Formulas for Econ 200, Microeconomic Analysis Fall 2014

Equilibrium

$D(p^*) = S(p^*)$

$Linear: x = a + bp$

$Loglinear: \ln(x) = a + b\ln(p) \quad b = \varepsilon_d$

$semiLog: \ln(x) = a + bp$

$\varepsilon_d = \left(\partial D/_{\partial P}\right)\left(P/_{Q}\right) = \left(\partial lnD/_{\partial lnP}\right)$

$\varepsilon_s = \left(\partial S/_{\partial P}\right)\left(P/_{Q}\right) = \left(\partial lnS/_{\partial lnP}\right)$

$CS = \int_0^{q^*} (D^{-1}(q) - p^*)\, dq$

$PS = \int_0^{q^*} (p^* - (S^{-1}(q)))\, dq$

Tax:

$Tax\ Rev = \int_0^{q^*} (p_d^* - p_s^*)\, dq = tq^*$

Unit Tax:

$p_d^* = p_s^* + tax,\ D(p_s^* + t) = S(p_s^*)$

Change in Consumer and Producer Price w/ Tax:

$\Delta p_d^* = \frac{t\mathcal{E}_s}{|\mathcal{E}_d| + \mathcal{E}_s}, \qquad \Delta p_s^* = -\frac{t|\mathcal{E}_d|}{|\mathcal{E}_d| + \mathcal{E}_s}$

Change in quantity w/Tax:

$\Delta D = \mathcal{E}_d \frac{Q_d}{p_d}\Delta p_d$

Welfare with tax:

$DWL = (1/_2)t\Delta D$

$\Delta CS_{tax} = (Q_d + \Delta D)\Delta p_d + [\frac{1}{2}((Q_d - (Q_d + \Delta D))\Delta p_d]$

$\Delta PS_{tax} = (Q_s + \Delta D)\Delta p_s + [\frac{1}{2}((Q_s - (Q_s + \Delta D))\Delta p_s]$

Consumer Behavior:

$MRS_{ij} = \left(-\frac{\partial u}{\partial x_i}\Big/\frac{\partial u}{\partial x_j}\right)$

$ERS = \left(-\frac{p_i}{p_j}\right) \quad (Economic\ Rate\ Substitution$

$= Slope\ of\ BC)$

Budget Constraint:

$p_1x_1 + p_2x_2 = m$

Expenditure Minimization Problem:

$e(p,\bar{u}) = \min px \quad s.t\ u(x) \geq \bar{u}$

EMP Lagrangian:

$\mathcal{L} = p_1x_1 + p_2x_2 + \lambda(\bar{u} - u(x))$

Hicksian Demand:

$x^h(p,u) = \frac{\partial\, e(p,u)}{\partial\, p_i}$

Utility Functions:

CES:

$U(x_1, \dots, x_n) = \frac{1}{\rho}\ln\left(a_1x_1^\rho + \cdots + a_nx_n^\rho\right)$

Quasilinear:

$U(x_1, x_2) = x_1 + g(x_2)$

Cobb − Douglas:

$U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}\ or\ a_1lnx_1 + a_2lnx_2$

Indirect utility identity:

$e(p, v(p,m)) = m$

Utility Maximization Problem:

Indirect Utility

$v(p,m^*) = \max \mathrm{U(x)} \quad s.t\ px \leq m^*$

UMP Lagrangian:

$\mathcal{L} = U(x) + \lambda(m - px)$

Marshalian Demand

${x^*}_m(p,m)$

Slutsky via Derivative and Elasticity:

$\frac{\partial\, x_j(p^*,m^*)}{\partial\, p_i} = \frac{\partial\, h_j(p^*,u^*)}{\partial\, p_i} - \frac{\partial\, x_i(p^*,m^*)}{\partial\, m}x_i^*$

$\varepsilon_{ij} = \varepsilon_{ij}^h - s_j\eta_j$

Own, Cross and Income Elasticities

$\varepsilon_{11} + \varepsilon_{12} + \varepsilon_{13} + \eta_1 = 0$

Roy's Identity:

$x_m^* = -\frac{\partial v}{\partial p_i}\Big/\frac{\partial v}{\partial m}$

Normal Good: $\uparrow$ *Income* $\Rightarrow\uparrow$ *Demand*

Inferior Good: $\uparrow$ *Income* $\Rightarrow\downarrow$ *Demand*

Firm Behavior:

Profit Maximization:

$\pi(p,w) = \max_{x\geq 0} pf(x) - w.x \quad \Rightarrow \quad MC = MR$

$Profit\ Function: py^* - c(y^*)$

$\frac{\partial\pi}{\partial p} = y^*(p,w) \quad (by\ Hotelling)$

$\frac{\partial\pi}{\partial w_i} = -x_i^*(p,w)\ (uncond.\ factor\ demand\ )$

Cost Minimization:

$c(w,y) = \min_{x_1,x_2} w_1x_1 + w_2x_2$

$s.t.\ f(x_1,x_2) = y$

Shepard's Lemma

$\frac{\partial c}{\partial w} = x^*(w,y)\ (conditional\ factor\ demand)$

$(ERS = TRS), -\frac{w_i}{w_j} = -\left(\frac{\partial f(x^*)}{\partial x_i}\Big/\frac{\partial f(x^*)}{\partial x_j}\right)$

Learning Curve:

$Given\ Y_t = \sum_{s=0}^{t} y_s$

$\ln(AC_t) = \ln(AC_0) - blnY_t$

Economies of Scope:

$C(y_1y_2; w) < C(y_1, 0; w) + C(0, y_2; w)$

Cost complimentarity:

$\frac{\partial^2 C}{\partial y_1 y_2} = \left(\frac{\partial}{\partial y_2}\left(\frac{\partial c}{\partial y_1}\right)\right) < 0$

Returns to Scale:

$IRS: f(tx) > tf(x)\ for\ all\ t > 1$

$CRS: f(tx) = tf(x) for\ all\ t \geq 0$

Envelope Theorem:

Given a profit function $\pi(p,w) = py^* - wx^*$

$\left(\partial\pi/_{\partial p^*}\right) = y^* + p\left(\partial y/_{\partial p}\right)$

Monopoly:

$p(y)\left[1 + \frac{1}{\varepsilon}\right] = c'(y) \quad (markup\ price)$

Markup Factor: $\left(\varepsilon/_{(\varepsilon + 1)}\right)$

$\frac{\partial p^m}{\partial c} = \frac{1}{2 + \left(\frac{yp''(y)}{p'(y)}\right)}$

$MR = p(y) + p'(y)y$