**Problem Set #5**

As usual, you are encouraged to discuss all problems with other members of your group, and other class members. Please turn in **your own individual writeup** of problems in Parts I and II. For Part III, please turn in only **one copy for the entire group**, with all members’ names written down.

Due in class Tuesday November 12.

**Part I. Problems.**

1. [Replicator dynamics spreadsheet exercise to be presented in class by guest lecturer.]
2. Player #1, the Serf, can either plant [P] his seed corn or hide it [H]. Player #2, the Duke, can either take the entire harvest [t] or share it evenly [s]. Payoffs of the stage game are:

|  |  |  |
| --- | --- | --- |
|  | t | s |
| P | 0, 3 | 2,2 |
| H | 1, 0 | 1, 0 |

1. Find a pure NE of the stage game, and compute its efficiency (total payoff as a fraction of the maximum total payoff over all strategy profiles).
2. Suppose the game is repeated indefinitely. What factors determine the discount factor d?
3. For which values of d can trigger strategies in the repeated game sustain an efficient stage game outcome?

3. Several companies are choosing the location of their storefront on a road (a bounded, finite marketplace, like the line below). There is a uniform distribution of customers along this street and each customer will buy at whichever storefront is closest to her. Each company selects a single location. All prices are identical, the inside of each store is identical, marginal costs are less than the common price by an amount m>0, so payoff is just ui = m\*Ti, where Ti <1 is size of the territory for which company i’s location xi is closer than the rivals’ locations.

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1. If there are two firms, find the best response function for each firm. [Hint: calculus is less helpful when the payoff function is discontinuous. Diagrams are usually helpful.] Also find a pure-strategy Nash equilibrium.
2. Find BR functions when there are 3 companies. Find a pure or [Extra credit] mixed NE, or show that none exists.
3. Find a pure strategy NE when there are 4 companies.

4. Question for HW

You must explain your game in one paragraph, and explain all the steps in the following set of steps (the logic of what you are doing).

Take your group project game. If you have a two-population game

First consider reducing the problem to a one-population game

of either two or three strategies. (If you have an extensive form game you must first turn it into a normal or strategic form game).

Next symmetrize the payoffs such that you have a symmetric (if your game had asymmetric payoffs).

one population, two or three strategy game.

Next use the existing spreadsheets (do not create your own)

[bio.research.ucsc.edu/~barrylab/classes/game\_theory/HawkDove.xls](http://bio.research.ucsc.edu/~barrylab/classes/game_theory/HawkDove.xls)

OR

[bio.research.ucsc.edu/~barrylab/classes/game\_theory/RPS\_game.xls](http://bio.research.ucsc.edu/~barrylab/classes/game_theory/RPS_game.xls)

and put in your payoffs into the 2 or 3 strategy game. Yes numbers into the payoff matrix is all you have to do, but you also have to justify the numbers and why a given strategy gets the payoff that you assign.

ALL numbers must be W\_i,j >= 0!!! (no negatives, reformulate so the problem has no negatives (take the exponent of your payoffs if you have negatives).

What happens during the evolution of your game?

Play with the starting frequencies, vary the payoffs, explore the evolution.

Is one of your strategies an ESS?

**Part II. Problems from Harrington.**

Look at all chapter-end exercises of Ch 13-14 of your textbook, and write out your solutions to the following.

Chapter 13: #6, 7.

Chapter 14: #3.

**Part III. Team Games.**

Turn in a progress report on your team project, in advance of the rough draft due next week. Mention the current state of all sections, and any serious obstacles you have encountered.