Computational Astrodynamics

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Exercise 1: Time in Earth's shadow

A spacecraft is in an elliptical trajectory around the Earth, with semi-major axis $a = 32000$ km and eccentricity $e = 0.2$. Moreover, the Sun lies on the $(1, 0, 0)$ direction in the satellite's perifocal reference frame. Assuming that Earth casts a cylindrical shadow (see sketch bellow), find the time that the satellite spends in Earth's shadow.

Parameters: $\mu_{Earth} = 398600.433$ km^3/s^2, $R_{Earth} = 6378$ km.

Exercise 2: Lunar orbiter

Task 1: Find the semi-major axis for a Lunar-synchronous orbiter. Compare that distance with Moon's Hill's radius[1]

$$r_H \approx a_{Moon}(1 - e_{Moon}) \left(\frac{M_{Moon}}{3M_{Earth}} \right)^{1/3}$$

where a_{Moon} and e_{Moon} are the Moon's semi-major axis and eccentricity of its orbit around the Earth.

Task 2: Assume a satellite around the Moon, with the following orbital elements at $t_0 = 0$:

semi-major axis	5737.4 km
eccentricity	0.61
inclination	57.82°
RAAN	0°
argument of pericenter	90°
true anomaly	0°

a) Draw the Keplerian ellipse that the satellite describes about the Moon. Compute the distance of the satellite from Moon's surface for an orbital period (assume the Moon as a spherical surface with radius R_{Moon}).
b) Make a propagation of the orbital state of the satellite and provide the orbital elements at time $t_{end} = 30$ days.
c) Compute the groundtrack of the satellite for 3-days and 30-days timespan. Assume that at $t_0 = 0$ the lunar prime-meridian coincides with the x-axis of the Moon Center Inertial frame.

Parameters: $\mu_{Moon} = 4902.8$ km^3/s^2, $R_{Moon} = 1738.1$ km, $\omega_{Moon} = 2.66186 \cdot 10^{-6}$ rad/s, $M_{Moon} = 7.3477 \cdot 10^{22}$ kg, $M_{Earth} = 5.972 \cdot 10^{24}$ kg, $a_{Moon} = 348400$ km, $e_{Moon} = 0.0549$.

[1] The Hill's radius is the distance from the Moon within which smaller bodies would tend to orbit around it.

Exercise 3: Earth satellite maneuvering

A satellite operator has an Earth satellite that at time $t = 0$ has the following orbital state in the ECI frame:

x	-9141.878 km
y	-1648.0758 km
z	4141.679 km
v_x	-1.153 km/s
v_y	-5.31 km/s
v_z	-2.898 km/s

The operator wants to transfer his satellite to the following orbital location:

semi-major axis	12940 km
eccentricity	0.2173
inclination	0.8692 rad
RAAN	1.448 rad
argument of pericenter	2.721 rad
true anomaly	2.827 rad

Using 1 of the 16 variations of the maneuver sequence of change of plane - change of perigee - change of shape, compute the total ΔV cost and the time-of-flight needed to perform the transfer.

Parameters: $\mu_{Earth} = 398600.433\ km^3/s^2$