**ECON 166A MIDTERM**

**SInervo And Musacchio**

**FALL 2012**

**?? points total**

1) On October 14, 1962, the US confirmed the presence of Soviet nuclear missiles in Cuba. It was the time of the Cold War and the US and USSR were archrivals. The sequence of decisions facing the US and USSR leaders is as follows.

The US moves first and has to decide to Blockade the island of Cuba so as to prevent Soviet ships from reaching the island, or perform an Air Strike.

If the US chose to Blockade, the USSR decides between the following two actions: Withdraw the missiles or Maintain the missiles.

If the US chose to launch an Air Strike, the USSR decides between the following 2 actions: launching a Nuclear Retaliation against the United States or using the incident to achieve a Political Victory by making the world see the US as the more belligerent of the two countries.

The outcome preferences of the US in order of most preferred to least preferred are:

* US chooses Blockade, followed by a USSR Withdraw
* US chooses Air Strike, followed by USSR Political Victory
* US chooses Blockade, followed by USSR choosing to Maintain
* US chooses Air Strike, followed by USSR Nuclear Retaliation

This is because the US cares most about not having a nuclear war, followed by succeeding in getting the missiles out of Cuba.

The outcome preferences of the USSR are, in order of most preferred to least preferred are:

* US chooses Air Strike, followed by USSR Political Victory
* US chooses Blockade, followed by a USSR Withdraw
* US chooses Blockade, followed by USSR choosing to Maintain
* US chooses Air Strike, followed by USSR Nuclear Retaliation

This is because the USSR also does not want a nuclear war. Maintaining the missiles in the face of a blockade would be very difficult, and could lead to Cuba (recently turned communist) switching back to the US sphere of influence. **[30 points]**

1. Draw the EFG for the US (Kennedy) and USSR (Kruschev), and using payoff least preferred = 1 and payoff most preferred =4 (and the intermediate payoffs of 2, 3). Write out the payoffs to the US and USSR on your EFG. **[5 points]**

US

USSR

B

AS

W

M

NS

PV

4,3

2,2

1,1

3,4

1. Solve the EFG by backwards induction (Show all the steps) and come up with the BI solution. BI should tell you what player(s) would do if any information set, including those off the equilibrium path. Be sure to specify what players would do in these info sets as well. **[5 points]**

US

USSR

B

AS

W

M

NS

PV

4,3

2,2

1,1

3,4

*The EFG solution is that the US Blockades and then the USSR Withdraws, If the USSR were to somehow reach the off-equilbirum information set after AS, it would play PV.*

1. What are the strategy spaces of both players in the EFG? **[5 points]**

US: {B, AS} USSR: { W+NS, W+PV, M+NS, M+PV }

1. Now convert the EFG into a normal form game **[5 points]**

USSR

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | W+NS | W+PV | M+NS | M+PV |
| B | 4,3 | 4,3 | 2,2 | 2,2 |
| AS | 1,1 | 3,4 | 1,1 | 3,4 |

US

1. Find all pure strategy Nash Equilibria of the NFG **[5 points]**

USSR

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | W+NS | W+PV | M+NS | M+PV |
| B | 4,3 | 4,3 | 2,2 | 2,2 |
| AS | 1,1 | 3,4 | 1,1 | 3,4 |

US

1. Consider any equilibria that you found in part e that do not correspond to the solution you found in part b. What game theoretic property do these additional equilibria lack that make them less likely to be played? **[5 points**]

*The additional equilibria are (B, W+NS) and (AS,M+PV). Both of these are not subgame perfect. In the first case, in the subgame after an Airstrike, the USSR would not want to do NS. In the second case, the USSR would not want to Maintain in the subgame after Blockade.*

**2)** Given the game matrix that follows **[30 pts total]**:

1. Categorize the game as symmetric or asymmetric, and zero sum or non-zero sum (**2pts)**.
2. Solve the pure NE (**3 pts**). *Make sure you list your steps (tell us what you are doing).*
3. During your solution, if the game reduces in # of strategies, categorize the sub-game (**3 pts**).
4. Label the Weakly Dominated Strategies.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a | b | c | b |
| A | 4, 4 | 0, 0 | 5, 2 | 2, 7 |
| B | 3, 4 | 0, 2 | 3, 3 | 7, 3 |
| C | 7, 2 | 0, 0 | 3, 7 | 3, 3 |
| D | 0, 0 | 2, 2 | 2, 2 | 0, 0 |

**2 cont)** Given the game matrix that follows:

1. Categorize the game as symmetric or asymmetric (**2pts)**.
2. Solve for all pure NE. (**4 pts**). *Make sure you list your steps (tell us what you are doing).*
3. During your solution, if the game reduces in strategies categorize the sub-game as symmetric or asymmetric (**3 pts**).
4. Provide (game-theoretic) justifications for choosing among NE. Define the concept. (**3pts**).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | w | x | y | z |
| W | 3, 3 | 1, 1 | 1, 1 | -1, 3 |
| X | 1, 1 | 1, 1 | 3, 3 | 1, -1 |
| Y | 1, 1 | 3, 3 | 4, 4 | 1, 5 |
| Z | 3, -1 | -1, 1 | 5, 1 | -1, -1 |

**2 cont)** Given the game matrix:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | m | n | o | p |
| M | 90, 90 | 80, 120 | 120, 80 | 120, 70 |
| N | 120, 80 | 90, 90 | 80, 120 | 140, 70 |
| O | 80, 120 | 120, 80 | 90, 90 | 70, 40 |
| P | 70, 120 | 70, 140 | 40, 70 | 70, 70 |

1. Categorize the game as symmetric or asymmetric (**2pts)**.
2. Solve for all pure NE. (**4 pts**). *Make sure you list your steps (tell us what you are doing).*
3. During your solution, if the game reduces in strategies categorize the sub-game as symmetric or asymmetric (**3 pts**).
4. Based on your analysis in j (e.g., of all pure NE), do you expect there will be a mixed NE? Justify your answer. **[1 point]**

3) Three software companies make software for personal computers. Software from all 3 companies resides on a given computer since the companies make the operating system, web browser, and document reader respectively. Each company decides a dollar amount *si* to invest in the security of its product. They can choose any real number (not just integers) between 0 and 1. (The units are billions of dollars.) The overall security of a PC depends on how good the weakest of the three products is since attackers will focus on the most vulnerable of the three. Therefore the payoff to each company *i* is

2 min*(s1,s2,s3) - si .*

Find any and all pure Nash equilibria. Give a clear and concise argument why there are not additional equilibria than the one(s) you identify. **[15 points]**

*Each players’ payoff takes the form:*

*2 min(si, s-i) - si*

*where s-i is the minimum investment from the opponents of i. This function is maximized when player matches the minimum investment s-i of his opponents. Thus BRi = s-i.*

*Everybody investing the same amount (that number can be any arbitrary number from 0 to 1) is a Nash equilibrium since everyone is playing a best response in this profile.*

*No other profile can be a NE. To see that. Consider a player that plays more than the minimum of his opponents. There must be such a player if all the investment levels are not identical. This player can deviate to his best response of playing the minimum of his opponents and improve his payoff. Thus these other profiles cannot be NE.*

4) It is 2020, and industry consolidation has led to having only 2 oil companies in the world – (B)otswana Petroleum, and (E)xxelon. The price per barrel of oil is determined by the total quantity Q produced according to the demand function

*p = (200 – Q)*

where Q is in millions of barrels produced per day, and p is in dollars.

Suppose that it costs each company $80 for each barrel they produce. (We are ignoring other factors like fixed costs, and differences in oil reserves the companies have access to.) Thus the profit of each company *i = b, e* is

*Vi = (200 - Q - 80)qi*

where *Q=qb + qe,* and the units of *Vi*are millions of dollars per day. **[20 points total]**

1. What is the best response function of company *i*? Express your answer in terms of *q-i* , the quantity the opponent of firm *i* produces. **[8pts]**

*i’s payoff with respect to qi is “hill shaped,” so to find the best response, we take the derivative with respect to qi and find when it is 0.*



1. Find a symmetric NE. **[4pts]**

*In any symmetric equilibrium, all companies produce the same amount so q-i must equal qi. Thus*

**

1. What is the profit of each company in the equilibrium you find in part b? (Leave your answer in millions of dollars per day as that’s easiest.) **[4pts]**

*Just plug the quantity found in part b into the payoff function.*

*(200 – 80 – 40 – 40) X 40 = 1600 million dollars per day.*

1. The CEOs of the two companies propose to merge into 1 mega-company because they claim it will be better for the environment. What is the most profitable output level of the mega-company? What will be its profits at this production level? **[4pts]**

*Now the payoff function for the mega company looks like*

*V=(120-Q)Q*

*To optimize, we find the derivative w.r.t. Q and set it to 0.*

*dV/dQ = 120 – 2Q*

*Q\* = 60*

*Plug this into the payoff function to get the profit*

*(120-60)\*60 = 3600 millions of dollars per day*