

# Example 1

Vaccine effectiveness

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## TND studies on vaccine effectiveness

- The objective of a TND study is to assess the effectiveness of vaccines by comparing the odds of testing positive among vaccinated patients with the odds of positive testing among unvaccinated patients.

## A TND study on Covid-19 effectiveness

A TND study conducted in Ontario, Canada, (Chung et al. 2021) investigated the effectiveness of mRNA Covid-19 vaccines (bnt162b2 and mrna-1273) against symptomatic SARS-CoV-2 infection.

	T+	T-
V-	51220	251541
V+	57	3817

$$OR = \frac{57/3817}{51220/251541} = 0.073$$

$$VE = (1 - OR) \cdot 100 = 92.7\%$$

## TND studies on vaccine effectiveness

- If a perfect diagnostic assay is available (i.e. sensitivity and specificity equal to 1), we can assume that the results of the test follow an independent binomial sampling distribution for both unvaccinated ( $V-$ ) or vaccinated ( $V+$ ) sub-groups:

$$y_{V+} \sim \text{Bin}(n_{V+}, p_{V+})$$

$$y_{V-} \sim \text{Bin}(n_{V-}, p_{V-})$$

- The Odds Ratio (OR) and the Vaccine Effectiveness (VE) are defined as follows:

$$OR = \frac{p_{V+}/(1 - p_{V+})}{p_{V-}/(1 - p_{V-})}$$
$$VE = (1 - OR) \cdot 100$$

- In the simple case of nondifferential misclassification bias, the following relationships hold:

$$Se_{V_+} = Se_{V_-} = Se$$

$$Sp_{V_+} = Sp_{V_-} = Sp$$

## Bayesian modeling for addressing misclassification bias

- We can therefore assume that for each of the two sub-groups (vaccinated and unvaccinated) the results of the test follow an independent binomial sampling distribution:

$$y_{*V_+} \sim \text{Bin}(n_{*V_+}, p_{*V_+})$$

$$y_{*V_-} \sim \text{Bin}(n_{*V_-}, p_{*V_-})$$

with

$$p_{*V_+} = p_{V_+} \cdot Se + (1 - p_{V_+}) \cdot (1 - Sp)$$

$$p_{*V_-} = p_{V_-} \cdot Se + (1 - p_{V_-}) \cdot (1 - Sp)$$

## Bayesian modeling for addressing misclassification bias

- This model is over-parameterized with four parameters ( $p_{V+}$ ,  $p_{V-}$ ,  $Se$ ,  $Sp$ ) but only two independent pieces of information provided by the data, i.e. the apparent prevalences in the vaccinated ( $y_{V+}/n_{V+}$ ) and unvaccinated ( $y_{V-}/n_{V-}$ ) sub-groups.
- As such, these models are only of practical use within a Bayesian framework of inference, as this allows for prior information on diagnostic test characteristics (sensitivity and specificity) to be used along with the observed data.



## Bayesian model assuming perfect classification

```
bm_1t_perf <- " model {  
  for (i in 1:2) {  
    # likelihood  
    y[i,1] ~ dbin(pi[i], N[i])  
    # priors for prevalence parameters  
    pi[i] ~ dbeta(2,2)}  
    # Computing OR/VE  
    OR <- (pi[2]/(1-pi[2])) / (pi[1]/(1-pi[1]))  
    VE <- (1-OR)*100  
  #data# N, y  
  #inits#  
  #monitor# pi, OR, VE  
}"
```

# Bayesian model for non-differential misclassification

```
bm_1t_nondif <- " model {  
  for (i in 1:2) {  
    # likelihood  
    y[i,1] ~ dbin(prob[i], N[i])  
    prob[i] <- pi[i]*Se + (1-pi[i])*(1-Sp)  
    # priors for prevalence parameters  
    pi[i] ~ dbeta(2,2)}  
    # priors for Se and Sp  
    Se~dbeta(HPSe[1], HPSe[2])  
    Sp~dbeta(HPSp[1], HPSp[2])  
    # Computing OR/VE  
    OR <- (pi[2]/(1-pi[2])) / (pi[1]/(1-pi[1]))  
    VE <- (1-OR)*100  
    #data# N, y, HPSe, HPSp  
    #inits#  
    #monitor# Se, Sp, pi, OR, VE  
  }"
```

## Gibbs sampling set-up

```
inits1 <- list(".RNG.name" = "base::Mersenne-Twister", ".RNG.seed" = 100022)
inits2 <- list(".RNG.name" = "base::Mersenne-Twister", ".RNG.seed" = 300022)

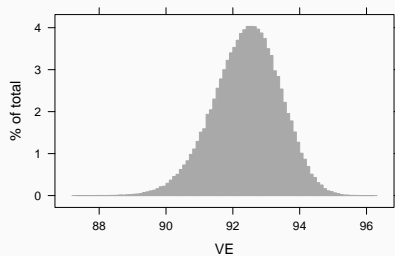
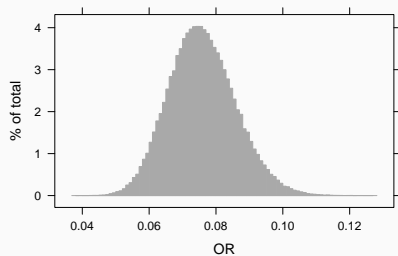
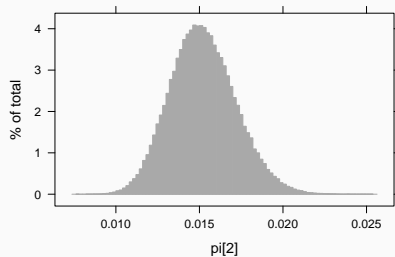
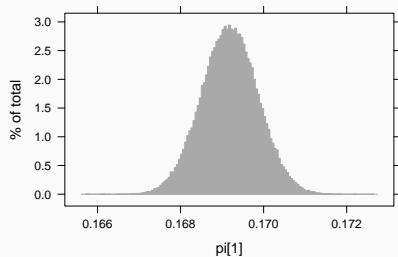
n_thin <- 25
n_burnin <- 50000
n_samples <- 100000
```

## Model 1: perfect classification

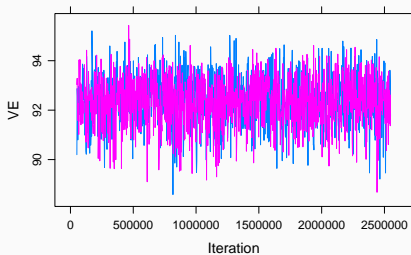
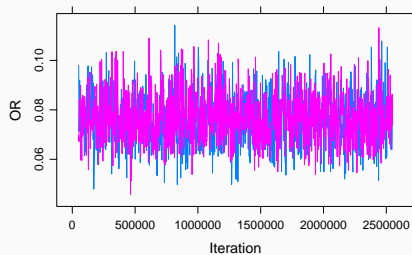
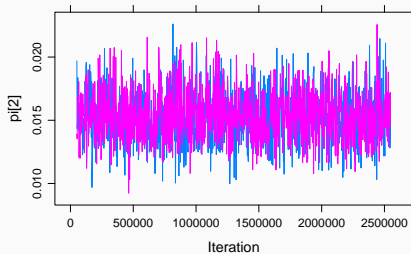
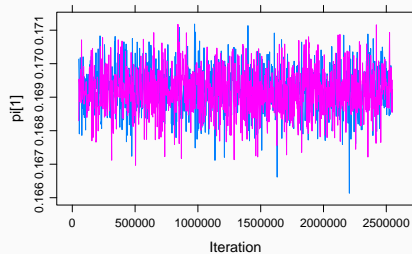
```
res_perfect <- run.jags(  
  bm_1t_perf,  
  n.chains = 2,  
  inits = list(inits1, inits2),  
  burnin = n_burnin,  
  sample = n_samples,  
  thin = n_thin  
)  
  
## Loading required namespace: rjags  
round(summary(res_perfect), 3) |>  
  kable()
```

	Lower95	Median	Upper95	Mean	SD	Mode	MCerr	MC%ofSD	SSeff	AC.250	psrf
pi[1]	0.168	0.169	0.171	0.169	0.001	NA	0.000	0.7	20000	0.002	1
pi[2]	0.012	0.015	0.019	0.015	0.002	NA	0.000	0.7	20405	0.002	1
OR	0.057	0.075	0.096	0.076	0.010	NA	0.000	0.7	20382	0.002	1
VE	90.409	92.460	94.269	92.404	0.996	NA	0.007	0.7	20382	0.002	1

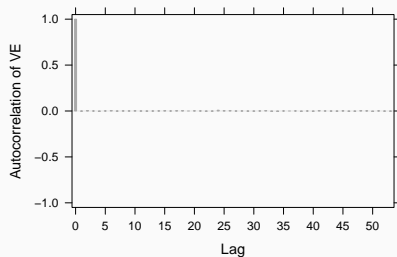
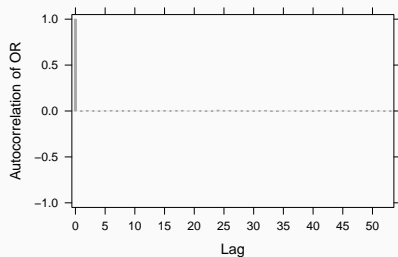
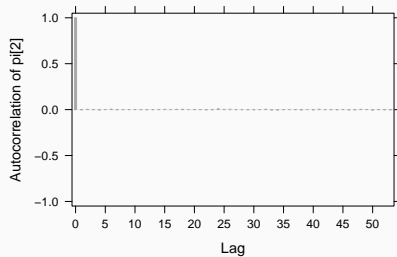
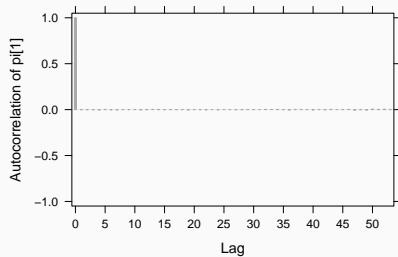
# Model 1: perfect classification



# Model 1: perfect classification



# Model 1: perfect classification



## Model 2: sensitivity and specificity from Kostoulas et al. (2021)

- Kostoulas and colleagues (Kostoulas, Eusebi, and Hartnack 2021) used a Bayesian latent class model to estimate the diagnostic accuracy of RT-PCR and lateral flow immunoassay tests for Covid-19.
- The sensitivity of RT-PCR was 0.68 (95% PrI=0.63-0.73), while the specificity was 0.99 (95% PrI=0.98-1.00).
- We plugged-in this prior information in our model by using

$$\text{Sensitivity} \sim \text{Beta}(226.16, 105.93)$$

$$\text{Specificity} \sim \text{Beta}(606.34, 6.45)$$



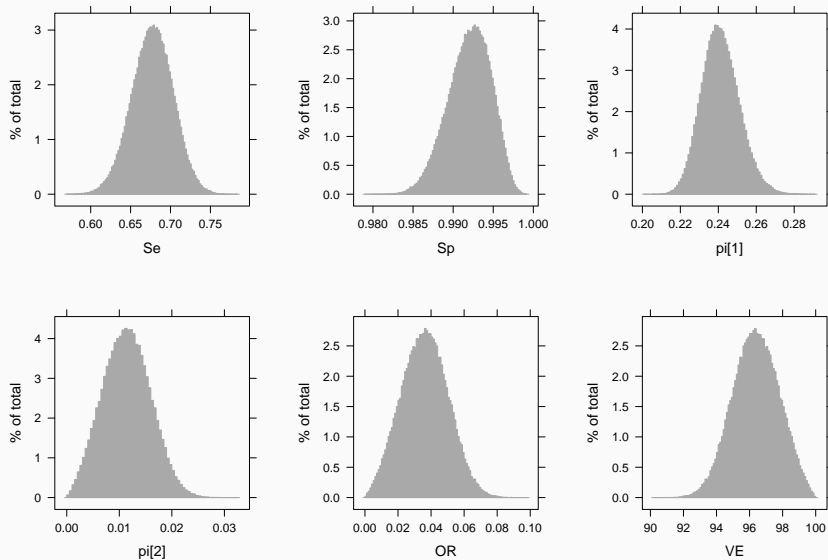
## Model 2: sensitivity and specificity from Kostoulas et al. (2021)

```
HPSe <- findbetaqq2(  
  percentile.value1 = 0.63,  
  percentile1 = 0.025,  
  percentile.value2 = 0.73,  
  percentile2 = 0.975  
)  
HPSe  
## [1] 226.16 105.93  
HPSp <- findbeta2(  
  themedian = 0.99,  
  percentile = 0.975,  
  lower.v = FALSE,  
  percentile.value = 0.98  
)  
HPSp  
## [1] 606.34 6.45
```

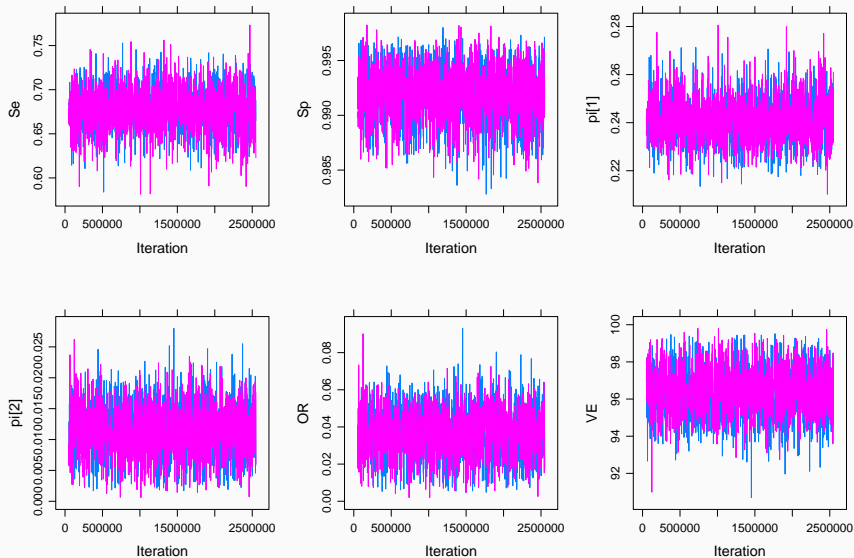
## Model 2: sensitivity and specificity from Kostoulas et al. (2021)

	Lower95	Median	Upper95	Mean	SD	Mode	MCerr	MC%ofSD	SSeff	AC.250	psrf
Se	0.626	0.678	0.728	0.677	0.026	NA	0.00	0.8	14469	0.160	1
Sp	0.987	0.992	0.997	0.992	0.003	NA	0.00	0.7	20000	0.004	1
pi[1]	0.223	0.241	0.262	0.241	0.010	NA	0.00	0.8	14725	0.167	1
pi[2]	0.002	0.011	0.020	0.011	0.004	NA	0.00	0.7	20000	0.012	1
OR	0.009	0.036	0.063	0.036	0.014	NA	0.00	0.7	20000	0.004	1
VE	93.700	96.405	99.051	96.384	1.391	NA	0.01	0.7	20000	0.004	1

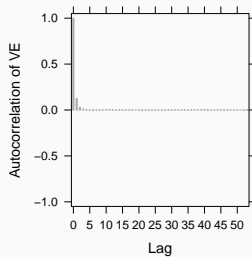
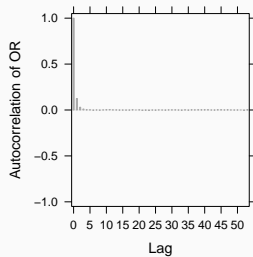
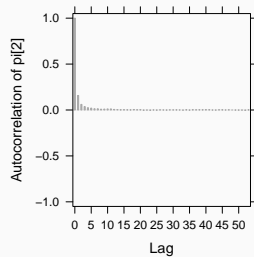
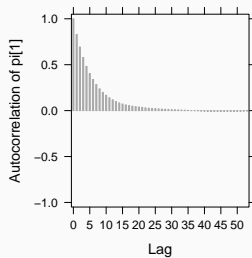
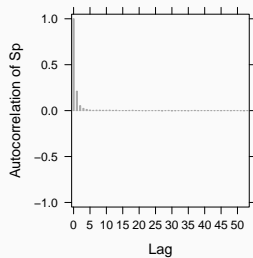
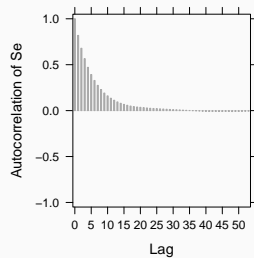
## Model 2: sensitivity and specificity from Kostoulas et al. (2021)



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### Model 3: sensitivity and specificity from Staerk et al. (2022)

- A recent report using Danish registries data used a Bayesian latent class model to estimate the diagnostic accuracy of RT-PCR and antigen tests for Covid-19. (Stærk-Østergaard et al. 2022)
- The specificity of RT-PCR was estimated to be close to 1.00.
- The sensitivity estimates were 0.957 (95% PrI=0.928-0.984).
- We plugged-in this prior information in our model by using:

$$\text{Sensitivity} \sim \text{Beta}(3040.61, 3.64)$$

$$\text{Specificity} \sim \text{Beta}(168.66, 6.84)$$

## Model 3: sensitivity and specificity from Staerk et al. (2022)

```
# Sp
HPSp <- findbetaqq2(
  percentile.value1 = 0.9973,
  percentile1 = 0.025,
  percentile.value2 = 0.9997,
  percentile2 = 0.975)
HPSp
## [1] 3040.61    3.64
round(qbeta(c(0.025, 0.5, 0.975), HPSp[1], HPSp[2]), 4)
## [1] 0.9973 0.9989 0.9997

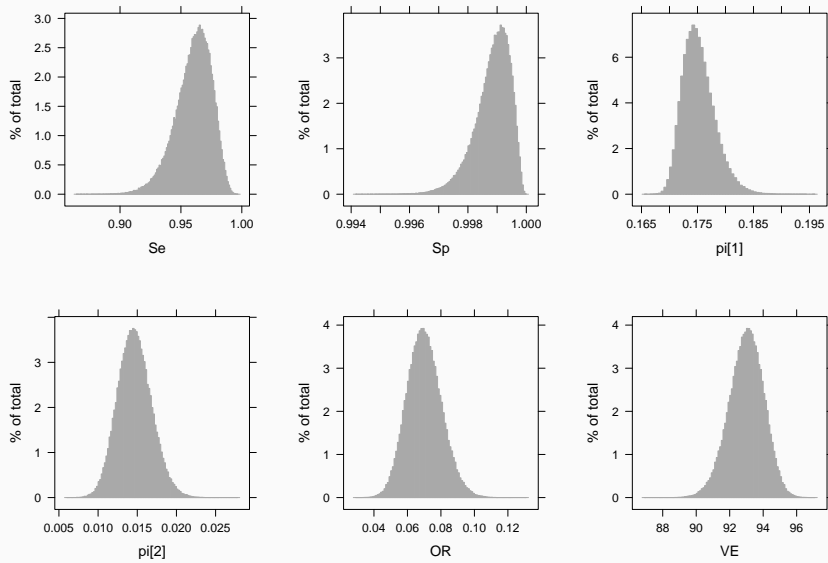
# Se
HPSe <- findbetaqq2(
  percentile.value1 = 0.9279,
  percentile1 = 0.025,
  percentile.value2 = 0.9843,
  percentile2 = 0.975)
HPSe
## [1] 168.66    6.84
round(qbeta(c(0.025, 0.5, 0.975), HPSe[1], HPSe[2]), 4)
## [1] 0.9279 0.9628 0.9843
```

### Model 3: sensitivity and specificity from Staerk et al. (2022)

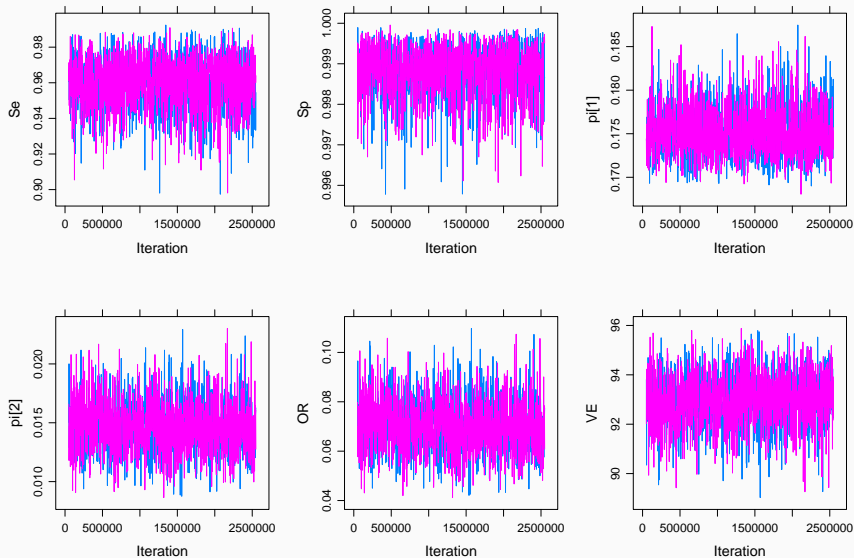
	Lower95	Median	Upper95	Mean	SD	Mode	MCerr	MC%ofSD	SSeff	AC.250	psrf
Se	0.931	0.962	0.987	0.960	0.015	NA	0.000	0.7	20000	0.000	1
Sp	0.998	0.999	1.000	0.999	0.001	NA	0.000	0.7	20598	0.000	1
pi[1]	0.170	0.175	0.181	0.175	0.003	NA	0.000	0.7	20000	0.002	1
pi[2]	0.011	0.015	0.019	0.015	0.002	NA	0.000	0.7	20000	0.000	1
OR	0.050	0.070	0.090	0.070	0.010	NA	0.000	0.7	20000	0.000	1
VE	90.965	93.030	95.006	92.993	1.033	NA	0.007	0.7	20000	0.000	1



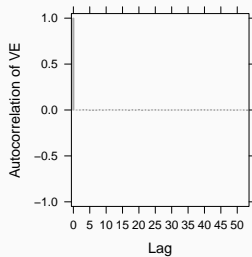
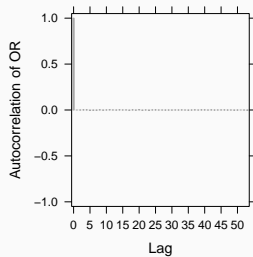
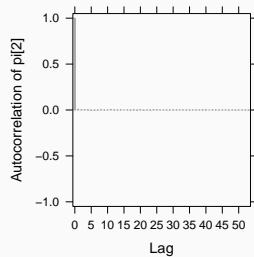
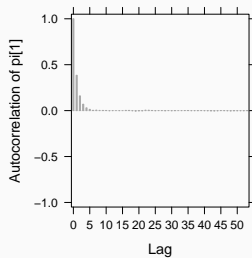
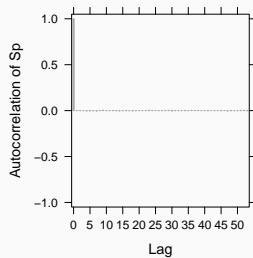
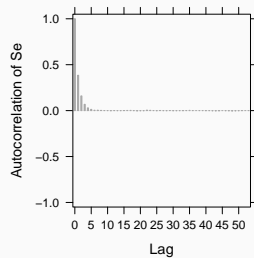
## Model 3: sensitivity and specificity from Staerk et al. (2022)



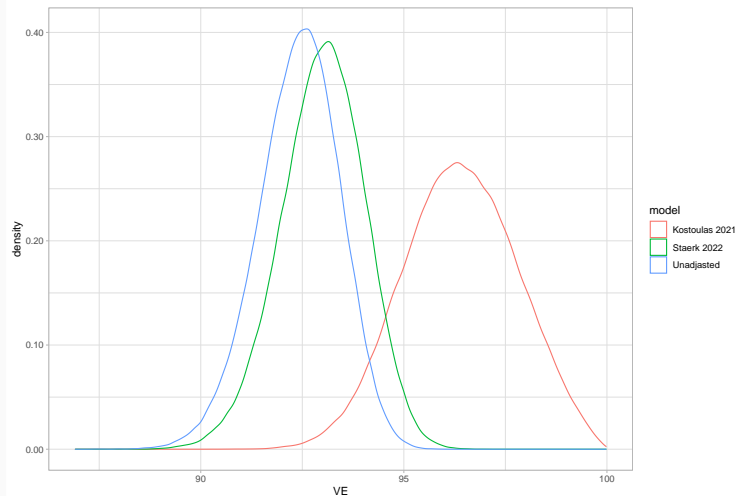
## Model 3: sensitivity and specificity from Staerk et al. (2022)



## Model 3: sensitivity and specificity from Staerk et al. (2022)



# Posterior: vaccine effectiveness



Addressing Misclassification Bias in Vaccine Effectiveness Studies with an Application to Covid-19 (Eusebi et al. 2022)

<https://www.researchsquare.com/article/rs-1799561/v1>

<https://github.com/paoloeusebi/tnd-vaccine-effectiveness/>

- Chung, Hannah, Siyi He, Sharifa Nasreen, Maria E Sundaram, Sarah A Buchan, Sarah E Wilson, Branson Chen, et al. 2021. "Effectiveness of BNT162b2 and mRNA-1273 Covid-19 Vaccines Against Symptomatic SARS-CoV-2 Infection and Severe Covid-19 Outcomes in Ontario, Canada: Test Negative Design Study." *BMJ*, August, n1943. <https://doi.org/10.1136/bmj.n1943>.
- Eusebi, Paolo, Nico Speybroeck, Sonja Hartnack, Jacob Staerk-Østergaard, Matthew D. Denwood, and Polychronis Kostoulas. 2022. "Addressing Misclassification Bias in Vaccine Effectiveness Studies with an Application to Covid-19." <http://dx.doi.org/10.21203/rs.3.rs-1799561/v1>.

- Koustoulas, Polychronis, Paolo Eusebi, and Sonja Hartnack. 2021. "Diagnostic Accuracy Estimates for COVID-19 Real-Time Polymerase Chain Reaction and Lateral Flow Immunoassay Tests With Bayesian Latent-Class Models." *American Journal of Epidemiology* 190 (8): 1689–95. <https://doi.org/10.1093/aje/kwab093>.
- Stærk-Østergaard, Jacob, Carsten Kirkeby, Lasse E. Christiansen, Michael A. Andersen, Camilla H. Møller, Marianne Voldstedlund, and Matthew J. Denwood. 2022. "Evaluation of Diagnostic Test Procedures for SARS-CoV-2 Using Latent Class Models." *Journal of Medical Virology* 94 (10): 4754–61. <https://doi.org/10.1002/jmv.27943>.