ANSWER KEY HOMEWORK 3

1.a.

At , which means objective maximized at by setting derivative equal to zero

In general, BR function: Set derivative equal to zero to find points where A’s choices are optimal

So A is maximizing at any point (given B’s quantity) for which the above is true. The symmetry of this game implies the the best response of B to A is the same but with the subscripts flipped.

b. A Nash Equilibrium is any combination of ( such that both of the above equations are satisfied so that both players are mutually best responding. (8,8) the unique NE.

c. This makes our 3 BR functions look like:

And likewise for B and C. A NE of this is any set of quantities that satisfies all 3 BR functions. An example is (6,6,6) and it is unique.

d. This makes C’s BR function look like:

Again, a NE is any combination of (which satisfies all three BR functions. This occurs at (6.5,6.5,4.5).

2.

|  |  |  |
| --- | --- | --- |
|  | t | b |
| L | 3,3 | 0,2 |
| R | 2,0 | 1,1 |

Pure NE: (L,t), (R,b)

Payoff dominant: (L,t) because both players receive a bigger payoff relative to (R,b)

Risk dominant: (R,b) because on the off chance that the other player does something unanticipated (i.e. you hit an off-equilibrium spot), you get 2 instead of 1 where if you were going for the payoff dominant equilibrium and this happens, you get 0 instead of 3. So (L,t) is a riskier equilibrium.

Prediction: open ended.

3. *ui*(*si*, *M*) = 50 + 20*M* −10*si*

where M = max(*s*) where *s* is vector of all player’s strategies.

7 M<7

BRi(M) =

1 M=7

Consider situation where si and M both less than 7. Then *i* can instead play 7 and receive a higher payout. So there can be no equilibrium for which this is true.

Consider a situation where M is 7 and si is greater than 1. Then *i* can instead play 1 and improve their own payout. So there can again be no such equilibrium.

Consider a situation where siis 7 and M is greater than 1. Then ere exists at least one other player that is playing something greater than one and that player can change their move to 1 and improve their own payout, so there can be no equilibrium with *i* playing 7 and everybody else playing anything other than 1.

Consider a situation where *i* is 1 and M is 7 and at least two of the other players are playing more than 1. Then any of the other people not playing 7 can switch to 1 and improve their own payout.

These narrow our equilibria down to *n* different configurations, each one involving one single player playing 7 and everybody else playing 1.

b. The above reasoning still holds. Everybody except one person plays the minimum (0) and one person plays the maximum (7).

c. The structure of the game is still intact; any NE must involve all but one person playing the minimum and one person taking one for the team and putting forth effort. Let that one person be person *i*.

The payoff function for *i* is

*ui*(*si*) = 50 + 20*M* −2*si*2

Take derivatives as usual:

20Mi - 4si = 0

where Mi is the partial derivative of M with respect to si. Note that this is:

1 si = M

Mi =

0 si < M

So, since we have already defined *i* to be the strongest link si=M and this derivative equals 1, so player *i*’s payoff is maximized at 20 - 4si = 0, or si=5.

There are again *n* NE, one for each configuration that has one person playing 5 and everybody else playing the minimum.

4.

|  |  |  |
| --- | --- | --- |
|  | Attack A | Attack B |
| Put file in A | -30, 10 | 0, -10 |
| Put file in B | 0,-20 | -30, 20 |

**BR**1(Attack A) = Put file in B **BR**2(Put file in A) = Attack B

**BR**1(Attack B) = Put file in A **BR**2(Put file in B) = Attack A

No pure strategy NE: none of those are mutually best responses.

How would people react? Defender knows that attacker gets a bigger bounty from B, so maybe he thinks attacker will choose B with higher probability, so he should hide file in A. But attacker knows that defender knows this, so he will think that the defender will hide it in A with higher probability, so he should attack A. But… etc.

Mixed strategy

Defender plays A with probably *p* and B with probability *1-p* and attacker likewise plays A with probability *q* and B with probability *1-q*. Both players must select probabilities that makes the other player indifferent between all their options:

Defender choosing *p*:

10*p* + (-20)(*1-p*) = -10*p* + 20(*1-p*)

*p* = ½

Similarly:

-30*q* + 0(*1-q*) = 0*q* + (-30)(*1-q*)

q=½

Book Problems

5.5)

Each diner can choose among one of three options. Define the choice of meal of player *i* to be mi {P,S,F} for pasta,salmon,filet respectively. They are trying to maximize surplus which is defined as value of a meal minus cost incurred to eat that meal:

Si(mi,m-i) = Value(mi) - Cost(mi,m-i)

where m-i is the vector of meal choices of all players other than *i*.

A) for n=2, the surplus for player 1 is

S1(m1,m2) = Value(m1) - ½(Cost(m1) - Cost(m2))

By choosing an arbitrary meal for player 2 and plugging in each of the options available to player 1, you can see that salmon is a dominant strategy. Since there are two players, you can create a 3x3 payoff matrix to verify that this is the case. And since both players are identical, there is a unique NE at (S,S).

B) What if there are 4 diners?

The trick to this problem is that the marginal values and costs to a player are **only** affected by their own choice:

S1(m1,m2) = Value(m1) - ¼(Cost(m1) - Cost(m2) - Cost(m3) - Cost(m4))

Since the costs are separable in this, we can rewrite this function as

S1(m1,m2) = [Value(m1) - ¼Cost(m1)] -¼[Cost(m2) - Cost(m3) - Cost(m4)]

This part is the *only* This part is *constant*

part that changes for all choices *i*

when *i* changes can make

So we only have to worry about the first part. Plugging in each of the three options, it can been seen that when a person only has to incur ¼ of the cost of their own meal choice, the optimal meal is Filet Mignon. Since each person faces identical incentives, the unique NE is (F,F,F,F) which is the worst outcome from a social perspective!

10. Protesting

First note that since *v1-c* < 0, nobody protesting is an equilibrium. This is because if nobody else is protesting, the costs of protesting are large enough that person 1 (who, by construction, is the person that gets the most benefit from protesting) are too large and so the first person won’t start.

Is there an equilibrium with protesting? Potentially yes, but not one with everybody protesting because the value to the *last* person is given to be 0 and costs are strictly positive.

We need to find conditions for which the *i*th person will want to protest. Consider the case that there are *m’* people protesting. Person *i* will want to join iff

*vi - c/m’* > 0

Note that for all *j < i* , person *i* wanting to join necessarily means that *j* will also want in by construction (person *j* has a bigger benefit than *i*). There must be a borderline person for which the following is true

*vm’  - c/m’ 0 vm’+1 - c/(m’+1)*

and it is at the value of *m’* that makes this true at which there is a NE involving protests; the first *m’* people protest and the last *n-m’* people do not take part.

6.1

$20 are found and split between two people, Jane and Tim. They can each make a request for some amount of it and if the sum of their requests are $20 or less, they both get what they asked for and if it is greater, neither gets anything. Any leftovers are split evenly if they add up to less than 20 and nether player is allowed to ask for more than 20. Denote Jane’s move as *j* and Tim’s *t*.

Can there be a NE were *j+t<20*?

No. If *j+t*<20 then *j* could instead have requested *j’>j* where *j’ =* 20-*t*. This would result in a better payoff than *j*, so *j* can not be a best response to *t* if *j+t*<20.

Can there be a NE where *j+t>20*?

Yes! If *t=*20, Jane gets 0 and Tim gets 20 if *j*=0 and Jane gets 0 and Tim gets 0 if *j>0*. But if *j<20*, Tim’s best response is 20-*j*. But if Jane requests 20, Tim gets zero no matter what. So if both of them request 20, both of them get zero and both are mutually best responding! So (20,20) is a NE!

But if either *j* or *t* are less than 20, the best response of the other player is always unique at 20 less than their opponent. So there is a continuum of NE at all combinations of *j* and *t* at which *j+t*=20 and one more NE at (20,20).

6.7

V1(x1,x2) = x1 + 10x1x2

V2(x1,x2) = x2 + 20x1x2

where 100 x 0 and 50 x2 0

Best response: Take x1 derivative of V1

= 1 + 10x2

This is strictly greater than zero for all x2 0! So this is strictly increasing in x1 and the objective will be maximized at a maximal value of x1.

Do the same thing for player 2 and you will see that player 2’s value function is also strictly increasing in x2 and so the same will hold. The unique NE is then at (100,50).