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An Empirical Analysis of Price Formation in Double Auction Markets

This paper uses data from 30 previously conducted laboratory markets to investigate the price-formation process in continuous double auctions. Our analysis makes use of all market information—bids, asks, and transactions—and is based primarily on the theoretical models of Wilson[18] and Friedman[7] and the zero-intelligence algorithm from Gode and Sunder.[9] The models make essentially the same predictions regarding efficiency and transactions order, and the data generally support these predictions, though not always strongly. The models differ in their predictions regarding bid-ask sequences and regarding serial correlation in price changes. Our analysis finds greater support for the bid-ask sequences predicted by the Friedman model and the negative autocorrelation in price changes predicted by the Gode and Sunder algorithm.

INTRODUCTION

When market conditions change—for any reason, the introduction of new technology, the removal of trade barriers, or even the emergence of new information about pre-existing conditions—any previously established equilibrium is upset. Standard economic analysis presumes that a new equilibrium will be achieved in due course, perhaps more rapidly in some markets than in others, at which point the body of economic theory again becomes relevant. The process of price formation—*how* the market finds its way to a new equilibrium—therefore is literally fundamental to the application of most economic theory.

Price formation has direct practical as well as theoretical importance since many policy interventions, ranging from job retraining programs to the SEC reporting requirements in securities markets, are intended to influence the adjustment to equilibrium. For example, the New York Stock Exchange recently changed its rules to forbid program trading after a 50-point price change, with the announced intention of reducing volatility and increasing market efficiency. Since price formation is not well understood, it is debatable whether the new rule will achieve its worthy goals.

Indeed, economists have remarkably little to say about what factors will speed or impair the price-formation process. The standard theoretical tools of optimization and equilibrium are difficult to deploy in the absence of market-clearing and well-defined budget constraints. Standard empirical techniques are frustrated by the absence of field data on individual preferences, costs, and information; typically equilibrium must be assumed to interpret the data.

Laboratory data are more promising than field data as a source of evidence on price formation. Continuous double auction markets in the laboratory possess strong convergence properties[12,14] and provide very detailed performance data with controlled (induced) preferences, costs, and information. Such markets have been the focus of some recent theoretical work on price formation, surveyed in the next section. However, most previous empirical analyses of laboratory markets have focused on tests of market-equilibrium theories or across-period market-convergence properties, and generally have ignored the within-period price dynamics that are directly relevant in field environments.

We fill this gap by analyzing individual behavior—bids, asks, and transactions—during the price-adjustment process in continuous double auction laboratory markets. Most existing laboratory markets are repetitive stationary. That is, traders have the same endowments and marginal cost and valuation schedules in successive trading periods. Since double auction markets converge very rapidly, most transactions in stationary markets after the first period are at, or near, the equilibrium price. In order to have a significant number of observations of disequilibrium actions, we need nonstationary markets, in which traders must seek a new equilibrium price in each period. As explained in Section 3, we located two series of previously conducted experiments satisfying this requirement.

The data analysis relies primarily on the theoretical models of Wilson[18] and Friedman[7] and uses the zero-intelligence algorithm discussed in Gode and Sunder[9] as a "non-strategic" benchmark. Our initial analysis results suggested the zero-intelligence algorithm because it can explain the persistent negative price change autocorrelation observed in the data. Section 2 provides an informal summary of the models as well as a brief discussion of related literature on price formation. Section 3 describes the data.

Section 4 collects the results. The theoretical models make the same predictions regarding efficiency and transactions order, and the data generally support these predictions, though not always strongly. The models differ in their predictions regarding bid-ask sequences, and our analysis finds greater support for the bid-ask sequences predicted by the Friedman model. The models also differ in their predictions regarding serial correlation in price changes, and the negative autocorrelation in price changes observed in the data provides support for the Gode and Sunder algorithm. The final section interprets the findings and proposes new experiments designed specifically to investigate price formation.

THEORETICAL LITERATURE

Price formation, broadly construed as processes that lead to market equilibrium, has been the subject of innumerable theoretical discussions from the time of Walras and Marshall. The dominant tradition, at least since the second world war, has been to assume that markets are organized by a fictitious non-self-interested agent called an "auctioneer" who raises (lowers) prices on goods in excess demand (supply) until equilibrium is achieved. Only recently have theorists begun to consider price formation in the context of a viable trading institution. We are concerned here with the continuous double auction (DA), the trading institution most widely used in laboratory markets and field markets for homogeneous goods. In the DA, self-interested traders themselves announce buying prices (bids) and selling prices (asks) and transact by accepting other traders' bids or asks at any moment during a trading period.

Except for a little-known precursor, Garcia,[8] Easley and Ledyard[2] offer the first theoretical model designed to explain price formation in simple laboratory double auction markets. Their model postulates a reservation price for each participant which is not linked to induced value/cost parameters. They make plausible but *ad hoc* assumptions as to how participants adjust their reservation prices and offers within and across trading periods. The primary result is that after sufficiently many trading periods in which costs and values are held constant, transaction prices will always lie in a (usually narrow) interval that brackets the competitive equilibrium price. They also offer three testable predictions regarding (a) the range of transaction prices in successive trading periods, (b) the trading sequence, and (c) a lower bound on the number of transactions. The data do not strongly disconfirm any of these predictions, but only (b) is sharp enough to be useful for present purposes. It states that in any given trading period a subset of buyers (sellers) with the largest potential gains from trade will all transact before the remaining buyers (sellers); however, the transaction order within these sets is left unspecified, so the rank-order correlation of buyer valuation (seller cost) and transaction order should be negative (positive) but can be greater than −1.0 (less than 1.0).

MODELS TO BE TESTED

Our main concern in this paper is with price formation within a continuous double auction trading period, and we are aware of only three directly relevant models. In order of decreasing rationality, these models are the waiting game/Dutch auction (WGDA) model of Wilson,[18] the Bayesian game against nature (BGAN) model of Friedman,[7] and the zero-intelligence (ZI) algorithm discussed in Gode and Sunder.[9]

Wilson[18] regards the price-formation process as a sequential equilibrium of an extensive-form game in which the private values of n single-unit buyers and m single-unit sellers are drawn from a commonly known joint distribution. The basic idea is that agents play a waiting game, with each buyer's (seller's) impatience arising from the possible preemption of gains by the other buyers (sellers). Eventually some buyer (or seller) finally makes a "serious" bid (ask)—one which has a positive probability in sequential equilibrium of being accepted. If her offer is not immediately accepted, the bidder (asker) will steadily improve the offer (while other traders remain passive) until it is accepted, as in a Dutch auction.[1] The transactors will be the highest value buyer and the lowest cost seller remaining in the market. The gains from a transaction are split between buyer and seller according to relative hazard rates ("impatience"), so the ratio of remaining buyers to remaining sellers determines the split. With high probability enough transactions will occur to exhaust most potential gains from trade.

Appropriate tests of Wilson's waiting game/Dutch auction (henceforth WGDA) model are not immediately obvious. One might first consider comparing the entire set of event predictions (e.g., the model predicts that trader 3 bids $1.87 at $t = 32.6$ seconds) to the events in an actual experiment. In principle the model is sufficiently precise, but there is a practical difficulty. Solutions to the WGDA model are defined implicitly by a nested set of partial differential equations (PDE's) whose boundary conditions ("continuation values") at each stage are derived recursively from the solutions to the subsequent stage PDE's, with some arbitrariness as to the final stage specification.[2]

No numerical algorithms presently are available to solve the equations, even for simple value distributions and auxillary assumptions. Hence explicit predictions regarding bid, ask, and acceptance behavior are not available at present for the WGDA model.

On the other hand, it would be inappropriate to begin with such specific tests even if the tests were feasible. The potential value of the WGDA model, or any specific model of price formation, should be apparent in its qualitative (or more aggregate quantitative) implications. Fortunately the WGDA model offers several striking general implications that are readily tested:

[1]In an unpublished alternative version, Wilson replaces the Dutch auction by a waiting game by the sellers (buyers). The alternative version seems less able to account for observed bid and ask behavior, but otherwise gives the same predictions as the original version. Hence, we confine our analysis to the original version.

[2]Continuation value is defined as the expected payoff to a player of participating in the remainder of the game if he or she does not transact in the current stage.

1W. Prices and profit distribution: The waiting game aspect implies that in each transaction the gains from trade in excess of continuation values will be split between buyer and seller in the ratio of the number of other remaining sellers to other remaining buyers. In particular, the buyer/seller ratio of realized profits will increase (decrease) in successive transactions when initially more sellers (buyers) are present. Furthermore, to preclude intertemporal arbitrage, the best predictor of future transaction prices is the current transaction price; i.e., prices follow a martingale. This martingale property implies that price changes are serially uncorrelated.

2W. Bid and ask behavior: The Dutch auction aspect typically produces successive improvements on a bid (ask) by a given buyer (seller) culminating in an acceptance by a seller (buyer). On the other hand, the waiting game aspect implies that successive improvements by *different* buyers (sellers) will be rare. These implications apply to "serious" bids and asks, i.e., those with *a priori* positive probability of acceptance.

3W. Transactions partners: Early (later) transactions will be between high-value (low value) bidders and low-cost (high cost) sellers. Assuming risk neutrality and symmetric expectations, the rank-order correlation of buyer valuation and transaction order should be -1.0, and the rank-order correlation of seller cost and transaction order should be 1.0. This is a stronger version of the Easley-Ledyard prediction (b).

4W. Efficiency: Most potential gains from trade will be exhausted; any unrealized profitable transactions will be those offering the smallest gains. Easley and Ledyard make the same prediction, but only for later trading periods under stationary repetition.

The second model draws on the Bertrand perspective of Friedman,[5] which assumes traders carry reservation prices for buying or for selling. A buyer seizes the market bid if she can do so at a price better than her reservation price, and she accepts the market ask whenever it exceeds her reservation price; sellers are analogous. To complete the model, the dependence of reservation prices on time and history must be specified. In Friedman,[7] this is done by means of a drastic simplification: buyers and sellers are assumed to ignore the impact that their own current bids and asks will have on subsequent offers by others. This "game against nature" assumption, together with Bayesian updating and some auxillary assumptions similar to Wilson's (e.g., risk neutrality and proportional time parameterization) gives reservation prices as solutions of the optimal stopping problem associated with current parameter estimates for "Nature's" bid- and ask-generating process. The reservation prices in turn permit numerical calculation of bids, asks, and acceptances for any set of induced value parameters, random or otherwise. More generally, this Bayesian game against nature (BGAN) model agrees with implications (3W) and (4W) of Wilson's WGDA model but differs sharply from (1W) and (2W):

1B. Successive price changes in the BGAN model will be positively correlated.[3] This effect will lessen as traders beliefs become more firmly established, i.e., later in a trading period.

2B. The Bertrand aspect of the BGAN model implies that successive improvements to the market bid (ask) by a given buyer (seller) will be rare, while successive improvements by different buyers (sellers) will be common.

Gode and Sunder,[9] and chapter 7 of this volume, convert an oral tradition regarding brainless trader behavior into a well-specified algorithm called zero-intelligence (ZI) traders. In the ZI model, each bid is uniformly distributed between zero and the buyer's redemption value, and seller's asks are similarly distributed between cost and an *a priori* upper bound on buyers' values. Under the usual "NYSE convention" for the DA, bids (or asks) which do not improve the current market bid (or ask) will not be observed. Buyers bid and sellers ask independently at random times. A transaction occurs at the current market ask (or market bid) whenever a new bid exceeds (or a new ask falls below) the current market ask (or market bid).

These ZI assumptions imply that transaction prices are independent draws from a distribution which changes over time as successful transactors exit from the market. If prices are independent draws from a *fixed* distribution, then successive price changes have a correlation coefficient τ of precisely -0.5.[4]

The changes in the distribution have only a small effect on τ. Hence, we have the following general implications for the ZI model:

1Z. Successive price changes will have correlation of about -0.5.

2Z. Successive improvements to the market bid (ask) by a given buyer (seller) will occur with a probability which depends on the level of the market bid (ask) and all buyers' values (sellers' costs). The probability can be readily estimated by Monte Carlo methods.

3Z. There will be a slightly greater transaction probability for higher valued buyers and lower cost sellers.

[3]The martingale property fails in the BGAN model because traders incorrectly assume that other traders' bids and asks arise as if from "natural" processes whose parameters are unknown but unchanging. To the extent that traders' beliefs change in response to a new bid or ask observation, their reservation prices will shift, and as a result, the actual data-generating process will also shift. It can be shown that such an unanticipated shift in the data-generating processes leads to positively correlated transaction price changes. This effect dies out fairly rapidly as traders accumulate observations and therefore change their beliefs less in response to new observations. A second possible reason for positive serial correlation (probably negligible in practice) is an asymmetry in variance estimates for bid versus ask distributions. We know of no arguments within the BGAN model for *negative* correlation in transaction price changes.

[4]**Proof:** Suppose transaction prices P_t are IID with (finite, positive) variance V. Normalize P_t so $EP_t = 0$. Then $V = E_t(P_{k+t})^2$ and $E_0 P_t P_{t+1} = E_0 P_t E_t P_{t+1} = 0$. The correlation coefficient is $\tau = E_0(P_{t+1} - P_t)(P_t - P_{t-1})/E_0(P_{t+1} - P_t)^2$. The numerator of τ is $E_0(-P_t)^2 = -V$ and the denominator is $E_0(P_{t+1})^2 + E_0(P_t)^2 = 2V$. Therefore $\tau = -0.5$.

Hypotheses:	WGDA	BGAN	ZI
1. Price Change Autocorrelation	Zero	Positive	Negative (near -0.5)
2. Bid/Ask Improvements	By same buyer/seller	By different buyers/sellers	Intermediate (estimated)
3. Transaction Order	In order of valuations	In order of valuations	Weakly in order of valuations
4. Market Efficiency	High	High	High

FIGURE 1 Summary of model implications.

4Z. Most potential gains from trade will be exhausted, but unrealized gains need not be the least profitable. The expected efficiency can be readily calculated from the arrival rate of bids and asks and from the value and cost parameters.

The four implications of each of the three models are summarized by Figure 1. The WGDA model implies serially uncorrelated price changes, while the BGAN and ZI models imply positive and negative serial correlation in price changes, respectively. Each model also has distinct predictions regarding the bid and ask behavior. The WGDA model implies that successive improvements on a bid (ask) will be typically made by a given buyer (seller), and the BGAN model implies that successive improvements on a bid (ask) will be typically made by different buyers (sellers). The ZI model predicts both kinds of bid and ask improvements. All three models predict that high-value buyers and low-cost sellers will transact earlier in the period than low-value buyers and high-cost sellers; however, the WGDA and BGAN models both imply a stronger form of this hypothesis than the ZI model. Finally, all three models predict that most potential gains from trade will be exhausted.[5]

THE DATA

Vast numbers of double auction market experiments have been conducted in many laboratories. Unfortunately for us, most of the experiments involve stationary repetition or fixed (deterministic) shifts: all traders receive the same endowments and induced preferences in most successive trading periods. Thus, the equilibrium price (or equilibrium price interval) also remains the same across trading periods. In this case the experimenter has only a single observation of price formation, often spread across many trading periods. To focus cleanly on the within-period process, we need data from experiments in which equilibrium prices shift unpredictably between trading periods. We found two series of experiments meeting this requirement, both conducted using the PLATO computerized double auction. All experiments employed inexperienced subjects.

In PLATO double auctions both buyers and sellers are free at any moment during the trading period to initiate price quotes (bids to buy and offers to sell) for a single commodity unit by typing in a number and then touching a box shaped area on their display screen. The market (highest) bid and market (lowest) ask are displayed continuously on each buyer and seller's screen. Any seller (buyer) is free to accept the market bid (ask) by first touching a box labelled "ACCEPT" and then touching another box labelled "CONFIRM" within five seconds. After the transaction is confirmed, it is recorded in both the buyer's and seller's private record sheet and the price is publicly displayed. All transactions and certain bids and asks are public information (i.e., appear on all subject's display screens). The double auction rules used for the experiments reported here included both the New York Stock Exchange "improvement rule" and a computerized "specialist book." The improvement rule requires each ask (bid) to be lower (higher) than the current market ask (bid) for it to be announced. An ask (bid) that is higher (lower) than the market ask (bid) is placed in a queue ("specialist book"), ordered with lower asks (higher bids) having priority. After acceptances of the market bid (ask), the highest priority bid (ask) in the queue automatically becomes the new market bid (ask). Additional details of the PLATO trading procedures are provided elsewhere; see, for example, Smith and Williams.[16]

[5] To be more precise, the WGDA model predicts that market inefficiencies should only be realized if some transactions offering the smallest gains do not occur, while the ZI model predicts that most market inefficiencies should occur when extra-marginal units trade.

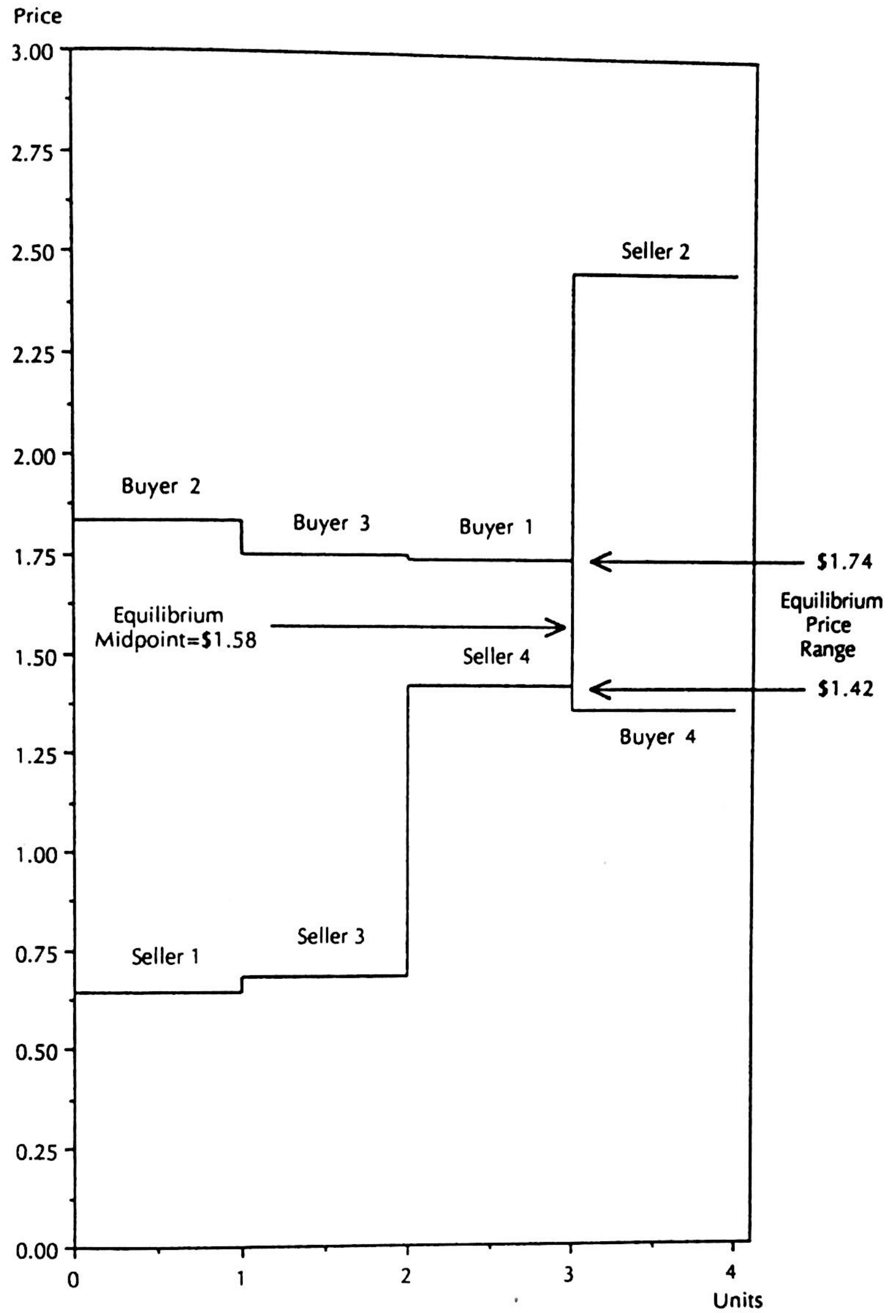

FIGURE 2 Example of experiment series 1 buyer and seller valuations: Period 1.

SERIES 1

The first series of experiments conforms closely to the assumptions of the WGDA model. In each period, each agent is endowed with a single unit whose valuation is random but whose distribution is common knowledge. To date, three such experiments have been conducted, all at the University of Arizona. Eight subjects (four buyers and four sellers) participated in each 15-period experiment, with 120 seconds per period. Sellers' unit costs were drawn independently from a uniform distribution with support [\$0.00, \$2.50], and buyers' redemption values were drawn independently from a uniform distribution with support [\$1.00, \$3.50].[6] Because valuations were drawn randomly, the equilibrium price range shifted each period; the support of the equilibrium distribution is clearly [\$1.00, \$2.50]. The midpoint of this range varied between \$1.21 and \$2.41 for the 15 periods. Figure 2 plots the induced supply and demand arrays for Period 1 of this series. The equilibrium price range is \$1.42 to \$1.74, with midpoint $P_m = \$1.58$.

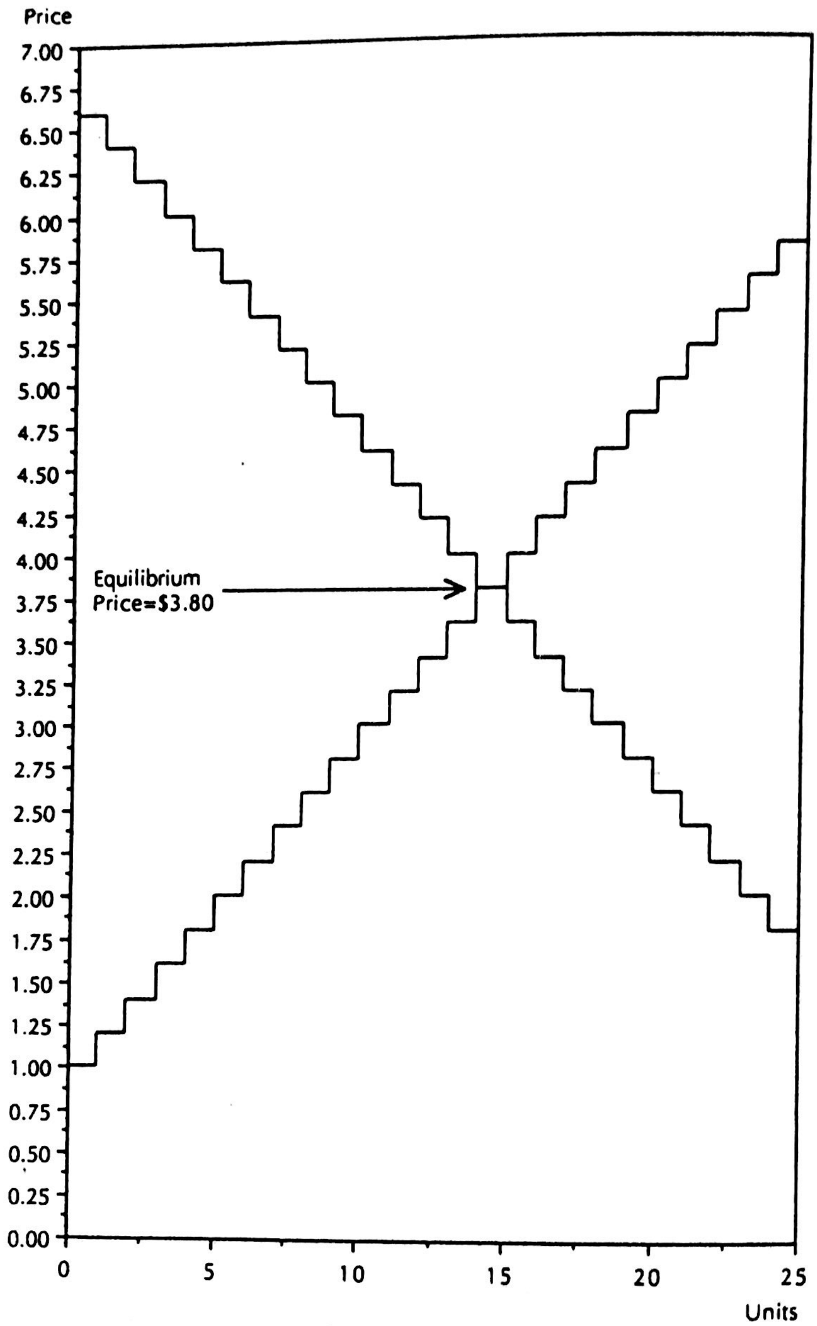

FIGURE 3 Example of experiment series 2 buyer and seller valuations: Period 1.

[6] All three experiments used the same random sequences of costs and values. For the third experiment (3pd206), all costs and values were shifted up by \$1.00.

SERIES 2

The second series of experiments has characteristics more typical of laboratory double auctions, but some characteristics violate assumptions of the WGDA and BGAN models. For example, each agent has a trading capacity of five units per period (although each typically traded only two or three units per period), and no information is provided regarding other traders' unit valuations. The unique feature of these experiments that make them useful for this research is that traders' valuations (and therefore the equilibrium price) shifted every period.

The 27 experiments were conducted by Jim Cox and Ron Oaxaca at the University of Arizona to train subjects for their research evaluating econometric estimators (see Cox and Oaxaca).[1] These trainer experiments were used to expose subjects to the randomly shifting induced supply and demand functions employed in their experimental design, and the results are not reported by Cox and Oaxaca.

Ten subjects (five buyers and five sellers) participated in each six-period experiment, with 360 seconds per period. Sellers' unit costs fit along the following induced inverse supply curve:

$$P_s = -0.2 + 0.2Q + 1.0X_s + U_s\,, \tag{1}$$

and buyers' redemption values fit along the following induced inverse demand curve:

$$P_d = 0.2 - 0.2Q + 1.0X_d + U_d\,, \tag{2}$$

where the P's and Q's are prices and quantities, respectively, X_s and X_d are supply and demand stocks, respectively, and U_s and U_d are IID random terms drawn from a uniform distribution with support $[-0.4, 0.4]$. Because of the random supply and demand shocks, the equilibrium price range shifted each period.[7] The midpoint of this range varied between \$3.20 and \$6.60 for the six periods. Figure 3 plots the induced supply and demand arrays for Period 1 of this series, with a equilibrium midpoint P_m = \$3.80. The "steps" on the supply and demand curves occupied by each trader were also changed each period. Each seller (buyer) had exactly one unit among the five lowest cost (highest-valuation) units on the supply (demand) curve, exactly one unit among the five cost (valuation) units ranking six through ten, and so on; see Figure 3. All 27 experiments used the same set of valuation parameters.

RESULTS

The results are presented in four subsections, each corresponding to the four implications described above in Section 2.

[7]The valuations were not shifted between periods five and six.

TRANSACTION PRICE CHANGES

WGDA HYPOTHESIS (1W): Transaction price changes will not be serially correlated because prices follow a martingale to preclude intertemporal arbitrage.

BGAN HYPOTHESIS (1B): Transaction price changes will be positively correlated.

ZI HYPOTHESIS (1Z): Transaction price changes will have an autocorrelation coefficient of -0.5.[8]

The null hypothesis from the WGDA model (1W) is that transaction prices P_t follow a martingale:[9]

$$E[P_t | P_{t-j}, j > 0] = P_{t-1}. \tag{3}$$

If we reject this hypothesis in favor of positive price change autocorrelation, we have support for the alternative Hypothesis (1B), and if we reject this hypothesis in favor of negative price change autocorrelation, we have support for the alternative Hypothesis (1Z). In particular, we shall test the hypothesis that prices follow a second-order martingale:

$$P_t = P_{t-1} + u_t, \text{ where } E[u_t] = 0 \text{ and Cov } (u_t, u_{t-s}) = 0 \text{ for all } s \neq 0. \tag{4}$$

Rewrite Eq. (4) in terms of price changes as follows:

$$u_t(T) = P_t - P_{t-1}, t = 1, 2, \ldots \tag{5}$$

where $u_t(T)$ is the price change over the interval T. For the following results, an interval T is given by one transaction.[10] The null hypothesis given above in Eq. (4) implies that[11]

$$\text{Cov } (u_j(T), u_{j-s}(T)) = 0 \text{ for all } s \neq 0. \tag{6}$$

[8] The model in Roll[13] also predicts that transaction price changes in an efficient market in equilibrium will be negatively correlated. This result occurs because, assuming no new information arrives, prices will oscillate in the bid-ask spread from accepted bid to accepted ask. However, the time series of price changes from each accepted ask to the next accepted ask (and accepted bid to accepted bid) should be serially uncorrelated.

[9] For similar tests using the time series of securities prices, see the classic paper by Fama,[3] or for surveys of related research on stock returns, see Granger and Morgenstern[11] or Fama.[4]

[10] Results are essentially unchanged, and thus not reported here, when examining price changes over nonoverlapping intervals of more than one transaction or fixed time intervals such as one minute.

[11] In practice, prices P_t are often transformed to $\hat{P}_t = \log(P_t)$ because (among other reasons) the series P_t is unbounded from above but is bounded below by zero, while $\log(P_t)$ is symmetrically unbounded. All of the tests reported below were also conducted for $\log(P_t)$ with no significant impact on the results.

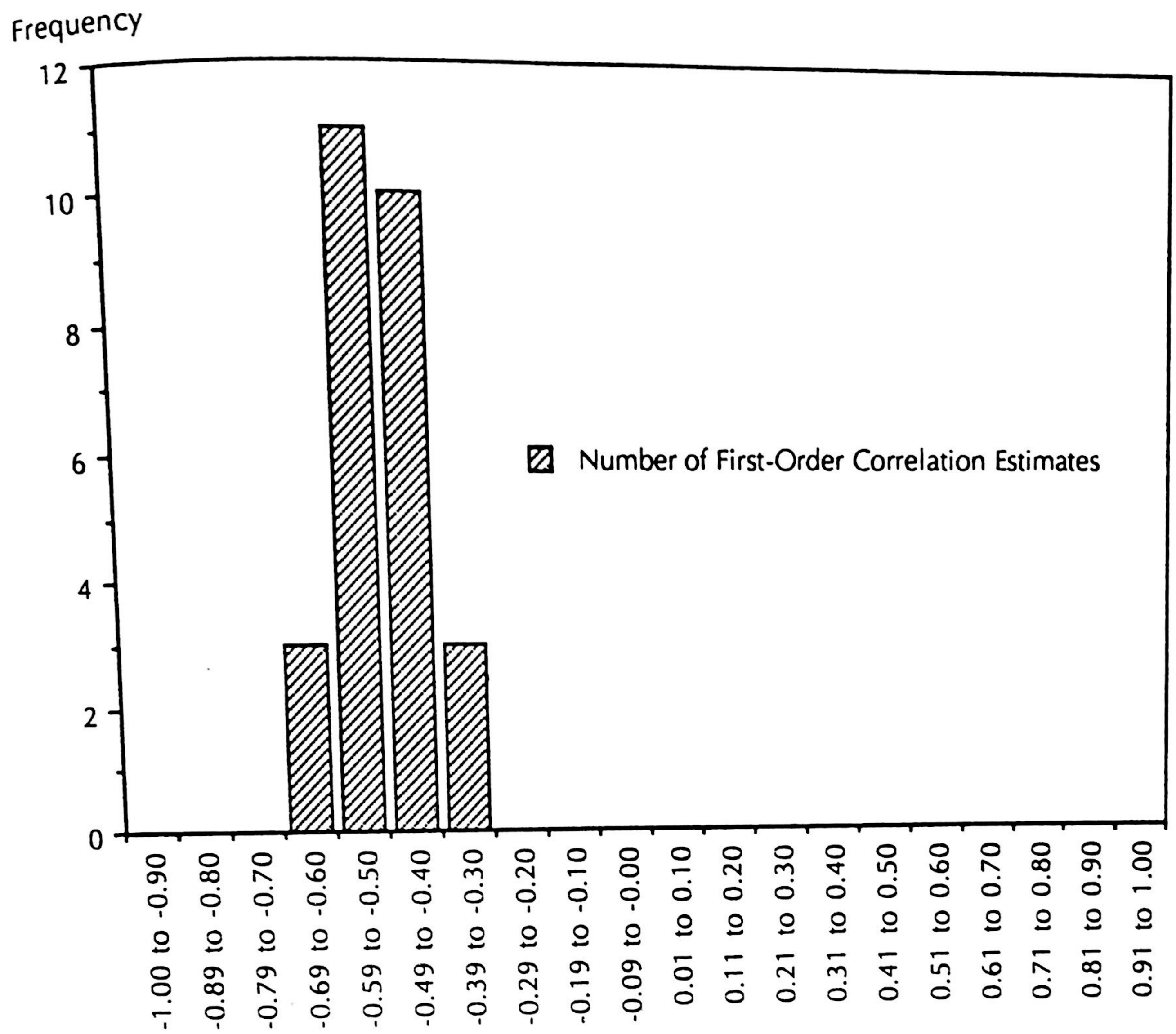

FIGURE 4 Frequency distribution of first-order correlation coefficients for price changes: Experiment series 2.

EXPERIMENT SERIES 1: Note that this hypothesis applies to *within*-period price changes. The models make no predictions for between-period price changes, so for the estimates below we exclude the price change that occurs at the first transaction each period. Because of this restriction, a period with n transactions contributes no more than $n-2$ observations for the correlation estimates. The Series 1 experiments typically had only two or three transactions per period, so they contributed only about one observation per period for this test. The estimated first-order correlation coefficients for price changes are 0.43 ($n = 11$) for experiment 3pd204, -0.14 ($n = 16$) for experiment 3pd205 and -0.43 ($n = 13$) for experiment 3pd206. None of

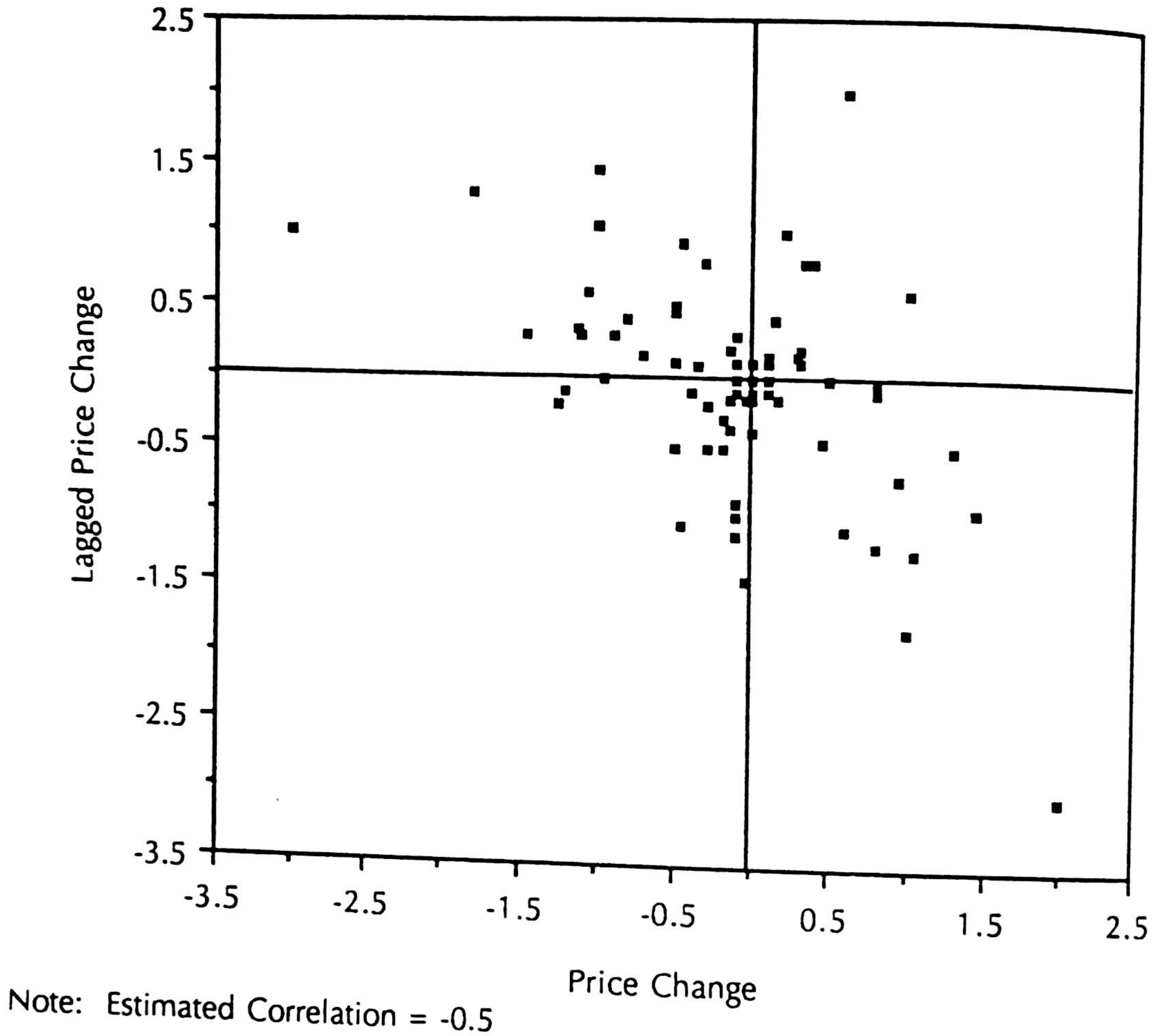

FIGURE 5 Scatter plot of price change against lagged price change: Experiment AZTRO8.

these estimates are significantly different from zero.[12] Because the small number of observations in the Series 1 experiments severely limits the power of the statistical tests, the remainder of this section reports results for the high-volume Series 2 experiments.

[12]This first-order correlation statistic was also calculated based on a simulation of 1000 markets populated with only ZI traders using the Series 1 parameters. Because the transaction prices are drawn from a distribution that changes slowly over time, the mean estimate of this correlation is greater than −0.5 (specifically, −0.34); 90% of the 1000 estimates are within the range −0.68 and 0.09. One of the three Series 1 experiments has an estimated correlation outside of this range.

EXPERIMENT SERIES 2: When estimated separately for each Series 2 experiment, the first-order correlation coefficient of price changes for successive transactions (Eq. (6) with $T = 1$) is always negative, significantly different from zero and near −0.5. Figure 4 presents the distribution of the first-order correlation coefficients for the 27 experiments. The estimated correlations range between −0.30 and −0.69, and nearly all fall within the range −0.4 and −0.6 and are not statistically different from −0.5. Figure 5 illustrates this negative correlation with a scatter plot of each price change against each *lagged* price change for experiment aztr08. This experiment has an estimated correlation coefficient of −0.50, and the correlation coefficient is also −0.50 when pooled across all 27 experiments ($n = 1980$). We estimated the first-order serial correlation by period to determine if it decreased later in the experiments due to learning or some other factor. The estimated coefficients for the pooled experiments are −0.50, −0.51, −0.49, −0.50, −0.44, and −0.55 for periods 1 through 6. All are *extremely* different from zero but not statistically different from −0.5.

As mentioned previously in footnote 8, the model in Roll[13] predicts negative serial correlation in price changes but zero serial correlation in price changes *from accepted ask to accepted ask* and *from accepted bid to accepted bid*. We test this prediction by estimating the first-order serial correlation coefficients for price changes between accepted asks and between accepted bids. Pooling across all Series 2 experiments, the estimated coefficients are *both* −0.44 for ask- and bid-acceptance price changes, and both are significantly different from zero at the one-percent level. According to the Roll model, these price changes should follow a martingale (i.e., have zero serial correlation), so the Roll model does not explain the price change negative autocorrelation result.

This strong negative correlation result for price changes provides support for the zero-intelligence model (Hypothesis (1Z)) and implies that price changes are not unpredictable; if the price just increased by \$0.50, a forecast that the next price will be about \$0.25 below the current price is more accurate than the (martingale) forecast that the next price will be equal to the current price (see Eq. (3)). Traders identifying this pattern can increase their expected trading profit. Because of the existence of these intertemporal arbitrage opportunities, we estimated the first-order serial correlation coefficient in price changes for some other computerized double auction markets to determine if this result is specific to these randomly shifting supply and demand experiments with inexperienced subjects. In particular, we examined the first period of seven experiments from Smith and Williams,[15] all using inexperienced subjects, the first two periods[13] of eight experiments from Williams and Smith,[17] all using experienced subjects, and all twelve periods of one asset market experiment with experienced subjects from Friedman.[6]

With the inexperienced subjects in the 1982 study, the negative and significant serial correlation persists (estimate equal to −0.43, $n = 53$, significantly different from zero at the one-percent level). With experienced subjects in the 1984 study

[13] The first two periods of these experiments provide useful data because the supply and demand shifted between periods.

and no "speculators" that can buy *and* sell units, the estimated coefficient is still negative but is no longer significantly different from zero (estimate equal to -0.26, $n = 32$). When adding speculators that can buy and sell units and carry them between periods, the estimated coefficient increases to -0.24 ($n = 57$), also not significantly different from zero. In the Friedman asset market, the first-order serial correlation of price changes was significantly negative at -0.26 ($n = 172$). This cursory examination of other double auction data suggests that subject inexperience may be one source of the negative serial correlation in price changes, and that adding speculators that can buy and sell units may further decrease the negative correlation.

CONCLUSION. Experiment Series 2 provides strong evidence that price changes are negatively serially correlated, so that both WGDA Hypothesis (1W) and BGAN Hypothesis (1B) can be rejected in favor of ZI Hypothesis (1Z).

BID AND ASK BEHAVIOR

WGDA HYPOTHESIS (2W) Successive improvements on a bid (ask) by a given buyer (seller) culminating in an acceptance by a seller (buyer) will be common, and successive improvements by different buyers (sellers) will be rare. This prediction applies to "serious" bids and asks.

BGAN HYPOTHESIS (2B) Successive improvements on a bid (ask) by a given buyer (seller) will be rare, and successive improvements by different buyers (sellers) will be common.

ZI HYPOTHESIS (2Z) Successive improvements on a bid (ask) by a given buyer (seller) will occur with a probability which depends on the level of the bid (ask) and all buyers' values (sellers' costs).

In order to test these alternative hypotheses, we first specify the criteria for regarding a transaction observation as inconsistent with the BGAN and WGDA models:

DEFINITION 1 ("NOT BGAN") A transaction is regarded as inconsistent with BGAN if any outstanding bid or offer since the previous transaction is updated by the same agent currently holding the bid or offer.

DEFINITION 2 ("NOT WGDA") A transaction is regarded as inconsistent with WGDA if it is an accepted bid (ask) and any bid (ask) revisions since the previous transaction are made by different buyers (sellers).

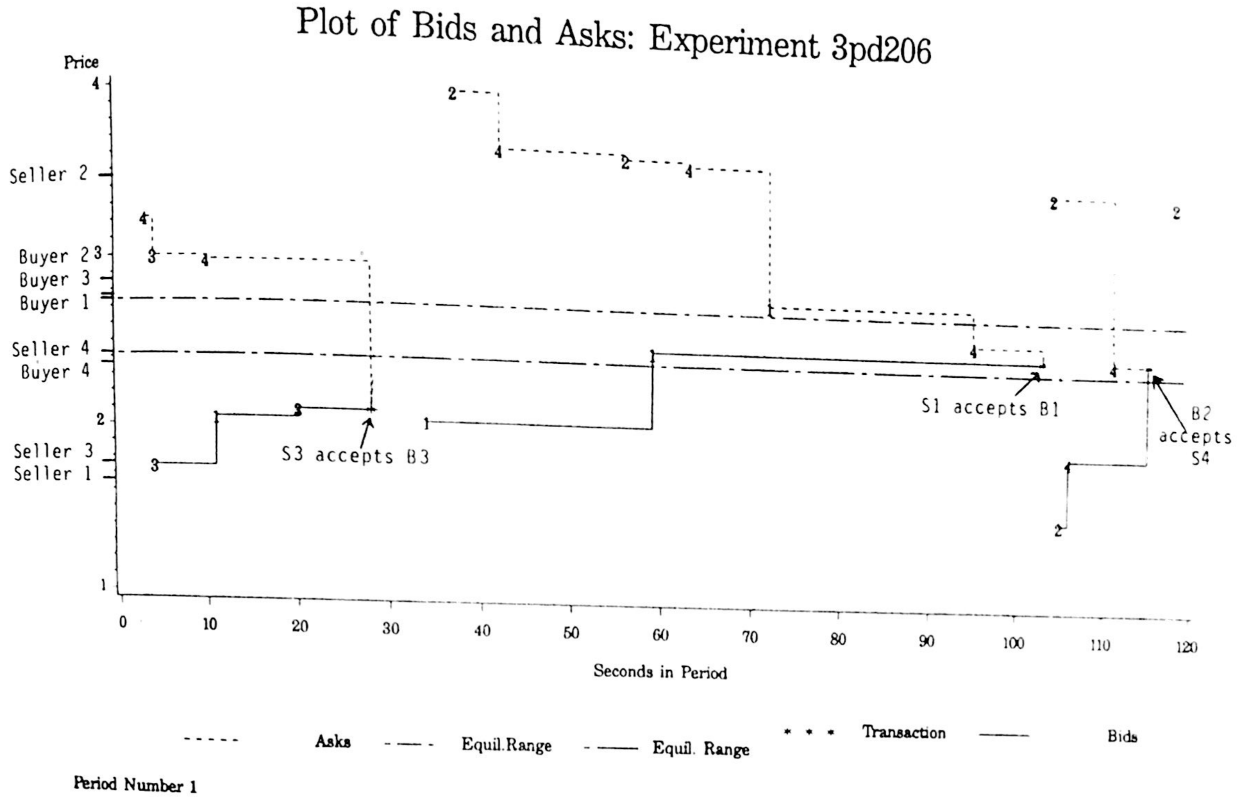

FIGURE 6 Experiment 3pd206 (Series 1), Period 1.

Figures 6 through 9 illustrate these definitions. These figures present the market state for each second of some typical periods. Figures 6, 7, and 8 present data from the Series 1 experiments, and Figure 9 presents data from a Series 2 experiment. The dotted lines represent the market asks leading up to each transaction, and the solid lines represent the market bids. Each action that improves the market bid or ask is flagged with the identification number of the buyer or seller posting the quote. The buyer and seller unit valuations are provided on the vertical axis. The dot-dash lines bracket the range of equilibrium prices.

Consider the second transaction in Figure 6. Since Buyer 1 revises her (standing market) bid at 59 seconds, this transaction is "Not BGAN" according to Definition 1. (In fact, the BGAN model does not rule out this behavior, but does suggest that it is relatively rare.) Since this transaction is an accepted bid and the only active bidder is Buyer 1, it is consistent with WGDA according to Definition 2. (Actually, the formal WGDA model rules out the frequent ask revisions prior to this transaction.) The final transaction of Figure 7 provides a better example of WGDA behavior: Seller 2 gradually improves her ask until Buyer 1 accepts. Next, consider the first transaction in Figure 8. The flurry of 8 bids and 11 asks prior to this transaction is entirely consistent with BGAN (because all revisions are made by different agents) and is "Not WGDA" (because the transaction is an accepted bid and the market bid is raised by different buyers). One observation indicated

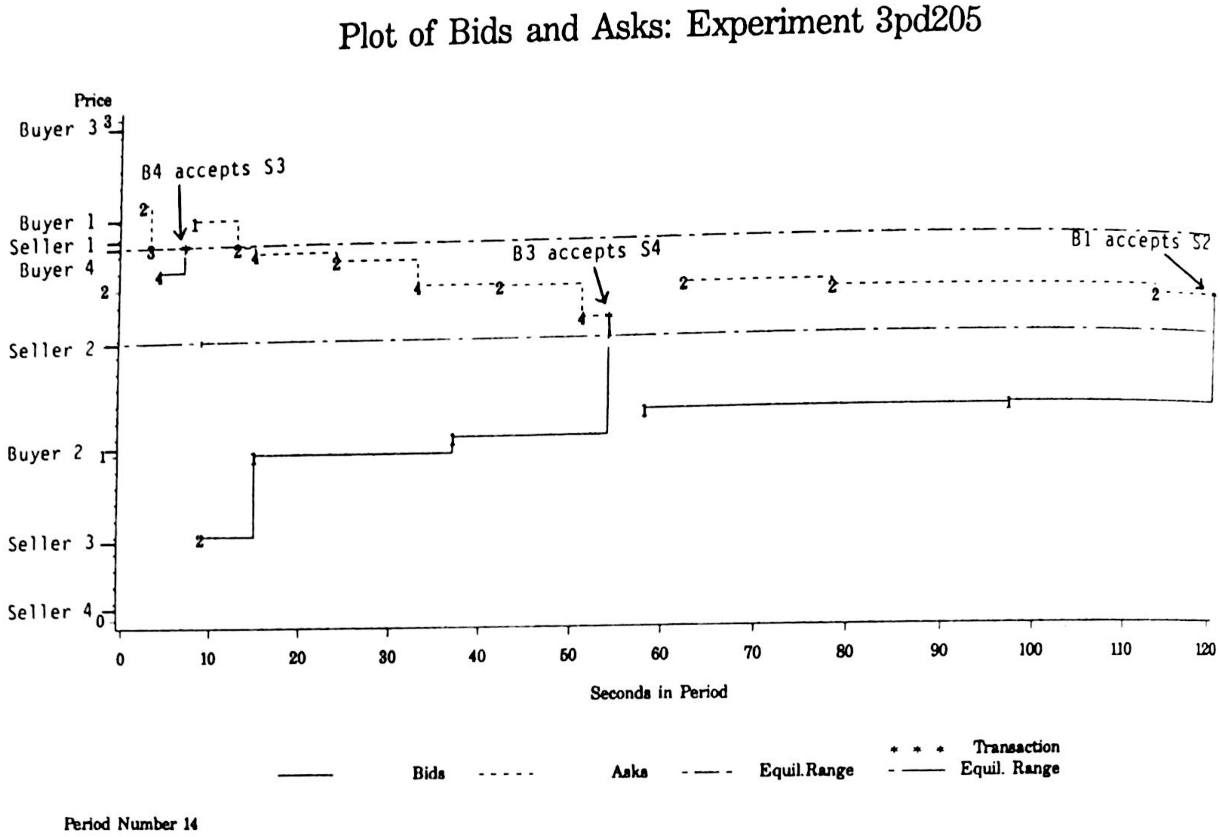

FIGURE 7 Experiment 3pd205 (Series 1), Period 14.

by these figures (and confirmed by the formal analysis below) is that quotes are more frequently revised by different agents (i.e., are "Not WGDA") rather than by the same agent that currently holds the quote (i.e., are "Not BGAN"). For that reason, we are comfortable with these simplistic definitions, even though Definition 1 is more strict than the formal BGAN model and Definition 2 is more lax than the formal WGDA model.

The null hypothesis for the statistical tests below is the zero-intelligence model. The "Not BGAN" and "Not WGDA" definitions are too complicated for an analytical derivation of their statistical properties; fortunately, the ZI model's simple rules and non-strategic behavior permit straightforward computer simulation of bids, asks, and transactions for any set of induced value and cost parameters. We conducted a Monte Carlo simulation of 1000 ZI markets using the Series 1 parameters to determine how many ZI algorithm transactions are Not BGAN and Not WGDA. Based on the empirical distributions from the simulation, we can determine the probability that a given "human" experiment's Not BGAN and Not WGDA frequencies come from the ZI model. Finding too few Not BGAN observations and too many Not WGDA observations would lead us to reject Hypothesis (2Z) in favor of the BGAN Hypothesis (2B), and finding too many Not BGAN observations and too few Not WGDA observations would lead us to reject Hypothesis (2Z) in favor of the WGDA Hypothesis (2W).

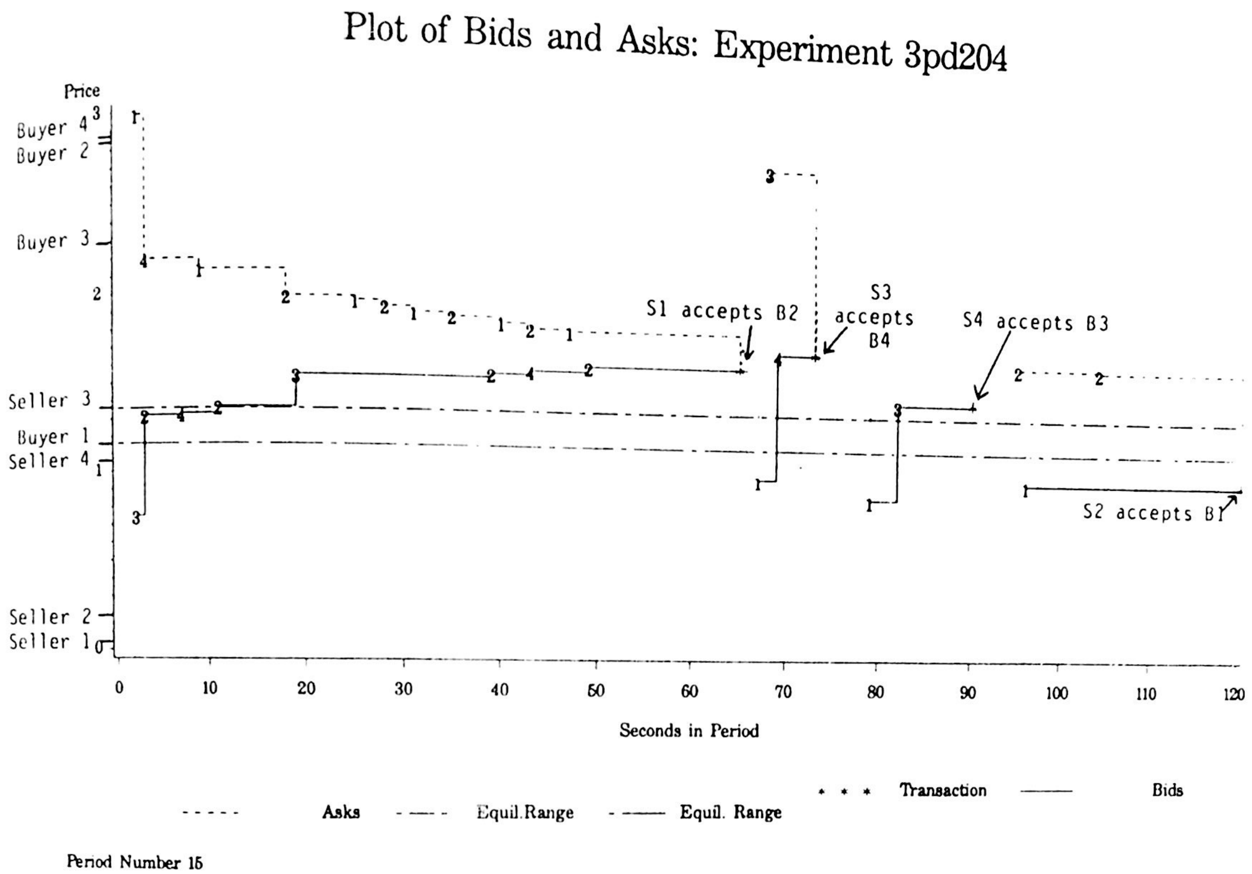

FIGURE 8 Experiment 3pd204 (Series 1), Period 15.

DEFINITION OF "SERIOUS" BIDS AND ASKS: Recall that Hypothesis (2W) applies only to "serious" bids and asks, i.e., those bids and asks that (in a sequential equilibrium) have a positive probability of acceptance. An empirical implementation could be cast in terms of either actual beliefs or beliefs in sequential equilibrium. Unfortunately, the former are unobservable and the latter are at present not computable. We examine two crude approximations for the Series 1 experiments. In the first alternative, we exclude no bids and asks as non-serious, and in the second alternative we exclude quotes outside the support of equilibrium prices, [$1.00, $2.50].[14] It turns out that neither alternative affects the conclusions.

EXPERIMENT SERIES 1: Results classifying the transactions from the three experiments of Series 1 are presented in Table 1. When all market quotes are considered (Table 1(a)), about 40% of the transactions are inconsistent with BGAN

[14]This criterion excludes 13.7% of the 1027 market actions in these three experiments. It also excludes some transaction prices (for example, the final transaction on Figure 8 is at $0.98). We are left uncertain as to whether this criterion is too strict or too lax.

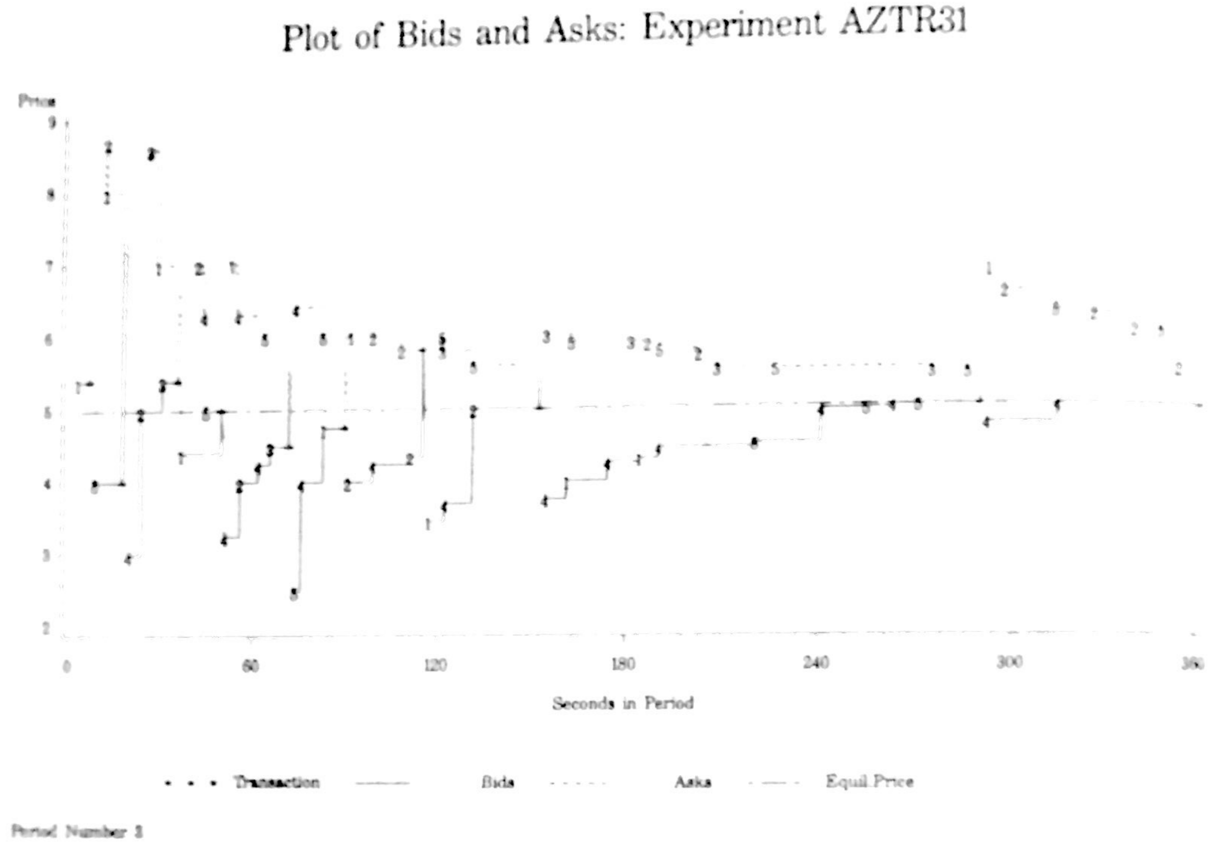

FIGURE 9 Experiment aztr31 (Series 2), Period 3.

and 75% of the transactions are inconsistent with WGDA. The simulation results indicate that Not BGAN and Not WGDA observations are equally likely in the ZI model. In all three experiments, the frequency of Not WGDA transactions exceeds the ZI critical value of 60% and the frequency of Not BGAN transactions is near the lower critical value of 38%. Furthermore, there are never enough "WGDA but Not BGAN" observations and the "BGAN but not WGDA" observations are always too frequent to be consistent with the ZI model.

When we exclude the quotes outside the support of equilibrium prices as suggested by the WGDA model (Table 1(b)), the percentage of transactions inconsistent with Hypotheses (2B) and (2W) decline slightly. However, the ZI simulation indicates that the distributions of these statistics should decrease more substantially. In general, the ZI model can be rejected in favor of the BGAN model even when excluding non-serious bids and asks to give WGDA a "better shot" at explaining the data. Furthermore, only 24% (12 of 51) of transactions inconsistent with BGAN can be classified as consistent with WGDA, while 59% (57 of 96) of transactions inconsistent with WGDA are consistent with BGAN. Put another way, nearly five times as many transactions are [BGAN but Not WGDA] than are [WGDA but Not BGAN]. When excluding quotes outside the support of equilibrium prices, over three times as many transactions are [BGAN but Not WGDA] than are [WGDA but Not BGAN].[15]

EXPERIMENT SERIES 2: For the Series 2 experiments, we have not excluded any bids and asks as non-serious. Figure 10 presents the distribution of the percentage of transactions inconsistent with BGAN and WGDA across the 27 experiments in Series 2. The percentage inconsistent with BGAN clusters around 10%, while the

TABLE 1 (a) Classification of observations as inconsistent with BGAN and/or WGDA: Exp. series 1—All observed quotes.

Exper. Number	Number of Observ.	Inconsist. with BGAN	Inconsist. with WGDA	Inconsistent with BGAN but consist. w/WGDA	Inconsistent with WGDA but consist. w/BGAN
3pd204	41	16 (39%)	34 (83%)[1]	3 (7%)[1]	21 (51%)[1]
3pd205	45	19 (42%)	30 (67%)[1]	5 (11%)[1]	16 (36%)[1]
3pd206	42	16 (38%)[1]	32 (76%)[1]	4 (10%)[1]	20 (48%)[1]
Pooled	128	51 (40%)	96 (75%)[1]	12 (9%)[1]	57 (45%)[1]
		ZI Simulation		1000 Markets	
Mean		50%	50%	24%	24%
5th percentile		38%	39%	15%	13%
95th percentile		61%	60%	33%	34%

[1] Indicates rejection of ZI Hypothesis (2Z) in favor of BGAN Hypothesis (2B) at the 5% level.

[15] A slightly different version of the ZI algorithm was also implemented in another Monte Carlo simulation. In the unreported version, each bid is uniformly distributed between the current market bid and the buyer's redemption value, and sellers' asks are similarly distributed between cost and the current market ask. Because the DA rules filter out the quotes that do not improve the current bid and ask, the results described above are basically unchanged when using this alternative algorithm.

TABLE 1 (b) Classification of observations as inconsistent with BGAN and/or WGDA: Exp. series 1—Excluding quotes from outside the equilibrium price range.

Exper. Number	Number of Observ.	Inconsist. with BGAN	Inconsist. with WGDA	Inconsistent with BGAN but consist. w/WGDA	Inconsistent with WGDA but consist. w/BGAN
3pd204	41	15 (37%)	25 (61%)[1]	5 (12%)[1]	15 (37%)[1]
3pd205	45	15 (33%)	26 (58%)[1]	7 (16%)	18 (40%)[1]
3pd206	42	15 (36%)	28 (67%)[1]	4 (10%)[1]	17 (41%)[1]
Pooled	128	45 (35%)	79 (62%)[1]	16 (13%)[1]	50 (39%)[1]
		ZI Simulation		1000 Markets	
Mean		32%	22%	23%	14%
5th percentile		23%	13%	14%	5%
95th percentile		43%	32%	33%	22%

[1] Indicates rejection of ZI Hypothesis (2Z) in favor of BGAN Hypothesis (2B) at the 5% level.

percentage inconsistent with WGDA clusters around 50–60%. Table 2 presents results numerically when all experiments of this series are pooled (a total of 2304 transactions). While we should reiterate that this series of experiments differs in several important ways from assumptions employed in the WGDA model—in particular, multiple units per trader and no public information regarding the distribution of unit valuations—it does indicate that behavior consistent with BGAN is much more common than behavior consistent with WGDA. Nearly twenty times as many transactions are [BGAN but Not WGDA] than are [WGDA but Not BGAN].

An additional feature of this series is that bid and ask behavior preceding each transaction changes for the later transactions in each period. This is illustrated by Figure 11, which plots the percentage of transactions inconsistent with each hypothesis for each transaction of the period. All 27 Series 2 experiments are combined for this figure. For the first five or six transactions in each period, almost no observations are inconsistent with BGAN (i.e., no traders are revising their own outstanding quote), and less than one-half of all transactions are inconsistent with WGDA. This occurs because early in each period bids and asks are accepted quickly, with little effort expended "bargaining" over prices with bid and ask improvements. The bottom line of this figure illustrates that on average only two or three market quotes precede these early transactions. Later in the period, however, the percentage of transactions inconsistent with both hypotheses increases substantially; there appears to be more "haggling" over prices. The bottom line in the figure shows that the average number of quotes preceding the later transactions increases to over eight. One feature of the Not WGDA and Not BGAN definitions is that increased bid and ask activity can only increase the likelihood of making a transaction inconsistent with one or both models.

In the Series 1 experiments, one buyer and one seller must exit the market after each transaction because each trader can transact only one unit. Consequently, only one or two traders are on each side of the market for the final transaction in each period, making Not BGAN observations much more likely than Not WGDA observations. This may partially explain the higher percentage of Not BGAN transactions observed in Series 1.

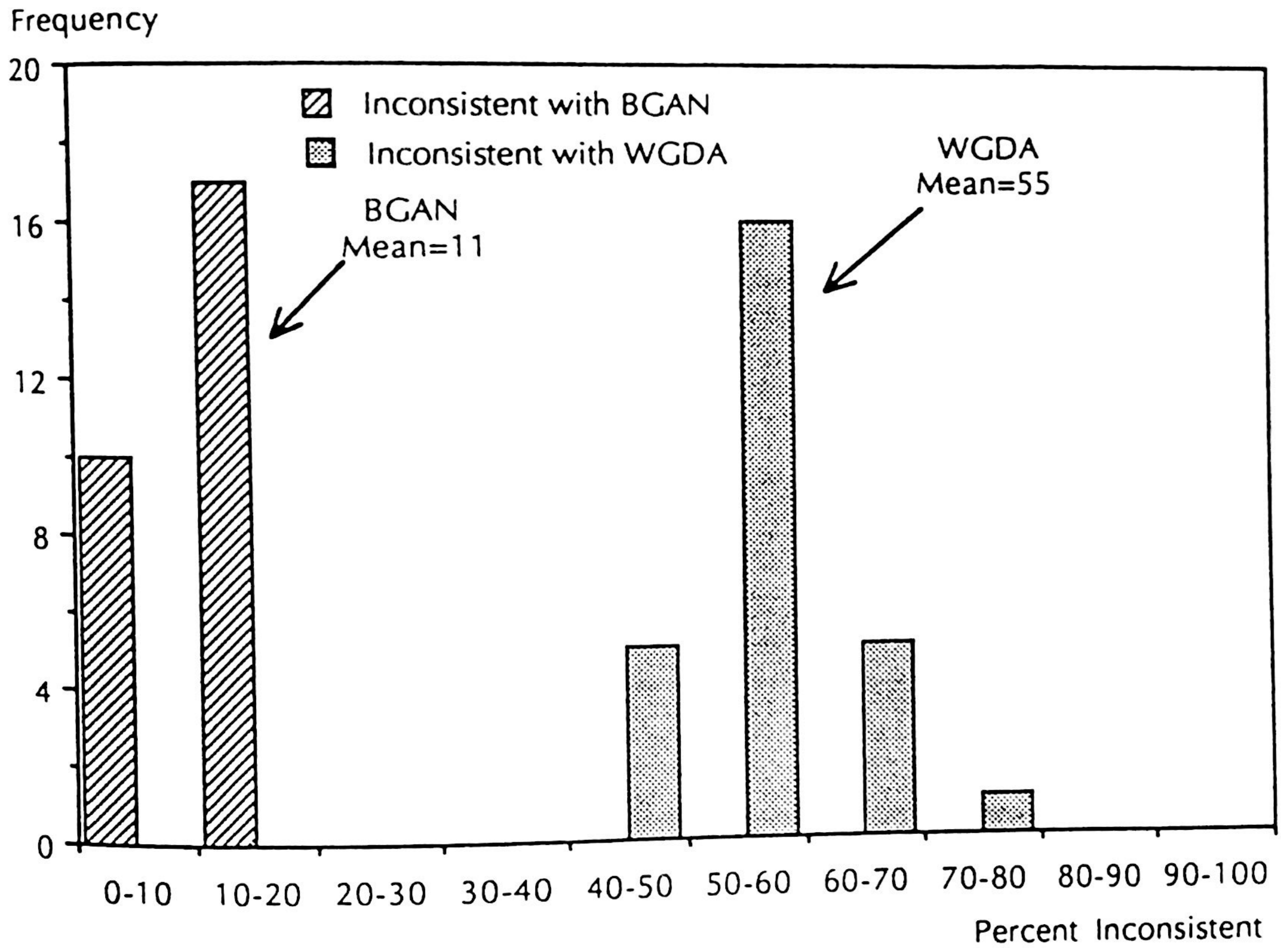

FIGURE 10 Distribution of percentages inconsistent with hypothesis 2 WGDA and BGAN models across experiment series 2 (27 observations).

TABLE 2 Classification of observations as inconsistent with BGAN and/or WGDA: experiment series 2.

Exper. Number	Number of Observ.	Inconsist. with BGAN	Inconsist. with WGDA	Inconsistent with BGAN but consist. w/WGDA	Inconsistent with WGDA but consist. w/BGAN
Pooled	2304	264 (11%)	1269 (55%)	52 (2%)	1057 (46%)

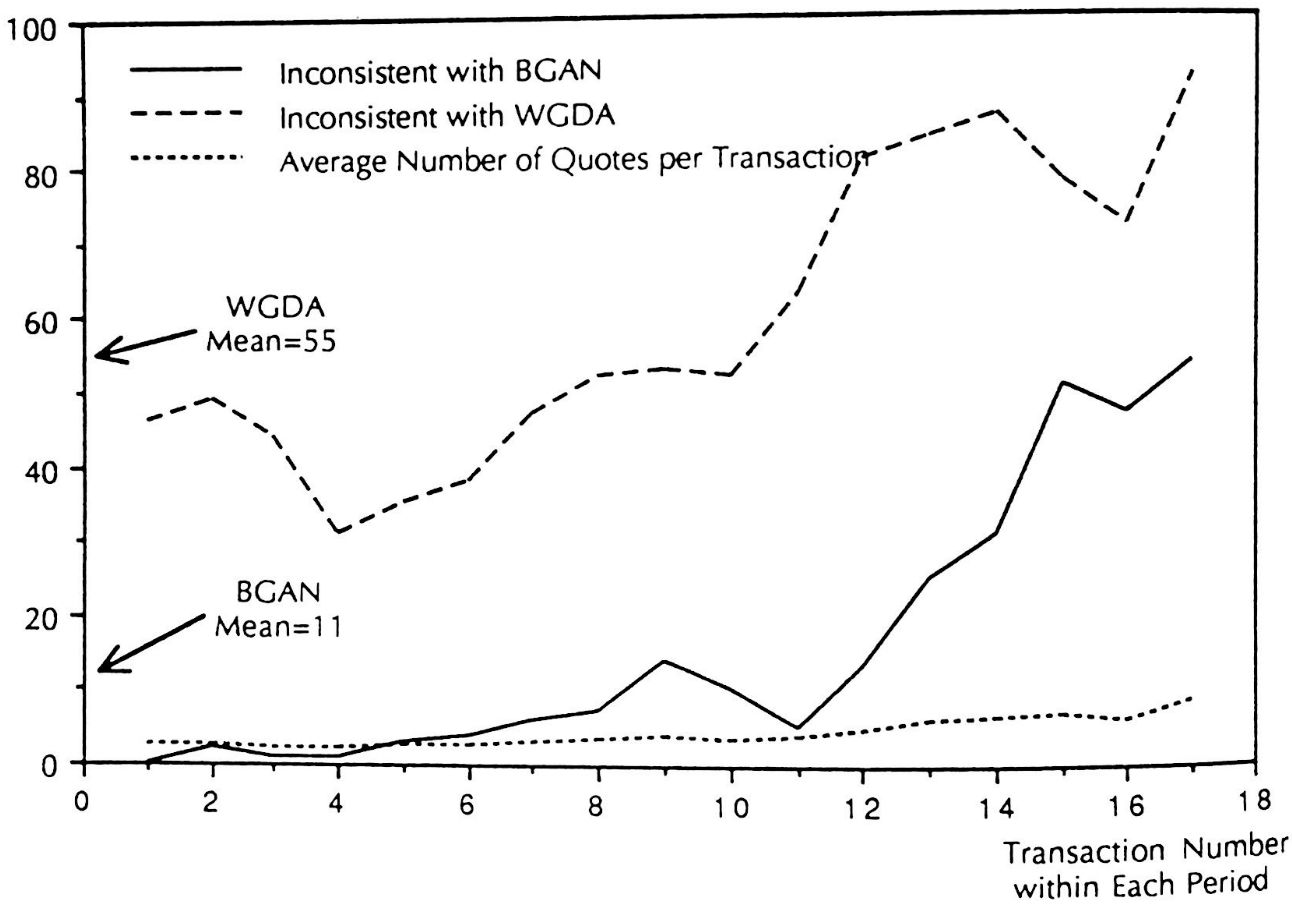

FIGURE 11 Percent of transactions inconsistent with hypothesis 2 WGDA and BGAN models: Experiment series 2. Percent inconsistent with each model and number of quotes per transaction.

CONCLUSION: In both series of experiments, improvements of the outstanding bids and offers by different buyers and sellers are more common than improvements by the same buyer and seller. Overall, the evidence clearly favors BGAN Hypothesis (2B) over WGDA Hypothesis (2W) and ZI Hypothesis (2Z).

TRANSACTIONS PARTNERS

All three models generally agree on the two remaining hypotheses, although the ZI model implies a somewhat weaker version of the following:

HYPOTHESIS (3): Early (later) transactions will be between high-value (low-value) bidders and low-cost (high-cost) sellers.

The weaker ZI model implication is that the high-value buyers and low-cost sellers are more *likely* to transact early.

Hypothesis (3) implies that the order in which buyers transact should be in decreasing order of their unit valuations, and the order in which sellers transact should by in increasing order of their unit costs. As a result, the Spearman Rank Correlation Coefficient between buyer (seller) valuation rank and transaction order should be significantly negative (positive).

EXPERIMENT SERIES 1: Table 3(a) presents estimates of this rank correlation coefficient for each of the three experiments separately and for the pooled data. For buyers, the estimated coefficients are negative (as predicted) for three of the four estimates, but only the estimate for experiment 3pd204 is significantly different from zero. For sellers, all four of the estimated coefficients are positive (as predicted), but only the estimate for experiment 3pd204 is significantly different from zero.

The rank correlation coefficients estimated for the 1000 simulated zero-intelligence markets ranged widely. For buyers, the estimated coefficients ranged from -0.59 to 0.35, with 80% of the estimates in the range $[-0.35, 0.06]$. For sellers, the estimated coefficients ranged from -0.26 to 0.75, with 80% of the estimates in the range $[0.09, 0.50]$. We cannot reject the hypothesis that the Table 3(a) estimates are generated by the ZI trader algorithm (the 5% lower critical value for sellers is 0.02).

A related implication of Hypothesis (3) is that gains from trade should be smaller for later trades in each period. Let the seconds left in the period at trade t be denoted as S_t, and define the gains from trade on trade at time t as $G_t = R_t - C_t$, where R_t is the valuation of the buyer in the transaction at time t and C_t is the cost of the seller in the transaction at time t. The implication that gains G_t should decrease as the period progresses can be tested by estimating the following equation:

$$G_t = \delta_o + \delta_1 S_t + e_t \,, \tag{7}$$

TABLE 3 (a) Rank correlation between transaction number and ranking of buyer and seller unit valuations: Experiment series 1.

Experiment Number	Buyer Rank Corr. Coeff.	p-value for 0 Corr.	Seller Rank Corr. Coeff.	p-value for 0 Corr.	N
3pd204	-0.33	0.037	0.33	0.037	41
3pd205	-0.07	0.652	0.08	0.580	45
3pd206	0.01	0.977	0.07	0.648	42
Pooled	-0.11	0.212	0.16	0.076	128

TABLE 3 (b) OLS estimation of Eq. (7): Experiment series 1.

Experiment Number	$\hat{\delta}_0$ (Intercept)	$\hat{\delta}_1$ (Seconds Left)	N	$\bar{R}^2$	D-W Stat
3pd204	1.00[1]	0.009[1]	41	0.22	2.0
	(0.18)	(0.002)			
3pd205	1.36[1]	0.002	45	0.00	2.5
	(0.19)	(0.003)			
3pd206	1.24[1]	0.004	42	0.02	2.4
	(0.21)	(0.003)			
Pooled	1.21[1]	0.005[1]	128	0.06	2.3
	(0.11)	(0.002)			

[1] Denotes significantly different from zero at the one percent level. (Standard errors in parentheses).

where e_t is the residual.[16] Hypothesis (3) implies that the gains from trade are decreasing as the seconds remaining in the period (S_t) are decreasing, which implies $\delta_1 > 0$. OLS estimation of this model is given in Table 3(b); all estimates are positive, and significant positive relationships are identified in two of the four regressions (experiment 3pd204 and all experiments pooled).

[16] The transaction ranking $T = 1, 2, 3, \ldots$ was also used instead of S_t as the explanatory variable in Eq. (7), with similar results.

EXPERIMENT SERIES 2: As explained previously, each agent had multiple units to trade in the experiments of this series. PLATO double auction rules require that buyers trade their highest valuation unit first, their second highest valuation unit second, and so on. Similarly, sellers are constrained to sell their lowest cost unit before selling higher cost units. For this reason, the institutional rules impose Hypothesis (3). Nevertheless, we tested Hypothesis (3) with these experiments *by only examining the first unit traded by each agent* to determine if the five buyers trade their first units in decreasing order of their resale values, and if the five sellers trade their first units in increasing order of their costs. When pooling across all 27 experiments, the rank correlation coefficients are significantly different from zero and have the sign predicted by Hypothesis (3) (although not impressively so, with estimates of -0.13 for buyers and 0.08 for sellers). An iterative maximum-likelihood GLS estimation of Eq. (7) using pooled data from all 27 experiments (GLS is used because of significant positive first-order disturbance autocorrelation) rejects the hypothesis that $\delta_1 = 0$ in favor of $\delta_1 > 0$, as implied by Hypothesis (3):

$$\underset{\text{(std. errors)}}{} \quad G_t = \underset{(0.47)}{2.21} + \underset{(0.001)}{0.007} S_t, N = 506, \text{estimated autocorrelation} = \underset{(0.04)}{0.50} \qquad (8)$$

CONCLUSION: Weak evidence exists that high-value buyers tend to transact before low-value buyers, and that low-cost sellers tend to transact before high-cost sellers. Weak evidence also exists that gains from trade decrease as more units transact, so we conclude that a weak version of Hypothesis (3), such as the version implied by the ZI model, is supported.

EFFICIENCY

HYPOTHESIS (4): Most potential gains from trade will be exhausted.

The efficiency of the three Series 1 random value markets is high and is similar to the efficiency of the initial periods in standard double auction markets (experiments 3pd204, 3pd205, and 3pd206 are 92.3%, 95.3%, and 89.9% efficient, respectively). Average efficiency is 93%, and 25 of the 45 periods (55%) achieved 100% efficiency. However, most of the ZI simulation markets achieved efficiency levels that exceeded the efficiency levels of the three experiments. The lower fifth percentile of the simulation market efficiencies is 93.9%, which is exceeded only by experiment 3pd205.

CONCLUSION: Hypothesis (4) is supported. These random value DA markets are also efficient, although they are not usually as efficient as comparable simulated ZI markets.

SUMMARY AND CONCLUSIONS

Because double auction markets are very important in field settings and because their strong equilibration properties have been extensively documented in the laboratory, we begin our study of price formation by looking at the most relevant existing data from double auction market experiments. The data analysis is shaped by three theoretical models that describe price formation within a single trading period: BGAN, WGDA, and ZI. The models agree in predicting allocational efficiency near 100% (Hypothesis 4), and the data broadly confirm this prediction. The models also agree in predicting that traders with larger potential gains from trade—the low-cost sellers and high-value buyers—will transact before those with smaller potential gains (Hypothesis 3). The data confirm that this is true on average, although there is also considerable randomness in the transaction sequences. The theoretical models disagree sharply in their characterizations of the bid and ask sequences. Here we find much stronger support for BGAN Hypothesis (2B), which calls for traders to improve the bids or asks of other traders à la Bertrand, than for WGDA Hypothesis (2W), which calls for traders to improve on their own bids or offers as in a Dutch auction, or for ZI Hypothesis (2Z), in which transactions consistent with BGAN and WGDA are about equally likely. The models also disagree in their predictions of transaction price change autocorrelation, and we find that the data support the ZI model's negative autocorrelation Hypothesis (1Z) and reject both the positive autocorrelation called for in BGAN Hypothesis (1B) and the zero autocorrelation called for in WGDA Hypothesis (1W). Data from other laboratory DA markets leads us to conjecture that this negative correlation may be explained (in part) by subject inexperience.

The results reported here have a more general possible interpretation that should be investigated with the additional experiments described below. Note that the model that relies most heavily on trader rationality (WGDA) has the least ability to describe market behavior, while the ZI model requires very little trader rationality and yet describes market behavior as well as or better than the strategic WGDA and BGAN models. This suggests that the DA institution's *rules* lead to its remarkable performance, rather than the rationality and strategic play of its traders.

Nevertheless, we do not interpret this evidence as grounds for dismissing the WGDA model. In retrospect, there are two reasons for regarding the available data as unfavorable to the WGDA model. First, the experiments used inexperienced subjects, so the presumed common knowledge (of other players' contingent strategies as well as parameter distributions) of WGDA is not given a fair chance. Second, the data do not permit testing one of WGDA's more striking implications, the dependence of the split in gains from trade on the numbers of buyers and sellers remaining, because the experiments all employed equal numbers of buyers and sellers.

New experiments would be useful to further evaluate the predictive ability of the alternative theories. The new experiments should feature single-unit transactors whose costs/values are drawn from known simple distributions, as in the Series 1 experiments. The experiments should also employ experienced subjects and the number of buyers and sellers should systematically be varied. Then the WGDA model will have a better opportunity to show its ability to explain the data. If the increased subject experience does not reduce the negative autocorrelation in price changes, it may be useful to allow traders to both buy and sell units and determine if the intertemporal arbitrage opportunities persist.

Several additional model implications can be tested in these future experiments. For example, one can examine the timing of transactions across a period. The WGDA model suggests substantial delay before the initial transaction and many trades toward the end of the period. In contrast, the BGAN and ZI models suggest greater uniformity of transactions across the period. Another discriminating test involves the source of market efficiency losses. The WGDA model only allows infra-marginal efficiency losses (i.e., untraded units with available gains from trade at the equilibrium price). The BGAN and ZI models also allow for infra-marginal efficiency losses, but can also allow inefficiencies from the trade of extra-marginal units (i.e., units that are not profitable to trade at the equilibrium price). Additional experiments can be used to test these implications.

Data from such experiments should make clear the strengths and weaknesses of the models. At that point the models can be modified to better explain the data. For example it might turn out that WGDA provides a good characterization of events in later trading periods of experiments with experienced subjects. In that case, theorists might fruitfully seek ways to weaken the model's common-knowledge assumptions without altering the basic view of the double auction as a waiting game punctuated by dutch auctions. Alternatively, if BGAN continues to do well, theorists might wish to incorporate a more sophisticated rationality into its game against nature view. Regularities in the data might also suggest completely new models of the price formation process.

We see price formation as a frontier territory for economists, rich in potential for policymakers as well as for theorists. The work presented here represents a necessary first step in exploring this territory.

ACKNOWLEDGMENTS

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