

§ 10-6 相互垂直的简谐振动的合成

两个同频率的相互垂直的分运动位移表达式

$$x = A_1 \cos(\omega t + \phi_{10})$$

$$y = A_2 \cos(\omega t + \phi_{20})$$

消时间参数，得

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - 2 \frac{x}{A_1} \frac{y}{A_2} \cos(\phi_{20} - \phi_{10}) = \sin^2(\phi_{20} - \phi_{10})$$

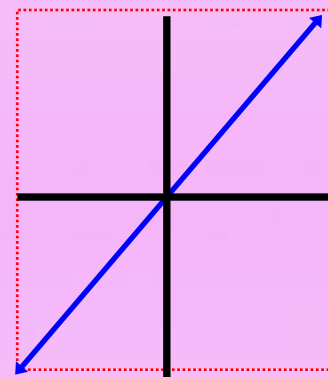
合运动一般是在 $(A_1 \text{向})$ 、 $(A_2 \text{向})$ 范围内的一个椭圆。

椭圆的性质(方位、长短轴、左右旋)在 A_1 、 A_2 确定之后,主要决定于 $\Delta\phi = \phi_{20} - \phi_{10}$

几种特殊情况:

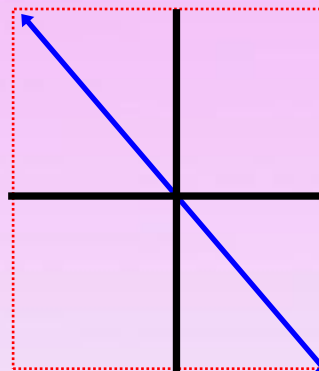
(1) $\phi_{20} - \phi_{10} = 0$, 两个分振动同相位, 得 $y = \frac{A_2}{A_1} x$
在任一时刻离开坐标原点位移为:

$$s = \sqrt{A_1^2 + A_2^2} \cos(\omega t + \phi)$$



(2) $\phi_{20} - \phi_{10} = \pi$, 两个分运动反相位, 得

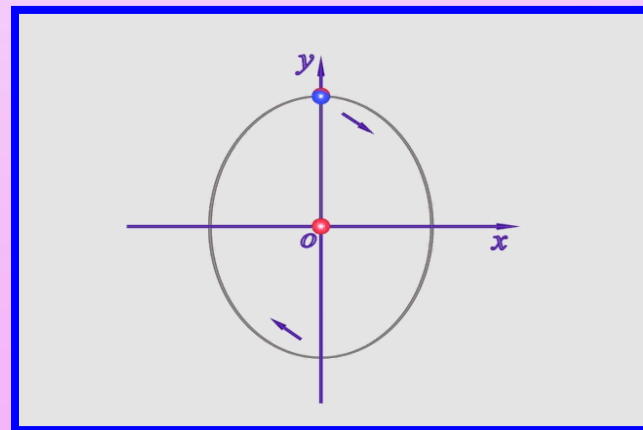
$$y = -\frac{A_2}{A_1} x$$



几种特殊情况:

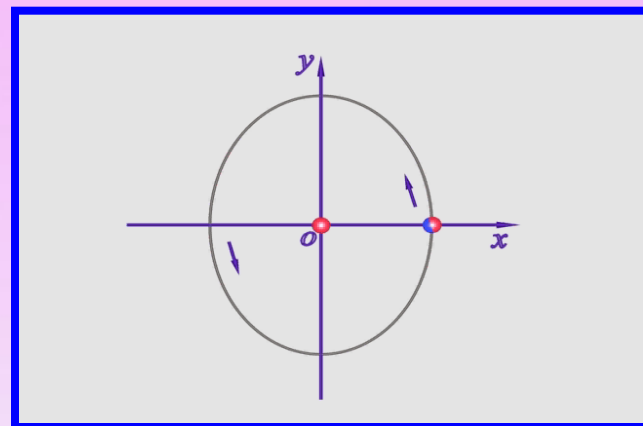
(3) $\phi_{20} - \phi_{10} = \pi/2$, 得 $\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$

这是坐标轴为主轴的椭圆，质点的轨迹是顺时针旋转。



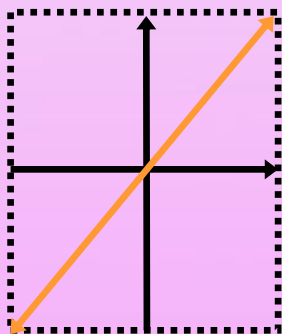
(4) $\phi_{20} - \phi_{10} = 3\pi/2$, 仍然得 $\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$

与(3)相同，只是质点的轨迹沿逆时针旋转。

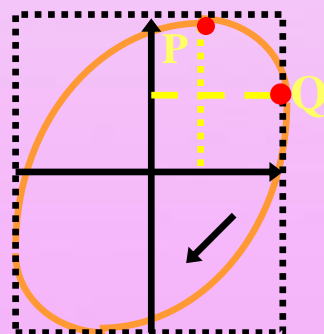


几种特殊情况:

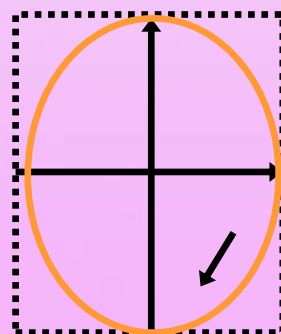
$$\Delta\phi = 0$$



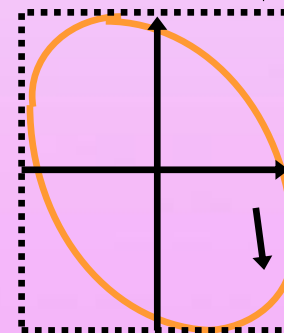
$$\Delta\phi = \pi/4$$



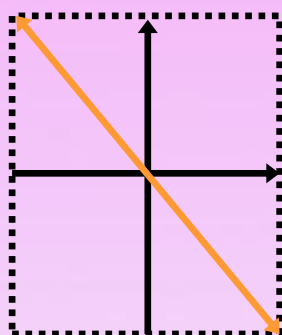
$$\Delta\phi = \pi/2$$



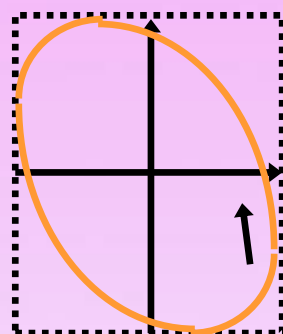
$$\Delta\phi = 3\pi/4$$



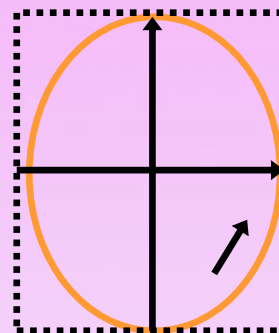
$$\Delta\phi = \pi$$



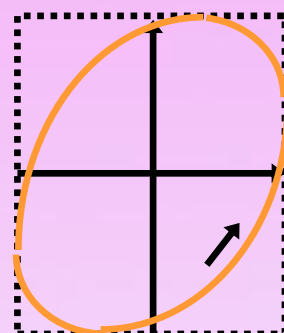
$$\Delta\phi = 5\pi/4$$



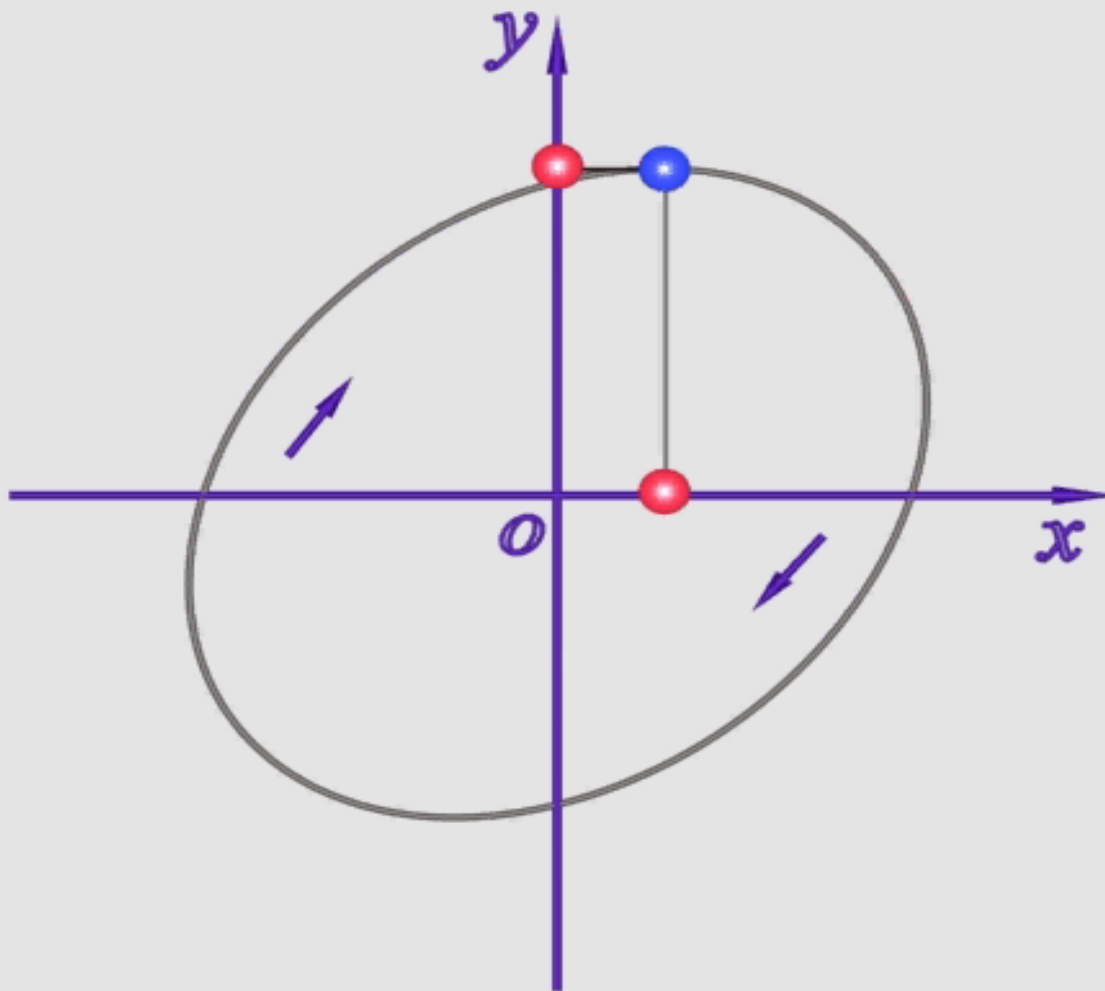
$$\Delta\phi = 3\pi/2$$



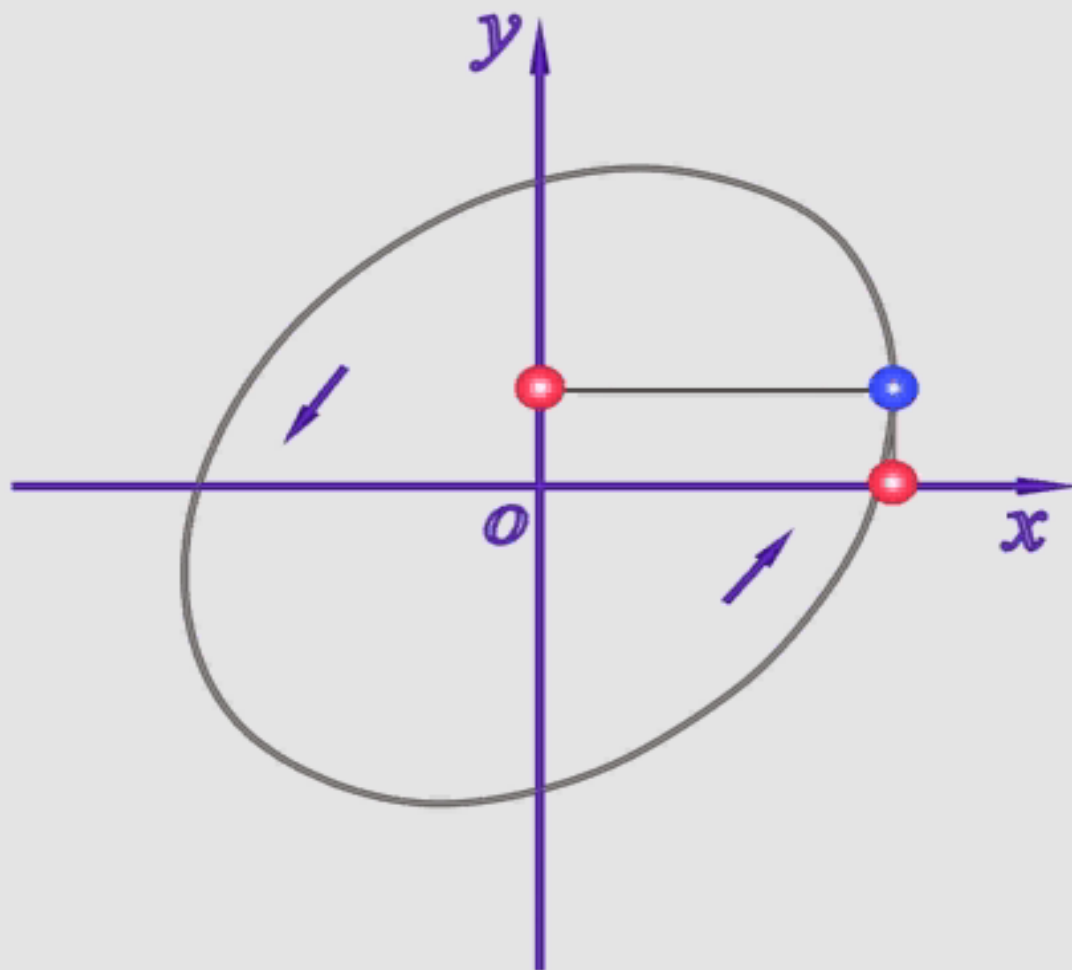
$$\Delta\phi = 7\pi/4$$



$$\Delta\phi = \phi_{20} - \phi_{10}$$



$$\Delta\phi = \pi/4$$



$$\Delta\phi = 7\pi/4$$

方向垂直的不同频率的简谐振动的合成

- 两分振动频率相差很小

$$\Delta\phi = (\omega_2 - \omega_1)t$$

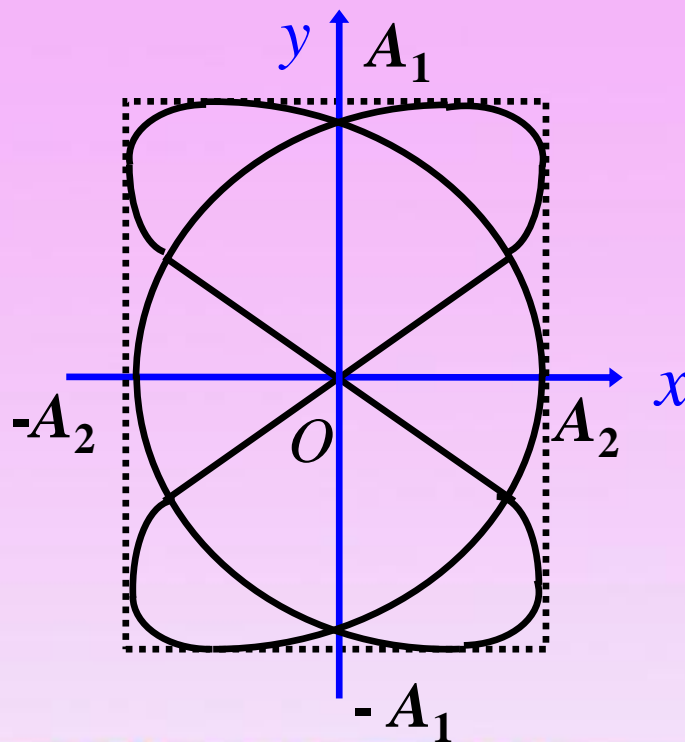
可看作两频率相等而 $\Delta\phi$ 随 t 缓慢变化，合运动轨迹将按上页图依次缓慢变化

- 两振动的频率成整数比

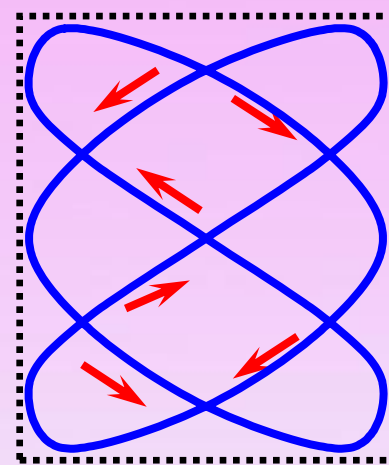
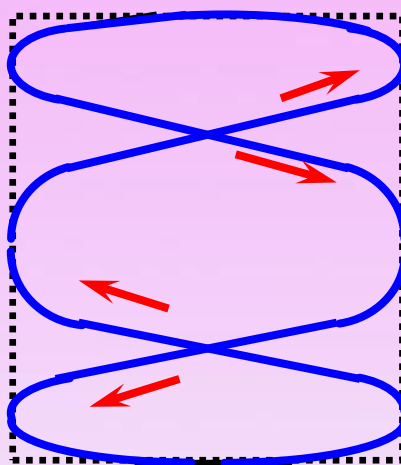
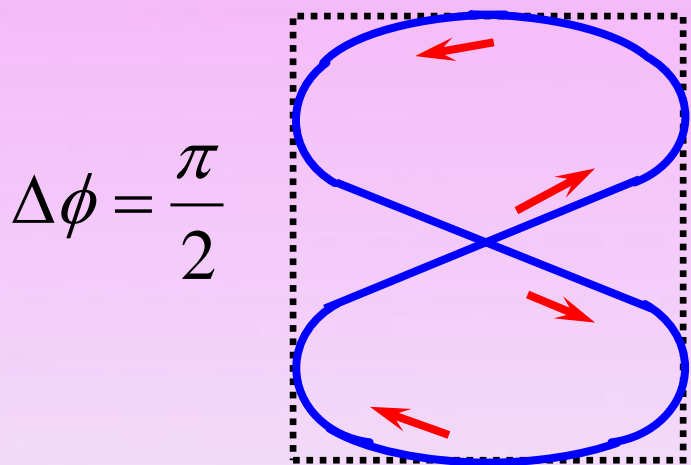
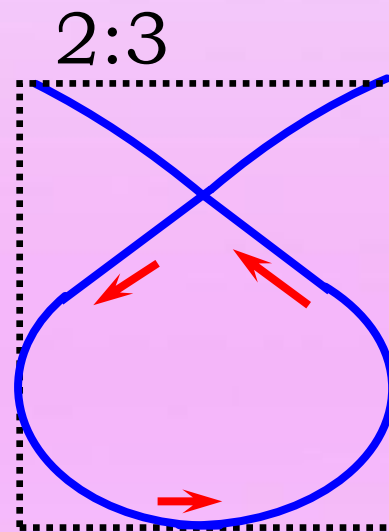
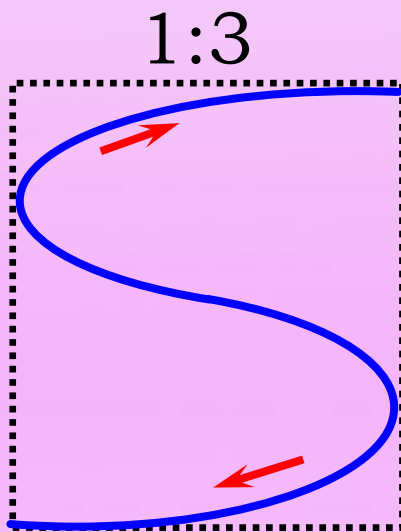
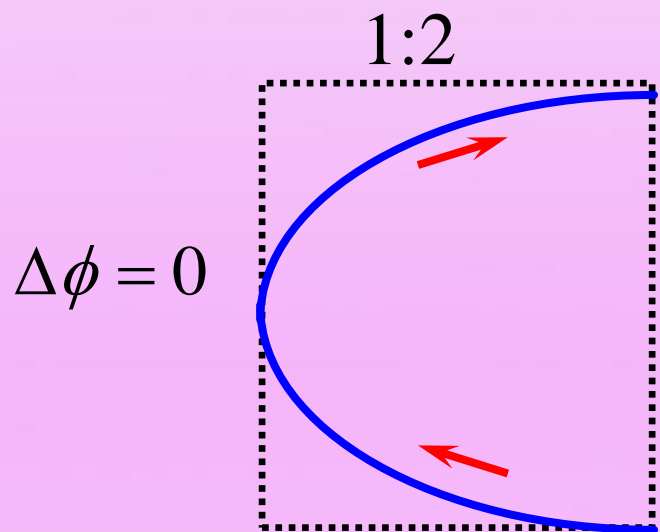
轨迹称为李萨如图形

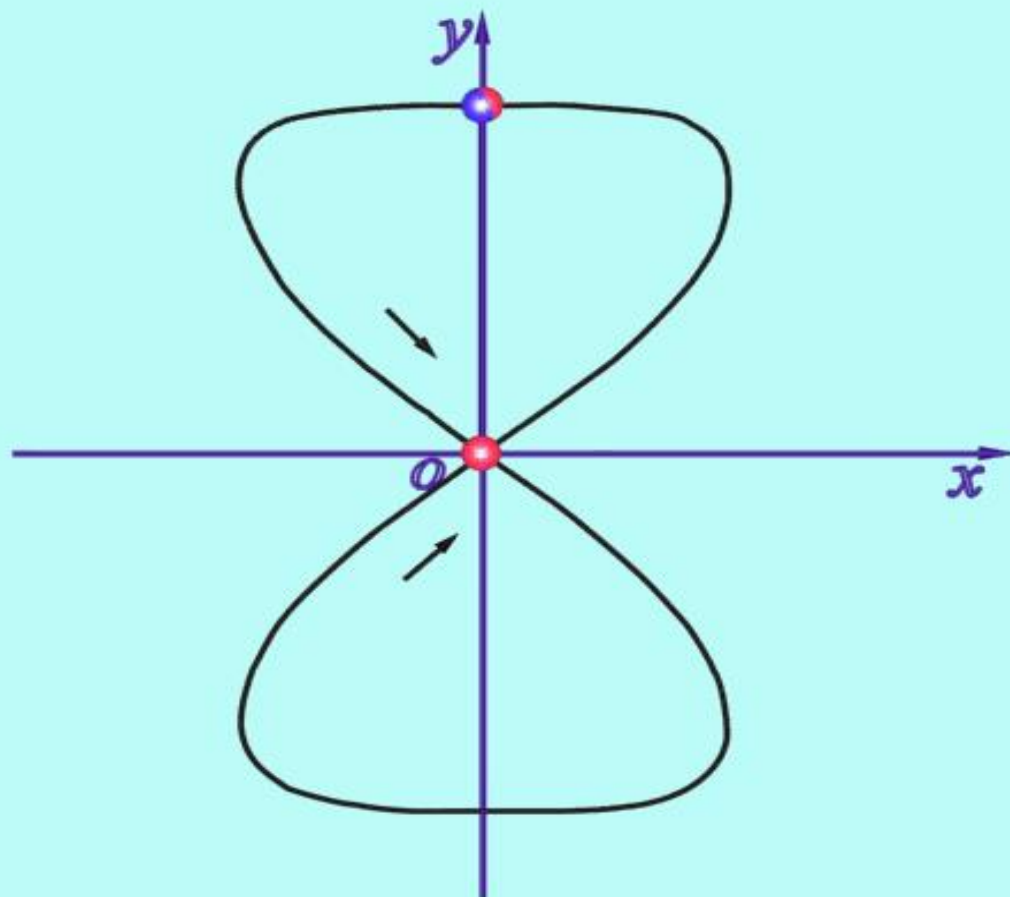
$$\omega_x : \omega_y = 3 : 2$$

$$\phi_{20} = \pi/4, \quad \phi_{10} = 0$$



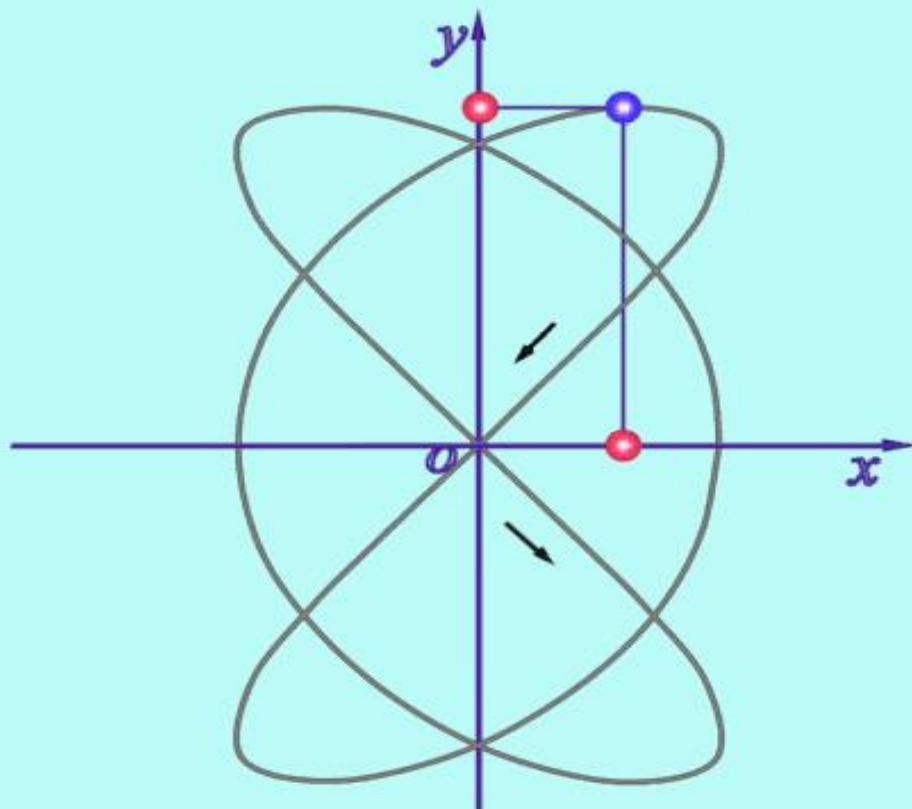
几幅典型的利萨如图形





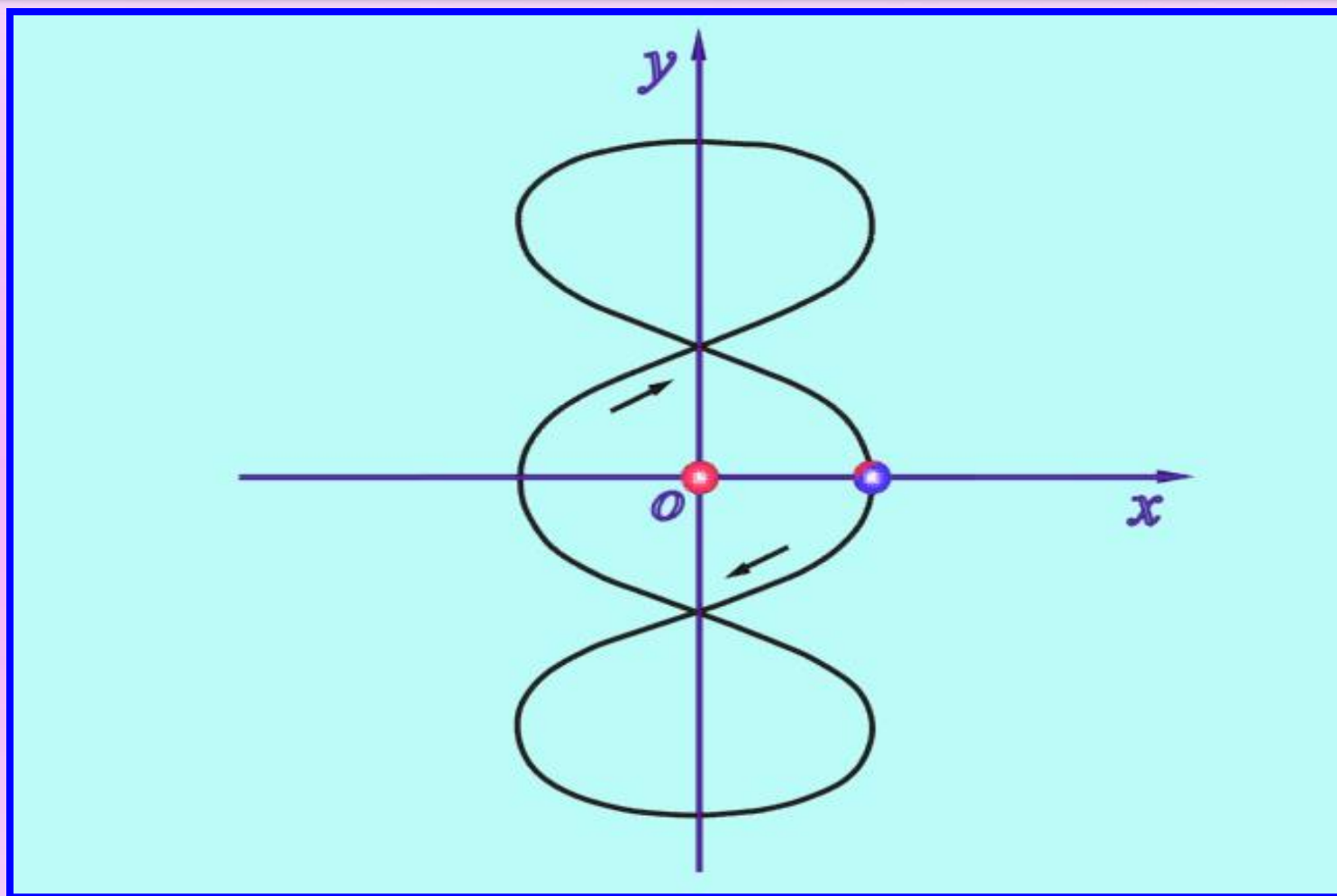
$$\omega_x : \omega_y = 2 : 1$$

$$\Delta\phi = \pi/2$$



$$\omega_x : \omega_y = 3 : 2$$

$$\Delta\phi = \pi/4$$



$$\omega_x : \omega_y = 3 : 1$$

$$\Delta\phi = \pi/2$$