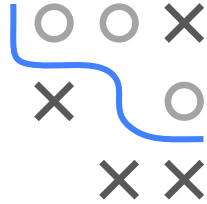
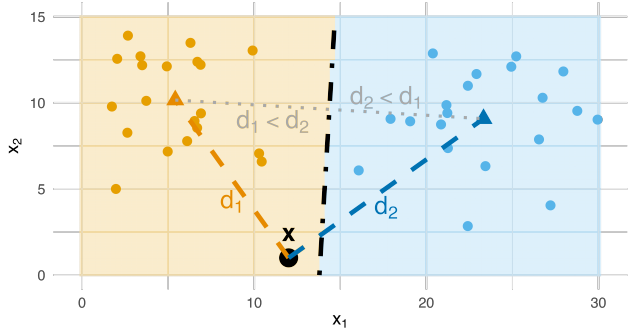
EXAMPLE: 2 CLASSES WITH CENTROIDS

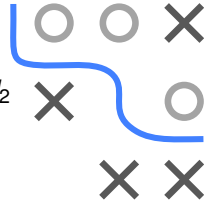
- Model binary problem with centroid $\boldsymbol{\mu_k}$ per class as "parameters"
- Don't really care how the centroids are estimated;
  could use class means, but the following doesn't depend on it
- Classify point **x** by assigning it to class *k* of nearest centroid





EXAMPLE: 2 CLASSES WITH CENTROIDS

Let's calculate the decision boundary:



$$d_1 = ||\mathbf{x}-\boldsymbol{\mu_1}||^2 = \mathbf{x}^\top\mathbf{x}-2\mathbf{x}^\top\boldsymbol{\mu_1}+\boldsymbol{\mu_1}^\top\boldsymbol{\mu_1} = \mathbf{x}^\top\mathbf{x}-2\mathbf{x}^\top\boldsymbol{\mu_2}+\boldsymbol{\mu_2}^\top\boldsymbol{\mu_2} = ||\mathbf{x}-\boldsymbol{\mu_2}||^2 = d_2$$

Where d is measured using Euclidean distance. This implies:

$$-2\mathbf{x}^\top\boldsymbol{\mu_1} + \boldsymbol{\mu_1}^\top\boldsymbol{\mu_1} = -2\mathbf{x}^\top\boldsymbol{\mu_2} + \boldsymbol{\mu_2}^\top\boldsymbol{\mu_2}$$

Which simplifies to:

$$2\mathbf{x}^\top(\boldsymbol{\mu_2} - \boldsymbol{\mu_1}) = \boldsymbol{\mu_2}^\top\boldsymbol{\mu_2} - \boldsymbol{\mu_1}^\top\boldsymbol{\mu_1}$$

Thus, it's a linear classifier!