Bayes' theorem - Predictive values

Polychronis Kostoulas

Bayesian thinking

Subjectivity - Objectivity

Who Is Who

- Presbyterian Pastor
- 1702-1761
- “An essay towards solving a problem in the Doctrine of chances”. *Philosophical Transactions*, 1763 - (R. Price)

LII. *An Essay towards solving a Problem in the Doctrine of Chances. By the late Rev. Mr.* Bayes, *F. R. S. communicated by Mr.* Price, *in a Letter to* John Canton, *A. M. F. R. S.*

Dear Sir,

Read Dec. 23, 1763.

I Now send you an essay which I have found among the papers of our deceased friend Mr. Bayes, and which, in my opinion, has great merit, and well deserves to be preserved. Experimental philosophy, you will find, is nearly interested in the subject of it; and on this account there seems to be particular reason for thinking that a communication of it to the Royal Society cannot be improper.

He had, you know, the honour of being a member of that illustrious Society, and was much esteem-

Bayes' Theorem

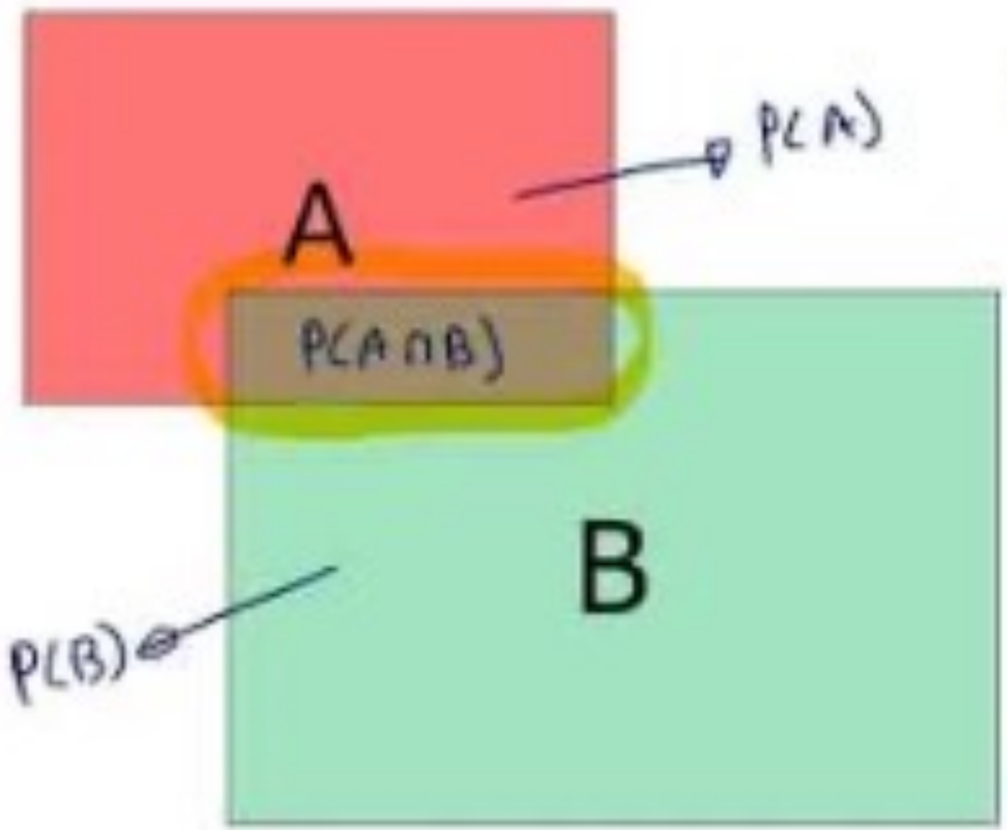
- Let us consider two possible outcomes A and B. The Bayes' theorem provides an expression for the conditional probability of A given B, which is equal to:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(A|B)=\frac{P(B|A)*P(A)}{P(B)}$$

$$P(D+|T+)=\frac{P(T+|D+)*P(D+)}{P(T+)}$$

Conditional probabilities



$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(B|A) = \underline{\qquad\qquad}$$

Example of diagnostic relevance

- A rapid test has been developed to detect a person infected with the new SARS-CoV-2 virus.
- This test is fairly reliable:
  - 95% of all infected individuals are detected and,
  - 95% of all healthy individuals are identified as such.
- It has been documented that at most one passenger of 100 aboard an airplane is infected.

Example of diagnostic relevance (cont.)

- The air company you are flying with, decides to scan all passengers, and the man sitting next to you tests positive.
- What are the chances that you are sitting next to someone who is really infected?

- P(T+|D+) = 0.95
- P(T-|D-) = 0.95 → P(T+|D-) = 0.05

- P(D+) = 0.01 → P(D-) = 0.99
- P(T+)= P(T+|D+)*P(D+)+ P(T+|D-)*P(D-) =0.059

$$P(A|B)=\frac{P(B|A)*P(A)}{P(B)}$$

$$P(D+|T+)=\frac{P(T+|D+)*P(D+)}{P(T+)}$$

$$P(D+|T+)=\frac{0.95*0.01}{0.059}=0.16$$

Example of diagnostic relevance (cont.)

- What if:
  - 60% of all infected individuals are detected and,
  - 99% of all healthy individuals are identified as such.

- $P(T+|D+)=0.60$
- $P(T-|D-)=0.99 \rightarrow P(T+|D-)=0.01$

- $P(D+)=0.01$ -> $P(D-)=0.99$
- $P(T+)= P(T+|D+)*P(D+)+ P(T+|D-)*P(D-)=0.016$

$$P(A|B)=\frac{P(B|A)*P(A)}{P(B)}$$

$$P(D+|T+)=\frac{P(T+|D+)*P(D+)}{P(T+)}$$

$$P(D+|T+)=\frac{0.60*0.01}{0.016}=0.37$$

Example of diagnostic relevance (cont.)

- What if:
  - 60% of all infected individuals are detected and,
  - 99% of all healthy individuals are identified as such.

- What if:
  - 10 passengers of 100 aboard an airplane are infected.

- P(T+|D+)=0.60
- P(T-|D-)=0.99 → P(T+|D-)=0.01

- P(D+)=0.10 -> P(D-)=0.90
- P(T+)= P(T+|D+)*P(D+)+ P(T+|D-)*P(D-)=0.14

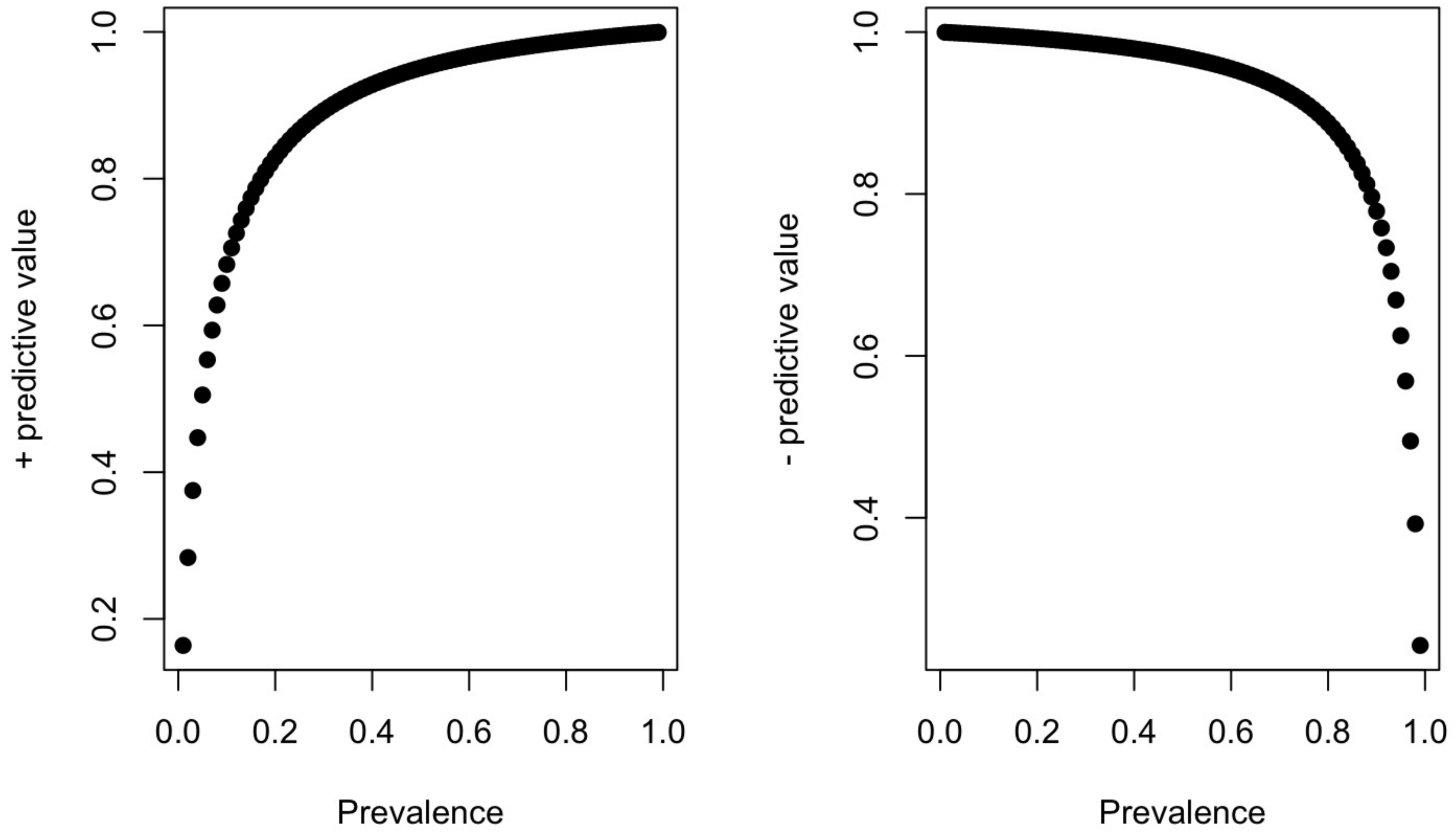
$$P(A|B)=\frac{P(B|A)*P(A)}{P(B)}$$

$$P(D+|T+)=\frac{P(T+|D+)*P(D+)}{P(T+)}$$

$$P(D+|T+)=\frac{0.60*0.1}{0.14}=0.68$$

Implementation in R - RStudio

- install.packages("BioProbability")
- library(BioProbability)
- p<-seq(0.01,0.99, by=0.01)
- predictive.value(p,Spe=0.95,Sen=0.97,plot.it=TRUE)


+ predictive value
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0
Prevalence
- predictive value
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0
Prevalence

HARMONY

Novel tools for test evaluation and disease prevalence estimation