Answer Key Homework 5

2. a. Looking at best responses, unique pure NE is at (H,t).

Maximum total payoff = 2+2 = 4

Total NE payoff = 1 + 0 = 1

Efficiency = ¼ = 25%

b. Things like life expectancy (does the serf expect to live very many more seasons?), does the serf expect to have children to feed in the future (might value future payouts more strongly), etc

c. Efficient outcome: (2,2)

Grim Trigger strategy profile:

Serf: Play P in round 1. In round 2+, if (P,s) has been the outcome of every round, play P. Otherwise, play H.

Duke: Play s in round 1. In round 2+, if (P,s) has been the outcome of every round, play s. Otherwise, play t.

Payouts for Serf for GT:

2 + 2ds + 2ds2 + … = 2/(1-ds)

Best deviation? There is no better outcome for the Serf than planting and having it shared, so there can be no profitable deviation.

Payout for Duke for GT:

2 + 2dd + 2dd2 + … = 2/(1-dd)

Best deviation for Duke? First round, play t and then play t in all other rounds

Payoff for this deviation:

3 + 0dd + 0 dd2 + … = 3

GT can be a SPNE iff

2/(1-dd) >= 3

2 >= 3-3dd

dd > ⅓

3. a. <---------|---------|---------|--------->

0 .25 .5 .75 1

This is known as the Hotelling model. Each player *i* chooses position xi between 0 and 1 and they win all the territory which they are closest to and get payoffs proportional to the size of their territory.

FIrst, the case with two firms and find the best response of player 1:

Player two is playing at x2 which some point between 0 and 1. First, consider the case where x2 > .5. That means that there exists an ε which x2 - ε > .5. If player one were to play at x2-ε, player 1 would capture more than half of the market. But this means that x1 > .5! So there exists an ε’ such that x1 - ε’ > .5 and player 2 can play x2 = x1 - ε’ and capture over half of the market. Thus any xi > .5 implies that the other player has a best response that ensures capturing over half of the market, so no NE can involve xi > 5.

You can apply this reasoning to eliminate any strategy profile that has any player *i* playing xi < .5, as well, leaving a unique NE at x1 = x2 = .5 resulting in each player capturing half of the market.

x2 - ε if x2 > .5

BR1(x2) = .5 if x2 = .5

x2 + ε if x2 < .5

where ε is the minimum amount of distance that can be attained and, if the strategy space is truly continuous, it would be a limit limε->0(x2 - ε). Play as close to player 2 as possible on the side of them that gives more territory. You can switch the 1s and 2s in that BR and player 2’s is identical. So from these BR functions, it is confirmed that players are only mutually best responding at (.5,.5)

b. For 3 players:

First: There can be no equilibrium for which all players 1,2,3 are playing at the same place. If all were playing at the same place, they split the whole map and each get a payoff of ⅓ of the total pie. But at any point that they could share, it must be the case that there exists a deviation that ensures a single player at least ½ of the pie. So all players playing the same spot can not be a NE.

Given they can’t all be playing at the same spot in equilibrium, it **must** hold that at least one person is playing at a higher spot than another. Let the person on the lower end be player 1 and the person at the higher end be player 2. The BR to choices x1 x2 by the other two players is always to see which of the following quantities is largest:

a) x1, b) x2-x1, c)1-x2; and, in case a) to locate one tick below x1, in case b) locate anywhere between x1 and x2, and in case c) locate a tick above x2.

There is no pure NE: suppose to the contrary x1 x2 x3 is one. But if x1>1/3 or x3<2/3 then the x2 player is not BRing [as just explained], while if x1 1/3 and ⅔ x2 0 then either the x1 player or the x2 player (or both) is not BRing because they could locate closer to x2 and increase payoff. Thus in every case someone is not BRing, so it is not a NE after all.

c. It is a NE for two agents to locate at ¼ and the other two to locate at ¾ giving ¼ of the pie to each player. If a member of each pair at were to move towards the edge, it would hurt their own payoff and if they were to move towards the interior it would give the other member of their pair an incentive to follow them to increase their own payoff.

Part 2

13.6) Payoff for both players starting with a period in which both choose miss given both playing tit for tat: both players play miss forever:

4 + 4d + 4d2 + … = 4/(1-d)

Alternatively, a player, for example player 1, could choose kill, and that will yield a payoff of 6 in the current period. According to their strategies, player 1 will choose miss and player 2 will choose kill in the next period. In the period after that, player 1 will choose kill and player 2 will choose miss. They will keep alternating in their actions in all periods. The payoff from choosing kill is then

6 + 0d + 6d2 + 0d3 + 6d4 + … = 6/(1-d2)

So choose miss IF

4/(1-d) 6/(1-d2)

or

½ d

Next, consider a history in which player 1 chose kill in the previous period and player 2 chose miss. Player 1’s prescribed action of miss is preferable to choosing kill if and only if

0 + 6d + 0d2 + 6d3 + … 2 + 2d + 2d2 + …

6d/(1-d2) 2/(1-d)

d ½

Last possible situation is both killed in the previous period: Tit-for-tat best iff

2 + 2d + 2d2 + … 0 + 6d + 0d2 + 6d3 + …

½ d

The only way that all of these conditions can hold is if d = ½ so that is the unique value of discount factor that makes tit-for-tat a SPNE

16.7)

V1(x1,x2) = 5 + x1 - 2x2

V2(x1,x2) = 5 + x2 - 2x1

a) If xi is any real number between 1 and 4 for both players, find NE

There is only one, where both players choose 4. This is because each player’s payoff is strictly increasing in their own play so they equilibrium requires that they both play the maximum value they can.

b) i. This a grim trigger. A player’s one round payout for sticking to the strategy given the other person does as well is

Vi(2,2) = 5 + 2 - 2(2) = 3

So the total GT payout for the infinitely repeated game is

3 + 3d + 3d2 + … = 3/(1-d)

The best deviation is to play 3 in the first round and then 3 forever after that, giving frst round payout

V1(3,2) = 5 + 3 - 2(2) = 4

and for the rest of forever

V1(3,3) = 5 + 3 - 2(3) = 2

so

4 + 2d/1-d

GT is SPNE iff

3/(1-d) 4 + 2d/1-d

d ½

ii) For subgames in which players are supposed to choose subgame perfect equilibrium requires that that is, the static Nash equilibrium. For subgames in which players are supposed to choose y, subgame perfect equilibrium requires

for **all** *x*

or

14.3) Denote expected payoff to A if they share:

VA = a[10s + 8(1-s)] + (1-a)[4s + (-1)(1-s)] + VA

Multiplying everything out and moving both VA terms to the same side, it follows that

VA = (-3as + 9a + 5s - 1)/(1-)

But if the sharing agreement is violated, A instead gets

WA = 10a + (-1)(1-a) + WA

WA = (11a-1)/(1-)

Spike’s equations are mirrors:

VS = (-3as + 9s + 5a - 1)/(1-)

WS = (11s-1)/(1-)

Fed A wants to share with hungry S iff

8 + (-3as + 9a + 5s - 1)/(1-) 10 + (11a-1)/(1-)

Which implies

Fed S wants to share with hungry A in the mirror