Midterm Answer Key 166A

1.a. No dominant strategies for either player

b. A is dominated by B for player 1 and a is dominated by b for player 2.

c. 1st round of IDSDS: Remove A and a

2nd round of IDSDS: C is dominated by B

3rd round of IDSDS: b is dominated by c

Only surviving strategy: (B,c)

d. Since there was only one strategy profile that survives IDSDS, it must be the unique NE.

2. a.

|  |  |  |
| --- | --- | --- |
|  | Call | Wait |
| Call | 0,0 | 1,2 |
| Wait | 1,1 | -1,-1 |

b. Best responses have been underlined. There are 2 pure NE, (Call, Wait) and (Wait, Call)

c. Let player 1 play Call with probability *p* and Wait with probability 1-*p*. P1 makes P2 indifferent by setting P2’s expected values equal to each other:

0*p* + 1(1-*p*) = 2*p* + -1(1-*p*)

1-*p* = 3*p* -1

2 = 4*p*

*p* = ½

Likewise, find P2’s mix that makes P1 indifferent

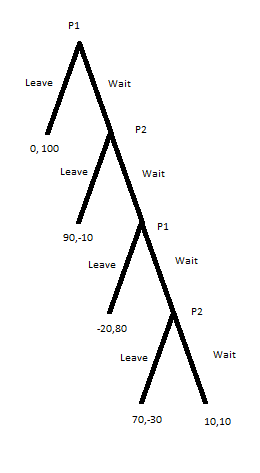
0*q* + 1(1-*q*) = 1*q* + -1(1-*q*)

*q* = ⅔

So there exists a mixed NE in which player one plays Call and Wait each with 50% probability and player two plays Call with a ⅔ probability and Wait with ⅓ probability

d. The most likely outcomes depends on the players; have they played this game before? If so, they likely have settled on a pure strategy outcome (pure strategies are payoff dominant over the mixed equilibrium) and (Call, Wait) is certainly preferred by P2. But if they have never had this happen, it’s conceivable that they have no expectation for what the other player will do and in this case it is arguable that they will mix. [Other sensible answers get credit.]

3. a.



b. Strategy sets: each player has two nodes, so a strategy has two components for each player. Denote “Leave” as L and “Wait” as W

P1 Strategy set: {LL, LW, WL, WW}

P2 Strategy set: {LL, LW, WL, WW}

c.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | WW | WL | LW | LL |
| WW | 10,10 | 70,-30 | 90,-10 | 90,-10 |
| WL | -20,80 | -20,80 | 90,-10 | 90,-10 |
| LW | 0 , 100 | 0 , 100 | 0 , 100 | 0 , 100 |
| LL | 0 , 100 | 0 , 100 | 0 , 100 | 0 , 100 |

d. Two approaches: First is IDSDS. Leaving at first opportunity is dominated by WW for both players, leaving 2x2 matrix with WW,WL remaining for both. In the reduced game, WW dominates WL for player 1 and after eliminating that, WW is dominant for player 2 as well leaving the unique (WW,WW) as the unique surviving profile. Alternatively, underline best responses leaves (WW,WW) as mutual best responses.

e. To find SPNE, use backward induction. At the last node, P2 will always Wait to receive 10 over -30. Given this, P1 will always wait to receive 10 over -20 at the 2nd to last node. Given this, P2 will always wait to receive 10 over -10 at P2’s first move and finally this means player 1’s opening move can be reduced to getting 0 by leaving or 10 by waiting, so P1 will Wait at the first node. Unique SPNE is (WW,WW).

4. a and b.

Find best response function:

ui = hi(1000-H) - 100hi = hi(900-H) = hi(900 - H-i - hi)

du/dh = 900 - H-i - 2hi = 0

solve for hi

hi\* = ½ (900-H-i)

If H-i = 600, person *i*’s best response is hi\*=½ (900-600) = 150

c. In a symmetric NE, all players are mutually best responding and are playing the same h, thus H-i = (n-1)hi = 8hi . Plug this into BR function:

hi = ½ (900-8hi)

10hi = 900

hi = 90

So the symmetric NE is for all 9 players each to fish for 90 hours

d. To find social optimal, note that the sum of utilities can be written

U(H) = H(1000-H) - 100H = H(900-H)

U’(H) = 0 = 900 - 2H\*

H\* = 450

Total social utility is maximized at a quota of H = 450 hours, which implies an average of 50 hrs per player.

e. There is a tragedy of the commons. If everybody has unrestricted access to the lake, everybody will fish for 90 hours each in equilibrium for a total of H = 810, which is more than is socially optimal, and gives a total social payoff of

U[NE] = 810(900-810) = 72900

At the social optimal level of H=450, total social payoff is

U[OPT] = 450(900-450) = 202500

So the efficiency loss due to this tragedy of the commons outcome is

(U[OPT] - U[NE]) / U[OPT] = 1- (72900 / 202500) = 0.64, or 64%.

Thus almost ⅔ of the potential payoff is lost in NE.