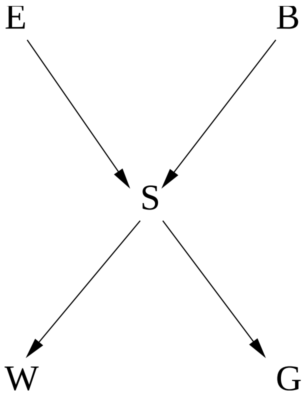
Tutorial Examples Uncertainty

November 27, 2020

Inference in Bayes Nets



$$P(E, S, B, W, G) = P(E)P(B)P(S|E, B)P(W|S)P(G|S)$$

Inference in Bayes Nets

P(E)	e	-e
	1/10	9/10

P(B)	b	-b
	1/10	9/10

P(S|E,B)	s	-s
e $\wedge$ b	9/10	1/10
e $\wedge$ -b	2/10	8/10
-e $\wedge$ b	8/10	2/10
-e $\wedge$ -b	0	1

P(W|S)	w	-w
s	8/10	2/10
-s	2/10	8/10

P(G|S)	g	-g
s	1/2	1/2
-s	0	1

Inference in Bayes Nets

- Given the alarm went off (s) what is the probability that Mrs. Gibbons phones you (g)?

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$$P(g|s) = 1/2$$

Inference in Bayes Nets

- Given that Mrs. Gibbons phones you (g) what is the probability the alarm went off (s)?

Inference in Bayes Nets

- Given that Mrs. Gibbons phones you (g) what is the probability the alarm went off (s)?

1. Bayes Rule says: $P(S|g) = P(g|S) * P(S)/P(g)$
2. $P(-s|g) = P(g|-s) * P(-s)/P(g) = 0 * P(-s)/P(g) = 0.$
3. Therefore $P(s|g) = 1$ ($P(s|g) + P(-s|g)$ must sum to 1.

$$P(s|g) = 1 \; P(-s|g) = 0$$

Alternatively: $-s \rightarrow -g$, so $g \rightarrow s$, so $P(s|g) = 1$.

Inference in Bayes Nets

- Say that there was a burglary (b) and but no earthquake (-e), what is the expression specifying the posterior probability of Dr. Watson phoning you (w) given the evidence. (You do not need to calculate a numeric answer, just give the probability expression).

Inference in Bayes Nets

- Say that there was a burglary (b) and but no earthquake (-e), what is the expression specifying the posterior probability of Dr. Watson phoning you (w) given the evidence. (You do not need to calculate a numeric answer, just give the probability expression).

$$P(w|b, -e)$$

Inference in Bayes Nets

- What is $P(G|S)$? (i.e., the four probability values) $P(g|s)$, $P(-g|s)$, $P(g|-s)$, $P(-g|-s)$.

Inference in Bayes Nets

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$$\begin{array}{ll} P(g|s) = 1/2 & P(-g|s) = 1/2 \\ P(-g|-s) = 0 & P(-g|-s) = 1 \end{array}$$

Inference in Bayes Nets

- What is $P(G|S \wedge W)$? (i.e., the 8 probability values $P(g|s \wedge w)$, $P(g|s \wedge -w)$, $\ldots$, $P(-g| - s \wedge -w)$).

Inference in Bayes Nets

- What is $P(G|S \wedge W)$? (i.e., the 8 probability values $P(g|s \wedge w)$, $P(g|s \wedge -w)$, ..., $P(-g|-s \wedge -w)$).

$$
\begin{array}{lclclcl}
P(g|s,-w) & = & P(g|s,w) & = & P(g|s) & = & 1/2 \\
P(-g|s,-w) & = & P(-g|s,w) & = & P(-g|s) & = & 1/2 \\
P(g|-s,-w) & = & P(g|-s,w) & = & P(g|-s) & = & 0 \\
P(-g|-s,-w) & = & P(-g|-s,w) & = & P(g|-s) & = & 1
\end{array}
$$

Inference in Bayes Nets

- What do these values tell us about the relationship between G, W and S?

  G is conditionally independent of W given S

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

Must do variable elimination.

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).
- Query variable is G.
- First run of VE, evidence is $W = w$.
- Second run of VE, evidence is $W = -w$.
- Use same ordering for both runs of VE: E, B, S, G.
- With same ordering some factors can be reused between the two runs of VE.

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).
  1. E: $P(E)$, $P(S|E, B)$
  2. B: $P(B)$,
  3. S: $P(w|S)$, $P(S|G)$
  4. G:

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$,
3. S: $P(w|S)$, $P(S|G)$
4. G:

$$\begin{aligned} F_1(S,B) &= \sum_E P(E) \times P(S|E,B) \\ &= P(e) \times P(S|e,B) + P(-e) \times P(S|-e,B) \end{aligned}$$

$$\begin{aligned} F_1(-s,-b) &= P(e)P(-s,e,-b) + P(-e)P(-s,-e,-b) \\ &= 0.1 \times 0.8 + 0.9 \times 1 = 0.98 \\ F_1(-s,b) &= P(e)P(-s,e,b) + P(-e)P(-s,-e,b) \\ &= 0.1 \times 0.1 + 0.9 \times 0.2 = 0.19 \\ F_1(s,-b) &= P(e)P(s,e,-b) + P(-e)P(s,-e,-b) \\ &= 0.1 \times 0.2 + 0.9 \times 0 = 0.02 \\ F_1(s,b) &= P(e)P(s,e,b) + P(-e)P(s,-e,b) \\ &= 0.1 \times 0.9 + 0.9 \times 0.8 = 0.81 \end{aligned}$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(w|S)$, $P(S|G)$
4. G:

$$\begin{aligned} F_2(S) &= \sum_B P(B) \times F_1(S,B) \\ &= P(b)F_1(S,b) + P(-b)F_1(S,-b) \end{aligned}$$

$$\begin{aligned} F_2(-s) &= P(b)F_1(-s,b) + P(-b)F_1(-s,-b) \\ &= 0.1 \times 0.19 + 0.9 \times 0.98 = 0.901 \\ F_2(s) &= P(b)F_1(s,b) + P(-b)F_1(s,-b) \\ &= 0.1 \times 0.81 + 0.9 \times 0.02 = 0.099 \end{aligned}$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(w|S)$, $P(S|G)$, $F_2(S)$
4. G:

$$\begin{aligned} F_3(G) &= \textstyle\sum_S P(w|S) \times P(S|G) \times F_2(S) \\ &= P(w|s)P(s|G)F_2(s) + P(w|-s)P(-s|G)F_2(-s) \end{aligned}$$

$$\begin{aligned} F_3(-g) &= P(w|s)P(s|-g)F_2(s) + P(w|-s)P(-s|-g)F_2(-s) \\ &= 0.8 \times 0.5 \times 0.099 + 0.2 \times 1 \times 0.901 = 0.2198 \\ F_3(g) &= P(w|s)P(s|g)F_2(s) + P(w|-s)P(-s|g)F_2(-s) \\ &= 0.8 \times 0.5 \times 0.099 + 0.2 \times 0 \times 0.901 = 0.0396 \end{aligned}$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(w|S)$, $P(S|G)$, $F_2(S)$
4. G: $F_3(G)$

Normalize $F_3(G)$:

$$\begin{aligned} P(-g|w) &= \frac{0.2198}{0.2198+0.0396} = 0.8473 \\ P(g|w) &= \frac{0.0396}{0.2198+0.0396} = 0.1527 \end{aligned}$$

Inference in Bayes Nets

- Now $P(G|-w)$?
    1. E: $P(E)$, $P(S|E,B)$
    2. B: $P(B)$,
    3. S: $P(-w|S)$, $P(S|G)$
    4. G:

Already computed as $F_1(S,B)$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(-w|S)$, $P(S|G)$
4. G:

Already computed as $F_2(S)$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(-w|S)$, $P(S|G)$, $F_2(S)$
4. G:

$$\begin{aligned} F_3(G) &= \sum_S P(-w|S) \times P(S|G) \times F_2(S) \\ &= P(-w|s)P(s|G)F_2(s) + P(-w|-s)P(-s|G)F_2(-s) \end{aligned}$$

$$\begin{aligned} F_3(-g) &= P(-w|s)P(s|-g)F_2(s) + P(-w|-s)P(-s|-g)F_2(-s) \\ &= 0.2 \times 0.5 \times 0.099 + 0.8 \times 1 \times 0.901 = 0.7307 \\ F_3(g) &= P(-w|s)P(s|g)F_2(s) + P(-w|-s)P(-s|g)F_2(-s) \\ &= 0.2 \times 0.5 \times 0.099 + 0.8 \times 0 \times 0.901 = 0.0099 \end{aligned}$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(-w|S)$, $P(S|G)$, $F_2(S)$
4. G: $F_3(G)$

Normalize $F_3(G)$:

$$
\begin{aligned}
P(-g|-w) &= \frac{0.7307}{0.7307+0.0099} = 0.9866 \\
P(g|-w) &= \frac{0.0099}{0.2198+0.00099} = 0.0134
\end{aligned}
$$

Inference in Bayes Nets

- What do these values tell us about the relationship between G and W, and why does this relationship differ when we know S?

Inference in Bayes Nets

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G and W are not independent of each other. But when S is known they become independent.