ANSWER KEY HOMEWORK 4

1.

a. 2 players, so write utility function

*u1*(*x1*, *x2*) = 0.5*T*0.5 - *x1*

where T = *x1 + x2*

Best Response function: take the *x1* derivative of 1’s utility function and set equal to zero

0 = .25(*x1 + x2*)-.5 - 1

solve for *x1 + x2*

*x1 + x2*  = 1/16

Do the same thing for player 2 and you find

0 = .25(*x1 + x2*)-.5 - 1

*x1 + x2*  = 1/16

b. These equations are the same, so they intersect at *all* points and thus any combination of *x1 + x2*  = 1/16 makes both players best responding to each other. There are then infinite NE at any combination of *x1 + x2*  such that their sum is 1/16

c. dV/dT = ½ T-½  - 1 = 0

T = ¼

Total contribution of ¼ maximizes social welfare

d. New payoff function

*u1*(*x1*, *x2*) = 10(*x1*/(*x1 + x2))* + 0.5*(x1 + x2)*0.5 - *x1*

Same as part a, take derivatives for both players and set equal to zero

Get system of equations:

Player 1: 0 = 10*x2*/(*x1 + x2)2* + .25(*x1 + x2*)-.5 - 1

Player 2: 0 = 10*x1*/(*x1 + x2)2* + .25(*x1 + x2*)-.5 - 1

This is a tough one to do analytically. Mathematica input/output is

Solve[1 == 10 (x/(x + y)^2) + 0.25/Sqrt[x + y] && 1 == 10 (y/(x + y)^2) + 0.25/Sqrt[x + y], {x, y}]

{{x -> 2.79557, y -> 2.79557}}

So both are optimizing at input of approx 2.8 and it’s symmetric, leading to net negative G

2. a. Wardrop equilibrium

Path 1

Start < > End

Path 2

Traffic on path 1 = *x1* and traffic on path 2 = *x2*

Path 1 delay: *d1* = *x15*

Path 2 delay: *d2 =* 1

If any path has a longer delay than another path, nobody would use it! Therefore, Wardrop equilibrium requires that all paths that are used have the same delay.

*d1 = d2*

*x15 =* 1

So in equilibrium

*x1 =* 1

*x2 =* 0

b. Avg delay = *d*

Optimize by taking derivative and setting equal to zero

*d = d1x1 + d2x2*

*d = (x15)x1 +* (1)(1-*x1*)

dd/d*x1* = ⅙ *x15 -*1 = 0

*x1 =* (⅙)⅕  ~ .7

*x2 =* 1 - (⅙)⅕ ~ .3

c. Add a toll to a road to get to the optimal traffic pattern?

In no-toll equilibrium, all take path 1. Optimal pattern requires that some take path 2, so we need to put toll on road 1 (not road 2) to convince some people to switch. Equilibrium is the same; delays (including toll cost) on both roads must be the same.

*d1 + t1 = d2*

*x15 + t1 =* 1

((⅙)⅕)5 + *t1 =* 1

*t1  = ⅚*

3. NE in mixed strategies

Player one mix: play strategy *t* with probability *p* and play strategy *b* with probability *(1-p)*

Player two mix: play strategy *L* with probability *q* and play strategy *R* with probability *(1-q)*

In NE, both players must be mutually best responding. So player 1 has to pick *p* such that player 2 does not have a pure strategy best response. This means that *p* must be chosen such that the expected value to player 2 of playing *L* must be equal to the expected value of playing *R*:

E[*L*] = 3*p* + 0(1-*p*) = 3*p*

E[R] = 0*p* + 1(1-*p*) = 1-*p*

3*p* = 1-*p*

*p =* ¼

Likewise, player 2 must pick *q* such that player 1 is indifferent between all options:

E[*t*] = 3*q* + 0(1-*q*)

E[*b*] = 0*q* + 1(1-*q*)

From the symmetry, it is clear that *q =* ¼

So the unique mixed NE is player 1 plays *t* with probability ¼ and *b* with probability ¾ while player 2 plays *L* with probability ¼ and *R* with probability ¾.

b. Expected payoff for mixed NE is ¾ . This is both risk dominated and payoff dominated by *(b,R)* pure strategy NE, so I wouldn’t expect to see people attempting to mix.

4. Holmes vs Moriarty

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Moriarty |  |
|  |  | c | n |
| Holmes | C | 0,2 | 1,1 |
|  | N | 2,0 | 0,2 |

No pure strategy NE.

Mixed NE: Holmes chooses C with probability p and N with probability 1-p. Moriarty chooses C with probability q and N with probability 1-q. Both must select their strategy to make other person indifferent between all options.

E[c] = E[n]

2p + 0(1-p) = 1p + 2(1-p)

2p = -p + 2

p=⅔

E[C] = E[N]

0q + 1(1-q) = 2q + 0(1-q)

q = ⅓

Most likely outcome?

Pr(C,c) = ⅔ \* ⅓ = 2/9

Pr(C,n) = ⅔ \* ⅔ = 4/9

Pr(N,c) = ⅓ \* ⅓ = 1/9

Pr(N,n) = ⅓ \* ⅔ = 2/9

Holmes exiting at C and Moriarty waiting at n is most likely.

Book problems

4.

Let *p* be probability that mugger chooses gun, hide and 1-p be probability that mugger chooses no gun while *q* is probability that victim chooses resist and 1-q is probability that victim chooses do not resist.

The mugger is indifferent when

3q + 5(1-q) = 2q + 6(1-q)

q = ½

The victim is indifferent when

6(1-p) + 2p = 3(1-p) + 4p

p = ⅗

These can only be a NE if the payout from mixing between the two is better than the other option, which is pure strategy gun, show.

Expected payout for gun, show = x, so:

½(3) + ½(5) >= x

4 >= x

If x is less than 4, the mugger will not want to gun,show because mixing over the first two options gives a better expected payout

6. First step: delete dominated strategies.

1st round:

*c* dominates *a*

*y* dominates *z*

2nd round:

*b* dominates *d*

Now this is a simple 2x2 game, solve it the same way as in previous three problems

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | P2 |  |
|  |  | x | y |
| P1 | b | 5,1 | 2,3 |
|  | c | 3,7 | 4,6 |

1p + 7(1-p) = 3p + 6(1-p)

p = ⅓

5q + 2(1-q) = 3q + 4(1-q)

q = ½