2022 16th ICME International Conference on Complex Medical Engineering (CME) | 978-1-6654-9699-5/22/$31.00 ©2022 IEEE | DOI: 10.1109/CME55444.2022.10063269

Medical ankle image denoising based on constrained least square filter

Zeng Jian
Mianyang Teachers College
Mianyang, Sichuan, Peoples R China
Zengjian0407@163.com

Li Yin*
China Aerodynamics Research and Development Center
Mianyang, Sichuan, Peoples R China
*Corresponding author's email: DIYLLLLY@163.com

Abstract—**In this work, the constrained least square filter is introduced to remove the noise of medical ankle image. Firstly, the algorithm principle regarding to the constrained least square filter is studied. On this basis, a medical ankle image with Gaussian noise is processed based on constrained least square filter and low pass filter. Finally, the peak signal-to-noise ratio, PSNR, is introduced to quantitatively estimate the post-processing medical ankle image quality. The obtained results show that the PSNR to the post-processing image obtained by constrained least square filter is larger than that obtained by low pass filter, which validates that the constrained least square filter can remove the medical ankle image noise effectively.**

Keywords- medical ankle image; constrained least square filter; remove the noise; PSNR

I. INTRODUCTION

In daily life, people are affected by the nature of work and surrounding things, such as the long-term ankle joint friction of soldiers, athletes and dance practitioners who need regular training, which makes the ankle joint subject to external forces, and it is easy to cause ankle disease [1]. Generally, medical photography is required when there is discomfort in the ankle. But, the medical image usually contains some noise due to the influence of equipment and environment and affects medical diagnosis [2].

This step of medical image noise removal is an extremely important step in medical image preprocessing. It is the primary guarantee for other subsequent operations of the image, such as image segmentation, and is a basis for us to extract effective information in the image. Therefore, it is essential to remove the noise from the medical image. Traditional denoising methods generally use window smoothing methods, such as median filtering, which can filter out the impact of impulse noise on the image, but the edge information structure is easy to be lost [3]. Constrained least square filter [4-6] is a great smoothing filter method with the following two advantages. One is that it can effectively highlight image details and edge information and the other is a good filtering and denoising function.

On this account, the constrained least square filter method is introduced to remove the noise of medical ankle image.

II. ALGORITHM PRINCIPLE

Constrained least square filter method is based on the least power filter restoration formula. The smoothness of the image is increased by selecting a reasonable smoothing criterion matrix, $\boldsymbol{Q}$, and optimizing $\| \boldsymbol{Qf} \|^2$, where $\boldsymbol{f}$ is the image containing the noise.

Laplace operator for image enhancement is as follows.

$$\nabla f = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{1}$$

As is known to us, Laplace operator has the function of highlighting edges, but $\iint \nabla^2 f dxdy$ restores the smoothness of the image. Therefore, the smoothness of the image obtained by Laplace operator can be taken as the main target in image restoration. In this case, how to express the image smoothness obtained by Laplace operator in the form of $\| \boldsymbol{Qf} \|^2$ becomes the key problem.

In the case of discretization, the Laplace operator can be realized by the following differential operation.

$$\begin{aligned} \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} &= f(x+1,y) \\ &+ f(x-1,y) + f(x,y+1) \\ &+ f(x,y-1) - 4f(x,y) \end{aligned} \tag{2}$$

Convolution of $f(x,y)$ with the template operator (in Eq. 3) can realize the operation of Eq. 2.

$$p(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{3}$$

In the process of discrete convolution, we can use the extension $f(x,y)$ and $p(x,y)$ to avoid overlapping error. Suppose the function after extension as $p_e(x,y)$. The block circulant matrix is established and the smoothing criterion is expressed in matrix form as shown in Eq. 4

$$C=\begin{bmatrix} C_1 & C_0 & C_{M-1} & \cdots & C_2 \\ C_2 & C_1 & C_0 & \cdots & C_3 \\ \vdots & \vdots & \vdots & & \vdots \\ C_{M-1} & C_{M-2} & C_{M-3} & \cdots & C_0 \end{bmatrix} \tag{4}$$

In Eq. 4, each submatrix $C_j\left(j=0,1,\cdots,M-1\right)$ is the $N\times N$ order cyclic matrix that is composed of the j-line of $p_e\left(x,y\right)$.

Thus

$$C_j=\begin{bmatrix} P_e\left(j,0\right) & P_e\left(j,N-1\right) & \cdots & P_e\left(j,1\right) \\ P_e\left(j,1\right) & P_e\left(j,0\right) & \cdots & P_e\left(j,2\right) \\ \vdots & \vdots & & \vdots \\ P_e\left(j,N-1\right) & P_e\left(j,N-2\right) & \cdots & P_e\left(j,0\right) \end{bmatrix} \tag{5}$$

According to the diagonalization of the cyclic matrix, it can be seen that the matrix can be used for diagonalization.

$$E=W^{-1}CW \tag{6}$$

Where E is the diagonal matrix and its element is shown in Eq. 7.

$$E\left(k,i\right)=\begin{cases} \left(P\left[\dfrac{k}{N}\right],k\mathrm{MODN}\right) & , \quad i\neq k \\ 0 & , \quad i=k \end{cases} \tag{7}$$

Where $E\left(k,i\right)$ is obtained by two dimensional Fourier transform of element $p_e\left(x,y\right)$ in matrix $\boldsymbol{C}$ and the $\iint \nabla^2 f dxdy$ can be transformed to $\boldsymbol{f}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{f}$. Letting $\boldsymbol{Q}=\boldsymbol{C}$ yields $\boldsymbol{f}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{f}=\|\boldsymbol{Q}\boldsymbol{f}\|^2$.

If the constraint conditions $\left\|g-Hf\right\|=\left\|n\right\|^2$ are required to be met, it would be obtained the following equation when $\boldsymbol{Q}=\boldsymbol{C}$

$$\begin{aligned} \hat{f} &= \left(H^T H+\gamma C^T C\right)^{-1} H^T g \\ &= \left(WD^* DW^{-1}+\gamma WE^* EW^{-1}\right)^{-1} WD^* W^{-1} g \end{aligned} \tag{8}$$

Both sides of the Eq. 8 are divided by W^{-1}, yields,

$$W^{-1}\hat{f}=\left(D^* D+\gamma E^* E\right)^{-1} D^* W^{-1} g \tag{9}$$

Where D^* and D are the conjugate matrix, thus

$$\begin{aligned} \hat{F}\left(u,v\right) &= \left[\frac{N^2 H^*\left(u,v\right)}{N^2\left|H\left(u,v\right)\right|^2+\gamma N^4\left|P\left(u,v\right)\right|^2}\right] G\left(u,v\right) \\ &= \left[\frac{H^*\left(u,v\right)}{\left|H\left(u,v\right)\right|^2+\gamma N^2\left|P\left(u,v\right)\right|^2}\right] G\left(u,v\right) \end{aligned} \tag{10}$$

Where $u,v=0,1,\cdots,N-1$, $\left|H\left(u,v\right)\right|^2=H^*\left(u,v\right)H\left(u,v\right)$.

III. Case Study

In this section, a case study is investigated to prove the the effectiveness of constrained least square filter in the image denoising. The processing procedures are listed as follows.

Step 1: add the noise to the medical ankle image

Gaussian noise is added to medical ankle image. The mean value and the variance of the Gaussian noise are 0 and 1e-3 respectively. The medical ankle image with Gaussian noise is shown in Fig. 1.

Figure 1. medical ankle image with Gaussian noise

Step 2: remove the noise of medical ankle image

In this step, the constrained least square filter is used to process the medical ankle image. Additionally, in order to carry out the comparative analysis, the low pass filter is introduced to process the medical ankle image. The processing results are shown in Fig. 2 and Fig. 3.

Figure 2. Processing results based on constrained least square filter

Figure 3. Processing results based on low pass filter

From the Fig. 1, Fig. 2 and Fig. 3, we can find that the Gaussian noise in the medical ankle image is weaken, which indicates that both the constrained least square filter and low pass filter can effectively remove the noise in the medical ankle image. But the low pass filter causes the overall brightness of the medical ankle image to be dark and the contrast of the ankle to be reduced. By contrast, the constrained least square filter can highlight the ankle details and edge information.

Step 3: quantitative estimation

In order to quantitatively estimate the applicability and superiority of the constrained least square filter for the medical ankle image, it is necessary to estimate the post-processing medical ankle image quality.

The common used image quality estimation indexes include the signal-to-noise ratio (SNR), peak signal-to-noise ratio (PSNR), mean square error (MSE), root mean square error (RMSE) [6]. In this work, the PSNR is used to quantitatively estimate the medical ankle image quality. The expression of PSNR is shown in Eq. 11.

$$PSNR = 10 \times \lg \left[\frac{M \times N \times [\max(I) - \min(I)]^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} [I(i,j) - I'(i,j)]^2} \right] \quad (11)$$

In Eq. 11, $I(i,j)$ is the medical ankle image with Gaussian noise, $I'(i,j)$ is the post-processing medical ankle image. M and N are the length and width of the medical ankle image respectively. The larger PSNR indicates the better the algorithm processing effect and the higher the image quality.

According to Eq. 11, The PSNR of the medical ankle image processed by constrained least square filter and by low pass filter is calculated respectively and the corresponding results are listed in Table I.

TABLE I. PSNR OF MEDICAL ANKLE IMAGE

Method	PSNR
constrained least square filter	8.8371
low pass filter	6.2749

As showed in Table I, the PSNR of post-processing medical ankle image obtained by constrained least square filter and by low pass filter are 8.8373 and 6.2749. The PSNR obtained by constrained least square filter is larger than that obtained by low pass filter, which indicates that the medical ankle image quality obtained by constrained least square filter is better than that obtained by low pass filter, proving the validity of constrained least square filter investigated in this work.

IV. CONCLUSION

The constrained least square filter is introduced to remove the noise of medical ankle image in this work. The algorithm principle algorithm principle regarding to the constrained least square filter is firstly analysed followed by a case study about medical ankle image to validate the effectiveness of the constrained least square filter. The obtained results show that the constrained least square filter can remove the medical ankle image noise effectively.

REFERENCES

[1] Chen J, Chen B and Wang Z. "Treatment of common diseases of foot and ankle," Chin J Orthop, vol. 24, no. 1, pp. 48 – 50, 2004.

[2] Wang S and Zeng F. "A difference-filtering way to smoothing image noise" Journal of Electronics & Information Technology, vol. 22, no. 3, pp. 411 – 415, 2000.

[3] Xie Y, Xia X, Ju S and Shao S. “Image smoothing denoising based on adaptive window,” Journal of Sichuan University (Natural Science Edition), vol. 50, no. 3, pp. 503-508, 2013.

[4] Zhang Y and Wu L. “Discussing on image restoration technology for constrained least square filter,” Journal of Nanchang University (Engineering & Technology), vol. 27, no. 4, pp. 85-87, 2005.

[5] Huang G, Zhang Y, Bi W, Gao G and Zhu Z. “Low illumination image decomposition and details enhancement under gradient sparse and least square constraint,” Acta Electronica Sinica, vol. 46, no. 2, pp. 424-432, 2018.

[6] Wang Z. “Review of no-reference image quality assessment,” Acta Automatica Sinica, vol. 41, no. 6, pp. 1062-1079, 2015.