-run equilibrium profit function is simply  $\pi_A(s, s^*) = (x_A^d)^2\beta + (x_A^e)^2\beta$  for home-country  $A$  firms, with analogous expressions for home-country  $B$  firms and for foreign firms.<sup>6</sup> Output quantities and profits for all firms depend on the overall state  $(s, s^*)$

<sup>5</sup> For the sake of simplicity, these demand functions assume perfect substitutability between domestic and foreign output by firms of the same organizational mode. Closer substitutability than for outputs of different mode firms suffices for the glut effects used in the general results in the next section. A justification for closer substitutability arises from the discussion of flexibility and batch size in Section 2. Given their comparative advantage for large batch production, the  $A$  firms typically will target the center of the market (e.g. basic cars), while  $B$  firms will target the entire market (e.g. hatchbacks, convertibles, small cars, etc.). The point that  $A$  ( $B$ ) firms compete more directly with each other than with  $B$  ( $A$ ) firms should be clear even in the absence of a more complex parametric model that incorporates characteristic spaces.

<sup>6</sup> To understand the simple quadratic relationship between profit and output in this Cournot model, first note that each output  $x$  is either zero or else satisfies a first-order condition of the form  $-\beta x + (P - c) = 0$ , i.e.  $x = (P - c)/\beta$  or  $= 0$ . Hence profit is of the form  $\pi = (P^d - c)x^d + (P^e - c)x^e = (x^d)^2\beta + (x^e)^2\beta$ .



because we have transnational glut effects as well as domestic skimming/network effects and glut effects in each country.

Evolution of the overall state is governed by the signs of the profit differences  $\pi_D = \pi_B - \pi_A$  in the home country and  $\pi_D^* = \pi_B^* - \pi_A^*$  in the foreign country. When  $\pi_D$  is positive (negative), the  $B$  mode is more (less) profitable at home and therefore  $s$  should increase (decrease). Likewise when  $\pi_D^*$  is positive (negative),  $s^*$  should increase (decrease). Some possibilities are illustrated in Fig. 2, Fig. 3 and Fig. 4.

The trade barrier parameter  $t$  is prohibitively large in Fig. 2, and all export quantities are zero. Hence in each country we have the two autarky equilibria  $s$  and  $s^* = 0$  or  $1$ , and the two-country system has an evolutionary equilibrium at each corner of the  $s - s^*$  square. Almost every point in the interior of the square evolves towards one of the corner equilibria. The lines  $\pi_D = 0$  and  $\pi_D^* = 0$  separate the sets of points that evolve towards each corner (the “basins of attraction”). Recall that the interior (unstable) equilibrium  $\hat{s}$  Fig. 1 is defined by the equal profit condition  $\pi_D$ , so Fig. 2 is a two-dimensional analogue; in each country either the  $A$  mode or the  $B$

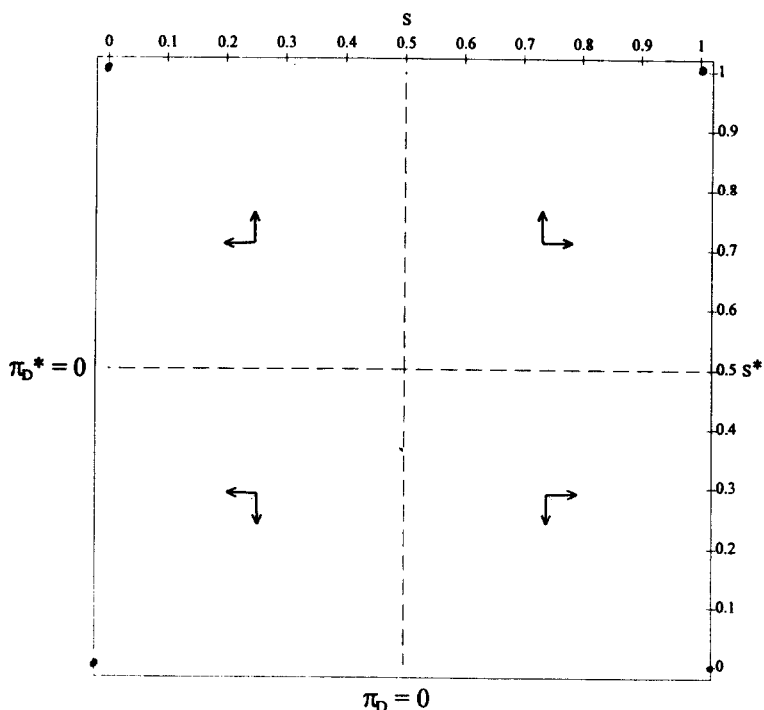


Fig. 2.

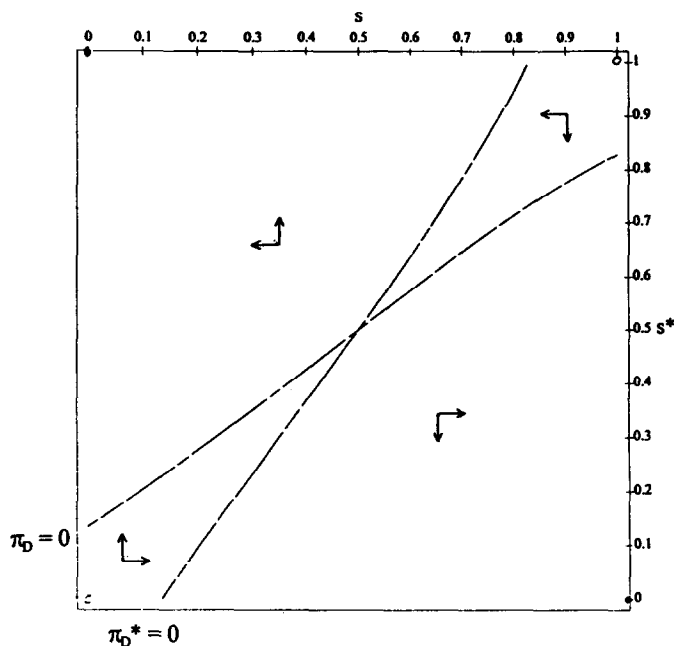


Fig. 3(a).

mode will dominate, and history will determine which of the four stable combinations prevails.

In Fig. 3(a) and Fig. 3(b), the trade barrier parameter is zero and exports are significant. The loci  $\pi_D = 0$  and  $\pi_D^* = 0$  still divide the square into four regions and for some parameter values each region still has an evolutionary equilibrium. However, asymmetries in size or cost or demand, or transnational glut effects can destabilize the symmetric equilibria (0,0) and (1,1). In Fig. 3(a), only the asymmetric equilibria (1,0) and (0,1) remain, due here to strong glut effects. That is, in the long run, each country is certain to specialize completely in a different organizational mode. Such specialization can also arise because of asymmetries in country size. Fig. 3(b) represents a stylized version of the US and Japan. Parameters are the same as in Fig. 2 (e.g.  $\gamma = 0.98$  so glut effects are weak, and cost conditions are the same in both countries) except that trade barriers are absent ( $t = 0$ ) and the home country is larger ( $N = 20$  and  $\alpha_A = 220$  versus  $N^* = 10$  and  $\alpha^* = 110$ ). The basins of attraction for the symmetric equilibria almost disappear.<sup>7</sup> The interpretation is that with low trade barriers for output, the two countries

<sup>7</sup> Since the loci are positively sloped, the basins of attraction for (0,1) and (1,0) strictly contain the areas NW and SE of both loci, so the basins of attraction for (0,0) and (1,1) are even narrower than the dashed lines suggest. See the Appendix for further discussion.

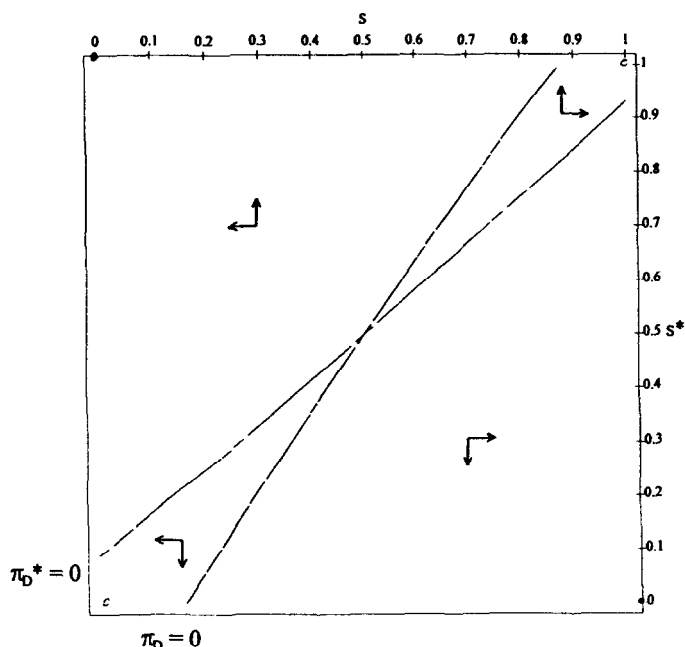


Fig. 3(b).

will adopt different organizational modes, for example all *A* mode in the US and all *B* mode in Japan. (This direction of specialization, of course, would be more likely than the opposite direction if we introduced appropriate cost asymmetries.)

Fig. 4 illustrates the case of completely free trade in firms' inputs as well as outputs. Setting the parameter  $t = 0$  indicates free trade in outputs. Free trade in inputs equalizes the cost parameters  $c_B = c^{*B} = \bar{c}_B$  and  $c_A = c^{*A} = \bar{c}_A$  across countries. It also equalizes the network/skimming effect, which now is transnational and depends only on the overall fraction  $\bar{s} = (Ns + N^*s^*) / (N + N^*)$  of *B* firms<sup>8</sup> at some combined rate  $\bar{b}$ . Hence with free trade in factors, the unit cost functions in both countries become  $\bar{c}_A$  for *A* firms and  $\bar{c}_B - \bar{b}\bar{s}$  for *B* firms. Note that the evolutionary equilibria now are at the opposite corners (0,0) and (1,1). The basins of attraction need not have the same size. For example, if we increase the marginal cost of the *B* mode slightly the basin of attraction for (0,0) becomes much larger. Fig. 4 shows for our stylized US/Japan example that a size asymmetry can skew the

<sup>8</sup> The simulations in Figs. 2 to 4 use the parameter  $y$  in the home country externality expression  $b(Ns + yN^*s^*) / (N + yN^*)$ . The value  $y = 0$  means no trade in inputs, and the value  $y = 1$  means free trade in inputs. The expression for the foreign country is analogous and uses the same value  $y$ .

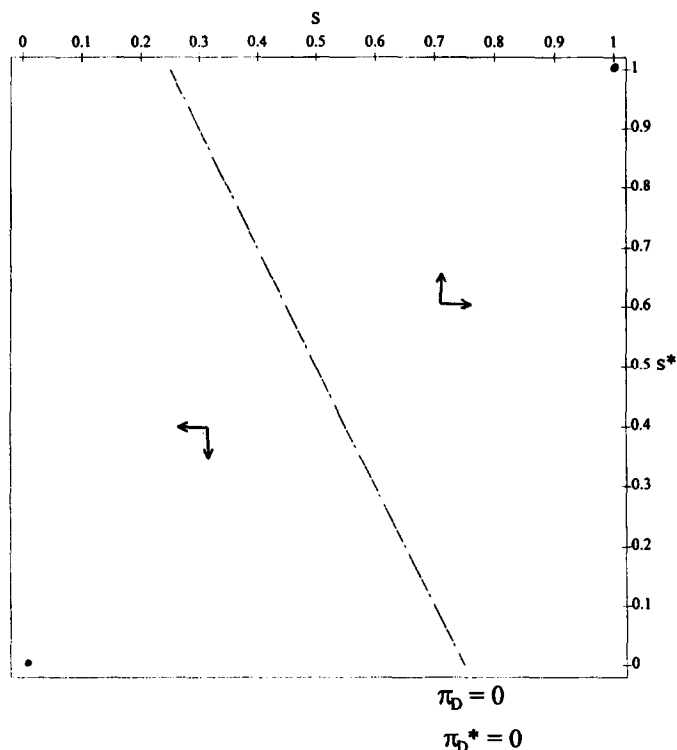


Fig. 4.

basins of attraction, even in the absence of cost asymmetries or strong glut effects.

These diagrams suggest an interesting story about trade between the US and Japan. Begin in autarky, at the point  $(0,1)$  in Fig. 2. Note that we are not assuming any cost asymmetries between the two countries; the specialization of Japanese firms in the  $B$  mode and US firms in the  $A$  mode at  $(0,1)$  is purely for historical reasons and is as likely (given the stylized parameters) as any other pattern. Next, open trade in outputs only. Fig. 3 suggests that the specialization pattern will persist, and indeed be reinforced in that the basin of attraction becomes larger for evolutionary equilibrium  $(0,1)$ . Finally, consider a new regime that now allows free (or nearly free) trade in inputs as well as outputs. The new regime will destabilize the existing pattern of specialization. Fig. 4 suggests that the system will evolve towards the highest cost and least efficient state  $(s,s^*) = (0,0)$ . That is, even though the  $B$  mode is relatively efficient, it will be completely displaced in the foreign country as well as in the home country.

Is this inefficiency result due to fortuitous parameter values or to details

of model specification? We have verified that it holds under a reasonably wide range of parameter values, but a parametric model cannot settle broader robustness questions. We will now construct a general model and find that the qualitative properties illustrated in the figures are indeed quite robust.

#### **4. A general evolutionary model**

Some readers may prefer a traditional Heckscher–Olin–Samuelson model (or a newer Romer-style growth model or something else) to our Nash–Cournot model. Some readers may regard linear demand and cost specifications as unrealistic or may object to an exogenous specification of the number of firms. In this section we present a general evolutionary trade model that can accommodate a large variety of specific parametric forms. The main limitation is not the functional form, but rather the relative adjustment speeds. Our evolutionary approach presumes that the state adjustment (i.e. entry and exit or differential growth or switching modes of internal organization) is not too rapid relative to the time required to establish short-run equilibrium at the current state, and that shocks or changes in the environment are reasonably small or infrequent.

We give general definitions of the environment and firms' short-run profitability, and introduce various sorts of long-run evolutionary adjustment processes and equilibria for a single country. After extending the definitions to cover a two-country trade model, we state three general propositions on equilibrium and trade.

##### *4.1. Model elements*

Begin with a single country called "home". Denote the fraction of firms choosing the  $B$  mode over the  $A$  mode by  $s \in [0,1]$ . Summarize the environmental variables affecting firms' profitability (the production technologies, inventory costs, workforce skill levels, wage rates, demand conditions, transportation and transaction costs, etc.) by an  $m$ -vector  $z \in Z \subset R^m$ , where the set  $Z$  of feasible environments is assumed to be convex and non-empty. For the moment assume that the home country is economically isolated so that firms' profitability does not depend on the environment or state in other countries. (Later we will be able to write this autarky assumption as a joint restriction on  $Z$  and on the feasible foreign environments  $Z^*$ .)

Now define short-run profits (i.e. smoothed per-period net earnings at the given state and environment) for the marginal firm adopting mode  $i = A, B$  by the functions  $\pi_i: [0,1] \times Z \rightarrow R$ . We assume that  $\pi_i(s; z)$  is continuously

differentiable in  $s$  and continuous in  $z$ . In the Nash–Cournot parametric model, a feasible environment (with a fixed number  $N$  of firms) is a six-component vector in the set  $Z = \{(\alpha, \beta, \gamma, c, b, t) \in R^6: \alpha > c > b \geq 0, \beta \geq \gamma > 0, t \geq 0\}$ , and the profit functions  $\pi_i$  were the ratios of polynomials mentioned in Section 3.1. An extension of that model allowing entry and exit (i.e. endogenous  $N$ ) would have somewhat different profit functions but the same set  $Z$  of feasible environments. In a general equilibrium growth parametric model, the feasible environments  $Z$  might be some thin subset of nine-dimensional Euclidean space and the profit functions  $\pi_i$  might be transcendental functions.

In the long run the state  $s$  evolves according to relative profitability  $\pi_D = \pi_B - \pi_A$  as firms enter and exit, expand and contract, and switch organizational modes. For convenience we express the evolutionary process in a continuous-time, deterministic framework.<sup>9</sup> The general specification is that the evolutionary path  $s(t)$  begins at some given state  $s(0) = s_0$  and thereafter the rate of change  $\dot{s} = ds/dt$  satisfies the ordinary differential equation

$$\dot{s} = F(s; z) \quad (2)$$

where  $F$  is a continuous function of the state  $s \in (0, 1)$  and the environment  $z \in Z$ . The dynamics  $F$  are *compatible* with the profit function  $\pi_D$  if the following three conditions hold for all  $s \in (0, 1)$  and  $z \in Z$ . (Recall that the sign function  $\text{sgn } v$  is 1, 0 or  $-1$  as  $v$  is positive, zero or negative.)

1.  $\text{sgn } F(s; z) = \text{sgn } \pi_D(s; z)$ ,
2.  $|F(s; z)| \geq \varepsilon |\pi_D(s; z)|$  for some  $\varepsilon > 0$ , and
3.  $\pi_D(0; z) < 0$  implies  $F(0; z) = 0$ , and  $\pi_D(1; z) > 0$  implies  $F(1; z) = 0$ .

Condition (1) is the conceptual key. It is the “survival of the fittest” requirement that the more profitable mode becomes more prevalent over time; the sign of  $\pi_D$  (e.g. positive if mode  $B$  is more profitable) is the sign of  $\dot{s}$  and  $F$  (e.g. positive if mode  $B$  is increasing). The other conditions are for technical convenience. Condition (2) ensures that evolution does not stagnate—the adjustment speed  $|\dot{s}|$  does not become arbitrarily slow relative

<sup>9</sup> The assumption of continuous time seems inessential; very similar results clearly can be obtained in discrete-time models. A natural stochastic extension of the model is to allow random perturbations of the environment  $z$ . We have not pursued such an extension but believe that our main results would survive in some form. Allowing  $\dot{s}$  to depend on lagged values of  $s$ , or non-autonomously on calendar time  $t$ , or on second or higher order time derivatives, can be handled at the cost of a higher-dimensional state space. We do not pursue such refinements here in the conviction that, for  $s$  in an appropriate state space, Eq. (2) is a sufficiently general specification to capture the most interesting systematic state adjustment processes.

to the profit advantage  $|\pi_D|$ . Condition (3) ensures that neither mode claims more than 100% of all firms by imposing the standard complementary slackness conditions at the endpoints  $s = 0, 1$  of the state space.

Examples of compatible dynamics include the linear specification  $F = a\pi_D$  for  $s \in (0, 1)$ , where the adjustment rate coefficient  $a > 0$  can be normalized to 1 by appropriate choice of the time scale. Much of the biological literature and some of the economic literature uses replicator (or Malthusian) dynamics  $f(s; z) = s(1 - s)\pi_D(s; z)$ . Obviously there are many other compatible dynamics that might apply to different sorts of learning or entry/exit processes. In deriving the results of the next subsection we need no specific functional form for  $F$ , and require only that the dynamics satisfy compatibility conditions (1–3).

There are several relevant concepts of long-run equilibrium. A state  $s$  is a *Nash equilibrium* (NE) if no firm can gain by switching modes at  $s$ . Implicitly regarding firms as small, we write the condition as  $\pi_D(s; z) = 0$  for interior states  $s \in (0, 1)$ , and as  $\pi_D(s; z) \leq 0$  for  $s = 0$  or  $\pi_D(s; z) \geq 0$  for  $s = 1$ . A state  $s$  is a *fixed point* (FP) or steady state if it has no tendency to change over time, that is, if  $F(s) = 0$ . It follows directly from the definitions that NE coincide precisely with the FP of any compatible dynamics  $F$ .

A fixed point  $s$  is an *evolutionary equilibrium* (EE) if it is locally asymptotically stable for any compatible dynamic  $F$ . For the present case of a one-dimensional state space, the stability condition simply is that  $F$  has the right sign in a neighborhood of the fixed point  $s$ , viz,  $\text{sgn}(s - r) = \text{sgn} F(r; z)$  for all  $r \in [0, 1]$  sufficiently close to  $s$ . Since the EE are a subset of the FP, they are a NE refinement.<sup>10</sup> Note that by definition, each evolutionary path starting at a point  $r$  near an EE  $s$  will converge to  $s$ . The basin of attraction of an EE  $s$  is the set of all such initial conditions, i.e. all points that eventually evolve to  $s$ .

Which long-run equilibrium concept is most relevant? As illustrated in Fig. 5, when there are several NE, typically some of them are unstable. For example, suppose the initial state is the NE  $s_3$ . It is a steady state, but even the tiniest positive (negative) shock will bring us to a state  $r$  where  $\pi_D$  is positive (negative). The deviation from  $s_3$  will then increase (at an exponential rate at first, in view of compatibility condition (2)) and soon the state will be near the neighboring EE  $s = 1$  (or  $= s_2$  for a negative shock). If shocks are small or occasional, the state we observe will usually be in a

<sup>10</sup> Much of the evolutionary games literature emphasizes a NE refinement called ESS, for evolutionarily steady strategy or state. In the present one-dimensional context the definition reduces to the condition  $\text{sgn}(s - r) = \text{sgn} \pi_D(r; z)$  for  $r \in [0, 1]$  sufficiently close to  $s$ . Hence ESS is equivalent here to EE when the state space is one-dimensional. See Friedman (1991) for an argument that for higher dimensional state spaces EE is not equivalent to ESS and is a more useful equilibrium concept.

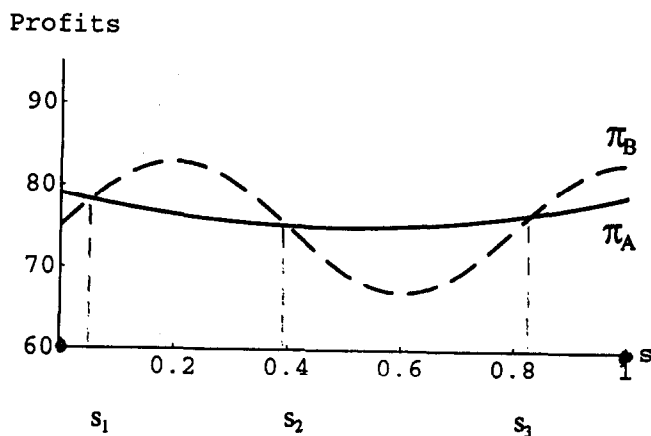


Fig. 5.

neighborhood of an EE ( $s = 0$ ,  $s_2$  and 1 in Fig. 5); the other NE ( $s_1$  and  $s_3$  in Fig. 5) merely separate the basins of attraction for the EE.

#### 4.2. Results

Given the discussion of network and skimming effects in Section 2, we are especially interested in environments  $z$  for which an increase in  $s$  enhances profits less for  $A$  firms than for  $B$  firms. Using the notation  $\pi'_i = d\pi_i(s; z)/ds$  and recalling that  $\pi_D = \pi_B - \pi_A$ , we can write the set of environments of interest as  $Z' = \{z \in Z: \pi'_D > 0 \forall s \in [0, 1]\}$ . Our first proposition shows that the long-run equilibria have a very simple structure in this case: only the “corner solution”  $s = 0$  and  $s = 1$  can be evolutionary equilibria (EE) and there is at most one interior Nash equilibrium (NE).

**Proposition 1.** *The parameter set  $Z'$  can be partitioned into three components,  $Z_1$ ,  $Z_2$  and  $Z_3$  such that*

- For all  $z \in Z_1$ , the state  $s = 0$  is the unique NE and is an EE.*
- For all  $z \in Z_2$ , the state  $s = 1$  is the unique NE and is an EE.*
- For all  $z \in Z_3$  there are three NE:  $s = 0$ ,  $s = 1$  and  $s = \hat{s}(z)$ , where  $\hat{s}$  is the unique solution to the equation  $\pi_D(s; z) = 0$ . The NE  $\hat{s}$  is never an EE, but as long as they are distinct from  $\hat{s}$ , the two endpoint NE are EE whose basins of attraction are separated by  $\hat{s}$ .*

The proof is straightforward and can be found in Appendix A along with the proofs of the other propositions.

It is not difficult to show even for parameters  $z$  outside  $Z'$  that an interior



NE  $\hat{s}$  is stable if  $\pi'_D(\hat{s};z) < 0$  and is unstable if  $\pi'_D(\hat{s};z) > 0$ . Hence the presence of network or skimming effects quite generally tends to produce corner solutions. Stable interior equilibria can only occur if  $\pi'_A > \pi'_B$  over some range of states  $s$ , perhaps because glut effects become very strong. In such cases, there are often several interior equilibria and empirical applications therefore seem problematic.

In the analysis to follow, we emphasize  $Z_3$  environments, where neither mode dominates overall. Here (as in Fig. 1) the specific environment  $z$  determines the equal profit state  $\hat{s}$  separating the basins of attraction, and the historically determined initial conditions determine which of the two corner (homogeneous mode) EE is applicable.

Conceptually it is straightforward to extend the basic single-country model to a two-country trade model in which variables pertaining to the second (foreign) country are indicated with asterisks. Autarky is ensured when the environments  $z$  and  $z^*$  preclude profitable exports. For example, in the parametric example of Section 3, the restrictions  $t, t^* > \max\{\alpha_A, \alpha_B, \alpha_A^*, \alpha_B^*\}$  and  $y = y^* = 0$  are sufficient. When there is trade between the two countries, the home-country profits depend on  $s^*$  as well as  $s$  (e.g. because of transnational glut effects) and on  $z^*$  as well as  $z$  (e.g. because of the foreign country's relative size and trade barriers). Hence for the general trade model we have profit functions  $\Pi_i(s, s^*; z, z^*)$ ,  $i = A, B, D$  in the home country, and  $\Pi_i^*(s, s^*; z, z^*) \equiv \Pi_i(s^*, s; z^*, z)$  in the foreign country. The point of the last identity is that any structural differences between the two countries that affect firms' profitability should be fully captured by differences in the environmental variables  $z$  and  $z^*$ , so the same functions  $\Pi_i$  apply to both countries. Again,  $\Pi_D = \Pi_B - \Pi_A$  is the profit difference, which is assumed to be differentiable in  $(s, s^*)$  on the state space  $[0, 1]^2$  and continuous in the environment  $(z, z^*) \in Z \times Z$ .

The definitions of compatible dynamics and equilibria extend directly. Compatibility conditions (1–3) now apply to  $\dot{s}^* = F(s^*, s; z^*, z)$  and  $\Pi_D(s^*, s; z^*, z)$  as well as to  $\dot{s} = F(s, s^*; z, z^*)$  and  $\Pi_D(s, s^*; z, z^*)$ . Likewise, the conditions for NE and steady states apply to  $s^*$  as well as  $s$ . The only real complication is for EE, the locally asymptotically stable steady states. Simple sign checks do not generally suffice for establishing stability in a two- (or higher) dimensional state space.<sup>11</sup>

Analysis of the two-country model is very simple for autarky. In the absence of trade or environmental interaction, home-country profits  $\Pi_i$  are independent of the foreign state and environment, i.e.  $\Pi_i(s, s^*; z, z^*) =$

<sup>11</sup> A relatively simple sufficient condition for a steady state  $(s, s^*)$  to be asymptotically stable under dynamics  $F$  is that the Jacobian matrix  $\begin{pmatrix} F_1^1 & F_1^2 \\ F_2^1 & F_2^2 \end{pmatrix}$  is negative definite (or at least its eigenvalues have negative real parts). The matrix components  $F_i$  and  $F_i^*$  are the  $i$ th partial derivatives of  $F$  evaluated at  $(s, s^*; z, z^*)$  and at  $(s^*, s; z^*, z)$  respectively.

$\pi_i(s; z)$ ,  $i = A, B, D$ . Nash equilibria in two-country autarky simply consist of states  $(s, s^*)$  whose component states  $s$  and  $s^*$  are NE in their own countries. For example, there are always nine NE when  $z$  and  $z^*$  both belong in  $Z_3$ , as in Fig. 2. The four corners then are EE, the four edge NE are saddle points, and the interior NE  $(\hat{s}, \hat{s}^*)$  is a source whose saddle paths separate the basins of attraction for the EE.

What can happen when international trade becomes feasible? Of course, for environments that permit only very small international trading volume, the EE and their basins remain qualitatively the same as in autarky. To see what else can happen as trade becomes more intense, we develop some new notation.

First consider environments  $z, z^*$  that permit trade in outputs but not inputs, e.g. in the parametric model  $y = 0$  but  $t$  not too large. Then we will have transnational output side (glut) effects but no cost side (network and skimming) effects. For such an environment, write  $\pi_i^o(s, s^*) := \Pi_i(s, s^*; z, z^*)$  for home firms' profits, and  $\pi_i^{*o}(s^*, s) := \Pi_i(s^*, s; z^*, z)$ . Our substitutability assumption – that increased output by a foreign firm decreases price (and profit) more for home firms of the same mode than for home firms of the other mode – then implies that  $\partial \pi_D^o / \partial s^* < 0$  and  $\pi_D^o(s, 0; z) > \pi_D(s; z) > \pi_D^o(s, 1; z)$ . Consequently the locus  $[\pi_D^o = 0]$  has a positive slope and intersects the (vertical) autarky locus  $[\pi_D = 0]$  somewhere in the interior of the square. Likewise the locus  $[\pi_D^{*o} = 0]$  has a positive slope and intersects the (horizontal) autarky line  $[\pi_D^* = 0]$  somewhere in the interior.

For environments that permit increasingly intense transnational trade effects (e.g. as  $t$  declines and as  $\gamma$  or the demand or cost symmetry decreases in the parametric model), the  $[\pi_D^o = 0]$  locus becomes less steep and eventually either (i) intersects the right edge of the square rather than the top edge, or (ii) intersects the left edge rather than the bottom edge. Similarly, the  $[\pi_D^{*o} = 0]$  locus eventually either (i\*) intersects the top edge of the square rather than the right edge, or (ii\*) intersects the bottom edge rather than the left edge. To analyze intense trade in outputs, define  $Z^o \subset Z_3 \times Z_3$  as the set of environments such that either (i) and (ii\*) hold (as in Fig. 3(a)) or (i\*) and (ii) hold.<sup>12</sup>

**Proposition 2.** *Let  $z, z^* \in Z_3$ . In autarky, the four corner states  $s = 0, 1$  and  $s^* = 0, 1$  are all evolutionary equilibria whose basins of attraction are separated by the vertical line at  $\hat{s}(z)$  and the horizontal line at  $\hat{s}^*(z^*)$ . Given trade in outputs with  $(z, z^*) \in Z^o$  the states  $(0, 0)$  and  $(1, 1)$  are unstable and*

<sup>12</sup> An anonymous referee points out that sufficient conditions for  $Z^o$  can be given in terms of the derivatives  $\pi_s = \partial \pi_D^o / \partial s$  etc. as follows: (a)  $\pi_s, \pi_{s^*}^* > 0$ ; (b)  $\pi_{s^*}, \pi_s^* < 0$ ; and (c)  $|\pi_{s^*}, \pi_s^*| > |\pi_s, \pi_{s^*}|$ . By manipulating only the cost parameters  $b$  and  $c_B^*$  and leaving the substitutability parameter  $\gamma$  at 0.98, it can be shown that  $Z^o$  is non-empty in the parametric example.

*the remaining EE (1,0) and (0,1) have basins of attraction separated by an unstable saddle path  $S$  that lies between the loci  $[\pi_D^o = 0]$  and  $[\pi_D^{*o} = 0]$ .*

The message here is that when there is a qualitative change from autarky due to trade in outputs only, the change typically will take the form of increased specialization in terms of organizational mode. The case  $(z, z^*) \in Z^o$  is perhaps the most important but is not the only possibility. For general  $z, z^* \in Z'$  one can get basically similar characterizations of the EE and their basins of attraction. The reader is invited to choose some combination of  $Z_1$ ,  $Z_2$  and  $Z_3$  for each country and trace through arguments similar to those in Appendix A for Proposition 2.

Finally, consider trade in inputs as well as outputs. In this case we have transnational network and skimming effects, as well as direct environmental interaction. The extreme case is perfect free trade (PFT), in which there is no environmental separation and no barriers of any sort, not even excess transportation costs for exports. In PFT the two-country model reduces to the one-country case. That is, for  $i = A, B, D$  the profit functions  $\Pi_i(s, s^*; z, z^*) = \Pi_i^*(s, s^*; z, z^*) = \pi_i(\bar{s}; \bar{z})$  where  $\bar{s}$  is a country-size weighted average of  $s$  and  $s^*$ , and  $\bar{z}$  is an appropriate smooth function of  $z$  and  $z^*$ . Our main interest is not in PFT but in approximations to it, which we refer to as almost free trade (AFT).<sup>13</sup>

*Proposition 3. Let  $\bar{z} \in Z_3$ . Under approximate free trade (AFT) in inputs as well as outputs, the only evolutionary equilibria (EE) are the pure corner states (0,0) and (1,1). Their basins of attraction are separated by a saddle path which may contain an unstable interior Nash equilibrium (NE).*

Suppose we begin in the output-only trade regime of Proposition 2 near the point (1,0). If the country specializing in  $A$  mode production is larger, then Proposition 3 suggests that opening free trade in inputs will destabilize the steady-state (1,0). The size asymmetry reinforces the presumption that initial conditions lie in the basin of attraction of the pure EE (0,0). In this case, the transition from the output-only trade regime to an approximate free trade regime will wipe out the smaller country's  $B$  firms. Intuitively, in approximate free trade the large country  $A$  firms dilute the positive externality supporting the small country  $B$  firms so that the  $B$  mode loses its cost advantage.

<sup>13</sup> The formal definition is that AFT is the set of environments in  $Z \times Z$  such that dynamics compatible with  $\Pi_D(s, s^*; z, z^*)$  and  $\Pi_D(s^*, s; z^*, z)$  give a phase portrait which is topologically equivalent to an arbitrarily small perturbation of PFT.

## 5. Discussion

Since the distinction between *A* and *B* modes arose from the contrast between traditional American and recent Japanese organizations, it seems appropriate to summarize our results in terms of US–Japan trade interactions. In our discussion we assume that environmental conditions in autarky permit either mode in either country (i.e.  $z, z^* \in Z_3$  in the notation of Section 4) but the basin of attraction for the *B* mode is larger in Japan than in the US. Then Proposition 1 is consistent with a predominance of *A* firms in the US and *B* firms in Japan when trade interactions are weak.

In recent decades, policy liberalization and lower ad valorem transportation costs have reduced (output) trade barriers between the two countries. Proposition 2 suggests that the effect is to reinforce the specialization. Our result stands in contrast to the common but unformalized belief that trade promotes convergence of technology and culture, presumably including organizational modes.

Our most interesting result concerns the opening of input or factor markets. Traditionally Japan's labor and capital markets have been closed to outsiders and her trade negotiators continue to resist liberalization measures which would allow US and other foreign firms to compete for ownership of Japanese firms and to bid for skilled Japanese workers. Our analysis suggests that the Japanese negotiators may have good reason to resist. Proposition 3 shows that under plausible conditions the *B* mode could become untenable in Japan if something approximating free trade were permitted in input markets. Transnational network and skimming effects could undermine the Japanese *B* firms and they could be supplanted by *A* firms if capital and labor markets were opened fully.

We did not develop a welfare analysis in this paper, but the positive externalities (cost reductions) generated in the *B* mode presumptively make it more efficient in environments for which both modes are viable. Therefore our last result suggests inefficiency; opening Japan's capital and labor markets might lower welfare for Japan and for the world as a whole.<sup>14</sup>

We close with an essential caveat. Our paper presents a highly simplified model in order to make some important theoretical points. For example, our analysis shows that, contrary to the reasoning in Alchian (1950), evolutionary pressures do not necessarily lead to efficient outcomes; the viability of a behavior depends on its susceptibility to corrosive influences as well as on its inherent efficiency. Our simplifications are less useful when it

<sup>14</sup> While this result may seem counterintuitive, it should be remembered that the inefficiency result is in the context of free trade in factors as well as goods and that there are cost externalities. It is well known that even free trade in goods only can produce paradoxical results in the presence of domestic distortions.

comes to making policy recommendations. The model does highlight some recent policy debates, but reliable policy recommendations should be based on less stylized and more empirically grounded models.

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## Appendix A

### 6. Mathematical details

*Proof of Proposition 1.* Recall that  $\pi_D := \pi_B - \pi_A$  is a continuous function of  $z$  and a smooth and strictly increasing function of  $s$  for all  $z \in Z'$ . Let  $Z_1 = \{z \in Z' : \pi_D(s; z) < 0 \forall s \in (0, 1)\}$ ,  $Z_2 = \{z \in Z' : \pi_D(s; z) > 0 \forall s \in (0, 1)\}$ , and  $Z_3 = Z' - (Z_1 \cup Z_2)$ . Parts (a) and (b) of the Proposition follow immediately from the definitions. To establish part (c), pick an arbitrary  $z \in Z_3$ . There is some  $s_1 \in (0, 1)$  such that  $\pi_D(s_1; z) \leq 0$  (otherwise  $z \in Z_2$ ) and some  $s_2 \in (0, 1)$  such that  $\pi_D(s_2; z) \geq 0$  (otherwise  $z \in Z_1$ ). Since  $\pi_D(s; z)$  is strictly increasing in  $s$  for every  $z \in Z'$  we can ensure that  $0 \leq s_1 < s_2 \leq 1$ . By the intermediate value theorem there is a unique solution  $\hat{s} \in [s_1, s_2]$  to the equation  $\pi_D(s; z) = 0$ . Hence  $\hat{s}$  is a NE and the only interior NE. If  $\hat{s} > 0$  then  $\pi_D(0; z) < 0$  and hence 0 is an NE. Indeed, for  $s \in (0, \hat{s})$  we have  $\text{sgn}(0 - s) = -1 = \text{sgn} \pi_D(s; z)$  so 0 is in fact an EE. A similar argument shows that 1 is an EE if  $\hat{s} < 1$ . On the other hand, we have  $\text{sgn}(\hat{s} - s) = -\text{sgn} \pi_D(s; z)$  for all  $s \in [0, 1]$ , so  $\hat{s}$  is definitely not an EE.  $\square$

*Proof of Proposition 2.* The autarky case follows from Proposition 1. For the trade case, fix  $(z, z^*) \in Z^o$  and define  $L$  and  $L^*$  as the equal-profit loci for home and foreign firms:  $L = \{s, s^* \in [0, 1]^2 : \pi_D^o(s, s^*) = 0\}$  and  $L^* = \{s, s^* \in [0, 1]^2 : \pi_D^{*o}(s, s^*) = 0\}$ . Define Region 1 (2) as the set of points in the

square  $[0,1]^2$  strictly northwest (southeast) of both  $L$  and  $L^*$ . The following facts can be verified directly from the definitions.

- (a) The only possible NE are at intersections of the edges and the loci  $L$  and  $L^*$ .
- (b) In Region 1 (Region 2) the signs of  $\pi_D^o$  and  $\pi_{D^*}^o$  are, respectively, negative and positive (positive and negative).
- (c) Region 2 contains the corner  $(0,1)$  and Region 1 contains the other three corners of the square, or else (as in Fig. 3) Region 2 contains the corner  $(1,0)$  and Region 1 contains the other three corners of the square.

Let  $F$  be any dynamic compatible with  $\Pi$  and  $\Pi^*$  and let  $S(t)$  be the evolutionary path under  $F$  beginning at an initial point  $s_o$ . The Poincare–Bendixson theorem implies that the limit set of the path  $S(t)$  must either be a fixed point or a closed orbit (e.g. Hirsch and Smale, 1974); it cannot be a strange attractor. The monotonicity of  $\Pi$  and  $\Pi^*$  for  $z \in Z'$  eliminates closed orbits, so we conclude that  $S(t)$  converges to some limit point  $s_\infty$  as  $t \rightarrow \infty$ . By the regularity part of the compatibility definition, we see that  $s_\infty$  must in fact be a NE.

Case 1:  $s_o$  is in Region 1. Then fact (b) implies that  $S(t)$  remains in a compact subset of Region 1. By facts (a) and (c),  $s_\infty$  must be one of three corners. Suppose  $s_\infty = (1,1)$ . Then a new evolutionary path beginning at an arbitrarily small perturbation of  $s_\infty$  towards  $(0,1)$ , say at  $s_o = (1 - \varepsilon, 1)$  will have  $\dot{s} < 0$  for all  $t \geq 0$ . Hence  $(1,1)$  is unstable (and indeed is not even a NE). The same argument and conclusion hold for  $s_\infty = (0,0)$ . Therefore for arbitrary  $s_o$  in Region 1 we get  $s_\infty = (0,1)$ . Thus this corner is an EE whose basin of attraction includes all of Region 1.

Case 2:  $s_o$  is in Region 2. A precisely analogous argument establishes that the corner  $(1,0)$  is an EE whose basin of attraction includes all of Region 2.

Case 3:  $s_o$  is not in Region 1 or 2. The previous arguments show that  $s_\infty$  is  $(0,1)$  if  $S(t)$  ever enters Region 1 and is  $(1,0)$  if it ever enters Region 2. Suppose then that  $S(t)$  always remains on or between  $L$  and  $L^*$ . Then  $s_\infty$  must be an intersection of  $L$ ,  $L^*$ , and/or the bottom or right edge of the square. It can be verified in any of these subcases that a small perturbation of any point  $s(t)$  to the northeast (southwest) will yield an evolutionary path that enters Region 1 (Region 2) – this is immediate at  $s(t) = s_\infty$  and can be verified for  $t < \infty$  (including  $t = 0$ ) using monotonicity. Hence  $S(t)$  lies along the saddle path separating the basins of the corner EE.  $\square$

*Proof of Proposition 3.* Each point  $(s, s^*)$  in the square belongs to some iso- $\bar{s}$  line. Since  $\bar{z} \in Z_3$ , Proposition 1 tells us that the unique solution  $\hat{s}$  to the equation  $\pi_D(s; \bar{z}) = 0$  is the critical value; compatible dynamics will drive

$\bar{s}(t) \rightarrow 1$  if  $\bar{s}(0) > \hat{s}$  and will drive  $\bar{s}(t) \rightarrow 0$  if  $\bar{s}(0) < \hat{s}$ . It is easily checked that the corner (1,1) is the only point on the  $\bar{s} = 1$  isoquant and that the corner (0,0) is the only point on the  $\bar{s} = 0$  isoquant. It follows that states  $(s, s^*)$  above (below) the  $\hat{s}$ -isoquant evolve toward the EE (1,1) (the EE (0,0)) in PFT.

In AFT, the environment  $(z, z^*)$  is slightly different from the PFT environment  $\bar{z}$  but except for an epsilon-band around the  $\bar{s}$ -isoquant, the basins of attraction for (0,0) and (1,1) remain the same. That isoquant is neutrally stable in PFT, but in AFT it becomes a saddle path, containing at least one saddle point. (Additional fixed points in the epsilon band, if any, come in saddlepoint–sourcepoint pairs.) See Abraham and Marsden (1978, chapter 7) for the standard theorems underlying this characterization of the bifurcation at PFT.  $\square$

Finally, returning to the parametric example of Section 3.2, we have maximization problems of the form

$$\max_{x_A^d, x_A^e} \left[ (\alpha_A - \beta(\hat{X}_A^d + x_A^d + X_A^{*e}) - \gamma(X_B^d + X_B^{*e}))x_A^d + (\alpha_A^* - \beta(X_A^{*d} + \hat{X}_A^e + x_A^e) - \gamma(X_B^{*d} + X_B^e))x_A^e - c_A(x_A^d + x_A^e) - tx_A^e \right] \quad (A.1)$$

Direct but tedious calculation from the resulting Kuhn–Tucker conditions yields explicit solutions. To conserve space, we use the notation  $\theta_B$  for the expression  $\alpha_B - c_B + bs$  and write out the solutions only for the case of two-way trade in the output of both kinds of firms and only for the home country. (The expressions for the foreign country are the same with asterisked and non-asterisked variables interchanged.)

$$\begin{aligned} x_A^d = \{ & \beta[\beta^2((1-s^*)N^* + 1)(sN + s^*N^* + 1) \\ & - \gamma^2(1-s^*)N^*(sN + s^*N^*)]\theta_A^d - \beta^2\gamma s N \theta_B^d + \beta(1 \\ & - s^*)N^*[\gamma^2(sN + s^*N^*) - \beta^2(sN + s^*N^* + 1)]\theta_A^{*e} \\ & - \beta^2\gamma s^* N^* \theta_B^{*e} \} / \Delta \end{aligned} \quad (A.2)$$

$$\begin{aligned} x_B^d = \{ & -\beta^2\gamma(1-s)N\theta_A^d \\ & + \beta[\beta^2(s^*N^* + 1)((1-s)N + (1-s^*)N^* + 1) \\ & - \gamma^2 s^* N^*((1-s)N + (1-s^*)N^*)]\theta_B^d - \beta^2\gamma(1-s^*)N^* \theta_A^{*e} \\ & + \beta s^* N^*[\gamma^2((1-s)N + (1-s^*)N^*) - \beta^2((1-s)N + (1-s^*)N^* \\ & + 1)]\theta_B^{*e} \} / \Delta \end{aligned} \quad (A.3)$$

$$\begin{aligned}
x_A^e = & \{ \beta(1-s^*)N^*[\gamma^2(sN + s^*N^*) - \beta^2(sN + s^*N^* + 1)]\theta_A^{*d} \\
& - \beta^2\gamma s^*N^*\theta_B^{*d} + \beta[\beta^2((1-s^*)N^* + 1)(sN + s^*N^* + 1) \\
& - \gamma^2(1-s^*)N^*(sN + s^*N^*)]\theta_A^e - \beta^2\gamma sN\theta_B^e \} / \Delta
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
x_B^e = & \{ -\beta^2\gamma(1-s^*)N^*\theta_A^{*d} \\
& + \beta s^*N^*(\gamma^2((1-s)N + (1-s^*)N^*) - \beta^2((1-s^*)N^* + (1-s)N \\
& + 1))\theta_B^{*d} - \beta^2\gamma(1-s)N\theta_A^e \\
& + \beta[\beta^2(s^*N^* + 1)((1-s)N + (1-s^*)N^* + 1) \\
& - \gamma^2 s^*N^*((1-s)N + (1-s^*)N^*)]\theta_B^e \} / \Delta
\end{aligned} \tag{A.5}$$

## References

- Abraham R. and J. Marsden, 1978, *Foundations of mechanics*, 2nd edn. (Benjamin/Cummings).
- Alchian, A., 1950, Uncertainty, evolution, and economic theory, *Journal of Political Economy* 58, 211–225.
- Aoki, Masahiko, 1988, *Information, incentives and bargaining in the Japanese economy* (Cambridge University Press, Cambridge, UK).
- Aoki, Masahiko, 1990, Toward an economic model of the Japanese firm, *Journal of Economic Literature* 28:1, 1–27.
- Arthur, W.B., 1989, Competing technologies, increasing returns, and lock-in by historical events, *Economic Journal* 99, 116–131.
- Benabou, R., 1993, Workings of a city: Location, education and production, *Quarterly Journal of Economics* 108:3, 619–652.
- Brander, J. and P. Krugman, 1983, A reciprocal dumping model of international trade, *Journal of International Economics* 15, 313–321.
- Ethier, W.J., 1979, International decreasing costs and world trade, *Journal of International Economics* 9, 1–24.
- Ethier, W.J., 1982, National and international returns to scale in the modern theory of international trade, *American Economic Review* 72, 389–405.
- Friedman, D., 1991, Evolutionary games in economics, *Econometrica* 59:3, 637–666.
- Fung, K.C., 1991, Characteristics of Japanese industrial groups and their potential impact on US–Japanese trade, *Empirical Studies of Commercial Policy*, NBER Conference Volume, University of Chicago Press, 137–294.
- Hashimoto, M., 1990, Employment and wage systems in Japan and their implications for productivity, in: Alan Blinder, ed., *Paying for Productivity* (Brookings Institute) 245–294.
- Helpman, E., 1984, Increasing returns, imperfect markets and trade theory, in: R. Jones and P. Kenen, eds., *Handbook of international economics*, Vol. 1, 325–365.
- Hirsch, M. and S. Smale, 1974, *Differential equations, dynamical systems, and linear algebra* (Academic Press, New York).
- Markusen, J., 1983, Factor movements and commodity trade as complements, *Journal of International Economics* 14, 341–356.



- Milgrom, P. and J. Roberts, 1990, The economics of modern manufacturing: Technology, strategy and organization, *American Economic Review* 80:3, 511–528.
- Weibull, J., 1994, An introduction to evolutionary games, Draft Textbook, Economics Department, Stockholm University.