ANSWER KEY HOMEWORK 2

1.

Game 1:

PLAYERS: 1,2. Let choices for both players be denoted “L” or “R” for left or right

P1 Strategy set: {L, R}

P2 Strategy set: {LL, LR, RL, RR}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | P2 |  |  |  |
|  |  | LL | LR | RL | RR |
| P1 | L | 1,-2 | 1,-2 | -2,10 | -2,10 |
|  | R | -3,5 | 2,-2 | -3,5 | 2,-2 |

Game 2:

P1 Strategy set: {L,R}

P2 Strategy set: {L,R}

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | P2 |  |
|  |  | L | R |
| P1 | L | 1,-2 | -2,10 |
|  | R | -3,5 | 2,-2 |

2. No IDSDS solutions for either. One pure NE where P1 plays R and P2 plays RL for game 1 and no pure NE for game 2.

3. Motivation game:

Players can choose xi = 0 or xi = 1 and receive payoff

ui = xj -5xi

There are *n* players in the game that all face the same decision. Define *m* as the number of *other* people (i.e. excluding player *i*) playing the game that have chosen xi = 1 (so *m* must be an integer between 0 and *n*-1). Then player *i* gets payouts as follows:

**xi = 1:** ui = (*m*+1)-5 = *m*-4

**xi = 0:** ui = *m*

It is clear that for all values of *m*, ui(0) > ui(1), so each player has a dominant strategy of picking 0 i.e. there is a IDSDS solution of everybody playing 0). If we define x-i to be the set of strategies of every other player in the game, we can write the BR of player *i* as

BRi(x-i) = 0

4. Strategy sets for both games:

P1: {RR, RD, DR, DD}

P2: {rr, rd, dr, dd}

QUAD A

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | P2 |  |  |
|  |  | rr | rd | dr | dd |
|  | RR | 3,3 | 4,2 | 2,0 | 2,0 |
| P1 | RD | 1,3 | 1,3 | 2,0 | 2,0 |
|  | DR | 0,1 | 0,1 | 0,1 | 0,1 |
|  | DD | 0,1 | 0,1 | 0,1 | 0,1 |

QUAD B

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | P2 |  |  |
|  |  | rr | rd | dr | dd |
|  | RR | 3,3 | 2,4 | 0,2 | 0,2 |
| P1 | RD | 3,1 | 3,1 | 0,2 | 0,2 |
|  | DR | 1,0 | 1,0 | 1,0 | 1,0 |
|  | DD | 1,0 | 1,0 | 1,0 | 1,0 |

Best responses:

QUAD A

BR1(rr) = RR BR1(rd) = RR BR1(dr) = {RR,RD} BR1(dd) = {RR,RD}

BR2(RR) = rr BR2(RD) = {rr,rd} BR2(DR) = {rr,rd,dr,dd} BR2(DD) = {rr,rd,dr,dd}

QUAD B

BR1(rr) = {RR,RD} BR1(rd) = RD BR1(dr) = {DR,DD} BR1(dd) = {DR,DD}

BR2(RR) = rd BR2(RD) = {dr,dd} BR2(DR) = {rr,rd,dr,dd} BR2(DD) = {rr,rd,dr,dd}

QUAD A IDSDS:

For P1: RR,RD strictly dominate DR,DD

With those eliminated, rr,rd strictly dominate dr,dd for P2

QUAD B: No strictly dominated strategies

QUAD A NE: Can find from best response correspondances. RR is 1’s BR to rr and rr is 2’s BR to RR, so (RR,rr) is a NE.

QUAD B NE: {DR,DD} are both best responses to {dr,dd} and likewise the other way around. There are 4 NE at (DR,dr),(DD,dr),(DR,dd),(DD,dd)

5. For *i*

ui = 10\***1**[xi = Z] + 3nxi

For *i*

ui = 7\***1**[xi = V] + 3nxi

Consider the case where each player owns the system that is idiosyncratically most appealing to them, i.e. the equilibrium (Z,Z,Z,Z,Z,V,V,V,V,V)

The payout to the first 5 players is

10 + 3(5) = 25

The payout they get from switching to the other system is

3(6) = 18

So they can not benefit by changing strategies.

The payout to the last 5 players is

7+3(5) = 22

The payout they get from switching to the other system is

3(6) = 18

So they can also not benefit by switching. Since nobody has incentive to deviate from their assigned strategies, (Z,Z,Z,Z,Z,V,V,V,V,V) is a NE. It also follows from the above that neither (Z,Z,Z,Z,V,V,V,V,V,V) nor (Z,Z,Z,Z,Z,Z,V,V,V,V) can be a NE (nor any permutations of those that leave constant the number of each type of player there is).

What about the corners, (Z,Z,Z,Z,Z,Z,Z,Z,Z,Z) and (V,V,V,V,V,V,V,V,V,V)?

Payout to players 1-5 for the first:

10+10(3) = 40

Payout to those players for switching:

0+1(3) = 3

They do not benefit by switching.

Payout to players 6-10:

0+10(3) = 30

Payout to them for switching:

7+1(3) = 10

So they also do not benefit. Therefore, all Z is a NE. A parallel argument holds for all V. All that is left is ruling out other configurations. Consider the situation where everybody plays Z except two people from 6-10. Those two can benefit from switching to Z, so this can not be a NE. Etc.

6.

Harrington problems

3.7:

a) 1. Eliminate a

2. Eliminate x

3. Eliminate c

b) All strategies that can be eliminated by IDSDS can be eliminated by IDWDS, so start from the end point of a).

4. Eliminate y

5. Eliminate b

6. Eliminate w

Down to only one pair of strategies that survives: (d,z)

3.10

Strategic form for this game

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | P2 |  |  |  |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 |
|  | 0 | 10,8 | 10,7 | 8,8 | 8,7 | 8,6 | 8,5 |
|  | 1 | 9,8 | 9,7 | 9,6 | 7,7 | 7,6 | 7,5 |
| P1 | 2 | 8,8 | 8,7 | 8,6 | 8,5 | 6,6 | 6,5 |
|  | 3 | 7,8 | 7,7 | 7,6 | 7,5 | 7,4 | 5,5 |
|  | 4 | 6,8 | 6,7 | 6,6 | 6,5 | 6,4 | 6,3 |
|  | 5 | 5,8 | 5,7 | 5,6 | 5,5 | 5,4 | 5,3 |

For both players, 0 effort strictly dominates 3,4,5. For player 2, 0 also strictly dominates 1.

Reduced game:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | P2 |
|  |  | 0 | 2 |
|  | 0 | 10,8 | 8,8 |
| P1 | 1 | 9,8 | 9,6 |
|  | 2 | 8,8 | 8,6 |

For player 1 in this reduced game, 1 strictly dominates 2, and that is as far as we can take it.

For part b, it is clear that for both players, 0 weakly dominates 2 in the reduced game, leading to the single surviving pair of strategies being both players exerting zero effort.

4.2

3 NE: Both players drive left, both players drive right, and both players zigzag

4.5

a)

1. Eliminate a

2. Eliminate x

3. Eliminate b

4. Eliminate y

5. Eliminate c

b) Only surviving pair of strategies is (d,z) which is the only NE