# Cache Memories

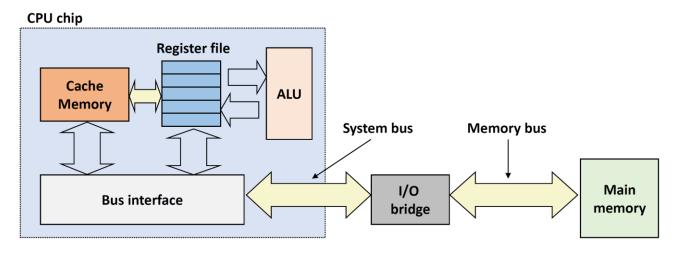
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송 인 식

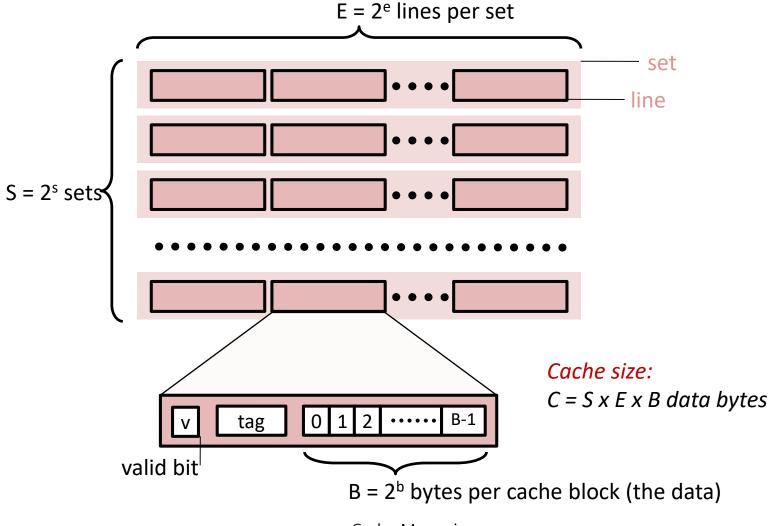
### Outline

- Cache memory organization and operation
- Performance impact of caches

- Small, fast SRAM-based memories managed automatically in hardware.
  - Hold frequently accessed blocks of main memory
- CPU looks first for data in caches (e.g., L1, L2, and L3), then in main memory.
- Typical system structure:

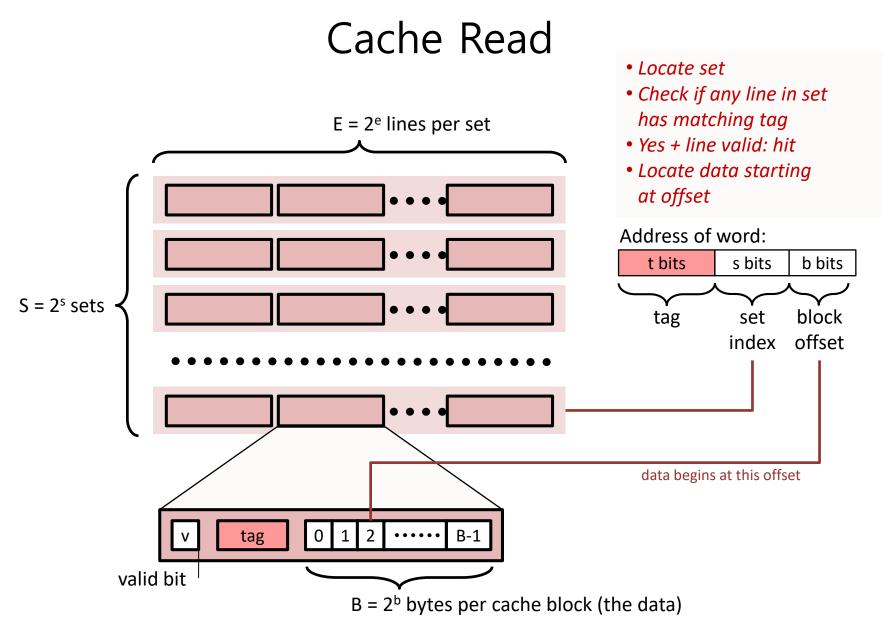


## General Cache Organization (S, E, B)



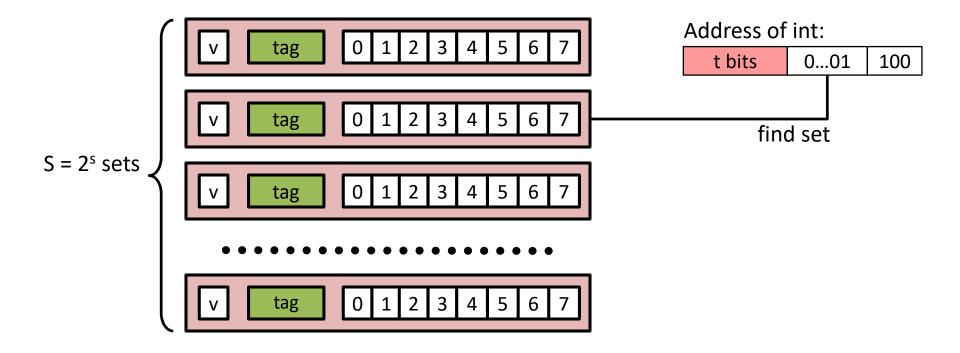
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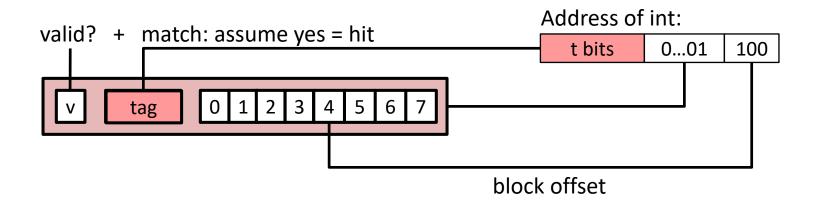
## Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set Assume: cache block size 8 bytes



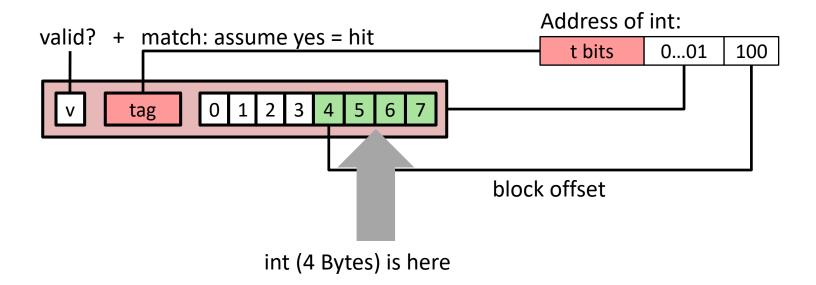
## Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set Assume: cache block size 8 bytes



### Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set Assume: cache block size 8 bytes



No match: old line is evicted and replaced

# Direct-Mapped Cache Simulation

t=1	s=2	b=1
Х	XX	Х

M=16 byte addresses, B=2 bytes/block, S=4 sets, E=1 Blocks/set

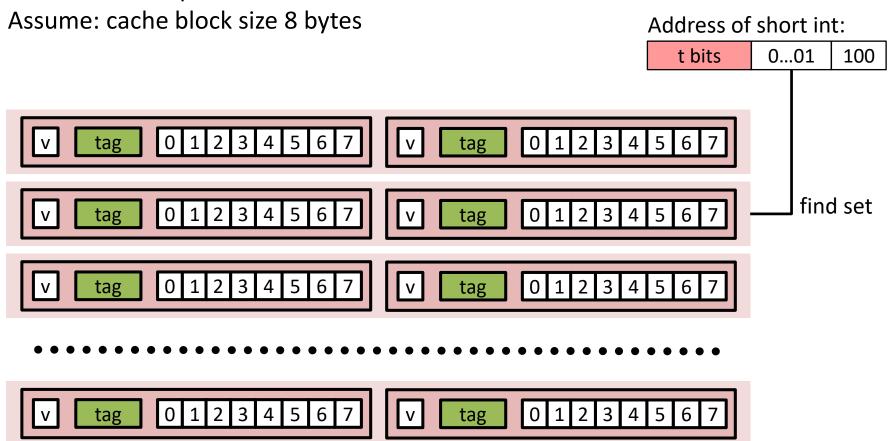
Address trace (reads, one byte per read):

0	$[0000_{2}],$	miss
1	$[0\underline{001}_{2}],$	hit
7	$[0\underline{11}1_2],$	miss
8	$[1\underline{00}0_{2}^{-}],$	miss
0	$[0000_{2}^{-}]$	miss

	V	Tag	Block
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

### E-way Set Associative Cache (Here: E = 2)

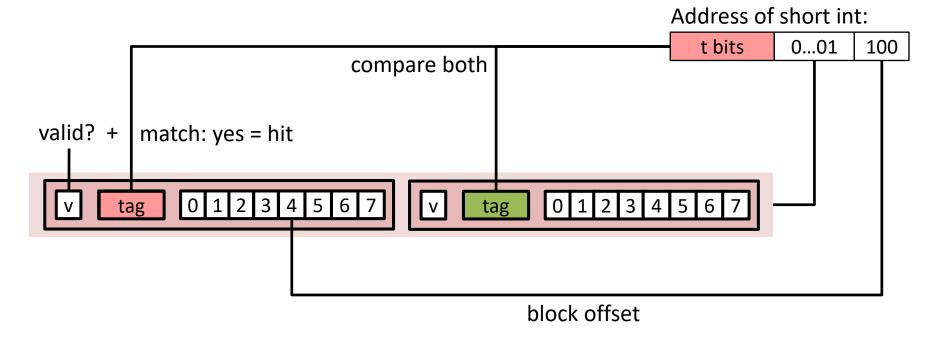
E = 2: Two lines per set



## E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes

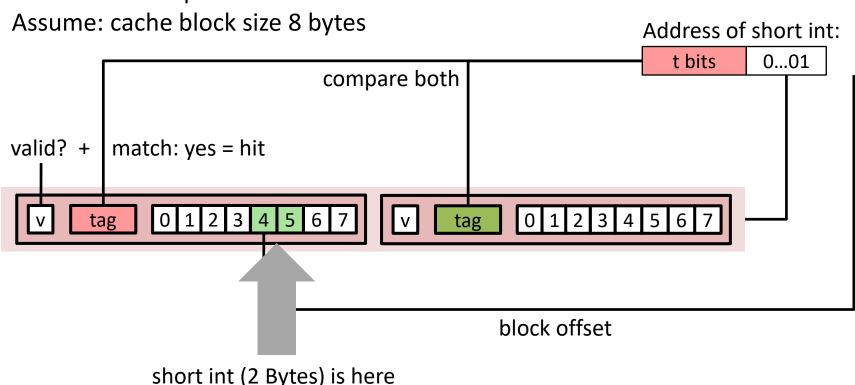


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## E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set



#### No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

# 2-Way Set Associative Cache Simulation

t=2	s=1	b=1
XX	Х	Х

M=16 byte addresses, B=2 bytes/block, S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0 
$$[00\underline{0}0_2]$$
, miss  
1  $[00\underline{0}1_2]$ , hit  
7  $[01\underline{1}1_2]$ , miss  
8  $[10\underline{0}0_2]$ , miss  
0  $[00\underline{0}0_2]$  hit

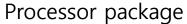
### What about writes?

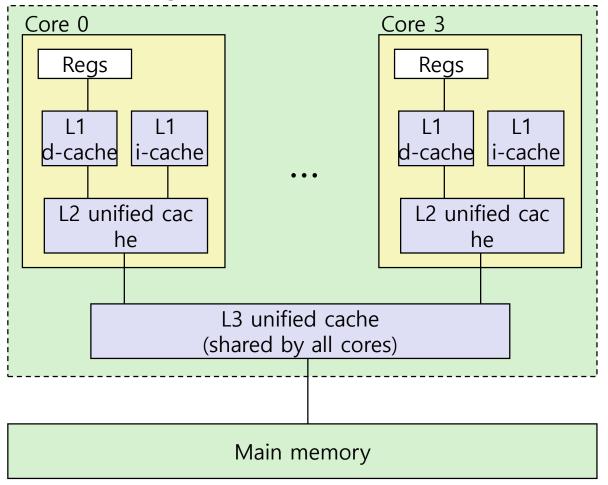
- Multiple copies of data exist:
  - L1, L2, Main Memory, Disk
- What to do on a write-hit?
  - Write-through (write immediately to memory)
  - Write-back (defer write to memory until replacement of line)
    - Need a dirty bit (line different from memory or not)
- What to do on a write-miss?
  - Write-allocate (load into cache, update line in cache)
    - Good if more writes to the location follow
  - No-write-allocate (writes immediately to memory)
- Typical
  - Write-through + No-write-allocate
  - Write-back + Write-allocate

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# Intel Core i7 Cache Hierarchy





L1 i-cache and d-cache: 32 KB, 8-way,

Access: 4 cycles

L2 unified cache:

256 KB, 8-way,

Access: 11 cycles

L3 unified cache:

8 MB, 16-way,

Access: 30-40 cycles

Block size:

64 bytes for all caches.

### Cache Performance Metrics

#### Miss Rate

- Fraction of memory references not found in cache (misses / accesses)
  - = 1 hit rate
- Typical numbers (in percentages):
  - 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.</li>

#### Hit Time

- Time to deliver a line in the cache to the processor
  - includes time to determine whether the line is in the cache
- Typical numbers:
  - 1-2 clock cycle for L1
  - 5-20 clock cycles for L2

### Miss Penalty

- Additional time required because of a miss
  - typically 50-200 cycles for main memory (Trend: increasing!)

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### Let's think about those numbers

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory
- Would you believe 99% hits is twice as good as 97%?
  - Consider:
     cache hit time of 1 cycle
     miss penalty of 100 cycles
  - Average access time:

```
97% hits: 1 cycle + 0.03 * 100 cycles = 4 cycles 99% hits: 1 cycle + 0.01 * 100 cycles = 2 cycles
```

This is why "miss rate" is used instead of "hit rate"

### Outline

- Cache memory organization and operation
- Performance impact of caches

## The Memory Mountain

- Read throughput (read bandwidth)
  - Number of bytes read from memory per second (MB/s)
- Memory mountain: Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

### Memory Mountain Test Function

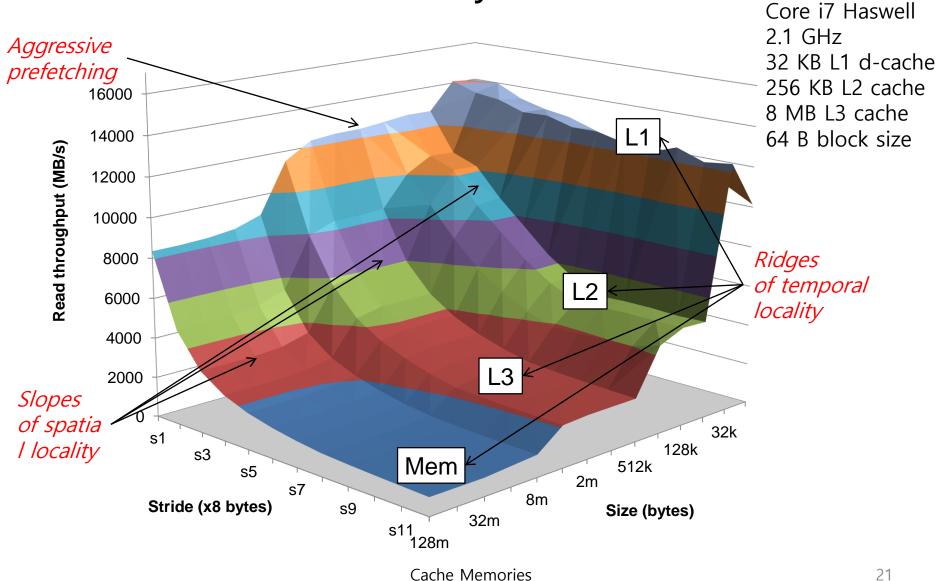
```
long data[MAXELEMS]; /* Global array to traverse */
/* test - Iterate over first "elems" elements of
      array "data" with stride of "stride", using
      using 4x4 loop unrolling.
int test(int elems, int stride) {
  long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
  long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
  long length = elems, limit = length - sx4;
  /* Combine 4 elements at a time */
  for (i = 0; i < limit; i += sx4) {
    acc0 = acc0 + data[i];
    acc1 = acc1 + data[i+stride];
    acc2 = acc2 + data[i+sx2];
    acc3 = acc3 + data[i+sx3];
  /* Finish any remaining elements */
  for (; i < length; i++) {
    acc0 = acc0 + data[i];
  return ((acc0 + acc1) + (acc2 + acc3));
```

Call test() with many comb
inations of elems
and stride.

For each elems and str ide:

- 1. Call test() once to warm up the caches.
- 2. Call test() again a nd measure the read th roughput(MB/s)

### The Memory Mountain



## Writing Cache Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions
- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

## Matrix Multiplication Example

### Description:

- Multiply N x N matrices
- O(N³) total operations
- N reads per source element
- N values summed per destination
  - but may be able to hold in register

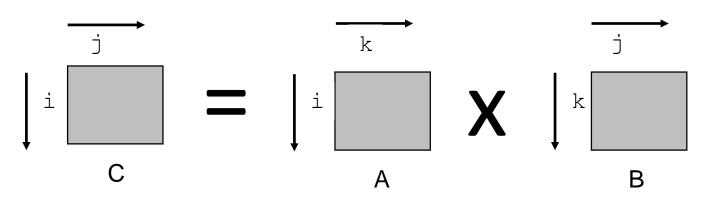
# Variable sum held in register

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}</pre>
```

# Miss Rate Analysis for Matrix Multiply

### Assume:

- Block size = 32B (big enough for four doubles)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



# Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:

```
- for (i = 0; i < N; i++)
    sum += a[0][i];</pre>
```

- accesses successive elements
- if block size (B) > sizeof(a<sub>ii</sub>) bytes, exploit spatial locality
  - miss rate = sizeof(a<sub>ii</sub>) / B
- Stepping through rows in one column:

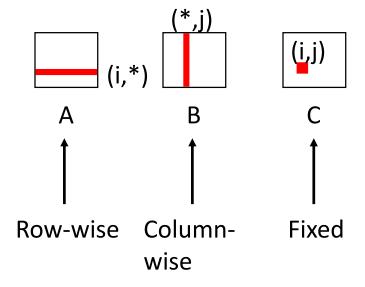
```
- for (i = 0; i < n; i++)
sum += a[i][0];</pre>
```

- accesses distant elements
- no spatial locality!
  - miss rate = 1 (i.e. 100%)

## Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
    c[i][j] = sum;
}
}
</pre>
matmult/mm.c
```

### Inner loop:



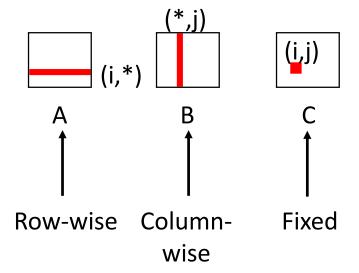
### Misses per inner loop iteration:

<u>A</u> <u>B</u> <u>C</u> 0.25 1.0 0.0

## Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
</pre>
matmult/mm.c
```

### Inner loop:

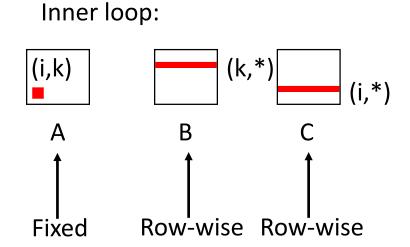


### Misses per inner loop iteration:

<u>A</u> 0.25 <u>В</u> 1.0

## Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
        c[i][j] += r * b[k][j];
  }
}
matmult/mm.c</pre>
```

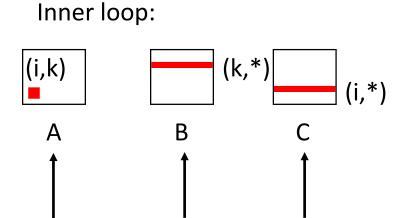


### Misses per inner loop iteration:

<u>A</u> <u>B</u> <u>C</u> 0.0 0.25 0.25

## Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
        c[i][j] += r * b[k][j];
  }
}
matmult/mm.c</pre>
```



Row-wise Row-wise

### Misses per inner loop iteration:

<u>A</u> 0.0 <u>B</u> 0.25

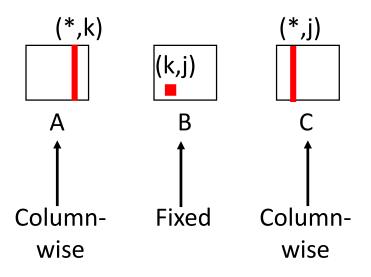
0.25

Fixed

# Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
        c[i][j] += a[i][k] * r;
  }
}
    matmult/mm.c</pre>
```

### Inner loop:

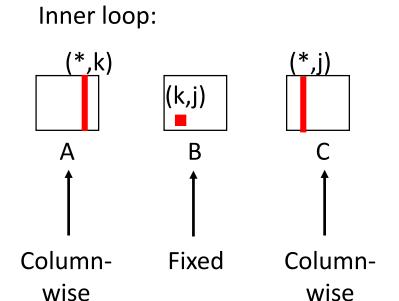


#### Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

## Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
        c[i][j] += a[i][k] * r;
  }
}
matmult/mm.c</pre>
```



### Misses per inner loop iteration:

<u>A</u> <u>B</u> 1.0 0.0

## Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
  for (k=0; k<n; k++)
    sum += a[i][k] * b[k][j];
  c[i][j] = sum;
}
}</pre>
```

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
    c[i][j] += r * b[k][j];
  }
}</pre>
```

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}</pre>
```

### ijk (& jik):

- 2 loads, 0 stores
- misses/iter = 1.25

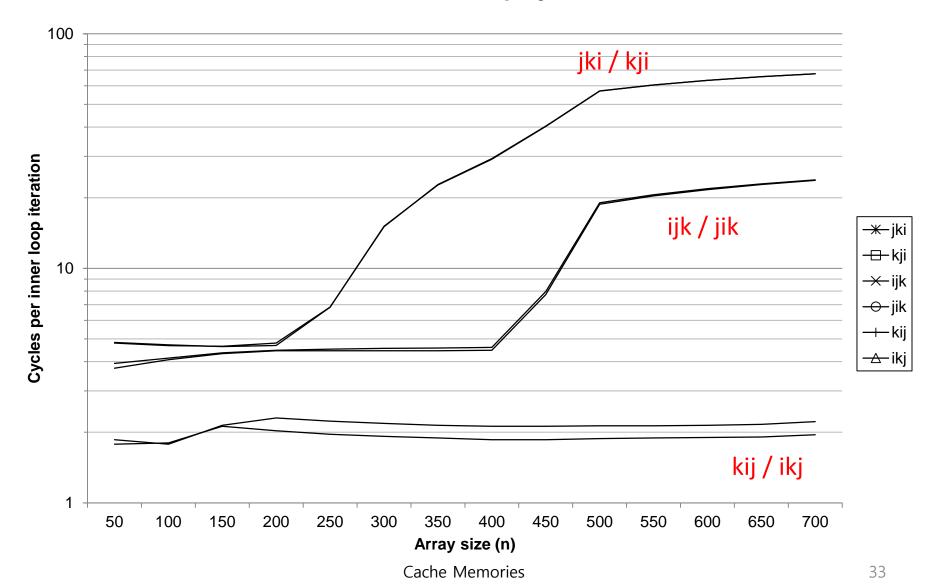
### kij (& ikj):

- 2 loads, 1 store
- misses/iter = 0.5

### jki (& kji):

- 2 loads, 1 store
- misses/iter = 2.0

# Core i7 Matrix Multiply Performance



## Cache Summary

- Cache memories can have significant performance impact
- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

# Questions?