

Building/Assessing Bayes Net Models

In the lecture notes we saw that in many real work examples it was possible to find conditional independencies/causations that allow us to construct good Bayes Net models.

We can also use our understanding of causation/independence to critique different Bayes net models.

Building/Assessing Bayes Net Models

Two astronomers in different parts of the world make measurements M_1 and M_2 of the number of stars N in some small region of the sky, using their telescopes. Normally, there is a small probability e or error of up to one star in each direction. Each telescope can also be badly out of focus with probability f . Let F_1 and F_2 be boolean variables with $F_i = \text{true}$ being that the i -th telescope is out of focus. If the telescope is out of focus then the scientist will always undercount by 3 or more stars (or, if N is 3 or less, fail to detect any stars at all).

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Variables

N —true number of stars in that region of the sky

M_1 measurement made by telescope one.

M_2 measurement made by telescope two

F_1 Telescope one is out of focus

F_2 Telescope two is out of focus

N, M_1, M_2 integers ≥ 0

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Probabilities:

Each Telescope

f probability of being out of focus $F_i = \text{true}$

If $F_i = \text{false}$,

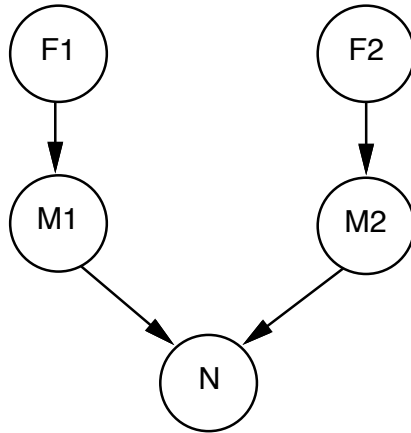
$N - M_i = -1$ probability e

$N - M_i = 1$ probability e

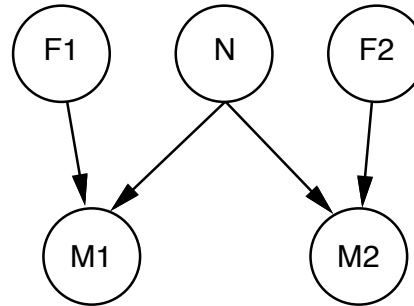
If $F_i = \text{true}$

$N - M_i \geq 3$ probability 1.

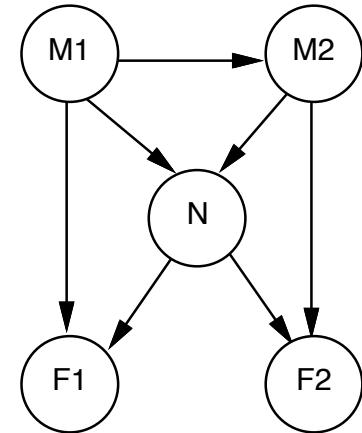
Building/Assessing Bayes Net Models



(i)



(ii)

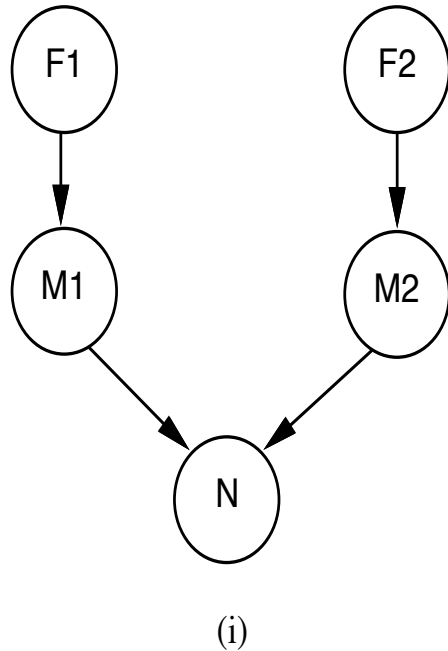


(iii)

Which of these Bayes Nets can correctly represent this example?

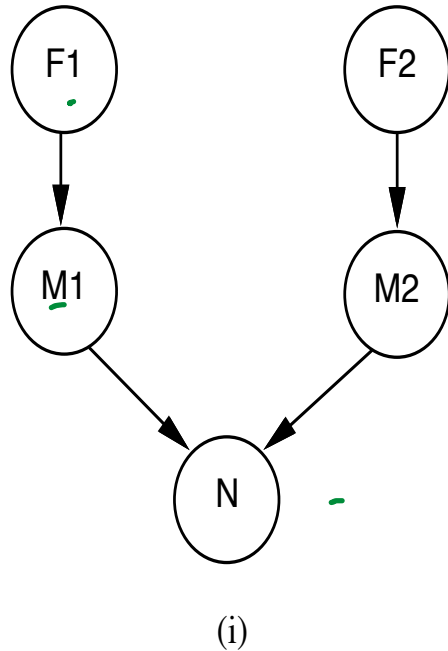
Which of the correct Networks is the best representation

Building/Assessing Bayes Net Models



- Choose ordering of variables such that parents come before children.
- Write chain rule decomposition of this ordering—know that the chain rule always produces a correct decomposition.
- Using our common sense intuitions ask if this chain rule decomposition can be simplified to be the same decomposition as the Bayes net.

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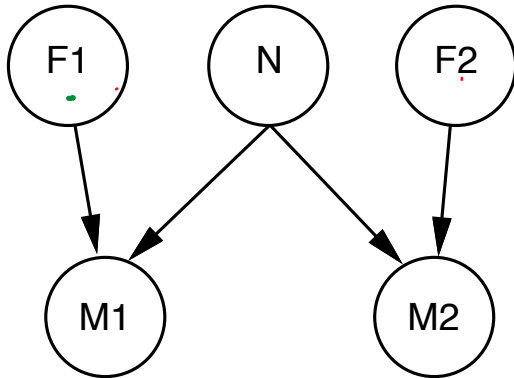
Chain Rule:

$$\begin{aligned} P(F1, F2, M1, M2, N) = & \\ & P(N \mid F1, F2, M1, M2)^* \\ & P(M2 \mid F1, F2, M1)^* \\ & P(M1 \mid F1, F2)^* \\ & P(F2 \mid F1)^* \\ & P(F1) \end{aligned}$$

Bayes Net decomposition:

$$\begin{aligned} P(F1, F2, M1, M2, N) = & \\ & P(N \mid M1, M2)^* \\ & P(M2 \mid F2)^* \\ & P(M1 \mid F1)^* \\ & P(F2)^* \\ & P(F1) \end{aligned}$$

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Chain Rule:

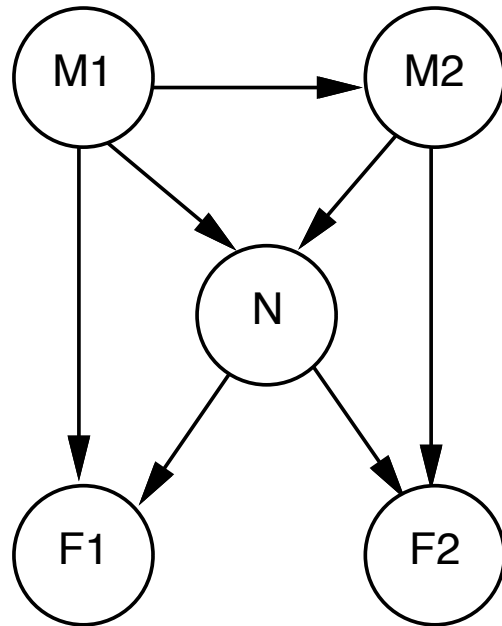
$$P(F1, N, F2, M1, M2) = \\ P(M2 \mid F1, N, F2, M1)^* \\ P(M1 \mid F1, N, F2)^* \\ P(F2 \mid F1, N)^* \\ P(N \mid F1)^* \\ P(F1)$$

Bayes Net decomposition:

$$P(F1, N, F2, M1, M2) = \\ P(M2 \mid N, F2)^* \\ P(M1 \mid F1, N)^* \\ P(F2)^* \\ P(N)^* \\ P(F1)$$

(ii)

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(iii)

Chain Rule:

$$\begin{aligned} P(M1, M2, N, F1, F2) = & \\ & P(F2 \mid M1, M2, N, F1)^* \\ & P(F1 \mid M1, M2, N)^* \\ & P(N \mid M1, M2)^* \\ & P(M2 \mid M1)^* \\ & P(M1) \end{aligned}$$

Bayes Net decomposition:

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